MORE THAN ONE AUTHOR WITH DIFFERENT AFFILIATIONS

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Abstract

The abstract is single-paragraph summary of approximately 150 to 250 words. Include a sentence about the focus of the paper and on the results, if applicable. Follow the abstract by three to five key words (see below).

Keywords: Keyword1, Keyword2, Keyword3

1 INTRODUCTION

Many websites (IO Tools, 2021; Codesansar, 2021) provide tools to determine an LU decomposition of a matrix; however, none of them can decompose *all* matrices. Using code, we developed a GeoGebra applet that can decompose square matrices for those that can be factored and decompose "nearby" matrices without an LU decomposition using perturbation. In this paper, we first present conditions for a matrix to have a unique LU decomposition. Several examples are included to illustrate this theorem. Next, we state and prove special cases for matrices with infinitely many and no LU factorizations.

2 LU DECOMPOSITION

For $A \in M_n$, the factorization $A = LU$, where L is unit lower triangular and U is upper triangular, is called the *LU decomposition*, or *LU factorization*. We can use such a factorization, when it exists, to solve the system $A\mathbf{x} = \mathbf{b}$ by first solving for the vector $\mathbf{y}$ in $L\mathbf{y} = \mathbf{b}$ and then solving $U\mathbf{x} = \mathbf{y}$. However, not every $n \times n$ matrix A has an LU decomposition. The following theorem provides conditions for the existence and uniqueness of an LU decomposition of a $n \times n$ matrix. A proof can be found in Johnson and Horn (1985, p. 160).

Example 1. The 3×3 matrix $A = \begin{bmatrix} 1 & 5 & 1 \\ 1 & 4 & 2 \\ 4 & 10 & 2 \end{bmatrix}$ has all non-zero principle minors, A_1, A_2 and A_3. Therefore, there is a unique LU factorization with both L and U nonsingular given by

$$\begin{bmatrix} 1 & 5 & 1 \\ 1 & 4 & 2 \\ 4 & 10 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 4 & 10 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 12 \end{bmatrix}.$$

Theorem 1 (Matrices with Infinitely Many LU Factorizations)**.** *For $A \in M_n$, if two or more of any first $(n-1)$ columns are linearly dependent or any of the first $(n-1)$ columns are 0, then A has infinitely many LU factorizations.*

Proof. We will prove only for the the case when $A \in M_3$.

$$dm + r = e \Rightarrow r = e - dm \tag{1}$$

$$dn + rp = f \Rightarrow p = \frac{f - dn}{r} \tag{2}$$

$$gm + s = h \Rightarrow s = h - gm \tag{3}$$

$$gn + sp + t = i \Rightarrow t = i - sp - gn \tag{4}$$

□

3 LISTS

NAGJ uses the `outline` package.

3.1 Enumerated List

The following code produces an enumerated list.

```
\begin{outline}[enumerate]
     \1 First Level
          \2 Second level
               \3 Third level
\end{outline}
```

1. First Level

   (a). Second level

      i. Third level

3.2 Itemized List

The following code produces an itemized list.

```
\begin{outline}
     \1 First item
          \2 Second level item
               \3 Third level sub-item
\end{outline}
```

- First Level
  - Second level
    - Third level

4 Tables and Figures

4.1 Figures

Figures should be high quality (1200 dpi for line art, 600 dpi for grayscale and 300 dpi for color, at the correct size). Figures should be supplied in one of our preferred file formats: EPS, PS, JPEG, TIFF, or Microsoft Word (DOC or DOCX) files are acceptable for figures that have been drawn in Word. For information relating to other file types, please consult our Submission of electronic artwork document.

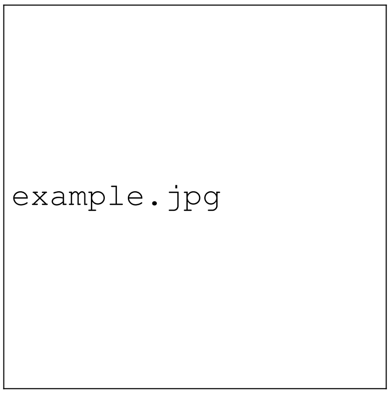

Figure 1. Provide a short caption description.

4.2 Tables

We use the `booktabs` package. Tables should present new information rather than duplicating what is in the text. Readers should be able to interpret the table without reference to the text. Please supply editable files.

5 Cross Reference

Figures, tables, and equations should be labeled (`label`) then referenced using the `ref`.

References

Codesansar (2021). Online LU Decomposition (Factorization) Calculator. *Retieved from: https://www.codesansar.com/numerical-methods/online-lu-decomposition-factorization-calculator.htm.*

IO Tools (2021). LU Factorization Calculator.

Johnson, C. R. and Horn, R. A. (1985). *Matrix analysis*. Cambridge University Press Cambridge.

Author is an Associate Professor at all the best universities.

Author, is a professor and has taught you everything you know.

Appendix - Code

```
for(int i = 0; i < dim - 1; i++){
    for(int k = i+1; k  < dim; k++ ){
        for(int m =  i+1; m < dim; m++){
            if(upper[i][i] == 0 && upper[m][i] != 0 ){
                'THERE IS NO LU FACTORIZATION'
            }
        }
        if(upper[i][i] != 0 && upper[k][i] != 0){
            multiplier =upper[i][i]/upper[k][i];
            lower[k][i] = multiplier;
        }else{
            if(upper[i][i] == 0 && upper[k][i] == 0){
                INFINITELY MANY LU FACTORIZATIONS
                multiplier = INPUT FROM USER;
                lower[k][i] = multiplier;
            }
        }
    for(int j = 0; j < dim; j++){
        row[j] = upper[i][j]*multiplier;
    }
    for(int r = 0; r < dim; r++){
        upper[k][r] = upper[k][r] - row[r];
    }
}
Display Lower And Upper
(Perform Matrix Multiplication on L and U)
Display L and U
}
```