Vectors

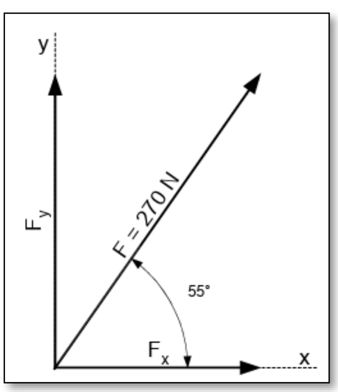
Contents

 No AI use

Time required: 120 minutes

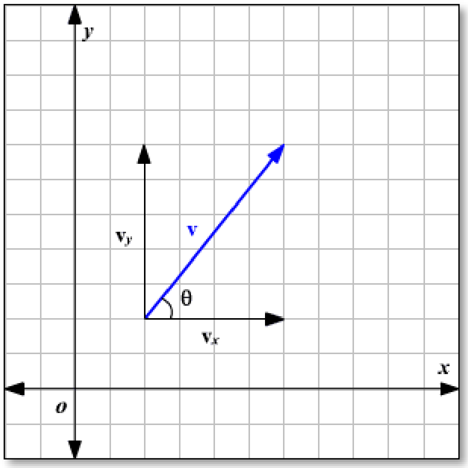
Problem 1

A force acts on a fixed pin with a magnitude of 270 N at an angle of 55° from horizontal. Find the horizontal and vertical components of the force.



Background on vectors and vector components

In a two-dimensional coordinate system, any vector can be broken into an $\boldsymbol{x}$-component and a y-component. For example, in the figure shown below, the vector $\vec{v}$ is broken into two components, v_x and v_y. Let the angle between the vector and its x-component be θ.



In the figure, the components can be quickly read. The vector in the component form is

$$\vec{v} = \langle 4, 5 \rangle$$

But if we only know the magnitude of the vector $\vec{v}$, and the angle θ, then using trigonometric ratios, we can calculate the components of the vector as below.

$$v_x = v\cos\theta \text{ and } v_y = v\sin\theta$$

We can write the vector $\vec{v}$ as

$$\vec{v} = \langle v_x, v_y \rangle = \langle v\cos\theta, v\sin\theta \rangle$$

In MATLab, you can use the following function to calculate sine and cosine. Notice that the function depends on the units of the angles.

- **sin(x)**: Calculates the sine of the angle 'x' in **radians**.
- **sind(x)**: Calculates the sine of the angle 'x' in **degrees**.
- **cos(x)**: Calculates the cosine of the angle 'x' in **radians**.
- **cosd(x)**: Calculates the cosine of the angle 'x' in **degrees**.
- **deg2rad(D)** converts angle units from degrees to radians for each element of D.
- **rad2deg(R)** converts angle units from radians to degrees for each element of R.

To read more about Trig function, click the link below.

https://www.mathworks.com/help/matlab/trigonometry.html?s_tid=CRUX_lftnav

Assignment:

A force acts on a fixed pin with a magnitude of 270 N at an angle of 55° from horizontal. Find the horizontal and vertical components of the force.

We need to write the Force vector in vector components.

$$\vec{\boldsymbol{F}} = \langle \boldsymbol{F_x}, \boldsymbol{F_y} \rangle$$

Where $F_x = F\cos\theta$ and $F_y = F\sin\theta$

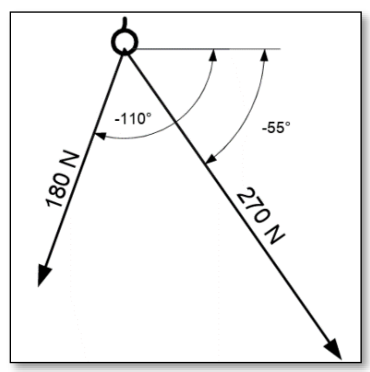
Find the component of the vector $\vec{F}$ and display the vector in vector form.

1. Assign the given values to variables.
2. Convert angle to radians.
3. Calculate the horizontal and vertical components.
4. Create the force vector.
5. Display the results.

Example run:

```
Horizontal Component: 154.87 N
Vertical Component: 221.17 N
Force Vector: [154.87, 221.17] N
```

Problem 2



Two boys are playing by pulling on ropes connected to a hook in a rafter. The bigger one pulls on the rope with a force of 270 N at an angle of 55° from horizontal. The smaller boy pulls with a force of 180 N at an angle of 110° from horizontal.

1. Assign the given values to variables.
2. Convert degrees to radians.
3. Calculate the x- and y- components of the first force ($F_1 = 270\ N$) and print F_1 as a vector using fprint.
   NOTE: To access individual elements of a vector use: F_resultant(i); where i represents the position of the element in the vector.
4. Calculate the x- and y- components of the second force ($F_2 = 180\ N$) and print F_2 as a vector using fprintf.
5. Calculate the resultant force on the hook? To find this, add the two vectors.

6. What is the magnitude of the resultant force? To do this, use the function **norm(v)**, where v is the vector.

7. What is the direction of the resultant force? To do this, use any of the following functions.

Function	*Argument(s)*	*Notes*
atan(abs(Fx / Fy))	one argument: abs(F_x / F_y)	Returns the included angle
atan2(F_y, F_x)	two arguments: F_x and F_y	Returns the coordinate direction angle Angle value is always between 0 and π radians (0 and 180°) A negative sign on the angle indicates a result in one of the lower quadrants of the Cartesian coordinate system
cart2pol (F_x, F_y)	two arguments: F_x and F_y	Returns the positive angle from the positive x-axis to the vector Angle value always between 0 and 2π radians (0 and 360°) An angle value greater than 180° (π radians) indicates a result in one of the lower quadrants of the Cartesian coordinate system

Display the resulting information using fprintf.

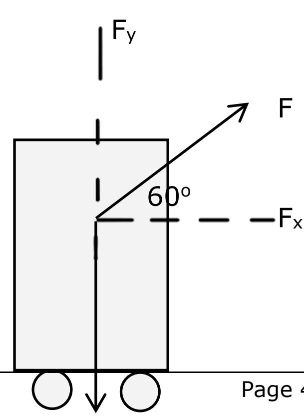
Example run:

```
F1 Vector: [154.87, 221.17] N
F2 Vector: [-61.56, 169.14] N
Resultant Force Vector: [93.30, 390.32] N
Magnitude of Resultant Force: 401.31 N
Direction of Resultant Force: 76.56°
```

Problem 3

A traveler at the airport is pulling her 15-kg suitcase.

Suppose the traveler is applying a force so that the net force on the vertical direction is at a static equilibrium. That is, the vertical component of the force is equal to the gravitational force of the suitcase. A force analysis to illustrate the scenario is given below.



If the traveler is applying a force F at a 60° angle from the floor, use projection of the force F to find the amount of force the traveler is using to pull the suitcase. Use g=9.8m/s^2.

The vertical component of the force is equal to the gravitational force of the suitcase,

$$F_y = mg$$

We can also write the vertical component of the vector F as

$$F_y = F \sin(\theta)$$

Combining these two equations, we get,

$$F \sin(\theta) = mg$$

Solve this equation to find the magnitude of the force (F) applied on the suitcase.

1. Assign the given values to variables.
2. Convert degrees to radians.
3. Write the force F in vector form.
4. As the suitcase moves along a straight line for 50 meters on the ground, find the work done to move the suitcase forward.

The suitcase is moving along the ground, which is same as x-axis. We can write the distance in the vector form as $d = [50,0]$.

Work done is calculated as $W = Fd$

When the force and distance is in vector form, we use the dot product to calculate the work done. $W = \vec{F} \cdot \vec{d}$

The dot product (or scalar product) can be calculated in two ways:

In trigonometric terms, the equation for a dot product is written as

$$A \bullet B = A\, B\, cos(\theta)$$

where θ is the angle between arbitrary vectors A and B.

In matrix form, the equation is written

$$A \bullet B = A_x\, B_x + A_y\, B_y + A_z\, B_z$$

MATLAB provides a dot product function, **dot(A,B)** to automatically perform the calculations required by the matrix form of the dot product.

5. The traveler had to lift the suitcase to the conveyor belt for a security check. Assume the initial position of the suitcase is at the origin, and the ending position is at <3, 1>. Use a dot product to find the work done.

If the origin is defined as $d_1 = [0,0]$ and ending as $d_2 = [3,1]$, the distance travelled will be,

$$d = d_2 - d_1$$

Use the dot product of the force vector and the distance travelled to find the work done.

Example run:

```
Force Vector: [84.87, 147.00] N
Work Done: 401.61 J
```

Problem 4

Consider a wrench being used to tighten a bolt. If the length of the wrench is represented by the position vector $\vec{r} = [2, -1, 3]$ and the force applied by the hand is $\vec{F} = [3,4,2]$, find the torque $\vec{\tau}$.

Background

Torque ($\vec{\tau}$) is a measure of the rotational force on an object. It is calculated using the **cross product** of the position vector $\vec{r}$ and the force vector $\vec{F}$.

The **cross product** (or vector product) can be calculated in two ways:

In trigonometric terms, the equation for a dot product is written as

$$\vec{\boldsymbol{C}} = \vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = \boldsymbol{AB}\,\mathbf{sin}(\boldsymbol{\theta})\,\vec{\boldsymbol{u}}_{\boldsymbol{c}}$$

where θ is the angle between arbitrary vectors $\vec{A}$ and $\vec{B}$, and $\vec{u}_c$ is a unit vector in the direction of $\vec{C}$ (perpendicular to $\vec{A}$ and $\vec{B}$, using right-hand rule).

In matrix form, the equation is written in using components of vectors $\vec{A}$ and $\vec{B}$, or as a determinant. Symbols $\vec{\imath}$, $\vec{\jmath}$, and $\vec{k}$ represent unit vectors in the coordinate directions.

$$\vec{\boldsymbol{\imath}} = [\mathbf{1}, \mathbf{0}, \mathbf{0}], \qquad \vec{\boldsymbol{\jmath}} = [\mathbf{0}, \mathbf{1}, \mathbf{0}], \qquad \vec{\boldsymbol{k}} = [\mathbf{0}, \mathbf{01}]$$

$$\boxed{\begin{aligned} \mathbf{A}\times\mathbf{B} &= \left(A_yB_z - A_zB_y\right)\mathbf{i} - \left(A_xB_z - A_zB_x\right)\mathbf{j} + \left(A_xB_y - A_yB_x\right)\mathbf{k} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \end{aligned}}$$

MATLAB provides a cross product function, **cross(A,B)**, to automatically perform the calculations required by the matrix form of the cross product.

Assignment

Consider a wrench being used to tighten a bolt. If the length of the wrench is represented by the position vector $\vec{r} = [2, -1, 3]$ and the force applied by the hand is $\vec{F} = [3, 4, 2]$, find the torque $\vec{\tau}$.

Example run:

```
Torque Vector: [-14.00, 5.00, 11.00] N·m
```

Assignment Submission

1. Submit properly named and commented script file.
2. Attach a screenshot of the Command Window showing the successful execution of each script.
3. Attach all to the assignment in Blackboard.