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REGULAR RESEARCH PAPER

An adaptive multiobjective evolutionary algorithm based on grid subspaces

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Abstract

The successful application of multi-objective evolutionary algorithms (MOEAs) in many kinds of multiobjective problems have attracted considerable attention in recent years. In this paper, an adaptive multi-objective evolutionary algorithm is proposed by incorporating the concepts of the grid system (denoted as AGMOEA). Based on grid, the objective space is divided into subspaces. Based on the quality and dominance relationship between subspaces, the evolutionary opportunities are dynamically allocated to different subspaces with an adaptive selection strategy. To improve the evolutionary efficiency, the evolutionary scheme and an external archive mechanism considering representative individuals are proposed. The experimental results on 21 benchmark problems demonstrate that the proposed algorithm is competitive or superior to the rival algorithms.

Keywords Grid · External archive · Multi-objective evolutionary algorithm · Dominance relationship

1 Introduction

There are a large number of multi-objective optimization problems (MOPs) in the fields of science and engineering. Different from the problems with a single objective, MOPs usually have several conflicting objectives to simultaneously optimize. Consequently, the solution set of MOPs is composed of some non-dominated solutions, which is called the Pareto solution set (PS). During optimization, both the convergence and distribution of the PS need to be guaranteed.

Multi-objective evolutionary algorithms (MOEAs), e.g., NSGA-II [1], MOEA/D [2], are global stochastic optimization approaches that are simple but effective. Due to the population-based evolution behavior, MOEAs are inherently better suited to solve MOPs, which have been widely used in cloud computing, wireless sensor allocation, production planning and scheduling, process optimization, and so on [3–7]. So, the framework and improvement of MOEAs have attracted extensive interest and research in recent years [8–10].

Generally, MOEAs can be divided into three categories. The first is based on dominance relationship to design selection mechanism, which is typically represented by SPEA [11], SPEA2 [12], PESA [13], PESA-II [14], PAES [15], and NSGA-II. The second is based on decomposition, represented by MOEA/D, which decomposes the MOPs into multiple single-objective subproblems. The subproblems are optimized in a collaborative manner within the neighborhood. The third is based on indicators, which directly evaluates the metrics as optimization objectives and is typically represented by IBEA [12], SMS-EMOA (Beume et al. 2007), and HypE (Bader and Zitzler 2011).

For MOEAs, it is very important to manage the diversity of the population. Based on grid system, objective space can be partitioned into different regions, and strategies can be designed to manage population diversity effectively. PAES introduced crowded region based on grid to describe the coverage of the solutions and maintain the diversity of the external archive. The archive was naturally clustered into small regions with grid in objective space, and a new crowding procedure was proposed in PAES. PESA-II used hypercubes to evaluate the density of the population and designed the strategies about the region-based selection and the update

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of the external archive. In the Pareto-adaptive epsilon-dominance algorithm [16], the hyper-grid is adopted in the archive to maintain diversity. GSEA [17] applied the construction strategy of the elitist population based on spatial grid to keep diversity and reduce computation cost. Moreover, Rachmawati and Srinivasain [18] presented a dynamic strategy of grid resizing which could shrink or expand the hyper-grid. Yuan et al. [19] proposed a novel multi-objective evolutionary based on hybrid adaptive grid algorithm (HAGA). Grid technology was also incorporated into decomposition MOEA [20, 21], multi-objective differential evolutionary algorithm [22, 23], multi-objective particle swarm optimization algorithm [24], multi-objective whale optimization algorithm [25], and so on. For the many-objective problems, it was very difficult to select elite solutions because a lot of solutions could not be compared using the Pareto dominance relationship. Based on the grid approach, designing new selection strategy could solve the issue of less selection pressure, and the convergence and diversity could be further balanced [26–29]. A new metric, Grid-IGD [30], was proposed to estimate both convergence and diversity of PF approximations for multi/many-objective optimization. Grid-IGD possessed desirable properties such as Pareto compliance, immunity to dominated/duplicate solutions and no need of normalization.

In the existing research, grids are mainly used to divide the objective space, and the concept of grid coordinates and density is used to design the diversity strategy and selection scheme. Moreover, from the perspective of the whole population, grid ranking can measure the convergence of different regions, and dominance relationship between girds can reflect the convergence and distribution of the whole population in objective space. Especially weakly grid-dominance relationship can be used to guaranteed the distribution information without losing convergence too much. Therefore, the grid ranking and grid dominance relationship can be used to balance the convergence and diversity and to improve the performance of an algorithm, as well. This paper proposes an adaptive MOEA based on grid, according to the concept of grid ranking and the dominance relationship. The main contributions are as follows:

The objective space is divided into subspaces based on the grid approach. According to the subspace ranking and subspace dominance relationship, an adaptive selection strategy of subspaces is proposed which can allocate evolutionary opportunities to different subspaces dynamically to achieve a good balance between convergence and diversity. To improve the evolution efficiency, the evolutionary scheme based on adaptive selection of solutions from the subspace and multiple crossover operators is developed. To well guide the search, the external archive is adopted and the grid-based local search is presented to improve the external archive.

2 Methodology

2.1 Multiobjective optimization

A MOP can be defined as the form of formula (1)–(4). Ω is the decision space defined by n variables, and $x = [x_1, x_2, \dots, x_n]^T$ is a decision vector.R^m is the objective space, and F(x) consists of m conflicting objective functions $g_i(x) \geq 0$ and $h_j(x) = 0$ are the inequality constraint and the equality constraint, respectively.

$$Minimize\ \mathrm{F}(x) = (f_1(x), \dots, f_m(x)) \tag{1}$$

$$s.t. g_i(x) \geq 0\ i = 1, \dots, I \tag{2}$$

$$h_j(x) = 0\ j = 1, \dots, J \tag{3}$$

$$x = [x_1, x_2, \dots x_n]^T \in \Omega \tag{4}$$

In general, Pareto dominance relationship can be used to compare solutions. Let $u, v \in \Omega$, u is said to dominate v, denoted by $u \prec v$ if and only if $f_i(u) \leq f_i(v)$, for every $i \in 1, \dots, m$ and $f_i(u) < f_i(v)$ for at least one index $j \in 1, \dots, m$. Given a set S in Ω, a solution in S is called non-dominated if no other solution in S can dominate it. A solution $x^* \in \Omega$ is called Pareto-optimal if $F(x^*)$ is non-dominated in the attainable objective set. $F(x^*)$ is then called a Pareto-optimal objective vector. The set of all the Pareto-optimal solutions is called the Pareto set (PS) and the set of all the Pareto-optimal objective vectors is called the Pareto Front (PF).

2.2 Subspace System

Grid system is a division of the objective space $i \in \{1, \dots, m\}$. Each objective dimension from the ideal solution $Z^* = (z_1^*, \dots, z_m^*)^T$ (defined by Eq. (5)) to the nadir point $Z^{nad} = (z_1^{nad}, \dots, z_m^{nad})^T$ (defined by Eq. (6)) is divided into K equal intervals, and then the grid interval vector $D = (d_1, \dots, d_m)^T$ for m objectives can be calculated by Eq. (7).

$$z_i^* = \min_{x \in \Omega} f_i(x)\ i = 1, \dots, m \tag{5}$$

$$z_i^{nad} = \max_{x \in \Omega} f_i(x)\ i = 1, \dots, m \tag{6}$$

$$d_i = (z_i^{nad} - z_i^*)/K \tag{7}$$

In the grid system, the grid coordinate $\vec{g} = (g_1, \dots, g_m)$ is defined as $g_i = \lfloor (f_i(x) - z_i^*)/d_i \rfloor$, $g_i \in [0, K-1]$. Based on the concept of grid, subspace system is defined. In subspace system, $K^{\vec{g}}$ denotes a subspace with the grid coordinate $\vec{g}$. That is, a solution x belongs to subspace $K^{\vec{g}}$, denoted as $x \in K^{\vec{g}}$, if and only if $f_i(x) \in [z_i^* + g_i \times d_i, z_i^* + (g_i + 1) \times d_i]$, i=1, …, m. In order to describe the relationship between subspaces, the following definitions based on grid system can be given based on Yang et al. [26] and Cai et al. [29].

Definition 1 *SD (subspace dominance)* For two subspaces $K^{\vec{a}}$ and $K^{\vec{b}}$, $\vec{a} = (a_1, \ldots, a_m)$, $\vec{b} = (b_1, \ldots, b_m)$. $K^{\vec{a}}$ is said to subspace dominate $K^{\vec{b}}$, denoted by $K^{\vec{a}} \prec_{sd} K^{\vec{b}}$, if and only if $a_i \le b_i$, for every $i \in \{1, \ldots, m\}$ and $a_j < b_j$ for at least one index $j \in \{1, \ldots, m\}$.

Definition 2 *SSD (strong subspace dominance)* For two subspaces $K^{\vec{a}}$ and $K^{\vec{b}}$, $\vec{a} = (a_1, \ldots, a_m)$, $\vec{b} = (b_1, \ldots, b_m)$. $K^{\vec{a}}$ is said to strongly subspace dominate $K^{\vec{b}}$, denoted by $K^{\vec{a}} \prec_{ssd} K^{\vec{b}}$, if and only if $a_i < b_i$, for every $i \in \{1, \ldots, m\}$.

Definition 3 *WSD (weak subspace dominance)* For two subspaces $K^{\vec{a}}$ and $K^{\vec{b}}$, $\vec{a} = (a_1, \ldots, a_m)$, $\vec{b} = (b_1, \ldots, b_m)$. $K^{\vec{a}}$ weakly subspace dominates $K^{\vec{b}}$, denoted by $K^{\vec{a}} \prec_{wsd} K^{\vec{b}}$, if and only if $a_i \le b_i$, for every $i \in \{1, \ldots, m\}$ and a_i=b_i for at least one index $j \in \{1, \ldots, m\}$.

Definition 4 *SCP (subspace corner point)* For subspace $K^{\vec{g}}$, $\vec{g} = (g_1, \ldots, g_m)$, the SCP can be denoted as $Z^{scp} = (z_1^{scp}, \ldots z_m^{scp})^T$, where $z_i^{scp} = z_i^* + d_i \times g_i, i = 1, \ldots, m$.

Definition 5 *SDV* (*subspace direction vector*) For subspaces $K^{\vec{g}}$, $\vec{g} = (g_1, \ldots, g_m)$, the subspace direction vector λ^g can be λ^g= $(\lambda_1^g, \ldots \lambda_m^g)^T$ denoted as, and λ_i^g is defined in Eq. (8) where ε is a very small positive number to avoid dividing by zero.

$$\lambda_i^g = 1.0/(\mathrm{g}_i + \varepsilon) \tag{8}$$

Definition 6 *SWS* (*subspace weighted sum*) For a solution x in subspace $K^{\vec{g}}$, its subspace weighted sum is denoted as $h^{sws}(x)$ that is calculated by Eq. (9).

$$h^{sws}(x|\lambda^g, Z^{scp}) = \sum_{i=1}^{m} \lambda_i^g (f_i(x) - z_i^{scp}) \tag{9}$$

Definition 7 SR (*subspace ranking*) For subspace $K^{\vec{g}}$, $\vec{g} = (g_1, \ldots, g_m)$, its subspace ranking is defined as the sum of all the coordinates, which can be defined in Eq. (10).

$$SR(\vec{g}) = \sum_{i=1}^{m} g_i \tag{10}$$

A subspace system for a bi-objective problem with K=3 is illustrated in Fig. 1. Based on the above definitions, the following dominance relationships between the subspaces can be obtained, such as $K^{(1,0)} \prec_{sd} K^{(2,0)}$, $K^{(1,0)} \prec_{sd} K^{(2,2)}$, $K^{(1,0)} \prec_{wsd} K^{(2,0)}$, $K^{(1,0)} \prec_{ssd} K^{(2,2)}$, and it is clear that the two subspaces $K^{(1,2)}$ and $K^{(2,1)}$ are subspace-nondominated with each other. Moreover, it can be seen that the relationship between the subspaces also determines the relationship between the solutions within them. For example, for two solutions $x_1 \in K^{\vec{a}}$ and $x_2 \in K^{\vec{b}}$, subspace dominates x_2, denoted by $x_1 \prec x_2$, if and only if $K^{\vec{a}} \prec_{sd} K^{\vec{b}}$. For example, it can be sai x_1 d that solution P_3 subspace dominates solution P_7 because of $K^{(1,1)} \prec_{sd} K^{(2,2)}$. That is, the quality of solutions can be measured with subspaces dominance relationship effectively.

3 Proposed AGMOEA

3.1 Algorithm overview

The overall framework of the proposed adaptive MOEA based on grid subspaces (denoted as AGMOEA) is presented in Algorithm 1. The main idea of AGMOEA is to divide the objective space into subspaces and adaptively select representative parent solutions from the subspaces according to the subspace ranking and the subspace dominance relationship so as to achieve a good balance of convergence and diversity. Besides, the multiple evolutionary operators and the local search of the external archive based on grid are also adopted to accelerate convergence speed. These strategies will be described in details in the following subsections.

3.2 Subspaces division method based on the grid

The initial population is generated randomly. After the individuals are evaluated, Z^* and Z^{nad} can be calculated by Eqs. (5) and (6), and the objective space can be divided into subspaces based on grid. All the individuals with the same grid coordinate belong to the same subspace. The maximum capacity of every subspace is set to N_{GBA}. When the number of individuals in a subspace exceeds N_{GBA}, the subspace will be processed by the capacity management method described in Sect. 3.5. After each generation, subspaces will be reconstructed because Z^* and Z^{nad} have been updated. For example, for a bi-objective problem, the number of intervals K is set to 2. With the current population, Z^* = (0.3, 1.0), Z^{nad} = (1.5, 3.0) and d = (0.6, 1.0) can be calculated according to Eqs. (5)–(7). As for the subspace $K^{(0,0)}$, the range in the first objective dimension should be from 0.3 to 0.9, and from 1.0 to 2.0 in the second objective dimension. The subspace $K^{(1,0)}$ takes values from 0.9 to 1.5 in the first objective dimension, and from 1.0 to 2.0 in the second objective dimension. For an individual X = (x_1, x_2), if $0.3 \le x_1 < 0.9$ and $1.0 \le x_1 < 2.0$, X is assigned to subspace $K^{(0,0)}$. With the same method, all the individuals in population can be assigned into certain subspaces.

Algorithm 1: Main Procedure of AGMOEA

Initialize parameters:
  NP: the population size;
  K: the interval number of each objective;
  N_{GBA}: the maximum size of each subspace;
  N_{EXA}: the maximum size of external archive;
Set the external archive *EXA* and the subspace *GBA[i]* ($i = 1, \ldots, K^m$) to be empty;
Generate the initial population P^0;
Evaluate the individuals in P^0;
Store the non-dominated solutions in P^0 into *EXA*;
While (termination criterion is not reached) **do**
  Construct subspaces with the subspaces division method in Section 3.2;
  Improve *EXA* using the external archive extension mechanism described in Section 3.6;
  Set *TP* to be empty;
  For $i = 1, \ldots, NP$ **do**
    Select a subspace using the adaptive selection strategy of subspaces in Section 3.3;
    Generate an offspring *x* with the evolutionary scheme considering the random or representative individual in the selected subspace (described in Section 3.4);
    Evaluate *x* and store *x* into *TP*;
  End For
  Update *EXA* with *TP*;
  $P' = TP \cup P^t$;
  Set P^{t+1} to be the set of the best *NP* individuals in P' based on fast non-dominated sorting;
End While

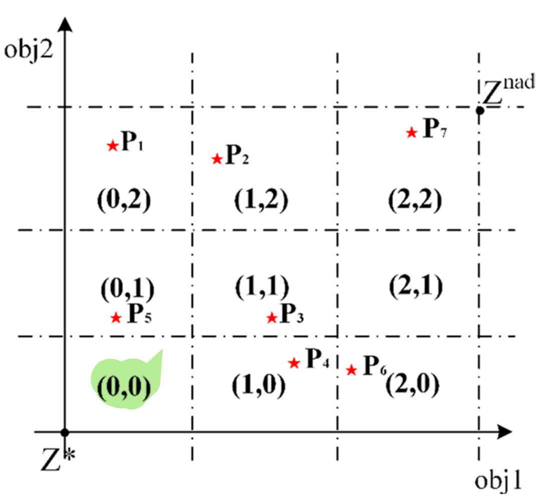

Fig. 1 A schematic View of the Subspace System

3.3 Adaptive selection strategy of subspaces

During the evolutionary process, good parents can usually generate high-quality offspring. So more evolutionary chances should be given to high-quality subspaces to accelerate convergence. However, some low-quality subspaces may help to maintain the diversity of population. Therefore, how to determine which kind of subspaces owns high-quality individuals, and how to balance the convergence and diversity of population using subspaces are the key issues in designing AGMOEA.

For a subspace, the subspace ranking (SR), calculated with Eq. (10), can be used to evaluate its quality. If a subspace has a smaller SR, then it will hold better convergence. Since each individual in the population is grouped into a certain subspace, the quality of individuals can be measured with SR. The smaller SR an individual has, the more evolutionary opportunities it will get. That is, SR can be used to select high-quality parents from the grid subspaces.

To balance the convergence and diversity of the whole population, the subspace dominance relationship is used in AGMOEA. The relationship between subspaces can be categorized into subspace non-dominance, SSD and WSD, which can reflect the convergence and distribution feature of the whole population in the objective space. If two subspaces do not dominate each other, the quality of the individuals in them will be similar. If there is an SSD relationship between two subspaces, individuals in the strong dominance subspace is relatively better. If two subspaces have WSD relationship, some objective values of individuals in the weak dominance subspace are poorer, and other objective values are similar.

As described above, it can be found that the SR and SD relationship can reflect the quality and distribution of different subspaces. By analyzing the SR and SD relationships between subspaces, an adaptive selection strategy of subspaces is proposed. Furthermore, in order to prevent selecting individuals from the same subspace or an inferior subspace repeatedly, a degraded set S is defined to save the currently selected subspace and the subspaces strongly dominated by it. The subspaces in S will be degraded and lose the evolutionary opportunity in the next generation. Let G denote the set of all subspaces, and then the subspaces selected from G–S will obtain evolutionary opportunities.

The proposed adaptive selection strategy of subspaces is shown in Algorithm 2. An example is given in Fig. 2, which is the illustration of a subspace system with two objectives and $K=3$. Let the firstly selected subspace is $\vec{K^2}$, and the individuals in $\vec{K^2}$ are more superior to the ones in $\vec{K^6}$ or $\vec{K^9}$ because of $\vec{K^2} \prec_{ssd} \vec{K^6}$ and $\vec{K^2} \prec_{ssd} \vec{K^9}$, respectively. In order to select good parents from distinct and convergent subspaces, $\vec{K^2}$, $\vec{K^6}$ and $\vec{K^9}$ will be added to set S. The individuals in $\vec{K^1}$, $\vec{K^4}$ and $\vec{K^7}$ are competitive to the ones in $\vec{K^2}$, because $\vec{K^1}$ weakly subspace dominates $\vec{K^2}$ and there are non-dominated relationships between $\vec{K^2}$ and $\vec{K^4}$, $\vec{K^2}$ and $\vec{K^7}$. There are weak dominance relationship between $\vec{K^2}$ and $\vec{K^3}$, $\vec{K^2}$ and $\vec{K^5}$, and $\vec{K^2}$ and $\vec{K^8}$. So, the subspace in the next turn will be selected from $\{\vec{K^1}, \vec{K^3}, \vec{K^4}, \vec{K^5}, \vec{K^7}, \vec{K^8}\}$. By such a selection strategy, solutions with high quality in $\vec{K^1}$, $\vec{K^4}$ and $\vec{K^7}$ will be selected and at the same time solutions with good diversity in $\vec{K^3}$, $\vec{K^5}$ and $\vec{K^8}$ will still have the chance to be selected as parents. At the beginning of each generation, S is set to be empty to ensure that all the subspaces can be selected again.

Algorithm 2: Adaptive Selection Strategy of Subspaces

input:

G: the set of all subspaces;

S: the set of degraded subspaces (at the beginning of each generation S is set to be empty);

output:

$K^{\vec{g}}$: the selected subspace;

S: the set of degraded subspaces;

1: Select the subspace $K^{\vec{g}}$ with the probability $p_{K^{\vec{g}}} = \dfrac{\frac{1}{SR(K^{\vec{g}})+\varepsilon}}{\sum_{i\in G, i\notin S} \frac{1}{SR(i)+\varepsilon}}$;

2: Set $S=\Phi, K^{\vec{g}} \to S$;

3: **For** each $K^{\vec{a}}$ in G **do**

4:   **If** $K^{\vec{g}} \prec_{ssd} K^{\vec{a}}$

5:     $K^{\vec{a}} \to S$;

6:   **End If**

7: **End For**

8: return $K^{\vec{g}}$ and S.

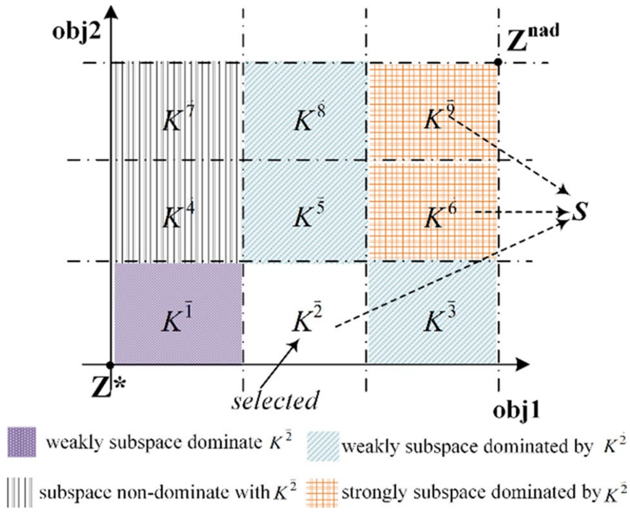

Fig. 2 An example for the adaptive selection strategy

3.4 Evolutionary scheme with adaptive solution selection and multiple operators

After a subspace is selected, one individual in it need to be selected to act as the parent. For individuals in each subspace, the subspace weighted sum (SWS) defined in the above Eq. (9) is used to evaluate their quality, and the individual with the minimum sum is taken as the representative individual of each subspace because a smaller SWS value means closer position to the ideal solution. To make the search focus of the AGMOEA change from the exploration at the early evolution process to the exploitation at the late evolution process, an adaptive selection probability of the repetitive individual (denoted as p_{RE}) is proposed in Eq. (11) [31], in which t is the current generation index and T_{max} is the maximum generations. Based on Eq. (11), the selection probability of the representative individual will gradually increase during the evolution process. If a random number generated in [0,1] is smaller than p_{RE}, then the representative individual will be selected; otherwise, a random individual from the subspace will be selected.

$$p_{RE} = 0.8 * \frac{1.0}{1.0 + \exp(-20 \times (\frac{t}{T_{\max}} - 0.25))} + 0.1 \tag{11}$$

Besides the representative individual or the random one from the subspace, other parent solutions from the external archive can further guarantee the convergence speed. That is, for each crossover operator, the parents in fact consists of two parts: one from the selected subspace and the other one or two (since some operator needs three parents) are selected from the external archive. Please note that if there are no individuals in the selected subspace, all the parents will be selected from the external archive.

Based on the selected parent individuals, the multiple operator strategy proposed in Tang and Wang [32] is adpoted to generate the offspring so that the robustness for different MOPs can be further guaranteed. More specifically, there are five crossover operators are adopted, i.e., SBX [33], PCX [34], SPX [35], BLX-α [36], and DE/rand/1. After crossover operation, the polynomial mutation operator is applied on the offspring. As suggested in [37], the mutation probability p_m is set as *1/n*, where *n* represents the number of decision variables.

After *NP* new offspring are generated, they are combined with current population and then the fast non-dominated sorting and crowding distance in NSGA-II is used to select the best *NP* individuals to construct the new population for the next generation.

3.5 Capacity management of subspaces

To avoid overcrowding in subspace, a maximum capacity N_{GBA} is set for each subspace. When the number of individuals in a subspace exceeds N_{GBA}, the SWS value of each

Table 1 The levels of parameters

Parameter	Levels
N_{GBA}	{10,20}
K	{5,10,15}

Table 2 The results of parameter sensitive analysis

Source	DF	Adj SS	Adj MS	F-value	*P* value
K	2	0.0031	0.0015	0.03	0.972
N_{GBA}	1	0.0017	0.0017	0.03	0.858
$K* N_{GBA}$	2	0.0015	0.0007	0.01	0.986
Error	120	9.2768	0.0533		
Total	125	9.2831			

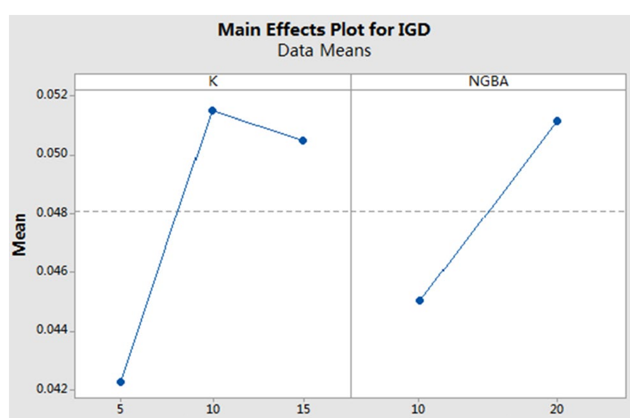

Fig. 3 Main Effect Plot for IGD of K and N_{GBA}

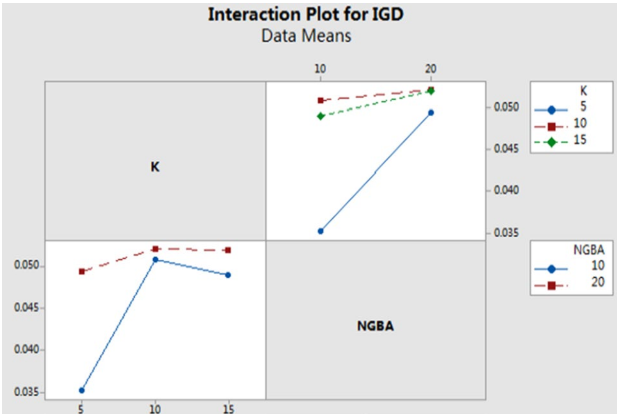

Fig. 4 Interaction Plot for IGD of K and N_{GBA}

individual in the subspace is calculated, and the individual with the largest subspace weighted sum is removed from the subspace until the number of individuals is equal to N_{GBA}. In this way, the loss of boundary information can be prevented by filtering and eliminating the individuals in the subspace [29].

3.6 External archive extension mechanism

External archive is widely used in many MOEAs. It can save the global best information during evolutionary process. Moreover, it can provide superior candidates to produce offspring as well. When the selected subspace is empty with the adaptive selection strategy of subspaces in Sect. 3.3, the individuals in external archive will be randomly selected as the parents. Since the quality of the external archive has great influence on population evolution, the method described in Sect. 3.2 is also adopt to construct subspaces for the external archive, and the representative candidates will be selected from the subspace to generate new solutions by the SBX crossover operator. Then the new solutions will be used to update the external archive. So, the external archive extension mechanism can be viewed as a kind of local search based on subspace on the external archive. When the capacity of the external archive exceeds the maximum limitation N_{EXA}, it will be maintained by removing solutions based on the crowding distance.

4 Experimental results

4.1 Test problems

In the following experiments, three series of benchmark problems are adopted, namely ZDT series [37], DTLZ series [38], and WFG series [39]. All the experiments are carried out on a personal computer with 3.6 GHZ CPU, 8 GB memory, and Windows 10 operating system.

4.2 Parameter setting and performance metric

The parameters of the proposed AGMOEA can be classified into three categories: (1) the parameters used in the evolutionary operators, such as the α of BLX_α, the distribution index η of SBX, the expansion rate ε of SPX, the σ_η, σ_ε of PCX, the control parameters CR and F of DE, and the minimum selection probability of each operator P_{min}; (2) the regular parameters of the general MOEA, such as the population size NP, and the maximum size of the external archive N_{EXA}; (3) the parameters of the subspace system, such as the interval number of each objective K, and the maximum size of each subspace N_{GBA}.

The suggested parameter setting from [32] is adopted for the first category. That is, $\alpha = 0.5$, $\eta = 20$, $\varepsilon = 1$, $\sigma_\eta = \sigma_\varepsilon = 0.1$, $CR = 1.0$, $F = 0.5$, and $P_{min} = 0.1$. For the second category of parameters, the following setting is used, i.e., $NP = 100$ and $N_{EXA} = 100$ for bi-objective problems, and $NP = 300$, $N_{EXA} = 300$ for tri-objective problems. For the third category of parameters, K is set to 5 and N_{GBA} is set to 10. The termination criterion is the maximum number of function evaluations, denoted as FET_{max}. For bi-objective problems, FET_{max} is set to 25,000, while for tri-objective problems, FET_{max} is set to 30,000.

To compare and analyze the performance of the proposed AGMOEA, the Invert Generational Distance (IGD), and Hypervolume (HV) metrics are adopted. The two metrics can measure both convergence and diversity of the obtained non-dominated solutions simultaneously. In all the following experiments, each algorithm is tested on each problem for 30 independent times, and the results are used for comparison and analysis.

4.3 Sensitivity analysis of parameters

Because the first category of parameters is set according to the suggested values, and the population size and the capacity of the external archive are set to the general values used in the literature. Only the third category of parameters is analyzed, i.e., K and N_{GBA}, in this subsection. The levels of the parameters are shown in Table 1 in which there are six parameter combinations. The termination criterion is set to 15,000 function evaluations. The sensitivity of parameters is analyzed using ANOVA with the confidence level of 95%. The analysis results are shown in Table 2, the main effect plot for IGD of N_{GBA} and K is shown in Fig. 3, and the interaction plot of K and N_{GBA} is shown in Fig. 4. From these results, it can be found that K, N_{GBA}, and their interaction

Table 3 IGD Comparison of adaptive selection strategy of subspaces

	MOEA-non					GMOEA-rand					GMOEA-adp			
	Min	Max	Mean	Std		Min	Max	Mean	Std		Min	Max	Mean	Std
ZDT1	5.62E−02	6.16E−02	5.95E−02	2.18E−03	+	5.28E−02	5.97E−02	5.66E−02	3.02E−03	=	**5.11E−02**	**5.84E−02**	**5.47E−02**	**2.89E−03**
ZDT2	8.82E−02	1.02E−01	9.84E−02	3.89E−03	+	**8.04E−02**	**1.01E−01**	**9.25E−02**	**4.00E−03**	−	8.16E−02	9.69E−02	9.38E−02	3.28E−03
ZDT3	4.67E−02	5.27E−02	5.09E−02	1.51E−03	+	4.28E−02	4.67E−02	4.49E−02	1.66E−03	+	**4.03E−02**	**4.59E−02**	**4.38E−02**	**2.93E−03**
ZDT4	1.49E+00	1.88E+00	1.68E+00	1.48E−01	+	1.43E+00	1.75E+00	1.56E+00	1.07E−01	=	**1.32E+00**	**1.43E+00**	**1.36E+00**	**3.68E−02**
ZDT6	1.87E−01	1.96E−01	1.90E−01	4.87E−03	+	1.84E−01	1.94E−01	1.90E−01	4.06E−03	+	**1.78E−01**	**1.91E−01**	**1.86E−01**	**5.00E−03**
DTLZ1	1.21E−01	2.62E−01	1.75E−01	3.55E−02	+	1.38E−01	2.27E−01	1.78E−01	4.23E−02	+	**8.27E−02**	**2.13E−01**	**1.68E−01**	**2.82E−02**
DTLZ2	2.67E−03	3.14E−03	2.79E−03	1.87E−04	+	**2.47E−03**	**3.00E−03**	**2.69E−03**	**1.32E−04**	=	2.59E−03	2.97E−03	2.74E−03	1.29E−04
DTLZ3	3.83E+00	5.01E+00	4.11E+00	6.71E−01	=	2.37E+00	4.97E+00	4.03E+00	7.08E−01	+	**3.14E+00**	**4.60E+00**	**3.68E+00**	**6.65E−01**
DTLZ4	4.84E−03	6.45E−03	5.51E−03	5.50E−04	+	**3.68E−03**	**5.14E−03**	**4.49E−03**	**3.39E−04**	=	3.69E−03	5.29E−03	4.60E−03	4.84E−04
DTLZ5	4.13E−04	5.62E−04	4.89E−04	3.74E−05	+	**3.59E−04**	**4.69E−04**	**4.18E−04**	**3.42E−05**	=	3.64E−04	4.97E−04	4.37E−04	4.04E−05
DTLZ6	4.70E−02	5.01E−02	4.90E−02	7.40E−04	+	4.43E−02	4.80E−02	4.60E−02	1.01E−03	+	**4.30E−02**	**4.77E−02**	**4.55E−02**	**1.10E−03**
DTLZ7	2.43E−01	2.85E−01	2.65E−01	1.51E−02	+	2.45E−01	2.88E−01	2.62E−01	2.06E−02	+	**2.15E−01**	**2.78E−01**	**2.49E−01**	**1.33E−02**
WFG1	6.87E−02	7.79E−02	7.37E−02	2.65E−03	+	6.25E−02	6.75E−02	6.53E−02	1.95E−03	+	**6.16E−02**	**6.63E−02**	**6.46E−02**	**1.71E−03**
WFG2	1.93E−02	2.86E−02	2.42E−02	2.27E−03	+	1.80E−02	2.54E−02	2.25E−02	6.20E−03	+	**1.69E−02**	**3.44E−02**	**2.15E−02**	**3.19E−03**
WFG3	6.86E−03	8.51E−03	7.64E−03	6.01E−04	+	**5.23E−03**	**6.44E−03**	**5.84E−03**	**4.98E−04**	=	5.16E−03	6.50E−03	5.89E−03	4.08E−04
WFG4	2.90E−03	3.56E−03	3.31E−03	1.98E−04	+	**2.37E−03**	**2.88E−03**	**2.53E−03**	**1.77E−04**	=	2.38E−03	2.85E−03	2.62E−03	1.36E−04
WFG5	6.78E−03	7.98E−03	7.33E−03	3.74E−04	+	4.78E−03	6.27E−03	5.47E−03	4.16E−04	+	**4.69E−03**	**6.14E−03**	**5.34E−03**	**4.03E−04**
WFG6	9.47E−03	1.21E−02	1.05E−02	7.98E−04	+	7.72E−03	9.20E−03	8.55E−03	5.56E−04	=	**7.31E−03**	**8.81E−03**	**8.38E−03**	**2.94E−04**
WFG7	8.82E−02	1.02E−01	9.84E−02	3.89E−03	+	8.94E−02	1.01E−01	9.38E−02	4.00E−03	=	**8.16E−02**	**9.69E−02**	**9.25E−02**	**3.28E−03**
WFG8	1.29E−02	1.68E−02	1.51E−02	1.08E−03	+	1.23E−02	1.43E−02	1.35E−02	7.03E−04	+	**1.17E−02**	**1.40E−02**	**1.29E−02**	**6.88E−04**
WFG9	1.33E−03	2.29E−03	1.80E−03	2.80E−04	+	1.04E−03	1.43E−03	1.24E−03	1.30E−04	=	**1.02E−03**	**1.31E−03**	**1.15E−03**	**9.96E−05**
Nos. +/=/−		20/1/0					10/10/1							

Table 4 IGD comparison of evolutionary scheme

	GMOEA-adp					AGMOEA-nopre					AGMOEA-pre			
	Min	Max	Mean	Std		Min	Max	Mean	Std		Min	Max	Mean	Std
ZDT1	5.11E−02	5.84E−02	5.47E−02	2.89E−03	+	1.48E−04	1.73E−04	1.60E−04	6.93E−06	+	**1.41E−04**	**1.55E−04**	**1.48E−04**	**4.09E−06**
ZDT2	8.16E−02	9.69E−02	9.38E−02	3.28E−03	+	1.50E−04	1.65E−04	1.58E−04	4.98E−06	+	**1.45E−04**	**1.55E−04**	**1.48E−04**	**2.41E−06**
ZDT3	4.03E−02	4.59E−02	4.38E−02	2.93E−03	+	1.74E−04	3.67E−04	2.29E−04	5.81E−05	+	**1.70E−04**	**2.12E−04**	**1.86E−04**	**1.07E−05**
ZDT4	1.32E+00	1.43E+00	1.36E+00	3.68E−02	+	7.07E−04	9.34E−02	3.40E−02	3.07E−02	+	**2.04E−04**	**6.61E−03**	**7.77E−04**	**1.16E−03**
ZDT6	1.78E−01	1.91E−01	1.86E−01	5.00E−03	+	1.27E−04	3.07E−04	2.19E−04	5.20E−05	+	**1.18E−04**	**1.54E−04**	**1.33E−04**	**1.10E−05**
DTLZ1	8.27E−02	2.13E−01	1.68E−01	2.82E−02	+	7.98E−03	1.08E−01	3.91E−02	2.15E−02	+	**3.22E−04**	**8.99E−02**	**3.84E−02**	**2.25E−02**
DTLZ2	2.59E−03	2.97E−03	2.74E−03	1.29E−04	+	7.84E−04	9.30E−04	8.45E−04	3.77E−05	+	**7.49E−04**	**8.97E−04**	**8.20E−04**	**3.77E−05**
DTLZ3	3.14E+00	4.60E+00	3.68E+00	6.65E−01	+	3.90E−01	2.58E+00	1.98E+00	4.74E−01	+	**3.27E−01**	**2.54E+00**	**1.77E+00**	**6.11E−01**
DTLZ4	3.69E−03	5.29E−03	4.60E−03	4.84E−04	+	**1.09E−03**	**1.63E−03**	**1.33E−03**	**1.40E−04**	=	1.17E−03	1.57E−03	1.34E−03	1.23E−04
DTLZ5	3.64E−04	4.97E−04	4.37E−04	4.04E−05	+	1.90E−05	2.80E−05	2.33E−05	2.53E−06	+	**1.80E−05**	**2.90E−05**	**2.16E−05**	**2.30E−06**
DTLZ6	4.30E−02	4.77E−02	4.55E−02	1.10E−03	+	2.90E−05	4.10E−05	3.27E−05	3.54E−06	+	**2.90E−05**	**3.50E−05**	**3.08E−05**	**1.45E−06**
DTLZ7	2.15E−01	2.78E−01	2.49E−01	1.33E−02	+	3.41E−03	7.65E−03	4.68E−03	1.36E−03	+	**3.26E−03**	**4.19E−03**	**3.61E−03**	**2.23E−04**
WFG1	6.16E−02	6.63E−02	6.46E−02	1.71E−03	+	3.10E−02	3.36E−02	3.22E−02	6.31E−04	+	**3.07E−02**	**3.31E−02**	**3.17E−02**	**5.11E−04**
WFG2	1.69E−02	3.44E−02	2.15E−02	3.19E−03	+	2.22E−03	4.30E−03	3.06E−03	5.72E−04	+	**1.66E−03**	**2.98E−03**	**2.32E−03**	**3.39E−04**
WFG3	5.16E−03	6.50E−03	5.89E−03	4.08E−04	+	6.06E−04	7.23E−04	6.51E−04	2.55E−05	+	**5.69E−04**	**6.63E−04**	**6.09E−04**	**2.35E−05**
WFG4	2.38E−03	2.85E−03	2.62E−03	1.36E−04	+	4.71E−04	5.83E−04	5.33E−04	3.51E−05	+	**4.49E−04**	**6.33E−04**	**5.27E−04**	**4.46E−05**
WFG5	4.69E−03	6.14E−03	5.34E−03	4.03E−04	+	**2.32E−03**	**2.35E−03**	**2.34E−03**	**9.43E−06**	=	2.32E−03	2.36E−03	2.34E−03	1.04E−05
WFG6	7.31E−03	8.81E−03	8.38E−03	2.94E−04	+	9.35E−04	1.68E−03	1.12E−03	1.90E−04	+	**8.54E−04**	**1.50E−03**	**1.01E−03**	**1.50E−04**
WFG7	8.16E−02	9.69E−02	9.25E−02	3.28E−03	+	1.50E−04	1.65E−04	1.58E−04	4.98E−06	+	**1.44E−04**	**1.56E−04**	**1.48E−04**	**2.33E−06**
WFG8	1.17E−02	1.40E−02	1.29E−02	6.88E−04	+	5.29E−03	1.10E−02	8.84E−03	1.52E−03	+	**5.15E−03**	**1.05E−02**	**8.42E−03**	**1.41E−03**
WFG9	1.02E−03	1.31E−03	1.15E−03	9.96E−05	+	3.73E−04	4.55E−04	4.00E−04	2.10E−05	=	**3.66E−04**	**4.55E−04**	**3.98E−04**	**1.99E−05**
Nos. +/=/−		21/0/0					18/3/0							

Table 5 IGD comparison of external archive extension mechanism

	AGMOEA-pre					AGMOEA-exten			
	Min	Max	Mean	Std		Min	Max	Mean	Std
ZDT1	1.41E−04	1.55E−04	1.48E−04	4.09E−06	=	**1.41E−04**	**1.48E−04**	**1.45E−04**	**1.94E−06**
ZDT2	**1.45E−04**	**1.55E−04**	**1.48E−04**	**2.41E−06**	=	1.45E−04	1.55E−04	1.48E−04	2.41E−06
ZDT3	1.70E−04	2.12E−04	1.86E−04	1.07E−05	+	**1.61E−04**	**1.79E−04**	**1.70E−04**	**3.16E−06**
ZDT4	2.04E−04	6.61E−03	7.77E−04	1.16E−03	+	**1.43E−04**	**1.88E−04**	**1.57E−04**	**9.32E−06**
ZDT6	1.18E−04	1.54E−04	1.33E−04	1.10E−05	+	**1.14E−04**	**1.25E−04**	**1.19E−04**	**3.38E−06**
DTLZ1	3.22E−04	8.99E−02	3.84E−02	2.25E−02	+	**3.19E−04**	**4.55E−02**	**2.27E−02**	**1.39E−02**
DTLZ2	7.49E−04	8.97E−04	8.20E−04	3.77E−05	+	**7.08E−04**	**8.33E−04**	**7.63E−04**	**3.32E−05**
DTLZ3	3.27E−01	2.54E+00	1.77E+00	6.11E−01	+	**4.76E−02**	**8.68E−01**	**2.18E−01**	**3.68E−01**
DTLZ4	1.17E−03	1.57E−03	1.34E−03	1.23E−04	=	**1.17E−03**	**1.50E−03**	**1.28E−03**	**8.86E−05**
DTLZ5	1.80E−05	2.90E−05	2.16E−05	2.30E−06	+	**1.20E−05**	**1.50E−05**	**1.37E−05**	**7.50E−07**
DTLZ6	2.90E−05	3.50E−05	3.08E−05	1.45E−06	+	**2.60E−05**	**3.10E−05**	**2.90E−05**	**9.10E−07**
DTLZ7	**3.26E−03**	**4.19E−03**	**3.61E−03**	**2.23E−04**	=	3.39E−03	4.70E−03	3.76E−03	2.71E−04
WFG1	3.07E−02	3.31E−02	3.17E−02	5.11E−04	+	**1.15E−02**	**2.37E−02**	**1.62E−02**	**3.01E−03**
WFG2	1.66E−03	2.98E−03	2.32E−03	3.39E−04	+	**1.24E−03**	**3.08E−03**	**1.61E−03**	**4.72E−04**
WFG3	5.69E−04	6.63E−04	6.09E−04	2.35E−05	+	**5.14E−04**	**5.84E−04**	**5.42E−04**	**1.63E−05**
WFG4	4.49E−04	6.33E−04	5.27E−04	4.46E−05	+	**3.97E−04**	**6.62E−04**	**4.82E−04**	**5.37E−05**
WFG5	2.32E−03	2.36E−03	2.34E−03	1.04E−05	=	**2.31E−03**	**2.36E−03**	**2.32E−03**	**1.21E−05**
WFG6	8.54E−04	1.50E−03	1.01E−03	1.50E−04	+	**7.88E−04**	**1.06E−03**	**8.43E−04**	**4.18E−05**
WFG7	**1.44E−04**	**1.56E−04**	**1.48E−04**	**2.33E−06**	−	2.98E−04	3.22E−04	3.09E−04	7.19E−06
WFG8	5.15E−03	1.05E−02	8.42E−03	1.41E−03	=	**3.70E−03**	**9.69E−03**	**8.10E−03**	**1.81E−03**
WFG9	3.66E−04	4.55E−04	3.98E−04	1.99E−05	+	**3.05E−04**	**4.12E−04**	**3.51E−04**	**2.63E−05**
Nos. +/=/−		14/6/1							

do not have significant impact on the performance of AGMOEA. Since $K=5$ and $N_{GBA}=10$ can result in the best IGD metric, K and N_{GBA} are set to 5 and 10, respectively.

4.4 Performance analysis of each improvement strategy

In order to analyze the effectiveness of each improvement strategy, IGD metric of the 21 benchmark problems are calculated and the minimum, maximum, mean and standard deviation of IGD metrics are taken as evaluation measures. Wilcoxon's rank sum test with the confidence level of 95% is used to analyze whether there is significant difference between algorithms with and without a certain strategy. In the comparison results, the best results of each problem are shown in bold type and has a gray-colored background. If two algorithms have the same best mean value, the one with the smaller standard deviation is considered to be better. The symbols "+" and "–" in the column represent that the algorithm with the improvement strategy is significantly better or worse than the algorithm without this strategy, while "=" means that there is no significant difference between them. "Nos. +/=/–" in the last line denotes the sum of instances for the algorithms, respectively.

4.4.1 Performance analysis of adaptive selection strategy of subspaces

To analyze the performance of the adaptive selection strategy of subspaces, the algorithm using this strategy is denoted by GMOEA-adp, the algorithm without subspaces is denoted by MOEA-non, and the algorithm using random selection strategy of non-empty subspaces is denoted by GMOEA-rand. To amplify their performance difference, all the three algorithms only use the SBX crossover operator, instead of the multiple operators. In MOEA-non, one of the two parents is the current individual in the population and the other one is an individual randomly selected from the external archive. In GMOEA-rand, one of the two parents is a random individual from a random selected non-empty subspace and the other one is an individual randomly selected from the external archive.

The comparison results are shown in Table 3, and according to this table, it can be found that GMOEA-rand and GMOEA-adp can obtain much better IGD metric than MOEA-rand in almost all the benchmark problems. It can verify that selecting individuals from subspaces based on grid has positive effect on the population evolution.

More specifically, for ZDT series, GOMEA-adp is significantly better than GMOEA-rand in ZDT3 and ZDT6

Table 6 IGD metrics comparison of algorithms

	NSGA−II			GrEA			CDG-MOEA			CMODE			MOEA/D-FRRMAB			AGMOEA	
	Min Max	Mean Std		Min Max	Mean Std		Min Max	Mean Std		Min Max	Mean Std		Min Max	Mean Std		Min Max	Mean Std
ZDT1	1.670E−04	1.889E−04	+	9.820E−04	1.191E−03	+	2.220E−04	4.479E−04	+	1.400E−04	1.458E−04	=	5.940E−04	1.165E−03	+	**1.410E−04**	**1.446E−04**
	2.080E−04	1.011E−05		1.628E−03	1.465E−04		9.500E−04	1.819E−04		1.540E−04	4.083E−06		2.022E−03	3.105E−04		**1.480E−04**	**1.938E−06**
ZDT2	1.430E−04	1.232E−03	+	1.594E−03	1.697E−03	+	3.340E−04	1.573E−03	+	1.770E−04	1.935E−04	+	5.950E−04	1.215E−03	+	**1.450E−04**	**1.480E−04**
	2.311E−02	4.492E−03		1.740E−03	4.250E−05		2.312E−02	4.205E−03		2.110E−04	8.541E−06		3.538E−03	7.102E−04		**1.550E−04**	**2.414E−06**
ZDT3	1.890E−04	3.042E−04	+	1.651E−03	1.926E−03	+	4.710E−04	7.497E−04	+	1.670E−04	1.706E−04	=	8.950E−04	3.383E−03	+	**1.610E−04**	**1.701E−04**
	2.932E−03	4.965E−04		2.199E−03	1.767E−04		1.432E−03	2.152E−04		1.900E−04	4.747E−06		4.983E−03	9.707E−04		**1.790E−04**	**3.155E−06**
ZDT4	1.880E−04	3.113E−04	+	1.143E−03	4.709E−03	+	8.070E−04	3.821E−01	+	1.690E−04	2.089E−02	+	3.034E−03	3.700E−02	+	**1.430E−04**	**1.574E−04**
	1.129E−03	2.000E−04		1.310E−02	3.437E−03		9.981E−01	3.317E−01		7.560E−02	1.440E−02		9.886E−02	2.624E−02		**1.880E−04**	**9.320E−06**
ZDT6	2.100E−04	2.465E−04	+	7.790E−04	1.069E−03	+	1.630E−04	5.264E−04	+	**1.130E−04**	**1.152E−04**	=	1.140E−04	1.415E−04	=	1.140E−04	1.190E−04
	2.880E−04	2.050E−05		1.291E−03	1.435E−04		1.845E−03	4.428E−04		**1.170E−04**	**1.215E−06**		3.440E−04	6.021E−05		1.250E−04	3.378E−06
DTLZ1	2.520E−04	2.981E−04	−	2.740E−04	6.510E−04	−	6.330E−04	1.302E−02	−	3.001E−03	1.967E−02	−	**2.910E−04**	**2.952E−04**	−	3.190E−04	2.268E−02
	5.150E−04	4.453E−05		1.651E−03	3.157E−04		4.094E−02	9.811E−03		8.742E−02	1.554E−02		**3.010E−04**	**2.388E−06**		4.552E−02	1.393E−02
DTLZ2	6.800E−04	7.426E−04	=	8.760E−04	1.015E−03	+	**4.470E−04**	**4.932E−04**	−	5.760E−04	5.963E−04	−	8.050E−04	8.169E−04	+	7.080E−04	7.631E−04
	8.180E−04	3.298E−05		1.069E−03	3.355E−05		**5.620E−04**	**2.621E−05**		6.310E−04	1.228E−05		8.290E−04	5.374E−06		8.330E−04	3.320E−05
DTLZ3	**1.180E−03**	**2.401E−03**	−	3.570E−02	1.657E−01	−	4.127E−02	1.910E−01	=	3.624E−02	1.861E−01	=	1.298E−03	5.946E−03	−	4.761E−02	2.181E−01
	1.683E−02	**3.618E−03**		4.120E−01	7.585E−02		8.631E−01	3.511E−01		8.254E−01	2.032E−01		1.382E−01	2.498E−02		8.685E−01	3.683E−01
DTLZ4	1.000E−03	1.424E−03	+	8.710E−04	1.143E−03	−	6.080E−04	8.912E−04	−	**7.250E−04**	**8.368E−04**	−	9.360E−04	1.325E−03	=	1.165E−03	1.280E−03
	9.254E−03	1.483E−03		1.210E−03	7.281E−05		1.720E−03	2.375E−04		**1.072E−03**	**8.407E−05**		1.590E−03	2.272E−04		1.495E−03	8.857E−05
DTLZ5	1.400E−05	1.557E−05	+	1.000E−04	1.296E−04	+	3.300E−05	4.013E−05	+	**1.200E−05**	**1.200E−05**	−	2.800E−05	2.863E−05	+	1.200E−05	1.370E−05
	1.900E−05	1.194E−06		1.430E−04	1.024E−05		5.500E−05	5.643E−06		**1.200E−05**	**0.000E+00**		2.900E−05	4.901E−07		1.500E−05	7.497E−07
DTLZ6	3.800E−05	1.120E−04	+	8.939E−03	9.575E−03	+	6.822E−03	9.956E−03	+	2.700E−05	2.913E−05	=	7.600E−05	7.673E−05	+	**2.600E−05**	**2.900E−05**
	5.410E−04	1.059E−04		1.010E−02	2.937E−04		1.394E−02	1.555E−03		3.100E−05	1.196E−06		7.700E−05	4.498E−07		**3.100E−05**	**9.097E−07**
DTLZ7	2.993E−03	3.982E−03	=	3.536E−03	4.674E−03	+	3.118E−03	4.124E−03	+	**2.383E−03**	**3.581E−03**	−	6.673E−03	1.004E−02	+	3.394E−03	3.762E−03
	1.916E−02	2.886E−03		5.362E−03	4.464E−04		6.311E−03	6.873E−04		**1.928E−02**	**4.254E−03**		4.020E−02	7.458E−03		4.703E−03	2.708E−04
WFG1	4.897E−03	2.080E−02	+	5.013E−03	1.040E−02	−	1.341E−02	1.962E−02	+	**5.465E−03**	**5.920E−03**	−	6.490E−03	2.108E−02	+	1.145E−02	1.622E−02
	3.565E−02	1.174E−02		1.545E−02	2.906E−03		2.689E−02	4.148E−03		**6.551E−03**	**2.845E−04**		3.312E−02	5.581E−03		2.370E−02	3.005E−03
WFG2	1.310E−03	1.346E−02	+	1.761E−02	2.227E−02	+	1.083E−03	1.616E−03	=	5.541E−02	5.544E−02	+	5.116E−03	5.358E−03	+	**1.239E−03**	**1.610E−03**
	1.830E−02	7.572E−03		2.511E−02	2.143E−03		2.272E−03	2.675E−04		5.548E−02	1.809E−05		5.599E−03	9.546E−05		**3.079E−03**	**4.723E−04**
WFG3	5.850E−04	6.649E−04	+	5.022E−03	5.813E−03	+	6.470E−04	7.996E−04	+	2.947E−02	2.947E−02	+	5.550E−04	5.593E−04	+	**5.140E−04**	**5.421E−04**
	7.670E−04	4.211E−05		6.055E−03	2.450E−04		1.450E−03	1.569E−04		2.947E−02	0.000E+00		5.640E−04	2.264E−06		**5.840E−04**	**1.634E−05**
WFG4	4.220E−04	4.844E−04	=	3.475E−03	4.414E−03	+	6.650E−04	8.762E−04	=	7.765E−03	7.773E−03	+	7.600E−04	9.559E−04	+	**3.970E−04**	**4.816E−04**
	6.020E−04	4.126E−05		4.755E−03	4.438E−04		1.096E−03	1.183E−04		7.785E−03	4.575E−06		1.410E−03	1.275E−04		**6.620E−04**	**5.365E−05**
WFG5	2.334E−03	2.369E−03	+	4.709E−03	6.985E−03	+	1.965E−03	2.438E−03	+	1.293E−02	1.343E−02	+	2.319E−03	2.334E−03	+	**2.308E−03**	**2.320E−03**
	2.427E−03	2.235E−05		7.185E−03	4.342E−04		2.696E−03	1.813E−04		1.346E−02	9.519E−05		2.522E−03	3.624E−05		**2.362E−03**	**1.208E−05**
WFG6	9.450E−04	1.428E−03	+	2.243E−03	2.978E−03	+	1.486E−03	2.144E−03	+	2.430E−02	2.436E−02	+	8.390E−04	8.495E−04	=	**7.880E−04**	**8.429E−04**
	4.277E−03	7.958E−04		3.494E−03	2.326E−04		2.977E−03	3.299E−04		2.458E−02	6.236E−05		8.590E−04	4.462E−06		**1.056E−03**	**4.182E−05**
WFG7	3.630E−04	3.901E−04	+	2.796E−03	4.084E−03	+	3.960E−04	4.811E−04	+	7.363E−03	7.369E−03	+	3.810E−04	3.822E−04	+	**2.980E−04**	**3.088E−04**
	4.670E−04	2.414E−05		4.279E−03	3.811E−04		6.440E−04	6.082E−05		7.375E−03	2.638E−06		3.830E−04	6.103E−07		**3.220E−04**	**7.190E−06**
WFG8	7.024E−03	8.882E−03	+	5.404E−03	9.596E−03	+	2.904E−03	7.877E−03	=	1.351E−02	1.372E−02	+	**3.275E−03**	**7.813E−03**	−	3.701E−03	8.098E−03

Table 6 (continued)

	NSGA–II			GrEA			CDG-MOEA			CMODE			MOEA/D-FRRMAB			AGMOEA	
	Min Max	Mean Std		Min Max	Mean Std		Min Max	Mean Std		Min Max	Mean Std		Min Max	Mean Std		Min Max	Mean Std
	9.809E−03	9.509E−04		1.136E−02	1.491E−03		1.172E−02	3.255E−03		1.535E−02	3.212E−04		**9.398E−03**	**1.828E−03**		9.692E−03	1.806E−03
WFG9	3.500E−04	4.079E−04	+	3.771E−03	3.888E−03	+	4.210E−04	5.681E−04	+	9.507E−03	9.695E−03	+	3.980E−04	4.147E−04	+	**3.050E−04**	**3.512E−04**
	4.850E−04	3.581E−05		4.140E−03	7.127E−05		7.260E−04	8.675E−05		1.029E−02	1.378E−04		4.290E−04	8.284E−06		**4.120E−04**	**2.631E−05**
	Nos. +/=/−			16/3/2			17/0/4			14/4/3			10/5/6			15/3/3	

problems. There is no significant difference between them in ZDT1 and ZDT4 problems. However, GMOEA-adp can get better IGD metrics. Only for the ZDT2 problem, GMOEA-rand is significantly better than GMOEA-adp.

For DTLZ series, GMOEA-adp gets significantly better statistical results of IGD metrics in 6 out of the 7 problems than MOEA-non. GMOEA-adp obtains four significantly better results than GMOEA-rand. In DTLZ2, DTLZ4, and DTLZ5 problems, GMOEA-rand can get better statistical results. Nevertheless, there is no significant difference between them.

For WFG series, GMOEA-adp is significantly better than MOEA-non in almost all the problems and can get the best statistical results for 7 out of the 9 problems. GOMEA-adp is significantly better than GMOEA-rand in 4 of the 9 problems. But there is no significant difference between them in the other five problems. Due to evolving population with subspaces, GMOEA-rand and GMOEA-adp outperform MOEA-non. Each subspace can be selected with the same probability in GMOEA-rand and the population can maintain good diversity. While GMOEA-adp adopts adaptive selection strategy of subspaces, it can further balance the convergence and diversity because the subspace ranking and the subspace dominance relationship are incorporated into GMOEA-adp. Through this strategy, the high-quality individuals can be selected from different region considering the distribution of the whole population, and the convergence and diversity of the population can be achieved.

4.4.2 Performance analysis of evolutionary scheme with multiple operators

To analyze the performance of the evolutionary scheme, three variants of our algorithm, all of which use the adaptive selection strategy of subspaces, are compared. The first one is the GMOEA-adp described in the above subsection and it only adopts the SBX operator. The second one is AGMOEA-nopre in which the multiple operators are used, but the individual is randomly selected from a subspace. The third one is the AGMOEA-pre whose difference from AGMOEA-nopre is that the adaptive selection strategy of representative individuals in a subspace (described in Sect. 3.4) is further adopted.

The minimum, maximum, mean and standard deviation of IGD metrics, as well as the Wilcoxon's rank sum test results, are shown in Table 4. According to the results, it appears that AGMOEA-nopre and AGMOEA-pre significantly outperform GMOEA-adp due to the adoption of multiple evolutionary operators. Compared with GMOEA-adp, GMOEA-pre can get all the significant metrics. Besides, AGMOEA-pre achieves significantly better results than AGMOEA-nopre for 18 out of the 21 problems. The main reason is that evolving population with multiple operators

Table 7 HV metrics comparison of algorithms

	NSGA−II			GrEA			CDG−MOEA			CMODE			MOEA/D−FRRMAB			AGMOEA	
	Min Max	Mean Std		Min Max	Mean Std		Min Max	Mean Std		Min Max	Mean Std		Min Max	Mean Std		Min Max	Mean Std
ZDT1	6.586E−01	6.593E−01	+	6.271E−01	6.318E−01	+	6.224E−01	6.466E−01	+	5.782E−01	6.151E−01	+	**6.612E−01**	**6.616E−01**	=	6.598E−01	6.606E−01
	6.601E−01	3.358E−04		6.370E−01	3.440E−03		6.559E−01	9.059E−03		6.372E−01	1.281E−02		**6.618E−01**	**1.150E−04**		6.613E−01	4.175E−04
ZDT2	0.000E+00	3.089E−01	+	2.786E−01	2.801E−01	+	0.000E+00	2.989E−01	+	3.256E−01	3.261E−01	=	2.089E−01	2.826E−01	+	**3.275E−01**	**3.279E−01**
	3.286E−01	7.453E−02		2.840E−01	1.447E−03		3.218E−01	5.858E−02		3.269E−01	3.176E−04		3.041E−01	2.280E−02		**3.283E−01**	**2.260E−04**
ZDT3	5.121E−01	5.135E−01	+	5.019E−01	5.042E−01	+	4.800E−01	4.991E−01	+	**5.155E−01**	**5.157E−01**	=	3.633E−01	4.128E−01	+	5.116E−01	5.145E−01
	5.146E−01	6.544E−04		5.065E−01	1.341E−03		5.082E−01	5.965E−03		**5.158E−01**	**6.421E−05**		4.867E−01	2.947E−02		5.158E−01	5.965E−04
ZDT4	5.146E−01	6.496E−01	+	4.797E−01	5.958E−01	+	0.000E+00	1.663E−02	+	0.000E+00	1.341E−01	+	0.000E+00	6.356E−02	+	**5.141E−01**	**6.552E−01**
	6.592E−01	2.570E−02		6.382E−01	4.554E−02		4.990E−01	9.111E−02		5.157E−01	1.429E−01		4.342E−01	1.287E−01		**6.616E−01**	**2.665E−02**
ZDT6	3.866E−01	3.894E−01	+	3.476E−01	3.576E−01	+	3.089E−01	3.752E−01	+	4.012E−01	4.013E−01	=	3.832E−01	3.992E−01	=	**4.008E−01**	**4.014E−01**
	3.920E−01	1.379E−03		3.699E−01	6.934E−03		3.981E−01	2.535E−02		4.014E−01	2.788E−04		4.013E−01	4.524E−03		**4.013E−01**	**1.274E−04**
DTLZ1	**7.234E−01**	**7.601E−01**	−	4.186E−01	6.781E−01	−	0.000E+00	1.594E−01	+	0.000E+00	2.562E−03	+	7.435E−01	7.458E−01	−	9.006E−02	2.941E−01
	7.713E−01	**8.577E−03**		7.715E−01	8.561E−02		9.227E−01	2.649E−01		7.686E−02	1.403E−02		7.489E−01	1.207E−03		7.489E−01	1.381E−01
DTLZ2	3.641E−01	3.771E−01	−	3.702E−01	3.767E−01	−	**4.189E−01**	**4.226E−01**	−	3.980E−01	4.055E−01	−	3.625E−01	3.660E−01	−	3.409E−01	3.587E−01
	3.854E−01	5.295E−03		3.963E−01	4.200E−03		**4.250E−01**	**1.367E−03**		4.108E−01	3.114E−03		3.692E−01	1.786E−03		3.725E−01	7.512E−03
DTLZ3	0.000E+00	3.146E−01	−	0.000E+00	0.000E+00	+	0.000E+00	0.000E+00	+	0.000E+00	0.000E+00	+	**0.000E+00**	**3.375E−01**	−	0.000E+00	4.155E−02
	3.715E−01	9.199E−02		0.000E+00	0.000E+00		0.000E+00	0.000E+00		0.000E+00	0.000E+00		**3.681E−01**	**6.888E−02**		6.185E−02	7.416E−03
DTLZ4	0.000E+00	3.646E−01	−	3.723E−01	3.764E−01	−	**4.237E−01**	**4.268E−01**	−	3.971E−01	4.038E−01	−	2.929E−01	3.369E−01	=	3.452E−01	3.580E−01
	3.850E−01	6.904E−02		3.895E−01	3.699E−03		**4.291E−01**	**1.398E−03**		4.100E−01	3.305E−03		3.755E−01	3.577E−02		3.669E−01	5.806E−03
DTLZ5	9.011E−02	9.257E−02	+	**1.880E−01**	**1.912E−01**	−	7.655E−02	8.156E−02	+	9.371E−02	9.389E−02	−	9.175E−02	9.192E−02	+	9.273E−02	9.305E−02
	9.288E−02	1.711E−04		**1.985E−01**	**2.549E−03**		8.506E−02	2.071E−03		9.399E−02	5.205E−05		9.202E−02	6.551E−05		9.335E−02	1.841E−04
DTLZ6	4.382E−02	8.435E−02	+	0.000E+00	0.000E+00	+	0.000E+00	0.000E+00	+	9.468E−02	9.479E−02	+	9.310E−02	9.311E−02	+	**9.486E−02**	**9.492E−02**
	9.440E−02	1.263E−02		0.000E+00	0.000E+00		0.000E+00	0.000E+00		9.488E−02	4.506E−05		9.312E−02	4.918E−06		**9.498E−02**	**3.637E−05**
DTLZ7	2.575E−01	2.737E−01	+	2.605E−01	2.722E−01	=	1.217E−01	1.872E−01	+	**2.632E−01**	**3.028E−01**	−	2.105E−01	2.518E−01	+	2.664E−01	2.836E−01
	2.807E−01	6.726E−03		2.864E−01	5.863E−03		2.259E−01	2.644E−02		**3.081E−01**	**1.074E−02**		2.604E−01	8.519E−03		2.925E−01	3.349E−03
WFG1	2.183E−01	4.530E−01	−	1.623E−01	2.584E−01	+	1.148E−01	1.428E−01	+	**4.890E−01**	**5.060E−01**	−	1.635E−01	1.794E−01	+	1.406E−01	2.972E−01
	6.114E−01	1.078E−01		4.112E−01	6.102E−02		1.609E−01	1.110E−02		**5.165E−01**	**5.977E−03**		1.955E−01	8.848E−03		5.917E−01	1.181E−01
WFG2	5.565E−01	5.606E−01	=	5.421E−01	5.471E−01	+	5.556E−01	5.577E−01	+	2.830E−01	2.831E−01	+	5.596E−01	5.614E−01	=	**5.617E−01**	**5.621E−01**
	5.624E−01	1.524E−03		5.528E−01	2.447E−03		5.595E−01	9.266E−04		2.831E−01	3.672E−05		5.641E−01	1.414E−03		**5.623E−01**	**1.331E−04**
WFG3	4.900E−01	4.914E−01	=	4.331E−01	4.386E−01	+	4.881E−01	4.902E−01	+	2.459E−01	2.460E−01	+	4.906E−01	4.915E−01	+	**4.929E−01**	**4.932E−01**
	4.930E−01	7.388E−04		4.486E−01	3.221E−03		4.914E−01	6.333E−04		2.462E−01	7.880E−05		4.923E−01	4.525E−04		**4.934E−01**	**9.572E−05**
WFG4	2.104E−01	2.141E−01	+	1.720E−01	1.766E−01	+	0.000E+00	2.014E−01	+	1.035E−01	1.037E−01	+	1.971E−01	2.068E−01	+	**2.162E−01**	**2.171E−01**
	2.167E−01	1.174E−03		1.887E−01	5.916E−03		2.131E−01	3.810E−02		1.039E−01	9.410E−05		2.127E−01	3.119E−03		**2.178E−01**	**3.718E−04**
WFG5	1.938E−01	1.947E−01	+	1.500E−01	1.517E−01	+	1.934E−01	1.940E−01	+	9.819E−02	9.854E−02	+	1.911E−01	1.945E−01	+	**1.952E−01**	**1.955E−01**
	1.954E−01	3.733E−04		1.730E−01	4.055E−03		1.948E−01	2.684E−03		1.033E−01	9.036E−04		1.947E−01	6.360E−04		**1.959E−01**	**1.824E−04**
WFG6	1.680E−01	2.001E−01	+	1.202E−01	1.488E−01	+	2.067E−01	2.073E−01	=	8.169E−02	8.420E−02	+	2.029E−01	2.072E−01	=	**2.086E−01**	**2.089E−01**
	2.091E−01	9.540E−03		1.717E−01	1.321E−02		2.082E−01	3.563E−04		8.490E−02	6.970E−04		2.086E−01	1.230E−03		**2.090E−01**	**8.514E−05**
WFG7	2.084E−01	2.089E−01	=	1.656E−01	1.677E−01	+	2.057E−01	2.068E−01	=	9.975E−02	9.999E−02	+	2.081E−01	2.087E−01	=	**2.088E−01**	**2.090E−01**
	2.096E−01	2.629E−04		1.783E−01	3.581E−03		2.074E−01	4.453E−04		1.001E−01	8.937E−05		2.092E−01	2.782E−04		**2.090E−01**	**6.533E−05**
WFG8	1.414E−01	1.459E−01	=	1.244E−01	1.333E−01	+	**1.515E−01**	**1.687E−01**	−	9.627E−02	9.724E−02	+	1.484E−01	1.547E−01	−	1.433E−01	1.513E−01

Table 7 (continued)

	NSGA−II			GrEA			CDG−MOEA			CMODE			MOEA/D−FRRMAB			AGMOEA	
	Min Max	Mean Std		Min Max	Mean Std		Min Max	Mean Std		Min Max	Mean Std		Min Max	Mean Std		Min Max	Mean Std
	1.505E−01	2.092E−03		1.733E−01	1.381E−02		**1.998E−01**	**2.066E−02**		9.842E−02	5.681E−04		1.995E−01	1.431E−02		1.913E−01	1.390E−02
WFG9	2.333E−01	2.364E−01	+	1.857E−01	1.887E−01	+	2.345E−01	2.354E−01	+	8.053E−02	8.793E−02	+	2.338E−01	2.347E−01	+	**2.323E−01**	**2.371E−01**
	2.386E−01	1.332E−03		1.918E−01	1.735E−03		2.393E−01	8.159E−04		9.096E−02	2.583E−03		2.361E−01	4.474E−04		**2.398E−01**	**1.919E−03**
Nos. +/=/−	12/4/5		16/1/4	16/2/3	13/3/5										11/6/4		

Table 8 Average and final ranking of algorithms on IGD and HV metrics

Algorithm	IGD ranking		HV ranking	
	Average	Final	Average	Final
NSGA-II	3.29	2	3.14	2
GrEA	4.61	6	4.28	6
CDG-MOEA	3.90	5	3.90	5
CMODE	3.62	4	3.95	4
MOEA/D-FRRMAB	3.48	3	3.57	3
AGMOEA	2.10	1	2.14	1

can increase the convergence and make the algorithm more robust. Moreover, the adaptive selection strategy of representative individuals from subspaces can further improve the search efficiency and maintain diversity.

4.4.3 Performance analysis of external archive extension mechanism

To analyze the efficiency of the external archive extension mechanism, the AGMOEA-pre (AGMOEA without the external archive extension mechanism) and AGMOEA-exten in this subsection are compared. Based on AGMOEA-pre, the external archive extension mechanism is further added to it, and represent the new variant as AGMOEA-exten.

The minimum, maximum, mean and standard deviation of IGD metrics, as well as the Wilcoxon's rank sum test results, are shown in Table 5. From this table, it can found that the external archive extension mechanism can improve the performance of the algorithm significantly. Among the 21 problems, AGMOEA-exten achieves 18 better values of IGD metric, among which the performance difference is significant for 14 problems. GMOEA-pre can only obtain better performance in DTLZ7 and WFG7 problems. The main reason is that the quality of external archive can be improved effectively, and it in turn can help to guide the generation of new offsprings to more promising regions since the individuals in external archive will be used in the evolutionary operators, which speeds up the convergence.

4.5 Comparison with other algorithms

To verify the performance of the proposed AGMOEA, it is compared with some other powerful MOEAs, such as NSGA-II, GrEA [26], CDG-MOEA [21], CMODE [40] and MOEA/D-FRRMAB [41]. In all experiments, the parameter setting of these MOEAs are set according to the original papers. To make a fair comparison, the termination criterion of all the algorithms is set to the maximum number of function evaluations, i.e., $FET_{\max}$. For bi-objective problems, $FET_{\max}$ is set to 25,000, while for tri-objective problems,

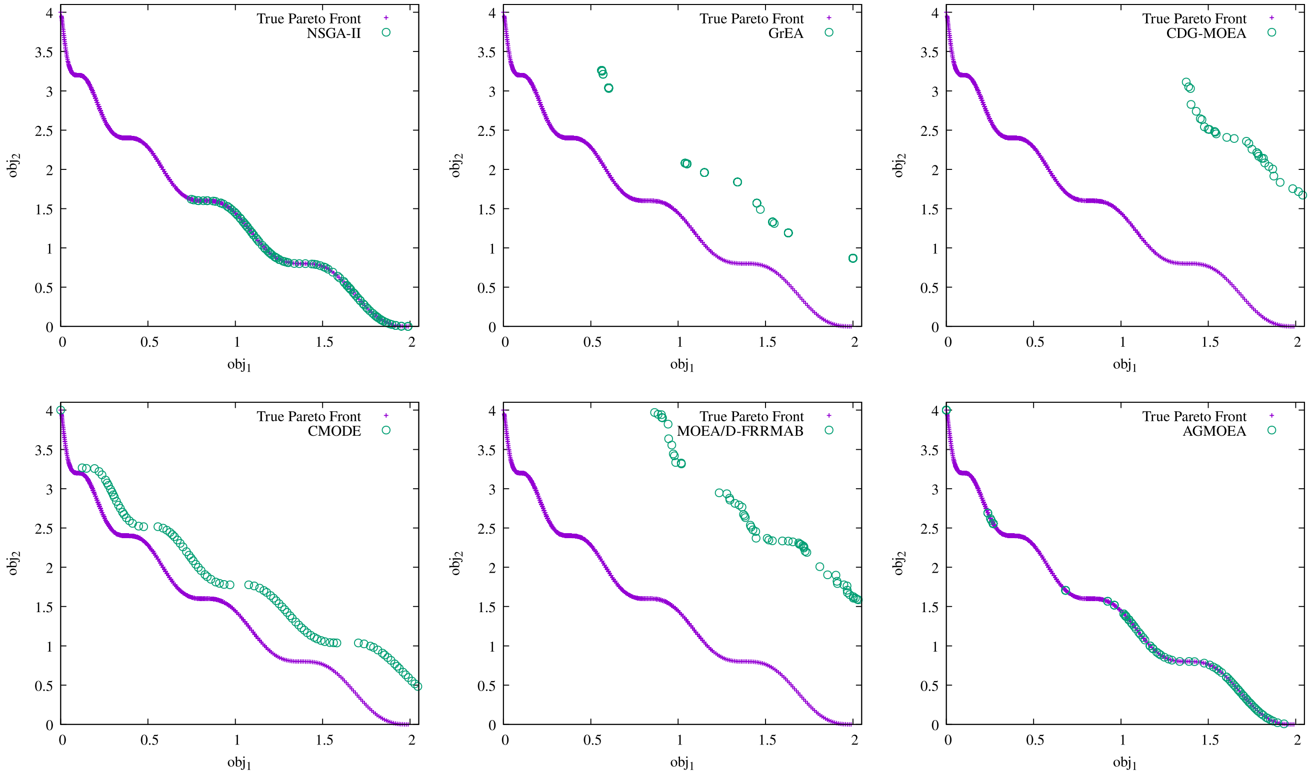

Fig. 5 Pareto Fronts Obtained by Different MOEAs on Problem WFG1

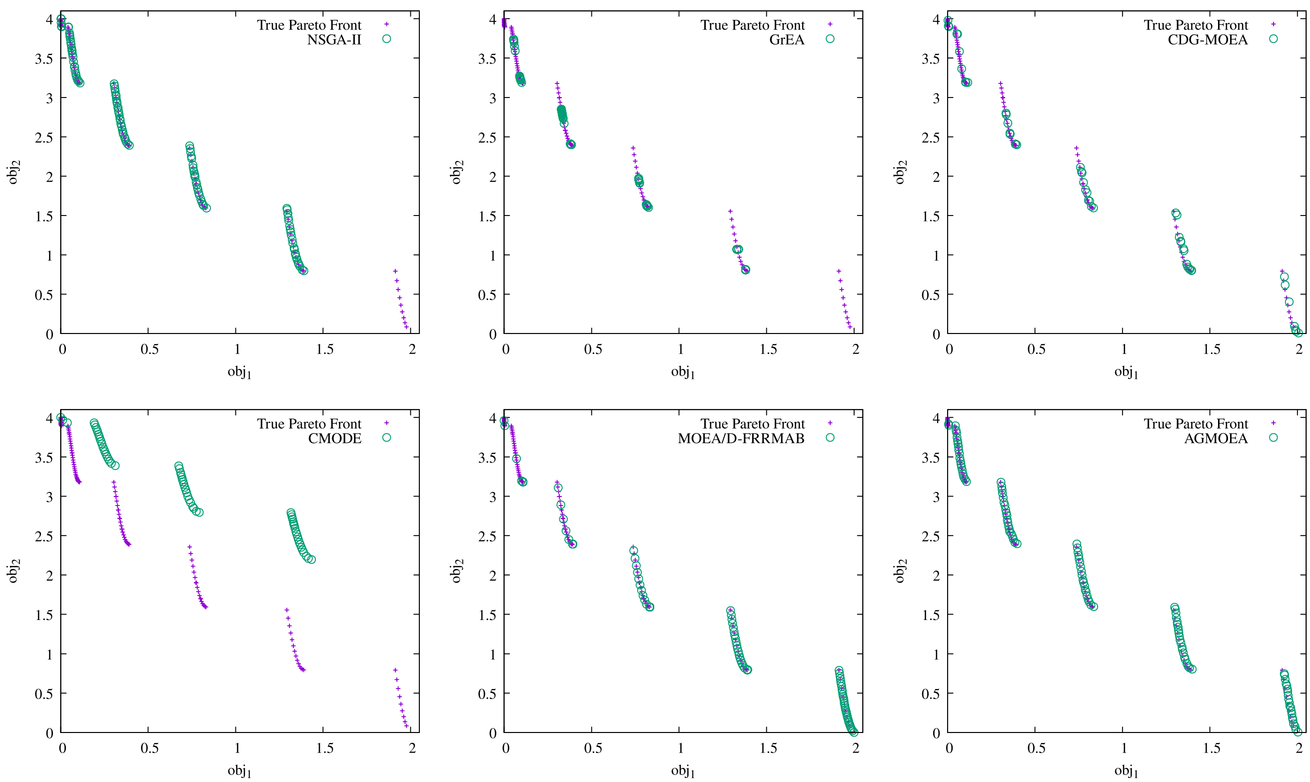

Fig. 6 Pareto Fronts Obtained by Different MOEAs on Problem WFG2

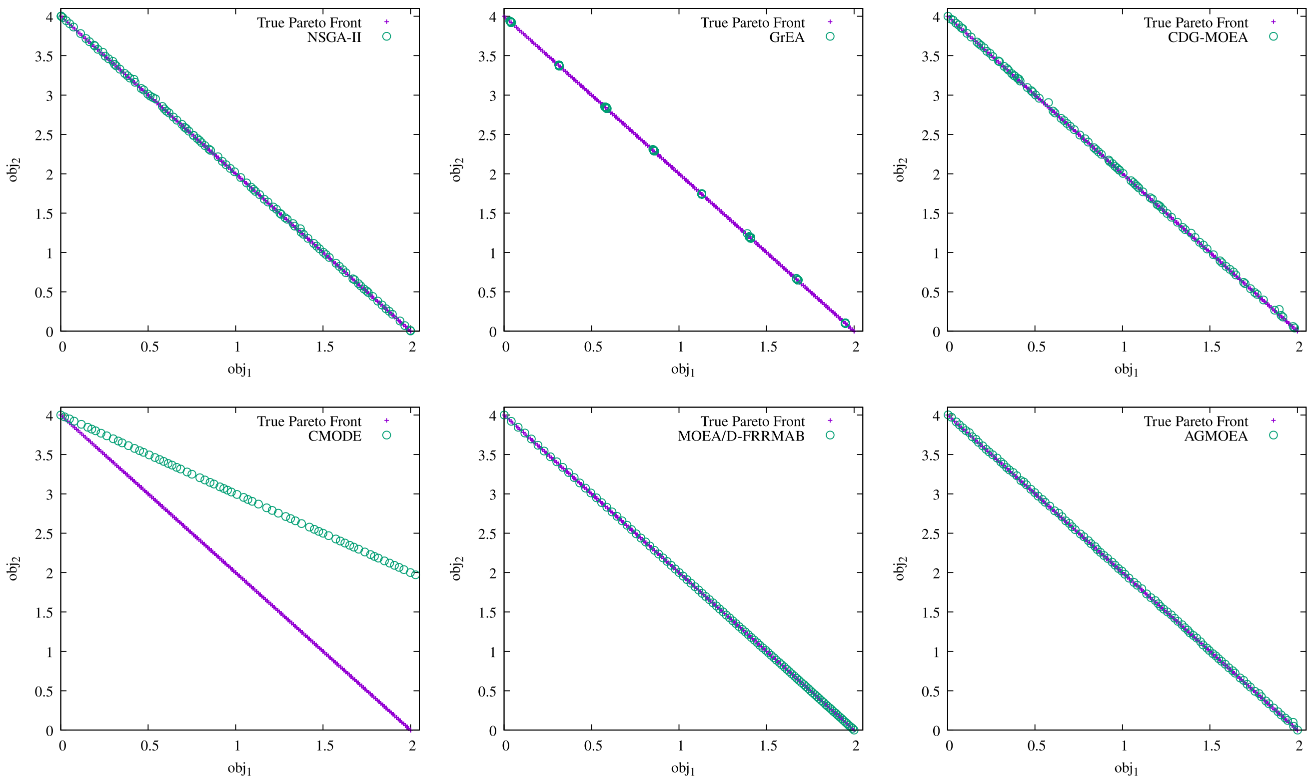

Fig. 7 Pareto Fronts Obtained by Different MOEAs on Problem WFG3

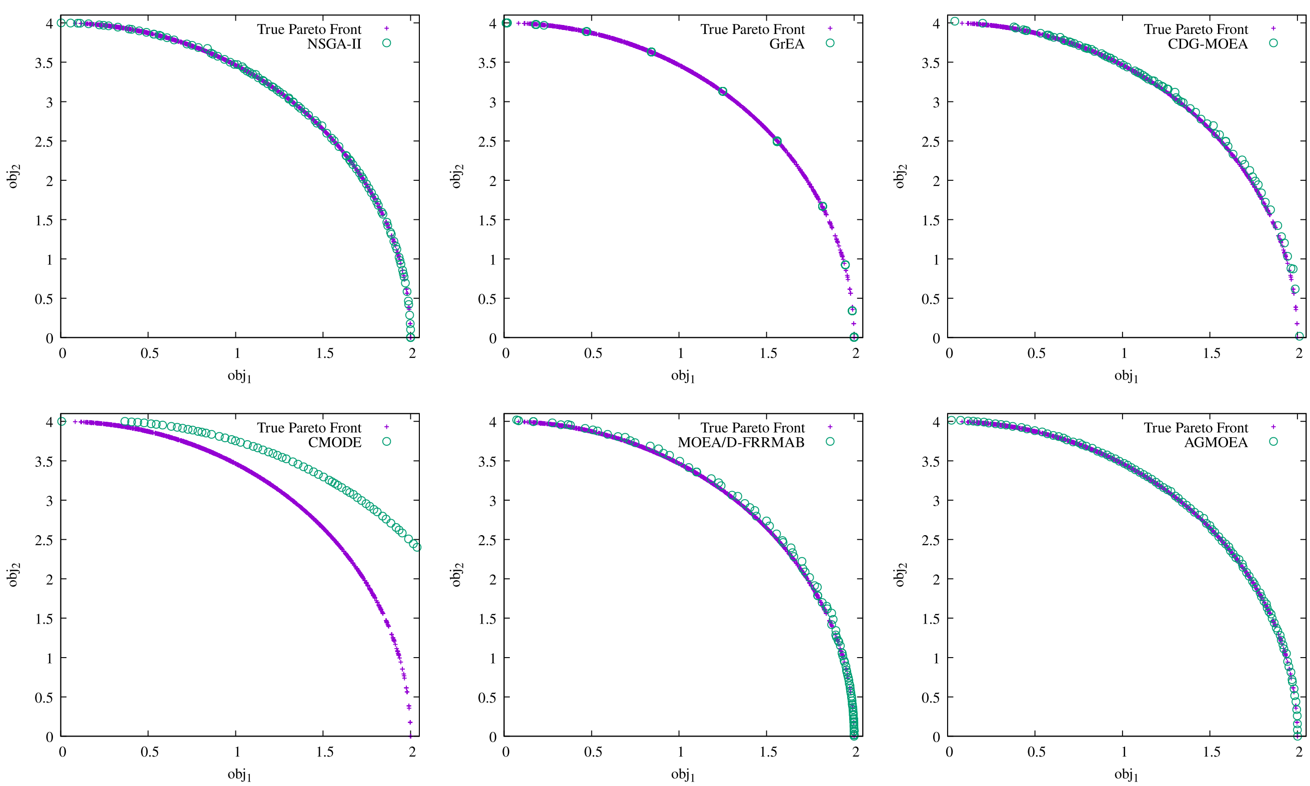

Fig. 8 Pareto Fronts Obtained by Different MOEAs on Problem WFG4

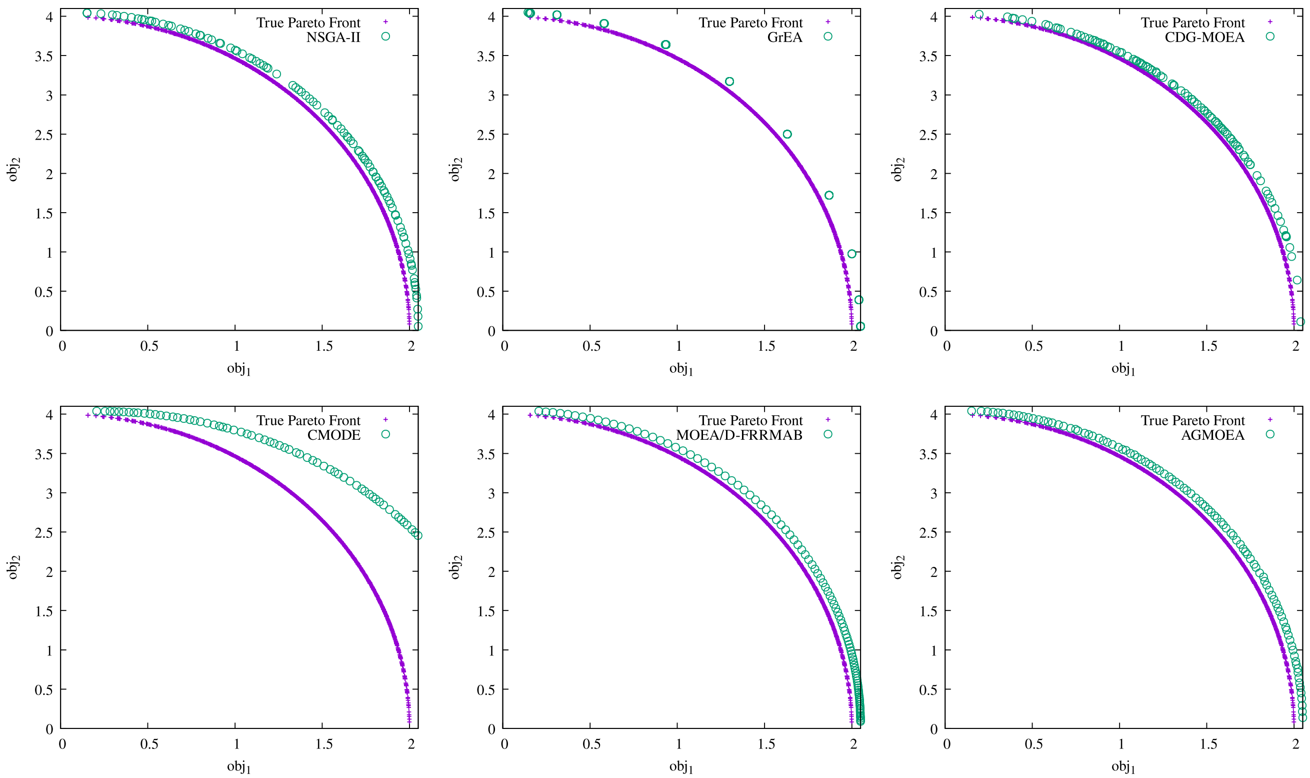

Fig. 9 Pareto Fronts Obtained by Different MOEAs on Problem WFG5

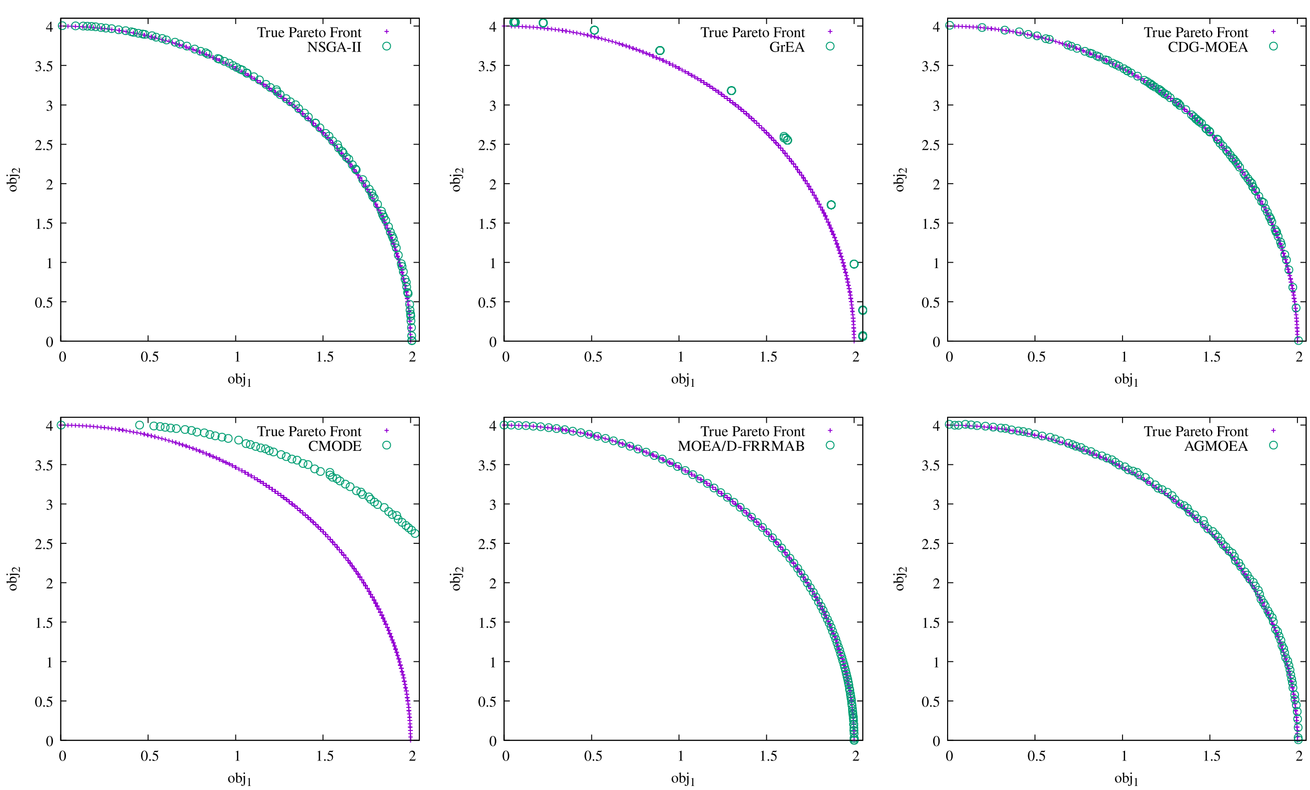

Fig. 10 Pareto Fronts Obtained by Different MOEAs on Problem WFG6

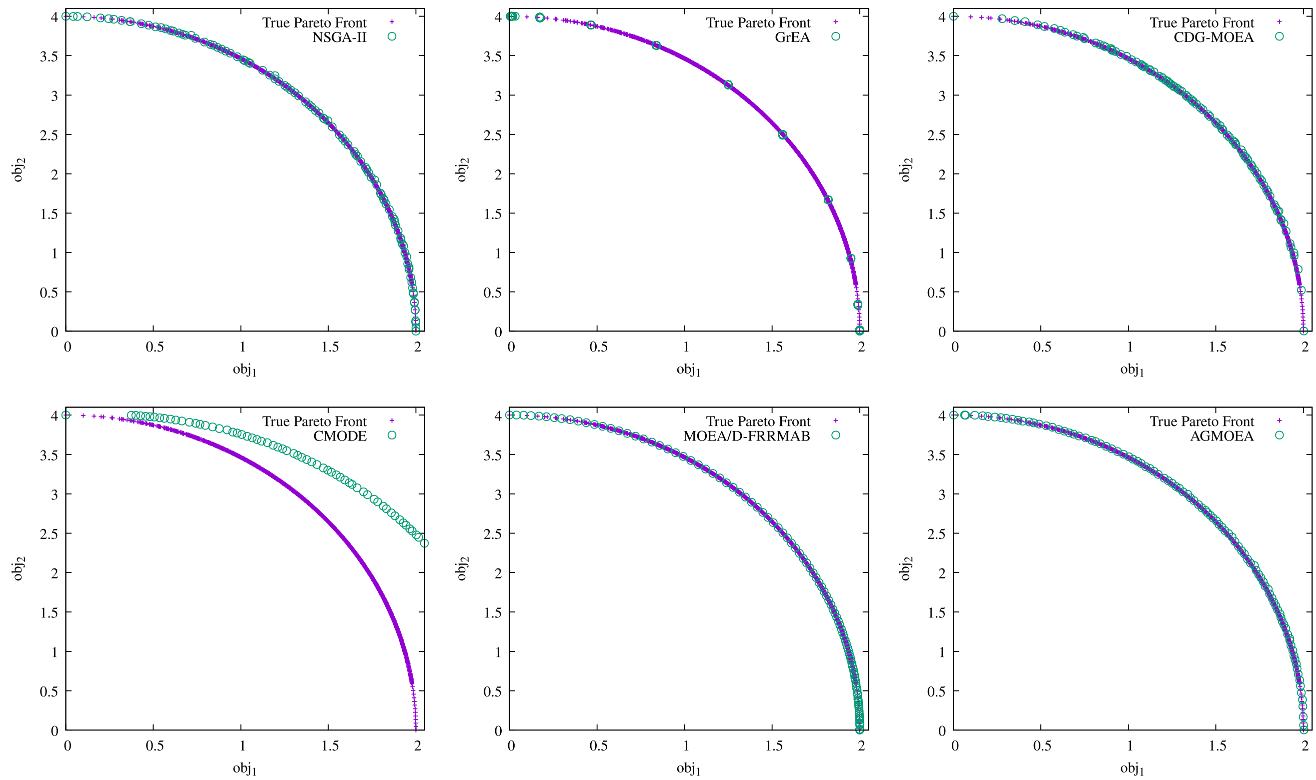

Fig. 11 Pareto Fronts Obtained by Different MOEAs on Problem WFG7

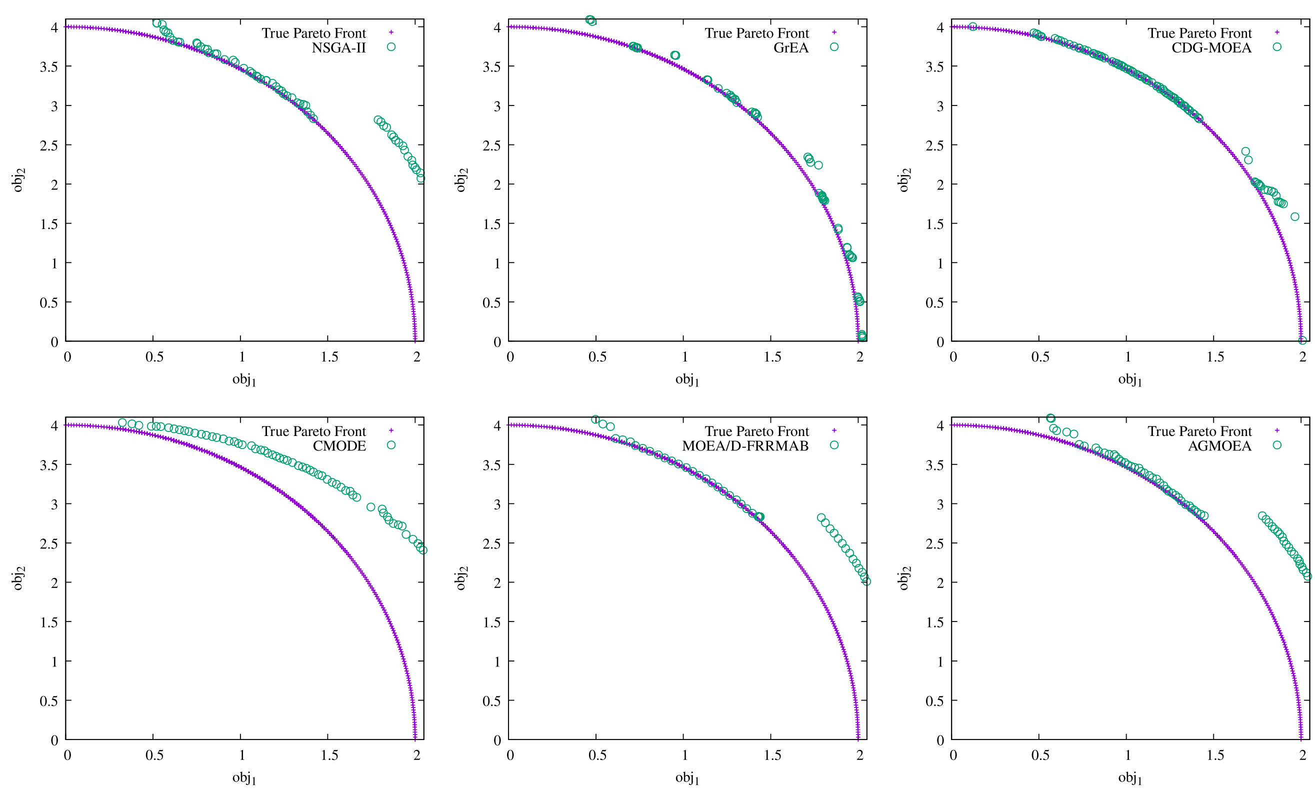

Fig.12 Pareto Fronts Obtained by Different MOEAs on Problem WFG8

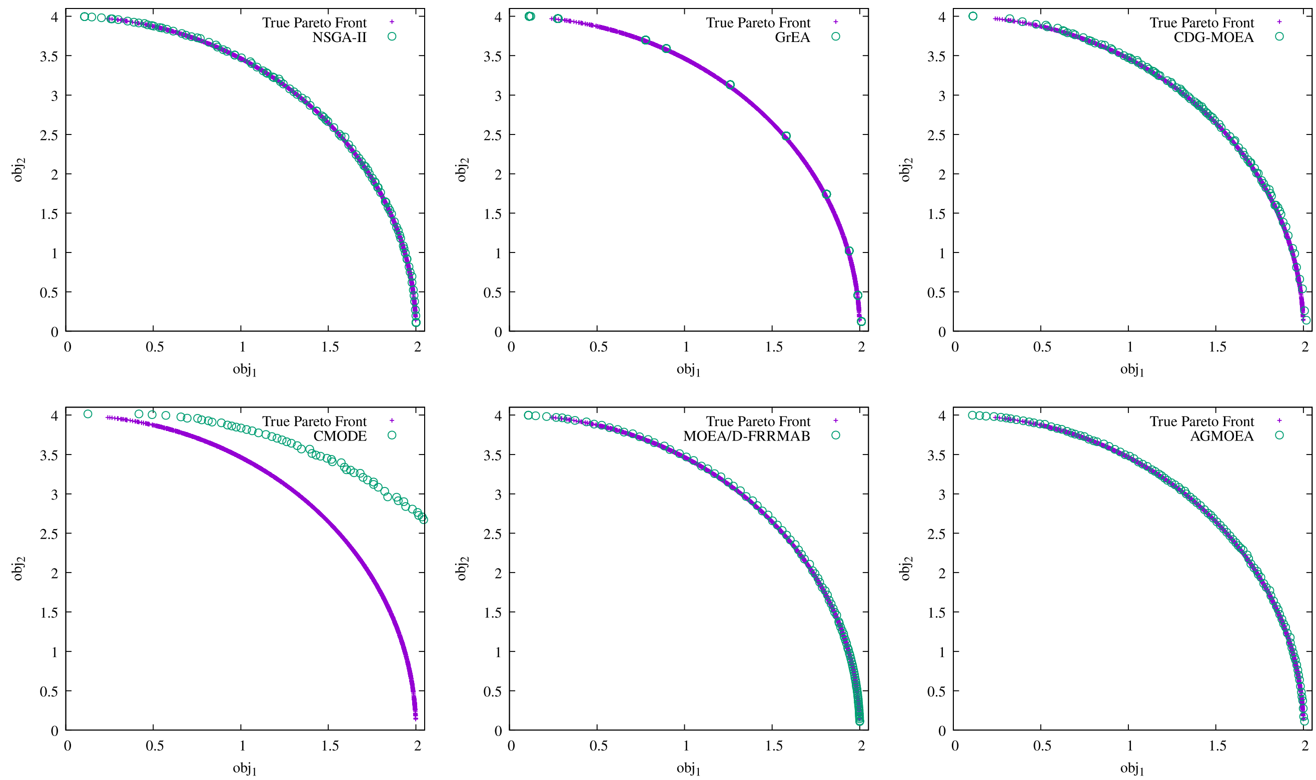

Fig. 13 Pareto Fronts Obtained by Different MOEAs on Problem WFG9

$FET_{\max}$ is set to 30,000. Each test problem is tested for 30 independent runs. The minimum, maximum, mean and standard deviation IGD and HV metrics are presented in Tables 6 and 7. Wilcoxon's rank sum test method is adopted to analyze the significant difference between AGMOEA and the other rival MOEAs and the confidence level is set to 95%. In the tables, the best result of each problem is shown in bold type and have a gray-colored background. If two algorithms have the same mean value of IGD or HV metric, the one with the smaller standard deviation is better. The symbols "+" and "–" in the column represent that the proposed AGMOEA algorithm is significantly better or worse than the rival algorithm, while "=" means that there is no significant difference between them. "Nos. +/=/–" in the last line denotes the sum of problems for the algorithm comparison, respectively.

From the results, it appears that AGMOEA is more superior or competitive than its rivals for both performance metrics, especially on the ZDT and WFG series of problems.

More specifically, based on these results of IGD metric in Table 6, it appears that AGMOEA can achieve 12 best median values of IGD metric out among the 21 problems, while CDG-MOEA, CMODE, and MOEA/D-FRRMAB can obtain the best results for only 1, 5, and 2 problems, respectively. Moreover, NSGA-II and GrEA don't obtain any best results. AGMOEA can obtain significantly better IGD for 16 problems than NSGA-II, for 17 problems than GrEA, for 14 problems than CDG-MOEA, for 10 problems than CMODE, and for 15 problems than MOEA/D-FRRMAB. For the HV metric shown in Table 7, it is clear that AGMOEA can achieve the best median values of HV metric for 11 out of the 21 problems, while NSGA-II, GrEA, CDG-MOEA, COMDE and MOEA/D-FRRMAB can only obtain the best results for 1, 1, 3, 3, and 2 out of the 21 problems, respectively. Furthermore, AGMOEA can get significantly better HV metrics for 12 problems than NSGA-II, for 16 problems than GrEA, for 16 problems than CDG-MOEA, for 13 problems than CMODE, and for 11 problems than MOEA/D-FRRMAB. The average and final rankings of each algorithms are shown in Table 8. Based on the results, it is clearly that AGMOEA gets the best rankings for both two metrics.

To give a graphical comparison, the best attainment Pareto fronts for the WFG series of problems are presented in Figs. 5, 6, 7, 8, 9, 10, 11, 12 and 13. From these figures, it can be observed that AGMOEA has better performance for many problems, especially for WFG2, WFG3, WFG4, WFG5, WFG6, WFG7 and WFG9. The main reasons can be analyzed as follows: (1) the adaptive selection strategy of subspaces can allocate evolutionary opportunities to different subspaces dynamically, and the convergence and diversity can be balanced better; (2) the evolutionary scheme based on adaptive selection of solutions from the subspace and multiple crossover operators can improve the evolution efficiency; (3) the external archive extension mechanism can improve the quality of non-dominated solutions and guide the evolutionary searching direction as well.

5 Conclusion

In this paper, the AGMOEA is proposed by incorporating the concepts of grid system. To balance the convergence and diversity, an adaptive selection strategy of subspaces is proposed. To improve the evolution efficiency, an evolutionary scheme considering representative individuals in subspaces is adopted. Moreover, five kinds of crossover operators with a self-adaptive selection strategy can make the algorithm more robustness to solve different kinds of MOPs. The external archive extension mechanism can further speed up the convergence of the algorithm. The AGMOEA is evaluated on 21 benchmark problems and compared with five powerful rival algorithms. The comparison results verify the effectiveness and competitiveness of the AGMOEA. The future research is to study the adaptive control of algorithm parameters and extend the algorithm to solve MaOPs.

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