50.012 Networks (2021 Term 6)

Homework 2

Transport Layer Overview

Hand-out: 7 Oct

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1. (2019 midterm exam question) Consider data communication over a link of RTT 100ms and transmission bandwidth 1Gbit/s. Assume 1G=10^9. Consider a pipelined transport protocol that uses ACKs to decide if packets were received successfully. Answer the following three questions:

1.1 After the protocol has sent a packet, what is the minimum amount of time needed for the protocol to infer that the packet was lost?

Answer: Assuming that the RTT remains constant, $\boldsymbol{t}_{\mathbf{min}} = \mathbf{100\ ms}$.

1.2 If the protocol uses a window size of 6 packets (each of size 1000 bytes), what is the maximum achievable data throughput?

Answer: Assuming that we have no packet loss, and that transmission delay is negligible relative to the RTT:

$$\text{Throughput}_{\text{max}} = \frac{\text{Number of bits}}{\text{Time taken}} = \frac{6 \times 1000 \times 8}{100 \times 10^{-3}} = \mathbf{480000\ bits/s}$$

1.3 To fully use the transmission bandwidth, estimate the minimum window size (in bytes) needed.

Answer: For a window size of N packets, we have:

$$\text{Throughput} = \frac{N \times 1000 \times 8}{100 \times 10^{-3}} = 80000N\ \text{bits/s}$$

Hence, for throughput = bandwidth, the minimum window size would be:

$$N_{\min} = \frac{10^9}{80000} = 12500 \text{ packets}$$

Since each packet is 1000 bytes large, we would have our minimum window size to be:

$$N_{\min} = 12500 \times 1000 = \mathbf{12500000\ bytes}$$

2. Consider the three 16-bit words (shown in binary) below.

01101001 11110110
11100011 00011100
10101010 10101010

What is the Internet checksum value for these three 16-bit words?

Answer: The sum of the three 16-bit words is: 1 11110111 10111100. By wrapping around the overflow, we get 11110111 10111101. Taking the 1s' complement of the wraparound sum, we would then get the Internet checksum value: **00001000 01000010**.

3. (textbook chapter 3, problem P44): Consider sending a large file from a host to another over a TCP connection that has no loss.

3.1 Suppose TCP uses AIMD for its congestion control without slow start. Assuming cwnd increases by 1 MSS every time a batch of ACKs is received and assuming approximately constant round-trip times, how long does it take for cwnd increase from 6 MSS to 12 MSS (assuming no loss events)?

Answer: Since we are assuming that there are no loss events that might decrease the cwnd, it would take **6 RTTs** to increase the cwnd from 6 MSS to 12 MSS. This is because it takes 1 RTT to increase the cwnd by 1 additional MSS, since cwnd would increase by 1 MSS every time a batch of ACKs is received, which would ideally return and be received after each RTT (instead of 1 MSS per ACK received adopted by the slow start mechanism).

3.2 Again, assume in the first RTT 6 MSS was sent, what is the average throughout (in terms of MSS and RTT) for this connection up through time = 6 RTT?

Answer: Total number of segments sent over the first 6 RTTs is:

$$6+7+8+9+10+11=51\text{ MSS}$$

Hence, the average throughput would be:

$$\frac{51\text{ MSS}}{6\text{ RTT}}=\mathbf{8.5\ MSS/RTT}$$

4. (textbook Chapter 3, problem 45 and 53) Recall the macroscopic description of TCP throughput. In the period of time from when the connection's rate varies from W/(2 · RTT) to W/RTT, only one packet is lost (at the very end of the period).

4.1 Show that the loss rate (fraction of packets lost) is equal to

$$L=\text{loss rate}=\frac{1}{\frac{3}{8}W^2+\frac{3}{4}W}$$

Answer: We define the loss rate, L, as the ratio of the number of packets lost over the number of packets sent. We also know that in a cycle, 1 packet is lost. The number of packets sent in a cycle is:

$$\begin{aligned}
\frac{W}{2}+\left(\frac{W}{2}+1\right)+\cdots+W &= \sum_{n=0}^{W/2}\left(\frac{W}{2}+n\right)\\
&= \left(\frac{W}{2}+1\right)\left(\frac{W}{2}\right)+\sum_{n=0}^{W/2} n\\
&= \left(\frac{W}{2}+1\right)\left(\frac{W}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{W}{2}\right)\left(\frac{W}{2}+1\right)\\
&= \frac{W^2}{4}+\frac{W}{2}+\frac{W^2}{8}+\frac{W}{4}\\
&= \frac{3}{8}W^2+\frac{3}{4}W
\end{aligned}$$

Hence, we have our loss rate to be:

$$\therefore \boldsymbol{L = \frac{1}{\frac{3}{8}W^2 + \frac{3}{4}W}} \ \textbf{(shown)}$$

4.2 Use the result above to show that if a connection has loss rate L, then its average rate is approximately given by

$$\approx \frac{1.22 \cdot MSS}{RTT\sqrt{L}}$$

Answer: For very large values of W, we can assume that:

$$\frac{3}{8}W^2 \gg \frac{3}{4}W$$

Because of this, we can have:

$$L \approx \frac{1}{\frac{3}{8}W^2} = \frac{8}{3W^2}$$

Rearranging the above equation, we get:

$$W \approx \sqrt{\frac{8}{3L}}$$

From the textbook, we have the equation for the average throughput rate:

$$\text{Throughput} = \frac{0.75 \times W}{RTT}$$

Hence, substituting the equation that we have obtained previously into the aforementioned textbook equation, we get:

$$\text{Throughput} \approx \frac{3}{4} \times \sqrt{\frac{8}{3L}} \times \frac{1}{\text{RTT}}$$

$$= \boldsymbol{\frac{\sqrt{6} \cdot MSS}{2 \cdot RTT \cdot \sqrt{L}}}$$

$$\approx \boldsymbol{\frac{1.22 \cdot MSS}{RTT \cdot \sqrt{L}}} \ \textbf{(shown)}$$

4.3 Let's assume 1500-byte packets and a 100 ms round-trip time. If TCP needed to support a 1Gbps connection, what would the tolerable loss rate be? How about 100Gbps?

Answer: Using the approximate equation obtained in 4.2, for the case of a 1Gbps connection, we can get:

$$1 \times 10^9 \approx \frac{1.22 \times 1500 \times 8}{100 \times 10^{-3} \times \sqrt{L}}$$

Rearranging, we can get a value of:

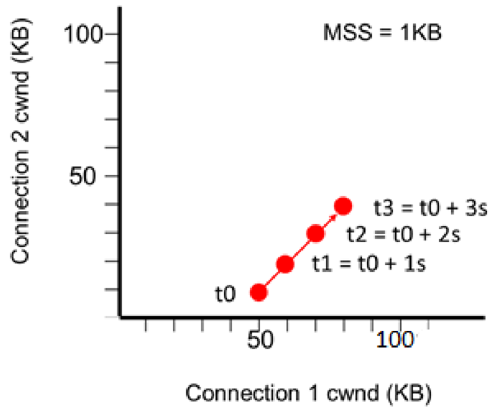
$$\therefore \boldsymbol{L \approx 2.14 \times 10^{-8}}$$

For the case of a 100Gbps connection, we have:

$$1 \times 10^{11} \approx \frac{1.22 \times 1500 \times 8}{100 \times 10^{-3} \times \sqrt{L}}$$

$$\therefore \boldsymbol{L \approx 2.14 \times 10^{-12}}$$

5. (2020 midterm exam question) Consider two TCP Reno connections that share one link. The figure below shows the evolution of the size of their respective congestion window (cwnd) over time. As shown, at time t0, connection 1's cwnd = 50KB and connection 2's cwnd=10KB. At time t1=t0+1s, connection 1's cwnd = 60KB and connection 2's cwnd=20KB. At time t2=t0+2s, connection 1's cwnd = 70KB and connection 2's cwnd=30KB. At time t3=t0+3s, connection 1's cwnd = 80KB and connection 2's cwnd=40KB. Assume the maximum segment size (MSS) for both connections is 1KB and both connections have constant round-trip time (RTT). We further assume that when the sum of the cwnd of the two connections reaches 120KB, both connections experience a packet loss event as indicated by triple duplicate ACKs. We also assume these are the only moments that the two connections experience packet losses.



5.1 From time t0 to t3, the two connections are in which state of the TCP congestion control? After the packet loss event at t3, what will be the cwnd size of connection 1 and connection 2 respectively?

Answer: Both connections are undergoing the AIMD mechanism (additive increase, in particular), and hence, the two connections are in the "**congestion avoidance**" state.

After the packet loss event at t3, the cwnd sizes of both connections are cut by half. Hence, connection 1's cwnd = **40KB** and connection 2's cwnd = **20KB** (assuming that we ignore the transient fast recovery state). After taking into account the fast recovery state, connection 1's cwnd = **43KB** and connection 2's cwnd = **23KB.**

5.2 What is the RTT for the two connections respectively? What is the respective average throughput of these two connections from t0 to t3?

Answer: The RTT for both connections is $\frac{1}{10} = \mathbf{0.1\ seconds}$.

Since the increase is linear from t0 to t3 (i.e., at a constant rate), we can simply get the average by averaging the start and end points.

For connection 1, the average throughput would be:

$$\text{Throughput} = \frac{\left(\frac{50+79}{2}\right)}{0.1} \times \frac{(8 \times 10^3)}{10^6} = \mathbf{5.16\ Mb/s}$$

For connection 2, the average throughput would be:

$$\text{Throughput} = \frac{\left(\frac{10+39}{2}\right)}{0.1} \times \frac{(8 \times 10^3)}{10^6} = \mathbf{1.96\ Mb/s}$$

5.3 Assume the two connections run for a long time. What will these two connections' respective average throughput converge to?

Answer: We will use this equation to calculate the average throughput rate:

$$\text{Throughput} = \frac{0.75 \times W}{RTT}$$

As $t \rightarrow \infty$, the cwnd sizes when loss occurs (i.e., when multiplicative decrease is executed) for both connections tend to 60 KB. This is because when we cut down the cwnd sizes by half, the absolute difference between the magnitudes of the two cwnd sizes decreases. Hence, using the aforementioned equation, we get the average throughput for both connections as:

$$\text{Throughput} = \frac{0.75 \times 60}{0.1} \times \frac{(8 \times 10^3)}{10^6} = \mathbf{3.6\ Mb/s}$$

5.4 Assume now connection 1's RTT reduces by 50% and connection 2's RTT remains unchanged. After a long time, what will these two connections' respective average throughput converge to?

Answer: As $t \rightarrow \infty$, the "maximum" cwnd size when loss occurs for connection 1 converges towards 80 KB, while the "maximum" cwnd size when loss occurs for connection 2 converges towards 40 KB.

Therefore, for connection 1, we have the average throughput:

$$\text{Throughput} = \frac{0.75 \times 80}{0.05} \times \frac{(8 \times 10^3)}{10^6} = \mathbf{9.6\ Mb/s}$$

Meanwhile, for connection 2, we have the average throughput:

$$\text{Throughput} = \frac{0.75 \times 40}{0.1} \times \frac{(8 \times 10^3)}{10^6} = \mathbf{2.4\ Mb/s}$$