e-ISSN: Journal of Balkan Science and Technology 2022: 1(1): 28-33

Journal of Balkan Science and Technology
journal homepage: www.jbst.nku.edu.tr/

Approximation Functions for the Analysis of Nonlinear Elements of an RC Circuit

Gülizar ALİSOY[1], Numan Derda ÖZKUTLU[2], Hafiz ALİSOY[3*]

[1]*Tekirdağ Namık Kemal University, Faculty of Arts and Sciences, Department of Mathematics, Tekirdağ, 59030, Turkey*
[2]*Ordu University Vocational School of Technical Sciences, Department of Electronics and Automation, 52200, Ordu, Turkey*
[3]*Tekirdağ Namık Kemal University, Faculty of Engineering, Department of Electronics, and Telecommunication, 59030, Tekirdağ, Turkey*

Research Article

Keywords:
Least squares method
Nonlinear circuit elements
Current-voltage Characteristics

Received: 08.04.2022
Accepted: 17.05.2022
Published:31.05.2022

DOI: 10.55848/jbst.2022

ABSTRACT

The present study aims to determine the current-voltage characteristics (CVCs) of nonlinear elements of an RC circuit as well as its functions which would allow one to approximate the experimental data. For this purpose, the well-known least squares method (LSM) has been used. In order to approximate the CVCs characteristics of elements of nonlinear circuits, which are often used in practice, the following functions have been used as approximating functions: exponential function, hyperbolic sine, and polynomial of the fifth degree. Algorithms and computer programs have been also developed to determine these functions.

Algorithms and computer programs for solving problems proposed in this paper can be used for computer analysis of nonlinear radio circuits based on active RCs.

1. Introduction

Nowadays, the production of information communication systems, computers and other electronic equipment includes highly complex electronic circuits. For this reason, the requirements for the quality of the designed electronic devices are also increasing. Therefore, each developed device must provide the required accuracy [1-9].

The examination of the RC based linear and nonlinear electronic circuits covers current and priority research areas. In practice, a problem often arises with the non-linearity of the current-voltage characteristics (CVCs) of electronic circuit elements. An electronic circuit which exhibits a non-linear response function to an action is called a “non-linear circuit” [10-12].

The current-voltage characteristics of the circuit elements are usually determined as a result of certain experiments. There are various graphs of current-voltage characteristics of various elements of electronic circuits in the literature [10, 13]. For example, for nonlinear resistive elements, the characteristics determine the relationship between instantaneous values of voltages and currents, and the nonlinear capacitance element is characterized by static capacitance and differential capacitance.

Graphical methods are widely used in practical calculations [10]. However, theoretical research and practical application of electronic circuits require an analytical expression of the current-voltage properties of nonlinear elements [12]. Therefore, determining the analytical formulas describing the relationship between current and voltage in various circuits emerges as an important problem. In other words, the approximation of experimental data with analytical functions is important and stands among the most stringent problems.

The main purpose of this study is to determine the expressions for the mathematical relationship between voltage and current in non-linear elements of electrical circuits by analyzing the existing relationships which would determine the current-voltage characteristics of various components used in electronic circuits.

2. Materials and Methods

As mentioned above, in order to develop a mathematical model of the transient phenomena that occur in an electronic circuit, the mathematical expressions of the current-voltage characteristics equations must be determined.

In the general case, an electronic circuit contains not only elements that can be considered linear with a sufficient degree of accuracy, but also elements that are clearly non-linear. The parameters of the nonlinear elements R, L, C differ in that they are a function of the voltage applied to them. Therefore, the equations of such circuits based on Kirchhoff's laws will not be linear. These functional relationships between the parameters of the circuit are determined from the experimental results. However, for computer analysis of electronic circuits with nonlinear elements, an analytical relationship between voltage and current is recommended. As a rule, approximation of experimental data is used for this purpose. Various types of approximation functions have been proposed in the literature dealing with the nonlinearity of the current-voltage characteristic of electronic circuit elements. It is necessary to

* Corresponding author: Tekirdağ Namık Kemal University, Faculty of Arts and Sciences, Department of Physics, Kampüs Cd. No:1, 59030 Tekirdağ/TURKEY
E-mail: halisoy@nku.edu.tr

give some information about these functions and their features [14-16].

In this part of the article, various variants of approximation functions are discussed, which are used to describe the functional relationship between current and voltage. In addition, the unknown parameters of these functions were determined.

As is known, experimental or statistical data are essential information for determining analytical relationships between the variables (indicators) studied. A function that describes such a relationship between these variables, which is in good agreement with the experimental data, needs to be determined. Before proceeding to the analysis of these functions, the first step is to define the dimensionless quantities. The reason for this is that the use of dimensionless quantities provides certain conveniences in calculating and modeling of the process under consideration. In order to switch to dimensionless variables, the characteristic sizes of the given problem must be selected. Then, the ratios of the quantities describing the process under consideration to the corresponding characteristic quantities are taken into account. This allows us to consider dimensionless quantities in subsequent calculations and then return to dimensional quantities.

In the given problem, let V_0 be a voltage characteristic for the given component. Then V_0/R, can be the characteristic value of the current for a fixed value of resistor R. In this case, the transition to dimensionless quantities is carried out using the following formulas.

$$x = \frac{V}{V_0} \quad , \quad y = \frac{iR}{V_0}$$

Here, x represents dimensionless voltage and y represents dimensionless current. Considering these formulas, the desired approximation function can be written as follows.

$$i = \frac{V_0}{R} f(x) \quad y = f(x)$$

Here f(x) is a function expressing the relationship between dimensionless current and dimensionless voltage.

From the analysis of available methods for approximation of experimental data, it is known that the following functions are often used in practice [16,17].

1. Linear function defined in the following form, where a, b are unknown parameters:

$$y = ax + b \tag{1}$$

The graph of this function represents a straight line.

2. A quadratic polynomial function with a, b, and c as unknown parameters:

$$y = ax^2 + bx + c \tag{2}$$

3. m-order polynomial function with $a_n, \dots, a_0$ as unknown parameters:

$$y = a_n x^m + a_{n-1} x^{m-1} + a_{n-2} x^{m-2} + \dots + a_2 x^2 + a_1 x + a_0 \tag{3}$$

4. Exponential function with a being an unknown parameter:

$$y = a\left[1 - \exp\left(-\frac{x}{a}\right)\right] \tag{4}$$

5. Hyperbolic function with a being an unknown parameter:

$$y = a\, sinh\, \frac{x}{a} \tag{5}$$

Linear and quadratic functions for the approximation of statistical or experimental data have long been successfully used in many fields of science. On the other hand, it is clearly known that the Eq. (1) is used to determine the functional relationship between current and voltage in linear electronic circuits, and this approximation expression cannot be used for circuits containing nonlinear elements.

For electronic circuits, which usually contain nonlinear elements, the functional relationship between current and voltage is defined by a second-order polynomial (see Eq. (2)). In this case, a system of linear algebraic equations, usually consisting of three equations, is solved to determine the unknown parameters a, b, c. Let us remember that in such problems, the use of a higher order polynomial allows us to achieve higher computational accuracy. On the other hand, the use of polynomials of order m requires solving a system of equations with a large number of unknowns, which is associated with certain difficulties.

The values of unknown coefficients contained in the mathematical expressions of the aforementioned approximation functions may be different for different components of the electronic circuit. Experimental data and well-known least squares method are used to determine the numerical values of these unknown parameters.

One of the reasons for choosing an exponential function or a hyperbolic sine function as the approximating is the fulfilment of a linear dependence for relatively small voltage values, i.e. Ohm's law. Indeed, the expansions of these functions to the Taylor series adjacent to x = 0 can be represented in the form of the following power series [18]

$$a\left[1 - \exp\left(-\frac{x}{a}\right)\right] = x - \frac{x^2}{2!\,a} + \frac{x^3}{3!\,a^2} + \cdots + (-1)^k \frac{x^k}{k!\,a^{k-1}} + \cdots \tag{6}$$

$$a\, sh\left(\frac{x}{a}\right) = x + \frac{x^3}{3!\,a^2} + \frac{x^5}{5!\,a^4} + \cdots + \frac{x^{2k-1}}{(2k-1)!\,a^{2k-1}} + \tag{7}$$

3. Results & Discussion

Finally, based on the experimentally measured current-voltage characteristics of the nonlinear element shown in Fig. 1, we find an approximation function that determines the analytical relationship between current and voltage for this element.

In this case, the solution of the problem consists of the following two steps.

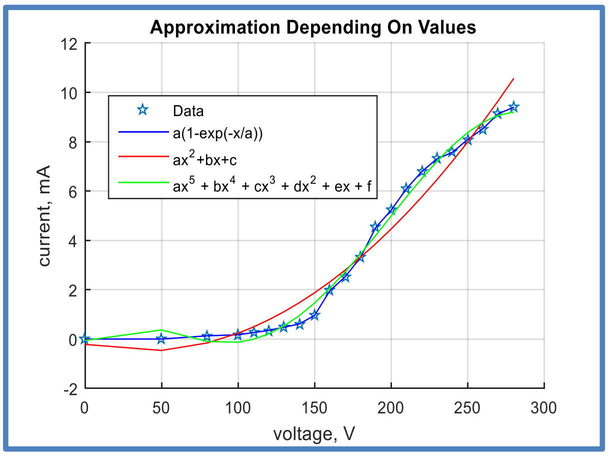

Fig. 1 - Current-voltage characteristic of nonlinear resistive material based on approximating functions in comparison with experimental data (i) $y(x) = a(1 - exp(-x/a))$; $a = 2.1177e-05$; ii) $y(x) = a_1x^2 + b_1x + c_1$; $a_1 = 0.00018852$; $b_1 = -0.014326$; $c_1 = -0.21854$; iii) $y(x) = a_2x^5 + b_2x^4 + c_2x^3 + d_2x^2 + e_2x + f_2$; $a_2 = 3.3701e-11$; $b_2 = -3.7966e-08$; $c_2 = 1.3294e-05$; $d_2 = -0.0015767$; $e_2 = 0.058586$; $f_2 = -0.05616$).

i) Selection of the general form of the approximation function, which would represent the experimentally determined current-voltage characteristic and contain unknown parameter(s)

ii) Determining the numerical values of the unknown parameters contained in the selected approximation function expression.

In general, the least squares method (LSM) is commonly used for mathematical processing of experimental results [15-17]. The essence of this method is as follows. First, the general form of the approximation function is chosen, whose values may depend on some parameters that are still considered unknown. Then, the values of these unknown parameters are determined on the condition that the graph of the approximation function passes very close to the points corresponding to the experimental data.

Suppose the approximation function is given as $y = f(x, a, b, c, \dots)$ with a, b, c being an unknown set of parameters.

$$U(a, b, c, \dots) = \sum_{k=1}^{n} [y_k - f(x_k, a, b, c, \dots)]^2 \rightarrow min \tag{8}$$

A distinctive feature of this method is that the signs of the numerical data determining the differences between the values of the approximation function and the experimental data do not affect the sum of these differences.

The requirement condition for the unconditional extremum of the expression defined by Eq. (8) is determined by setting the first-order partial derivatives of this function in terms of the relevant parameters to zero [17]

$$\frac{dU}{da} = 0 \ ; \ \frac{dU}{db} = 0 \ ; \ \frac{dU}{dc} = 0 \ ; \ \dots \tag{9}$$

In the next step, starting from Eq. (9), a system of algebraic or transcendent equations is obtained that allows the determination of unknown parameters a, b, c,… . In general, these equations have no analytical solutions. To solve such equations, it is necessary to use existing numerical methods. Consequently, two characteristic cases will be discussed here.

1.Approximation of test results with a polynomial of order m;

If m-order polynomials are used as approximation functions, the solution to the problem is reduced to the solution of the m+1 system of linear algebraic equations. For a large enough value of m, a system of equations with many unknowns must be solved.

Let's choose the m-order polynomial given in the following form as the approximation function. Then the equation will be $y = b_1x^m + b_2x^{m-1} + \dots + b_mx + b_{m+1}$.

If we use the least squares method (LSM) to determine the unknown coefficients of the polynomial, then;

$$U = \sum_{k=1}^{n} [y_k - b_1x_k^m + b_2x_k^{m-1} + \dots + b_mx_k + b_{m+1}]^2 \tag{10}$$

Note that the function $U = U(b_1, b_2, b_3, \dots, b_m, b_{m+1})$ contains m+1 parameters. As stated above, the requirement condition for the minimum of the Eq. (10) will be expressed as follows.

$$\frac{dU}{db_i} = -2\sum_{k=1}^{n}\left[y_k - \sum_{i=1}^{m+1} b_ix_k^{m+1-i}\right]x_k^{m+1-i} = 0 \tag{11}$$

In this formula, $i = 1,2, \dots, m+1$. Therefore, the number of equations is equal to m+1. Thus, we obtain a system of linear algebraic equations with m+1 unknown parameters $b_1, b_2, \dots, b_{m+1}$

$$\sum_{j=1}^{m+1} a_{ij}b_j = a_{i\,m+2} \quad , i = 1,2, \dots, m+1 \tag{12}$$

The coefficients and free terms of this algebraic system of equations are determined using the formulas below

$$a_{ij} = \sum_{k=1}^{n} x_k^{2m+2-i-j} \ , a_{i\,m+2} = \sum_{k=1}^{n} y_kx_k^{m+1-i} \ , a_{m+1\,m+i} = n \tag{13}$$

2.Approximation of the experimental results with the Transcendent function

If the transcendental functions are chosen as hyperbolic sine or exponential functions, transcendental equation must be solved. In this part of the article, the iteration method is used to solve the transcendental equations. The choice of the iteration method depends, first of all, on the following conditions: first, the iteration method allows the solution of an equation to be determined with a certain accuracy, and in most cases it is a stable method; second, the algorithm required for this solution method is easy to program.

In this study, the iteration method was used to solve such transcendent equations in subsequent calculations.

The following two special cases are discussed here.

i) The case of considering the function $y = a\left(1 - \exp\left(-\frac{x}{a}\right)\right)$ for the approximation of experimental data.

In this case, according to the least squares method (LSM), we obtain the following expression.

$$U(a) = \sum_{k=1}^{N} \left[y_k - a\left(1 - \exp\left(-\frac{x_k}{a}\right)\right)\right]^2 \rightarrow min \tag{14}$$

x_k and y_k represent the experimental data and n is the number of experimental points. As you can see, the U(a) function only depends on one argument. The requirement condition for the minimum of this function U(a) is determined by setting its first derivative with respect to a equal to zero.

$$\frac{dU(a)}{da} = -2\sum_{k=1}^{N} \left[y_k - a\left(1 - \exp\left(-\frac{x_k}{a}\right)\right)\right]\left[1 - \frac{\exp -x_i}{a} - \frac{x_i}{a}\exp\left(\frac{-x_i}{a}\right)\right] = 0 \tag{15}$$

After some transformations, we get the following equation.

$$\sum_{k=1}^{N} y_k\left[1 - \left(1 + \frac{-x_k}{a}\right)\exp\left(-\frac{x_k}{a}\right)\right] - a\sum_{k=1}^{N}\left[1 - \exp\left(-\frac{x_k}{a}\right)\right]\left[1 - \frac{x_k}{a}\exp\left(-\frac{x_k}{a}\right)\right] = 0 \tag{16}$$

Let us accept the following markings in this expression.

$$P_1 = \sum_{k=1}^{N} y_k\left[1 - \exp\left(-\frac{x_k}{a}\right)\right]$$
$$P_2 = \sum_{k=1}^{N}\left[1 - \exp\left(-\frac{x_k}{a}\right)\right]^2$$
$$P_3 = \sum_{k=1}^{N} x_k y_k \exp\left(-\frac{x_k}{a}\right) \tag{17}$$
$$P_4 = \sum_{k=1}^{N} x_k \exp\left(-\frac{x_k}{a}\right)\left[1 - \exp\left(-\frac{x_k}{a}\right)\right]$$

Taking these notations into account, we obtain the following transcendent equation for the unknown parameter a.

$$P_1 + P_4 - nP_2 - \frac{1}{a}P_3 = 0 \tag{18}$$

As can be seen, this equation is a transcendent equation with respect to the unknown a- parameter, and obviously such an equation cannot be solved analytically. The following recurrent expression is proposed to develop an algorithm needed to solve Eq. (18).

$$a_i = \frac{\left[P_1(a_{i-1}) + P_4(a_{i-1}) - \frac{P_3(a_{i-1})}{a_{i-1}}\right]}{P_4(a_{i-1})} \tag{19}$$

where a_{i-1} is the value of parameter a after $(i-1)$ iteration. The solution algorithm consists of the following steps.

1. x_k, y_k ;k=1,2,3,…,N definition
2. Selecting the approximate value of the a-parameter
3. Assigning the calculated value of the required parameter a as a previous approximation.
4. Calculation of P_1 , P_2 , P_3 , P_4
5. Determining the new value of the a_i parameter according to the Eq. (18)
6. Checking condition $|a_{k+1} - a_k| \leq \varepsilon$
7. Determining the final value of the a_i parameter

ii) Case of considering the function $y = a\sin\left(\frac{x}{a}\right)$ for the approximation of experimental data.

In this case, according to the least squares method (LSM), we obtain the following expression.

$$U(a) = \sum_{k=1}^{n}\left[y_k - a\, sh\left(\frac{x_k}{a}\right)\right]^2 \rightarrow min \tag{20}$$

As in the previous case Eq. (20) only the parameter a is unknown. In this case, the requirement condition for the minimum of the function U(a) is determined by setting its first derivative with respect to a equal to zero.

$$\frac{dU}{da} = -2\sum_{k=1}^{n}\left[y_k - a\, sinh\left(\frac{x_k}{a}\right)\right]\left[\frac{x_k}{a}\, cosh\left(\frac{x_k}{a}\right) - sinh\left(\frac{x_k}{a}\right)\right] = 0 \tag{21}$$

After simple transformations, the following equation is obtained.

$$\sum_{k=1}^{n} y_k\left[sinh\left(\frac{x_k}{a}\right) - \frac{x_k}{a}cosh\left(\frac{x_k}{a}\right)\right]\left[\frac{x_k}{a}cosh\left(\frac{x_k}{a}\right) - sinh\left(\frac{x_k}{a}\right)\right] = 0 \tag{22}$$

For convenience, the following definitions have been made.

$$S_1 = \sum_{k=1}^{n} y_k\, sh\left(\frac{x_k}{a}\right) \;;\; S_2 = \sum_{k=1}^{n} sh^2\left(\frac{x_k}{a}\right) \;;$$
$$S_3 = \sum_{k=1}^{n} x_k y_k\, ch\left(\frac{x_k}{a}\right) \;;\; S_4 = \sum_{k=1}^{n} x_k\, sh\left(\frac{x_k}{a}\right) ch\left(\frac{x_k}{a}\right) \tag{23}$$

Taking these notations into account, the following formula can be obtained for the unknown parameter a from Eq. (22).

$$S_1 + S_4 - aS_2 - \frac{1}{a}S_3 = 0 \tag{24}$$

The following recurrent expression is proposed to develop an algorithm needed to numerically solve the transcendental Eq. (22).

$$a_i = [S_1(a_{i-1}) + S_4(a_{i-1}) - S_3(a_{i-1})/a_{i-1}\ ]/S_2(a_{i-1}) \tag{25}$$

The use of the exponential function or the hyperbolic sine function as an approximation function to describe the current-voltage properties of the circuit elements requires the solution of the transcendent equations defined by the (Eqs. (18) and (24), respectively. As one can see, these equations are of the same type. Therefore, the algorithm for solving the recurrent expression defined by Eq. (25) will be the same as for solving the previous problem by using Eq. (19). Only function S_1, S_2, S_3, S_4 will be used instead of P_1, P_2, P_3, P_4 functions.

As an example, let us consider an approximation of the current-voltage characteristic of a nonlinear resistive material based on polypropylene and single-crystal silicon [19]. To select an approximating function $I = f(U)$, we study the graph built on points with coordinates (U_k, I_k). The smooth curve of the graph should pass as close as possible to the experimental points, and there should be approximately the same number of points on both sides of the curve. If some points lie far from the curve, then this would often indicate a measurement error. In this case, a break in the curve near a single outlier point is not allowed. Therefore, before carrying out the approximation, it is necessary to exclude errors from the experimental data.

In order to determine the unknown coefficients that are contained in the selected formulas of the approximating functions, the well-known method of least squares is used. In this case, the system of Eq. (12) is solved on the basis of previously measured experimental data. Note that, from the minimum condition for the function U (a) (see Eqs. (14) or (20)), a transcendental equation is obtained with respect to the unknown parameter a which can be solved by an iterative method.

According to this algorithm, a computer program was developed in the Matlab for solving a system of algebraic equations.

4. Conclusion

In order to determine the current-voltage characteristics (CVCs) of the non-linear elements of the RC - circuit, functions that allow performing the approximation of the experimental data are determined. For this purpose, the well-known least squares method (LSM) was used.

To approximate the CVCs characteristics of elements of nonlinear circuits, which are often used in practice, the following functions were used as approximating functions: exponential function, hyperbolic sine, polynomial of the fifth degree. Algorithms and computer programs have also been developed to determine these functions.

The use of the transcendental functions as approximating functions leads to the solution of a transcendental equation, for the solution of which it is advisable to use the iteration method. Using the hyperbolic sine does not always give positive results. One of the difficulties in using transcendental functions as approximating circuit elements for determining the current-voltage characteristics is the choice of the initial approximation of the value of the unknown parameter included in the approximation function.

For any kind of non-linear element, a polynomial of the highest degree can be successfully used, in particular, it can be limited to a polynomial of the fifth degree, which describes the experimental data quite well for various types of non-linear circuit elements.

The results presented in this study can be used in other areas of science and technology where non-linear phenomena take place.

Decleration

Author Contribution: Conceive-G.A.,N.D.,H.A.; Design-H.A.,N.D; Supervision-HA; Experimental Performance, Data Collection and/or Processing-H.A; Analysis and/or Interpretation-G.A., H.A; Literature Review-N.D.,H.A; Writer-N.D; Critical Reviews –G.A., N.D

Conflict of Interest : The authors have declared no conflicts of interest.

Orcid ID

Gülizar Alisoy https://orcid.org/0000-0003-2114-6669
Numan Derda Özkutlu https://orcid.org/0000-0002-9540-3165
Hafiz Alisoy https://orcid.org/0000-0003-4374-9559

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