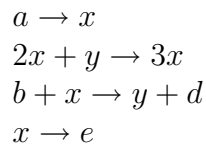


Due December 3, 2010

MTH 2140 Homework 5

Instructions: You may consult any sources, human or otherwise. Each student should write up his or her own work. Please cite your sources.

1. Reading: Sections 5.1-5.2
2. Section 5.1 #5
3. Section 5.2 #4,15
4. Consider the following chemical reaction:



in a well-mixed vat where concentrations of a , b , d , and e are kept constant. The associated ODE model is

$$\begin{aligned}\dot{x} &= a + x^2y - bx - x \\ \dot{y} &= bx - x^2y.\end{aligned}$$

The constants a and b , which represent concentrations, are assumed to be positive.

- What is the equilibrium?
- What is a qualitative description of the near-equilibrium dynamics (i.e. use the technique of linearization) and how does it change as the parameters a and b are varied? (the trace-determinant plane might be useful here)
- In the region of parameter space where the equilibrium point is not stable, what is the long time behavior of solutions with initial conditions close to the equilibrium point? (technology might be useful here)

5. The following system of nonlinear equations provides a model for the situation where tension between the behavior near the origin and the behavior far from the origin is resolved in the creation of a periodic orbit:

$$\dot{x} = \beta x - y - x(x^2 + y^2); \quad \dot{y} = x + \beta y - y(x^2 + y^2).$$

Make the transformation $r^2 = x^2 + y^2$. Find and analyze the equation for r . Describe what happens as β passes through the value $\beta = 0$. Relate this to the chemical reaction above.