Solution

$\int_0^1 \frac{x}{1-x^2+2\sqrt{1-x^2}}dx=\ln(3)-\ln(2)$ (Decimal: $0.40546\ldots$)

Steps

$\int_0^1 \frac{x}{1-x^2+2\sqrt{1-x^2}}dx$

Apply u − substitution — Show Steps

$=\int_1^0 -\frac{1}{2(u+2\sqrt{u})}du$

$\int_a^b f(x)\,dx=-\int_b^a f(x)\,dx,\ a<b$

$=-\int_0^1 -\frac{1}{2(u+2\sqrt{u})}du$

Take the constant out: $\int a\cdot f(x)\,dx=a\cdot\int f(x)\,dx$

$=-\left(-\frac{1}{2}\cdot\int_0^1\frac{1}{u+2\sqrt{u}}du\right)$

Apply u − substitution — Show Steps

$=-\left(-\frac{1}{2}\cdot\int_0^1\frac{2}{v+2}dv\right)$

Take the constant out: $\int a\cdot f(x)\,dx=a\cdot\int f(x)\,dx$

$=-\left(-\frac{1}{2}\cdot 2\cdot\int_0^1\frac{1}{v+2}dv\right)$

Apply u − substitution — Show Steps

$=-\left(-\frac{1}{2}\cdot 2\cdot\int_2^3\frac{1}{w}dw\right)$

Use the common integral: $\int\frac{1}{w}dw=\ln(|w|)$

$=-\left(-\frac{1}{2}\cdot 2\left[\ln|w|\right]_2^3\right)$

Simplify $-\left(-\frac{1}{2}\cdot 2\left[\ln|w|\right]_2^3\right)$: $\left[\ln|w|\right]_2^3$ — Show Steps

$=\left[\ln|w|\right]_2^3$

Compute the boundaries: $\ln(3)-\ln(2)$ Show Steps

$=\ln(3)-\ln(2)$