Due October 29, 2010

MTH 2140 Session II Homework 1

Instructions: The standard of writing for this assignment is that writing must be neat and clear enough for an expert to easily read and assess. You will not be graded based on exposition or quality of technical writing for this assignment. (On later assignments you may be asked to write for an audience of mathematical peers.) Please keep track of the amount of time that you spend on this assignment.

1. Read sections 1.1-1.3; 1.5
2. Read problems Section 1.1 #4,10-14,17, 21-22 and find a classmate to discuss these problems with. You don't have to write them up or hand them in. If you regard this as new material please let me know.
3. Do problems Section 1.2 #18,39; Section 1.3 #15,17,18 ; Section 1.5 #9ab,15
4. (from Hubbard & West p159; Strogatz p41) Imagine a cylindrical bucket with cross-sectional area A and with a hole of area $a < A$ cut out of the bottom. Imagine that at time t there is water in the bucket from the bottom up to height $h(t)$. Let $v(t)$ denote the velocity of the water passing through the hole at the bottom.
   (a) Derive a differential equation for h as follows: First use conservation of mass to relate $\dot{h}$ to v; then use conservation of energy to relate h to v (assuming that all the potential energy that is lost between t and Δt is transferred to kinetic energy); finally use the two equations together to relate $\dot{h}$ to h.
   (b) Solve this equation via separation of variables. What happens to your solution as $t \to -\infty$? Does this make physical sense?
   (c) Given that the bucket is empty at time $t = 0$, can you determine the height of the water in the bucket at time $t = -10$? why or why not? (give an explanation for the model differential equation as well as the real world situation)
   (d) Interpret your findings in terms of the uniqueness theorem.