Due November 5, 2010

MTH 2140 Homework 2

1. Reading: Sections 1.6-1.9
2. Section 1.6 #1,8,30, Section 1.7 #4,8,18,20 Section 1.8 #9
3. Consider the equation

$$\dot{p} = kp(1 - \frac{p}{N}) - a(1 + \sin(bt)). \tag{1}$$

This is a logistic population model with seasonal harvesting, i.e. when $\sin(bt) = 1$, the population is harvested (killed and eaten) at a rate of 2a, whereas when $\sin(bt) = -1$ the population in not harvested at all. In this exercise we will fix $k = 1/4$, $N = 4$, and $b = 1$. We will examine the behavior of this model as a varies.

   (a) Use MATLAB (or your favorite program) to solve this equation (e.g. with Euler's method) for several different values of a between .1 and .3 and for several different initial conditions.

   (b) Describe the behavior of the system when a is small.

   (c) Describe the behavior of the system when a is large.

   (d) Explain, as well as you are able, what happens as a increases and why it happens.

4. (Extra Credit) I think that this problem is fun and illuminating. If you have time left in your MTH 2140 budget and feel in control of the book problems, I think that you will find it a worthwhile challenge. Its more important though to master the fundamentals than to explore tangents, so don't feel compelled to proceed just for the extra credit.

Consider the equation

$$\dot{x} = a(t)x \tag{2}$$

with a periodic in the sense that there is a period T so that $a(t+T) = a(t)$ for any value t. Imagine that (2) admits a periodic solution $x_{per}(t)$ with period T, i.e. $x_{per}(t + T) = x_{per}(t)$ for any value t. I claim that the quantity $\frac{1}{T}\int_0^T a(t)dt$ tells us a lot about what happens to solutions to the initial value problem for (2) with initial values $x(0) = x_0$ close to $x_{per}(0)$.

(a) What does the quantity $\frac{1}{T}\int_0^T a(t)dt$ tells us about what happens to solutions to the initial value problem for (2) with initial values $x(0) = x_0$ close to $x_{per}(0)$?

(b) Give a condition on the coefficient $a(t)$ which classifies periodic solutions as either "sources" or "sinks."

(c) Consider a nonlinear equation $\dot{x} = f(x, t)$ that admits a periodic solution x_{per} such as (1). Give a condition on $a(t) := \frac{\partial f}{\partial x}(x_{per}(t), t)$ which classifies x_{per} as a "source" or "sink."

(d) Revisit (3d) above with your new-found knowledge.