MTH 2140 Homework 5

Instructions: You may consult any sources, human or otherwise. Each student should write up his or her own work. Please cite your sources.

- 1. Reading: Sections 5.1-5.2
- 2. Section 5.1 # 1,24
- 3. Section 5.2 # 4,12
- 4. Consider the following chemical reaction:

$$\begin{aligned} a &\to x \\ 2x + y &\to 3x \\ b + x &\to y + d \\ x &\to e \end{aligned}$$

in a well-mixed vat where concentrations of a, b, d, and e are kept constant. The associated ODE model is

$$\dot{x} = a + x^2y - bx - x$$

$$\dot{y} = bx - x^2y.$$

The constants a and b, which represent concentrations, are assumed to be positive. Write a mini-report which addresses the following issues:

- What is the equilibrium?
- What is a qualitative description of the near-equilibrium dynamics (i.e. use the technique of linearization) and how does it change as the parameters a and b are varied? (the trace-determinant plane might be useful here)
- How much information can we obtain about global dynamics from nullcline analysis? How do the nullclines and the direction fields along them vary as the parameters a and b vary?
- In the region of parameter space where the equilibrium point is not stable, what is the long time behavior of solutions with initial conditions close to the equilibrium point?

Your report should be easy for a mathematical peer to read. You can use technology, as appropriate, to simplify computations and generate pictures.