1) Linear stability analysis of the deterministic logistic equation :

$$\frac{dN}{dt} = \mu N \left(1 - \frac{N}{K}\right) = F(N)$$

$$\frac{dN}{dt} = 0 \quad \Leftrightarrow \quad \begin{matrix} N = 0 \\ N = K \end{matrix}$$

Starting from an equilibrium point "N^*" we add a little perturbation and we check if the system come back to N^* (stable) or not (unstable).

$$N = N^* + \delta N$$

$$F(N) = F(N^*) + \nabla F(N)\big|_{N^*} \delta N + \nabla^2 F(N)\big|_{N^*} \cdot \frac{1}{2!} \cdot \delta^2 N + o(\delta^3 N)$$

$$F(N) \sim F(N^*) + \nabla F(N)\big|_{N^*} \delta N \qquad (1^{st} \text{ order})$$

The logistic equation then becomes, around N^*:

$$\frac{dN}{dt} = F(N) \quad \Rightarrow \quad \dot{\delta N} = \nabla F(N)\big|_{N^*} \delta N$$

But.. $\nabla F(N)\big|_{N^*} = \mu - 2\frac{\mu}{K} N \Big|_{N^*}$

$\boxed{N^* = K}$ $\nabla F(N)\big|_{N^*} = \mu - 2\mu = -\mu$

$$\dot{\delta N} = \underbrace{-\mu}\, \delta N$$

$$-\mu = \nabla F(N)\big|_{N^*} < 0 \Rightarrow N^* = K \text{ è max}$$

è eq. stabile

$\boxed{N^* = 0}$ $\nabla F(N)\big|_{N^*=0} = \mu > 0 \Rightarrow$ not stable eq. point.