Due December 10, 2010

MTH 2140 Homework 6

Instructions: You may consult any sources, human or otherwise. Each student should write up his or her own work. Please cite your sources. This work will be graded based on effort each student will get a 100, a 50, or a 0. A score of 125 is possible if problem 4 is done as extra credit. A solution set will be provided for the required (book) problems.

1. Reading: Sections 5.3-5.4
2. Section 5.3 #9,19
3. Section 5.4 #9,13
4. Do something: Either
   (a) Review to solidify your understanding of the course material (come up with a reasonable deliverable if you want extra credit, otherwise just enjoy your review); OR
   (b) Go beyond the course by doing something you find cool and interesting. I've listed some examples of things I find cool and interesting (and perhaps challenging) below.
      i. Study the following mass-spring system of n identical masses coupled via n identical springs:

$$m\ddot{x}_n = k(x_{n+1} - x_n) - k(x_n - x_{n-1}); \qquad n = 1, \cdots, N$$

      Here you should interpret x_0 as x_N and x_{N+1} as x_1. This corresponds to coupling the N^{th} mass to the first mass by a spring.
      Decouple the system by making the Fourier Transform: $\hat{x}_j(t) := \sum_{k=1}^{N} e^{\frac{2\pi i k j}{N}} x_k(t)$, find a differential equation for $\hat{x}_j$ and say as much as you can about the solution.
      ii. # 24 in section 5.4
      iii. lab 5.4 in on page 556

iv. Numerically study the nonlinear mass-spring system

$$m\ddot{x}_n = F(x_{n+1} - x_n) - F(x_n - x_{n-1}); \qquad n = 1, \cdots, N$$

where the restoring force $F(x)$ is not simply $-kx$ but rather the more pendulum-like $F(x) = -kx + \beta x^3$. What can you learn about this system? The keywords Fermi-Pasta-Ulam will be a useful reference for research.