Due November 12, 2010

MTH 2140 Homework 3

Instructions: The standard of writing for this assignment is that writing must be neat and clear enough for an expert to easily read and assess. You will not be graded based on exposition or quality of technical writing for this assignment. (On later assignments you may be asked to write for an audience of mathematical peers.) Please keep track of the amount of time that you spend on this assignment.

1. Read sections 1.9, 2.1-2.4
2. Section 1.9 #7,19
3. Section 2.2 #11
4. Review exercises for chapter 2: #19-24
5. The forced and damped oscillator: Consider the differential operator

$$L = m\frac{d^2}{dt^2} + b\frac{d}{dt} + k$$

in the case that $b^2 < 4km$. In class we found the general solution of $Lx = 0$ to be

$$x(t) = e^{\alpha t}\left(c_1 \cos(\omega t) + c_2 \sin(\omega t)\right)$$

where $\alpha = \frac{-b}{2m}$ and $\omega = \frac{\sqrt{4km-b^2}}{2m}$.

   (a) Find the general solution of the inhomgeneous equation

$$Lx = \cos(\omega_0 t).$$

   Hint: Depending on how much you like complex numbers I would recommend either guessing a particular solution of the form $x_p(t) = ae^{i\omega_0 t}$ *and making use of the fact that* $\cos \omega_0 t = \frac{1}{2}(e^{i\omega t} + e^{-i\omega t})$ *or guessing a particular solution of the form* $x_p(t) = a_1 \cos(\omega_0 t) + a_2 \sin(\omega_0 t)$.

   (b) Show that as $t \to \infty$, the general solution approaches a periodic orbit.

(c) Find the amplitude of this limiting periodic orbit. This amplitude is called the **gain**. *Hint: the amplitude of a periodic orbit of the form* $ae^{i\omega_0 t}$ *is* $|a|$*, the modulus of* a*; the amplitude of a periodic orbit of the form* $a_1 \cos(\omega_0 t) + a_2 \sin(\omega_0 t)$ *is* $\sqrt{a_1^2 + a_2^2}$.

(d) Show that for b small enough and ω chosen carefully, the gain can be made as large as you want.

(e) Describe an engineering application of resonance and relate it to the rest of the problem.