MTHE 493 - Engineering Mathematics Project

List of Projects 2013-2014

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A Projects in Control

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A-1 Distributed algorithms for deployment of autonomous mobile networks (Gharesifard)

In this project, we will study a class of geometric optimization problems where the objective is to place a group of mobile vehicles in an environment of interest so that they maximize, in a proper sense, the information gathered; this is often known as the continuous p-center problem in the literature on facility location. The challenging part is that we would like the vehicles to start from some arbitrary locations and figure out such placements only using their limited sensing and communication capabilities. In other words, although the objective that the group of vehicles are trying to achieve is a global one, each vehicle only observes the environment from a local point of view.

The students will learn various techniques as the technical part of the project, including geometric optimization, distributed control design, stability of system using LaSalle invariance principle, and some elementary notions of gradient flow of nonsmooth systems. On the application side, the students will implement various deployment algorithms under different objective functions. Students will be encouraged to come up with new deployment schemes for classes of problems with dynamic constraints and to explore new applications for this class of deployment algorithms.

A-2 Robustness issues in formation dynamics (Gharesifard)

Multi-agent formation problem refers to a collection of agents, in 2- or 3-dimensional space, that wish to keep a constant inter-agent distance. The objective is to design control strategies which ensure that the agents can maintain their inter-agent distance over time. Formation dynamics has variety of applications in environmental monitoring, ocean sampling, and in the analysis of schooling and herding of groups of animals. One main challenge is that the control inputs to each agent can only depend on its distance to the neighboring agents. If neighboring agents can measure the locations of each other in a bidirectional manner, we say that the scenario is ”undirected”; otherwise, we use the term ”directed”.

In this project, students will simulate some of the existing algorithms, including the infinitesimally rigid formation and the consensus-based flocking protocols, for different applications. One main objective is to demonstrate that almost all existing control strategies for undirected scenarios are sensitive to perturbations. Students will be encouraged to explore possible ways to overcome such robustness issues. For ”directed” formation problems with more than three agents, students will become familiar with a very recently shown fundamental limitation of formation dynamics, namely that there exists no twice differentiable controller that globally stabilizes the directed formation. Depending on the time available, we might explore implications of Morse Theory that shed light about such limitations.

A-3 Optimal control of the heat equation in a non-homogeneous medium (Mansouri)

Consider a (1-dimensional) metal rod of length L with one end (at $x = 0$) thermally insulated from the environment and suppose that by heating or cooling the other end (at $x = L$), you can control how much heat you add at that end, this being your only means of influencing the system, i.e. your only control. The governing equation for this system is given by the partial differential equation

$$\begin{aligned}
\frac{\partial T}{\partial t}(x,t) &= \frac{\partial}{\partial x}(c(x)\frac{\partial T}{\partial x})(x,t), \quad 0 < x < L,\ t > 0, \\
\frac{\partial T}{\partial x}(0,t) &= 0,\ \ t \geq 0, \\
T(L,t) &= u(t),\ \ t \geq 0, \\
T(x,0) &= f(x),\ \ 0 \leq x \leq L,
\end{aligned}$$

where $T(x,t)$ is the temperature of the rod at position x and time t, c is the (spatially varying) coefficient of thermal diffusion, u is the control function, and f is the initial temperature profile of the rod. Note that the state of the system at time t is the function $x \mapsto T(x,t)$, i.e. an element of an *infinite-dimensional* real vector space. The problem we wish to consider is that of finding a control function $u : [0,\tau] \to \mathbb{R}$ which would steer the temperature function T from f at time $t = 0$ to some other temperature profile g at time $t = \tau$ while minimizing a cost function such as

$$\eta(u) = \int_0^\tau u^2(t)dt.$$

The main goal of this project is to approach this problem using tools from finite-dimensional linear control theory by “approximating” the above system by linear control systems of the form

$$\dot{\mathbf{x}}_h = \mathbf{A}_h\mathbf{x}_h + \mathbf{B}_h u_h,$$

with the approximation getting “finer” as $h \to 0$. The key questions then are whether the optimal controls u_h thus obtained do converge (in some sense) as $h \to 0$, and, if they do, whether their limit yields an optimal control for the original system. The specific goals of this project are meant to address these questions by

(i) understanding the mathematics of this problem (control of partial differential equations),

(ii) designing optimal controls for this problem by adapting tools from finite-dimensional linear control theory,

(iii) performing computer simulations to verify the theoretical observations.

A-4 Robot-based therapy task for subjects with stroke (Scott)

Stroke is a major cause of motor disability resulting in weakness and the inability to perform many activities of daily living. Traditional rehabilitation involves a therapist working with the individual one-on-one, which is time consuming and expensive. Robots provide a potential cost-effective approach to provide therapy permitting the subjects to perform many movements in a highly controlled environment.

The goal of this project is to make a goal-directed upper limb task using the KINARM robot. The task will require the subject to move their arm in the horizontal plane to virtual targets displayed in the work environment and the robot will measure their arm movements. Task complexity could be increased by adding visual or mechanical disturbances requiring subjects to practise making rapid corrections. The project will require the subjects to develop this rehabilitation task and assess behavioural performance of a group of healthy individuals.

B Projects in Computer Vision and Robotics

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B-1 Design of region merging algorithms for image segmentation (Alajaji and Mansouri)

Image segmentation is the process of subdividing an image into semantically meaningful parts based on color, intensity, texture, shape, or other features. The image parts obtained through such a subdivision can in turn help in automatic analysis and interpretation of the image content, for use in tasks such as robot navigation, medical image analysis, or automated surveillance. Numerous approaches have been proposed for image segmentation, one of these being region merging. In this approach, the desired image subdivision is obtained by starting from an overly fine subdivision of the image, and iteratively merging neighboring regions based on some merging criterion. Recently, region merging criteria have been proposed that are based on information theoretic ideas and results: Pixel intensity distributions in each region are modeled using first-order Markov models, and various measures of distance between their transition probability matrices are used to determine whether or not the regions should be merged.

The key drawback of this approach is that starting from an overly fine initial subdivision, the immediate subsequent merging operations become susceptible to modeling errors; it may be preferable therefore to start from a coarser initial subdivision, yet allow the possibility of further splitting during the iterations. Furthermore, these region merging algorithms may be sensitive to noise which corrupts the image in practical settings.

The goal of this project is to pursue one of the following directions:

(a) To investigate the advantages of incorporating region splitting within region merging algorithms for image segmentation and to design an image segmentation algorithm based on region merging and splitting; this entails defining criteria for determining when and how to split regions;

(b) To investigate the impact of correlated noise (e.g. burst noise) on region merging algorithms that assume an independent identically distributed noise model, and to design a region merging algorithm based on a correlated noise model, in addition to improving the probabilistic models that have been used in their development.

Both of these projects will require understanding and implementing on computer recent region merging criteria that make use of information theoretic statistical measures. A comparison of all these criteria and algorithms will be performed through computer experimentation.

B-2 Stochastic Image Models, Stochastic Gradient Descent, and Applications to Image Restoration (Mansouri)

The stochastic approach to image modeling consists in modeling an image as a family of random variables indexed by the vertices of an undirected graph; the graph structure defines the neighborhood relation between these random variables, and the dependencies between these random variables are modelled by conditional probabilities. For a certain class of probabilistic image models known as Markov/Gibbs Random Fields, these conditional probabilities take a relatively simple form and lead to tractable algorithms. As an illustration, suppose given a particular noisy image O, say a page of text rendered unreadable through noise or a noisy satellite image: We wish to remove the noise and restore the image by finding the most likely uncorrupted image that gave rise to that noisy observation O. Modeling an observed image as a family of random variables $\mathbf{O}$ and similarly modeling an uncorrupted image as a family of random variables $\mathbf{I}$, and modeling their relation by an image formation process of the form:

$$\mathbf{O} = f(\mathbf{I}, \eta),$$

with η a family of random variables modeling noise, and the function f modeling possible deterministic image transformations (such as blur), the mathematical problem becomes that of finding I which maximizes the "posterior" probability $P(\mathbf{I} = I|\mathbf{O} = O)$, which in turn is equivalent to finding I which maximizes the product

$$P(\mathbf{O} = O|\mathbf{I} = I)P(\mathbf{I} = I).$$

The probability function $P(\mathbf{O} = O|\mathbf{I} = I)$ is determined by the image formation model above, whereas the probability function $P(\mathbf{I} = I)$ (also called image prior) depends on the class of images one is considering (e.g. satellite images versus images of urban scenery). The problem is now two-fold:

- How to "learn" the prior probability distribution $P(\mathbf{I} = I)$ for a given class of images ?
- Once the prior probability distribution has been established, how to find the global maximizer of the posterior probability $P(\mathbf{I} = I|\mathbf{O} = O)$?

Note that these problems are far from trivial since the space of images one considers is typically huge and hence brute-force search is not feasible (even restricting ourselves to binary images of size 64×64, the set of all possible values that the random variables $\mathbf{I}$ and $\mathbf{O}$ can take – i.e. the set of all images – has cardinality 2^{4096}). It happens however that, under the assumption that $\mathbf{I}$ is a Markov Random Field and with a simplified image formation model, both of these problems can be approached using simple but powerful tools from Markov Chain theory.

The main goal of this project is to use this probabilistic approach in order to design an algorithm for restoring noisy or degraded images. More specifically, the key objectives are to:

(i) Learn the required mathematical tools (Homogeneous and Non-Homogeneous Markov Chains, Markov Random Fields, Gibbs Sampling, Stochastic Gradient Descent) that form the foundations of this project,

(ii) design an algorithm for learning the prior probability distribution for a given class of images and applying it to image restoration,

(iii) validate the designed algorithm through computer implementation.

B-3 Region Tracking in an Image Sequence (Mansouri)

Suppose given an image sequence with thousands of frames, and suppose a region of interest is defined in the first frame of the sequence (say a car); the goal is to have the computer "track" this region as accurately as possible in the remaining frames. This is the problem of automated region tracking, and it has numerous applications, e.g. in biomedical imaging, where the heart is tracked from frame to frame to analyze its behaviour, or in traffic monitoring, where individual cars are tracked to identify possible traffic violations (just Google "region tracking" to see the diversity of applications). The variational approach consists in formulating this problem as a minimization problem over a suitable space. More precisely, letting $I_0, I_1 : \Omega \to \mathbb{R}$ be two images, and letting $R_0 \subset \Omega$ be the subset of the image domain Ω corresponding to the region of interest in image I_0, we define a function (also called functional) $E_{(R_0,I_0,I_1)} : \mathcal{C} \to \mathbb{R}$, where $\mathcal{C}$ is a suitable subset of the set of all subsets of Ω (i.e. the set of all candidate regions), and we declare the global minimizer of this functional to be the desired region $R_1 \in \mathcal{C}$. The problem of "tracking" region R_0 from frame I_0 to frame I_1 is then reformulated as the problem of minimizing the functional $E_{(R_0,I_0,I_1)}$ over the set $\mathcal{C}$. Clearly, for this approach to be of any use, this functional must embed some information on the image characteristics and the regions' invariants. Once the mathematical issues of existence and uniqueness of a minimizer for the functional $E_{(R_0,I_0,I_1)}$ are resolved, the search for such a minimizer can proceed using tools from the calculus of variations, leading to partial differential equations which are then solved numerically.

The objective of this project is to investigate the variational approach to region tracking by:

(i) Learning the required mathematical tools (Calculus of Variations and Partial Differential Equations),

(ii) designing a region tracking functional and deriving the partial differential equations leading to a minimizer using the calculus of variations,

(iii) validating the tracking functional through computer implementation and evaluation of tracking performance on various image sequences.

B-4 Automated Face Recognition (Mansouri)

Face recognition by computer has seen numerous applications in recent years. An N by N gray scale image can be considered as a mapping $\mathbf{x} : \{1, 2, \cdots, N\}^2 \rightarrow \mathbb{R}$, and, equivalently, as a vector $\mathbf{x} \in \mathbb{R}^{N^2}$. Note that for $N = 256$ (a typical value for a small image) $\mathbb{R}^{N^2}$ is already a huge-dimensional real vector space. Assume now given a set

$$\mathcal{C} = \{(\mathbf{x}_1, l_1), (\mathbf{x}_2, l_2), \cdots, (\mathbf{x}_K, l_K)\}$$

of images of faces of various people (taken possibly under various lighting conditions or with various facial expressions), where for each $k \in \{1, 2, \cdots, K\}$, l_k denotes the label (i.e. identity) attached to image $\mathbf{x}_k$. Suppose now a new face image $\mathbf{z} \in \mathbb{R}^{N^2}$ is given; which individual does it correspond to ? Most approaches to face recognition assume that face images cluster around a low-dimensional linear subspace of $\mathbb{R}^{N^2}$ and that the non-essential features (e.g. lighting condition, facial expression) are in the image component orthogonal to that subspace. Face recognition can then proceed by dimensionality reduction and subspace projection. Although such face recognition algorithms have shown very good performance, the linear subspace assumption is clearly an idealization, and there is no reason for face images to cluster in such a linear way.

The main objective of this project is to investigate alternative approaches to face recognition by relaxing the linear subspace assumption; more specifically, the specific objectives are to:

(i) Understand the required mathematical tools for solving this problem (Principal Component Analysis, Fisher Discriminant),

(ii) design an algorithm for automated face recognition by formulating it as a problem of dimensionality reduction via a general submanifold of $\mathbb{R}^{N^2}$,

(iii) validate the designed algorithm through computer implementation.

All of the projects listed above will involve some basic literature search, an understanding of the key concepts, some computer programming, and some computer experimentation.

C Projects in Information Theory and Communications

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C-1 Image Communication over Wireless Channels (Alajaji and Linder)

The efficient compression and reliable transmission of multimedia signals (such as data, image, video, speech signals) over uncoded and error-control coded wireless and communication channels (such as satellites, smart phones and the wireless Internet) is a fundamental problem in information theory and digital communications. The goal of signal compression – also known as *source coding* – is to represent the signal in a compact way to reduce storage costs and bandwidth requirements. Reliable transmission is achieved through *channel coding*, a procedure that protects the signal against distortion and noise phenomena induced by the physical communication channel.

The main objective of this project is to investigate, design and implement high data rate robust coding techniques which realize the *dual* roles of source and channel coding in a single *combined* step. This will provide an interesting and substantially less complex alternative to the traditional approach which treats the source and channel coding operations separately. Note that reducing the system complexity (while guaranteeing an appropriate quality of the reproduced signal) is a vital requirement for the development and commercial success of multimedia technology. Several methods have been proposed recently to achieve this goal. In particular, the project will focus on making an effective high data rate image compression method, known as *vector quantization*, resilient to errors introduced by channel noise. The use of *energy efficient* modulation constellations and the judicious incorporation of the channel's soft information in the design of the quantizers is a key objective. Two communication wireless channel environments will be considered: additive white Gaussian noise and correlated Rayleigh fading channels used in conjunction with (appropriately designed) M-ary PSK or QAM modulation constellations. Performance comparisons with traditional (tandem) coding methods will also be undertaken.

C-2 Performance of Uncoded Modulation Systems over Gaussian Networks (Alajaji)

Investigate the problem of transmitting *non-uniformly* distributed modulated signals over a two-user noisy communication network. The objective is to design an optimal demodulation scheme at the receiver that minimizes the probability of decoding error. The channel model that will be considered is the two-user Gaussian multiple-access channel used in conjunction with coherent M-ary phase-shift keying (MPSK) modulation.

In this project, you are required to perform the following tasks:

- Given that a binary non-uniformly distributed source is modulated via MPSK and sent over the channel, design an optimal receiver scheme for this system.
- Derive closed-form expressions for the symbol error probability of the system.
- Derive upper and lower bounds to the bit error probability of the system.
- Compare the proposed demodulation scheme with the traditional *Maximum Likelihood* (ML) scheme.
- Evaluate the performance of the communication system by means of software simulations and compare the results with the analytically derived expressions.

C-3 Coding Strategies for Communication Networks (Alajaji)

The overall objective of this project is to design effective coding strategies for reliably sending information sources over noisy communication networks. The binary multiple-access channel will first be used to model a simple two-sender one-receiver network (although other more general models can also be adopted) and the sender information sources will be modeled as a two-dimensional binary memoryless source. The objective is to analyze, design and implement coding techniques using principals from data compression and error-correcting channel coding for the following system setups:

- A noisy version of the information source is available at the receiver as side information.
- Each sender benefits from the availability of feedback about the past received channel outputs.
- Each sender has some partial information about the other sender's source and can thus cooperate with the other user in constructing a code that will improve system performance.
- Basic system with no side information/feedback.

The performance of the above coding schemes will be assessed vis-a-vis Shannon's information-theoretic limit and available schemes in the literature.

C-4 Game Theory and Information (Linder and Yüksel)

Consider a patient who visits a doctor to find out about her health. The patient, however, knows that the doctor has an incentive to make her purchase a certain drug, yet the only way she can reach truth is through the doctor. The doctor knows that the patient is aware of the potential bias. How do the doctor and the patient act?

This is what we call a game. The above is a special case of an 'information transmission' game, where an encoder picks a coding policy mapping his knowledge to a set of measurements, and a decoder maps the measurement to an action. Game theory studies such decentralized optimization problems where multiple decision makers are present in a system and have typically different goals (or cost functions to minimize), but whose actions collectively affect each of the goals/cost functions. A collection of policies which are locally stable (that is, given other decision makers' policies, an optimal policy of each agent is consistent with the collection; this being so for every decision maker) is called a Nash equilibrium.

Our goal in this project is to study information games of the types described above. We will seek to understand if an equilibrium or an approximate equilibrium exists in a given setup, how to characterize such equilibria, how to reach such equilibria and how to approximate them. We will investigate quantizers as an important special class of information transmission policies. We will also study the setting where the goals are aligned, which is a well-studied communication and control theoretic problem. Connections with learning, either through repeated instances or through external data, will also be investigated.

On the societal impact side, the group is to make the connection of the mathematical analysis with the information-rich world around us where our access to truth is occasionally distorted. Applications to economic systems will also be studied.

Students are expected to understand the engineering and practical as well as mathematical significance of the problem, provide a rigorous analysis on the performance of the proposed schemes, and simulate their analytical results using a software of their choice. They have the flexibility to modify the problem, so long as the assumptions and the relaxations are reasonable.

C-5 Data Analysis for Communications Engineering (Thomson)

Many communications systems fail to perform as designed because of unanticipated climate, ionospheric, or solar effects. This project consists of reading relevant papers and analyzing some real-world data. There are several data sets ranging from induced voltages on transatlantic cables and power systems, dropped calls in cell phone systems, and various satellite data. Some experience with statistics, discrete time series, scientific programming, and Fourier transforms is desirable.