① Dist. of $\hat{p}$.

$$\hat{p} \sim N\left(p, \sqrt{\frac{p \times q}{n}}\right).$$

※ $q = 1 - p$, n = sample size.

② Dist of $\bar{y}$ population mean.

population s.d,

$$\bar{y} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

③ C.I. for p (pop. prop.).

$$\hat{p} \pm Z_{CL\%} \times \sqrt{\frac{\hat{p} \times \hat{q}}{n}}$$

※ $\hat{q} = 1 - \hat{p}$

④ C.I. for μ.

$$\bar{y} \pm t_{n-1} \times \frac{s}{\sqrt{n}}$$

s: sample s.d,

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⑤ Testing for p.
Testing for μ.

$$\text{Test statistic} = \frac{\text{statistic} - \text{parameter}}{SD(\text{statistic})}.$$

⑥ sample size
- proportion. (easy).
- mean (hard, 2 steps)

Problem 1. "type ①"

$$P\left(\hat{P} > \frac{80}{500}\right)$$

$$= P(\hat{P} > 0.16)$$

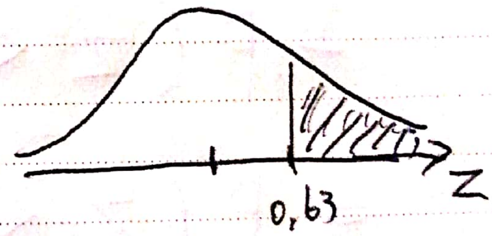
$p = 0.15$, $n = 500$, $\hat{P} \sim N\left(0.15, \boxed{\sqrt{\frac{0.15 \times 0.85}{500}}}\right)$ → 0.016

standardation

$$\approx P\left(Z > \frac{0.16 - 0.15}{0.016}\right)$$

$$= P(Z > 0.625)$$

$$\approx P(Z > 0.63)$$



$$= 1 - P(Z < 0.63)$$

$$= 1 - 0.7357 = 0.2643$$

$\therefore$ 26.43 %

Problem 2&3.

$\mu = 2$ | $p = 0.08$

$\sigma = 0.4$ | $n = 250$

"type ①"

2. $P(\hat{p} < 0.05) = ?$

$\hat{p} \sim N\left(0.08,\ \boxed{\sqrt{\frac{0.08 \times 0.92}{250}}}\right)$

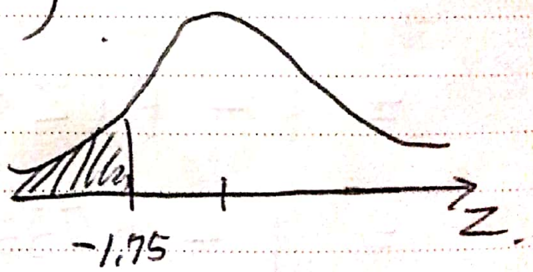
$\rightarrow 0.01716$

standardization.

$P\left(Z < \frac{0.05 - 0.08}{0.01716}\right)$

$= P(Z < -1.74845)$

$\approx P(Z < -1.75)$



$= 0.0401$

$\therefore$ 4.01%

"type ②"

3. $P(\bar{y} > 2{,}05)$

$\to 0{,}0253$

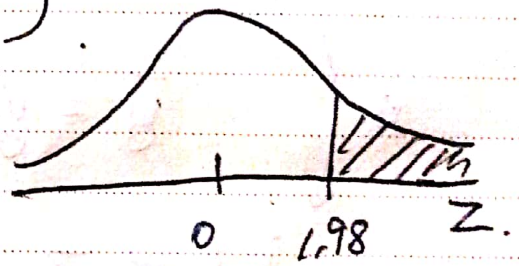
$\bar{y} \sim N\left(2, \boxed{\frac{0{,}4}{\sqrt{250}}}\right)$

※ standardization,

$P\left(Z > \frac{2.05 - 2}{0.0253}\right)$

$= P(Z > 1{,}97642)$

$\approx P(Z > 1{,}98)$



$= 1 - P(Z < 1{,}98)$

$= 1 - 0{,}9761 = 0{,}0239$

$\therefore$ 2,39%

Problem 4.

Joe $\mu_J = 22$. $\sigma = 2.5$

Mike. $\mu_M = 18$ $\sigma = 2$

a). $n = 1$. "type ②"

$P(y > 20)$.

Joe's

$\bar{y} = y \sim N\left(22, \frac{2.5}{\cancel{\sqrt{1}}}\right)$.

$P\left(Z > \frac{20-22}{2.5}\right)$

$= P(Z > -0.8)$.

$= 1 - P(Z < -0.8)$.

$= 1 - 0.2119 = 0.7881$.

"type ②" with probability chapter.

b) $P(JW > 20 \text{ and } MW > 20)$

$= \underbrace{P(JW > 20)}_{\text{from (a)}} \times \underbrace{P(MW > 20)}_{= 0.1587}$

$= 0.7881 \times 0.1587 = 0.12507$

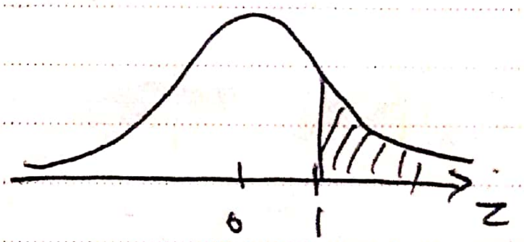
$\therefore 12.51\%$

b/c $P(MW > 20) = ?$

$\bar{y} = y \sim N(18, 2)$

$P\left(Z > \frac{20-18}{2}\right)$

$= P(Z > 1)$



$= 1 - P(Z < 1)$

$= 1 - 0.8413 = 0.1587$

Problem 5. "type ⑤" for P.

$p = 0.8$. $\hat{p} = \frac{131}{150} = 0.87333$

$n = 150$

a) $H_0 : p = 0.8$.

$H_A : p > 0.8$.

b) Test statistic $= \frac{\text{Statistic} - \text{parameter}}{SD(\text{statistic})}$.

$$= \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{0.87333 - 0.8}{0.03266}$$

$$= 2.245$$

$$SD(\hat{p}) = \sqrt{\frac{0.8 \times 0.2}{150}} = 0.03266$$

c)

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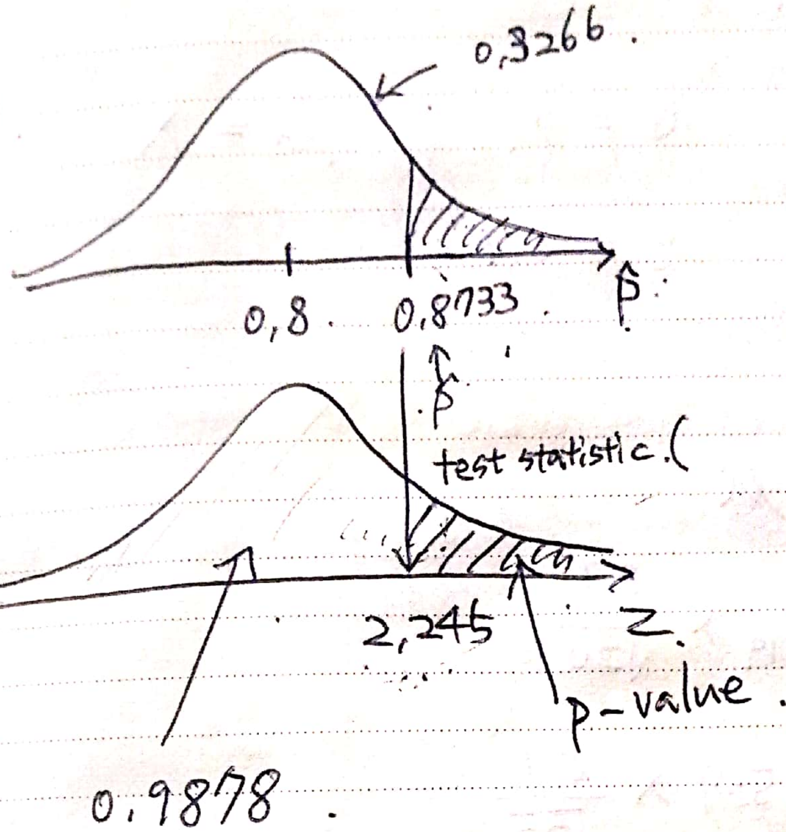
c)



p - value = 1 - 0,9878
= 0,0122

d) $\boxed{\begin{array}{l}\text{if your p-value} > 0{,}05 \text{, retain } H_0 \text{ (can not reject)} \\ \text{'' p-value} < 0{,}05 \text{, retain } H_A \text{ (can reject } H_0\text{)}\end{array}}$

Since p-value is 0,0122 < 0,05,

Reject H_0, there is strong evidence in support of H_A.

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Problem 6. "type ④"

$n = 20$. $\sigma = X$ $\mu = X$.

$\bar{y} = 0.5$ $s = 0.1$.

$C.L = 98\%$

a) 98% C.I. for μ.

$$\bar{y} \pm t_{19} \times \frac{0.1}{\sqrt{20}}$$

$$= 0.5 \pm 2.539 \times \frac{0.1}{\sqrt{20}}.$$

$$\Rightarrow (0.4432 , 0.55677).$$

$$\approx (0.443 , 0.557)$$

b) $n = 30$, $\bar{y} = 0.49$, $s = 0.11$.

98% C.I.

$$\bar{y} \pm t_{29} \times \frac{0.11}{\sqrt{30}}.$$

$$0.49 \pm 2.462 \times \frac{0.11}{\sqrt{30}}. \Rightarrow (0.441, 0.539).$$

c) We are 98% sure that
true average weight of the 1000 potatoes
is within this interval.

d) It would be narrower,
since the critical value of t_{29} with
95% confidence level will ~~tha~~ be smaller
than ~~that~~ the one with 98% confidence level.

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"type ⑥" hard !

Problem 7. sample size (mean),

ME = 0.7. C.L = 95%.

$s = 1$.

$$ME = \text{critical value} \times SE(\text{statistic}).$$

$$= t_{n-1} \times \frac{s}{\sqrt{n}}.$$

1 step. (Since we don't know t_{n-1})

using $Z_{95\%} = 1.96 \approx \boxed{2}$

$$0.7 = 2 \times \frac{1}{\sqrt{n}}.$$

$$\Rightarrow \frac{0.7}{2} = \frac{1}{\sqrt{n}}.$$

$$\Rightarrow \sqrt{n} = \frac{2}{0.7}.$$

$$\Rightarrow n = \left(\frac{2}{0.7}\right)^2 = 8.16326.$$

if $n > 60$, we can go with round up.

$n < 60$, we round down, go to step 2.

Step 2. (now we want to use t_{n-1} by using $n=8$)

$n=8$.

$$0,7 = t_7 \times \frac{1}{\sqrt{n}}$$

95% of $t_7 = 2,365$.

$$0,7 = 2,365 \times \frac{1}{\sqrt{n}}.$$

$$\Rightarrow n = 11,4147.$$

round up. $\Rightarrow$ $n = 12$.

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Problem 8. "type ⑤" for μ

$\mu = 110$.

$n = 12$

$\bar{y} = 109.2$, $s = 0.9$

a) $H_0 : \mu = 110$

$H_A : \mu < 110$

(~~sin~~ H_A can be set from the sentence

" determine whether a metal lost any of its mass after being exposed ")

b) Test statistic $= \dfrac{\text{statistic} - \text{parameter}}{SD(\text{statistic})}$

$$= \frac{\bar{y} - \mu}{\sigma / \sqrt{n}}.$$

☆ but we don't know σ, so we use s instead of σ

$$= \frac{109.2 - 110}{0.9/\sqrt{12}}$$

$$= -3.0792$$

$$\approx -3.08$$ (this follows t_{11} dist.)

Since we adjust the values for σ, test statistic no longer follow normal dist. It follows t-distribution w/ df = n-1.

c) because $-3.106 < -3.0792 < -2.718$.

t_{11} value corresponding 0.005 one tail

test statistic

t_{11} value corresponding 0.01 one tail

we can say

$$0.005 < \text{p-value} < 0.01.$$

d) Reject H_0, there is strong evidence in support of H_A.

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Problem 9. "type ③"

$\hat{p} = \frac{130}{200}$, $n = 200$.

a) 95 % C.I.

$\hat{p} \pm Z_{95\%} \times \sqrt{\frac{\hat{p}\hat{q}}{n}}$.

$= 0.65 \pm 1.96 \times \boxed{0.03373}$.

$\Rightarrow (0.5839, 0.7161)$

b) 90 % C.I.

$\hat{p} \pm Z_{90\%} \times \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 0.65 \pm 1.645 \times 0.03373$.

$\Rightarrow (0.5945, 0.7055)$

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Problem 10. "type ③"

$\hat{p} = \frac{195}{300}$, $n = 300$.

Note that $\hat{p} = \frac{195}{300} = 0.65$. which is the same value with $\hat{p}$ from the previous problem.

Thus, same $\hat{p}$, but $n = 300$, larger sample size.

95% C.I. for P.

$$\hat{p} \pm Z_{95\%} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= 0.65 \pm 1.96 \times \boxed{0.027537}$$

$(0.5960, 0.703974)$

Since $\boxed{SE(\hat{p})}$ is less than the one in problem (9a), C.I, Should be narrower.

Problem 11. "type ⑥ sample size (proportion)". easy

target ME = 0.02

C.L. = 90% $\Rightarrow Z_{90\%} = 1.645$

ME = critical value × SE($\hat{p}$)

$$0.02 = 1.645 \times \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}$$

1) $p = 0.65$

$$0.02 = 1.645 \times \sqrt{\frac{0.65 \times (1-0.65)}{n}}$$

$$\Rightarrow \frac{0.02}{1.645} = \sqrt{\frac{0.2275}{n}}$$

$$\Rightarrow \left(\frac{0.02}{1.645}\right)^2 = \left(\sqrt{\frac{0.2275}{n}}\right)^2$$

$$\Rightarrow 0.000147818 = \frac{0.2275}{n}$$

$$\Rightarrow n = 1539.05 \qquad \therefore n = 1540$$

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2) when $P = 0.5$

$$0.02 = 1.645 \times \sqrt{\frac{0.5 \times 0.5}{n}}$$

$$\Rightarrow n = 1691.265$$

$$\therefore n = 1692.$$