

Due November 11, 2010

MTH 2140 Session 02 Quiz 2

Instructions: This is a self-scheduled quiz. You can work for as long as you like, but you can't take breaks. You are not allowed to work in groups or discuss problems with other people and you are not allowed to use notes, books, web browsers etc.

After you have gotten as far as you can in a closed-book environment, you can change pen color and work in an open notes, open book, open internet environment for half credit.

Please place the quiz in my mailbox (in MH250) or in the box outside of my office (MH 257) sometime before Thursday at 5 PM.

This question is taken from MIT's open courseware, specifically from Prof. Mattuck's differential equations class.

The autonomous ODE

$$\dot{y} = .25 - ay + y^2$$

models a population of Australian lovebugs, which infest pomagranites in Farmer Jones' orchard. y is measured in megabugs, or millions of bugs. The term .25 reflects a constant immigration into Mr Jones' orchard from the neighboring orchards (where the pomagranites are inferior). These pests can be kept in check using an expensive bioengineered spray. Application at a rate a moves the natural rate of growth of the lovebug population from y (corresponding to $\dot{y} = y^2$) down to $y - a$ (corresponding to $\dot{y} = (y - a)y$).

1. Determine the smallest rate a which will contain the lovebug population at a finite level provided that it starts at a sufficiently low level. What is that level?
2. Better will be a choice of a which brings the lovebug population down to $y = 1/4$. What rate a will lead to that result? How large an initial population will this rate of application control?