Measures of Position

Section 3-3

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MAT 110

Lesson #7

Objectives

- Find the z score for a value in a data set
- Find the percentile rank for a value in a data set
- Find data values corresponding to given percentile ranks
- Find Q_1 and Q_3 for a data set
- Identify outliers in data sets

Measures of Position

Measures of position locate the relative position of a data value.

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We will be looking at two measures of position in this lesson:

- Standard (z) score
- Percentile rank

Standard Score

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The standard score (also known as the z score) tells how many standard deviations a data value is above or below the mean.

Formulas:

$z = \dfrac{X - \overline{X}}{s}$ for samples

$z = \dfrac{X - \mu}{\sigma}$ for populations

Standard Score

A student scored 85 on an English test while the mean score of all the students was 76 and the standard deviation was 4. She also scored 42 on a French test where the class mean was 36 and the standard deviation was 3. Compare the relative positions on the two tests.

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$$\begin{aligned} z &= \frac{X - \overline{X}}{s} \\ &= \frac{85 - 76}{4} \end{aligned}$$

$$\begin{aligned} z &= \frac{X - \overline{X}}{s} \\ &= \frac{42 - 36}{3} \end{aligned}$$

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$$\begin{aligned} z &= \frac{X - \overline{X}}{s} \\ &= \frac{85 - 76}{4} \\ &= 2.25 \end{aligned}$$

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Based on the results, our student had a higher relative position on the English test.

Standard Score

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$$z = \frac{X - \overline{X}}{s} = \frac{13 - 29.4}{8.6}$$

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$$\begin{aligned} z &= \frac{X - \overline{X}}{s} \\ &= \frac{13 - 29.4}{8.6} \\ &= \frac{-16.4}{8.6} \end{aligned}$$

Standard Score

If the average number of vacation days for a selection of various countries has a mean of 29.4 days and a standard deviation of 8.6 days, find the z score for the United States, which has an average number of 13 days per year.

$$
\begin{aligned}
z &= \frac{X - \overline{X}}{s} \\
&= \frac{13 - 29.4}{8.6} \\
&= \frac{-16.4}{8.6} \\
&= -1.91
\end{aligned}
$$

Standard Score

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\end{aligned}
$$

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-16320 &= X - 54166
\end{aligned}
$$

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z &= \frac{X - \mu}{\sigma} \\
-1.6 &= \frac{X - 54166}{10200} \\
-16320 &= X - 54166 \\
X &= 37846
\end{aligned}
$$

Standard Score

The average teacher's salary in a particular state is \$54,166. If the standard deviation \$10,200, find the salary associated with a z score of -1.6.

$$\begin{aligned} z &= \frac{X-\mu}{\sigma} \\ -1.6 &= \frac{X-54166}{10200} \\ -16320 &= X - 54166 \\ X &= 37846 \end{aligned}$$

So a z score of -1.6 corresponds to a salary of \$37,846.

Percentile Rank

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For example, say you score in the 80th percentile on a physical fitness test.

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Notation: $P_1, P_2, \ldots, P_{99}$

Percentile Rank

Often, graphs and tables showing percentiles for various measures have already been created, and we use them to check the percentile rank of a given measurement.

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Important Note: A *percentile* and a *percentage* are not the same thing!

Percentile Rank

Often, graphs and tables showing percentiles for various measures have already been created, and we use them to check the percentile rank of a given measurement.

Examples: standardized test score, height/weight of children at a given age, etc. (see pp. 150-151)

Important Note: A *percentile* and a *percentage* are not the same thing!

Example: Say you get a score of 72/100 on a math test. This means your *percentage* score was 72%. This does *not* mean that you scored in the 72nd percentile. Perhaps the median score was 85; in this case, you actually scored below the 50th *percentile*.

Percentile Ranks

To calculate the percentile rank for a given value X in a data set, use the formula

$$\text{Percentile} = \frac{(\text{number of values below } X) + 0.5}{\text{total number of values}} \cdot 100$$

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Note: This means we must order the data values from lowest to highest before calculating the percentile.

Note: We round up to the nearest whole number when calculating percentages. Example: 28.21 is considered the 29th percentile.

Percentile Ranks

The data show the population (in thousands) for a recent year of a sample of cities in South Carolina.

29	26	15	13	17	58
14	25	37	19	40	67
23	10	97	12	129	
27	20	18	120	35	
66	21	11	43	22	

Find the percentile rank for 58.

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Find the percentile rank for 58.

First, sort from lowest to highest:

$10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 29, 35, 37, 40, 43, 58, 66, 67, 97, 120, 129$

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Find the percentile rank for 58.

First, sort from lowest to highest:
$10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 29, 35, 37, 40, 43, 58, 66, 67, 97, 120, 129$

There are 21 values less than 58, so the percentile rank is $\frac{21.5}{27} \cdot 100 = 79.62$. We round this up, and say that 58 is in the 80th percentile.

Percentile Rank

Refer to the previous data set. Find the percentile rank of 21.

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$120, 129$

Percentile Rank

Refer to the previous data set. Find the percentile rank of 21.

$10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 29, 35, 37, 40, 43, 58, 66, 67, 97, 120, 129$

There are 10 values less than 21, so the percentile rank is $\frac{10.5}{27} \cdot 100 = 38.89$. We round this up, and say that 21 is in the 39th percentile.

Percentile Rank

Refer to the previous data set. Find the percentile rank of 21.

$10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 29, 35, 37, 40, 43, 58, 66, 67, 97,$
$120, 129$

There are 10 values less than 21, so the percentile rank is $\frac{10.5}{27} \cdot 100 = 38.89$. We round this up, and say that 21 is in the 39th percentile.

Percentile Rank

Now that we know how to calculate percentile ranks, we move to finding data values corresponding to a given percentile rank.

Procedure Table

Finding a Data Value Corresponding to a Given Percentile

Step 1 Arrange the data in order from lowest to highest.

Step 2 Substitute into the formula

$$c = \frac{n \cdot p}{100}$$

where n = total number of values
p = percentile

Step 3A If c is not a whole number, round up to the next whole number. Starting at the lowest value, count over to the number that corresponds to the rounded-up value.

Step 3B If c is a whole number, use the value halfway between the cth and $(c + 1)$st values when counting up from the lowest value.

Refer to the data set below. Find the value corresponding to the 75th percentile and the 90th percentile.

Percentile Rank

Refer to the data set below. Find the value corresponding to the 75th percentile and the 90th percentile.

$10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 29, 35, 37, 40, 43, 58, 66, 67, 97, 120, 129$

Percentile Rank

Refer to the data set below. Find the value corresponding to the 75th percentile and the 90th percentile.

$10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 29, 35, 37, 40, 43, 58, 66, 67, 97, 120, 129$

$$c = \frac{27 \times 75}{100}$$

$$c = \frac{27 \times 90}{100}$$

Percentile Rank

Refer to the data set below. Find the value corresponding to the 75th percentile and the 90th percentile.

$10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 29, 35, 37, 40, 43, 58, 66, 67, 97, 120, 129$

$$c = \frac{27 \times 75}{100}$$
$$c = 20.25 \uparrow 21$$

$$c = \frac{27 \times 90}{100}$$
$$c = 24.3 \uparrow 25$$

Percentile Rank

Refer to the data set below. Find the value corresponding to the 75th percentile and the 90th percentile.

$10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 29, 35, 37, 40, 43, 58, 66, 67, 97,$
$120, 129$

$c = \dfrac{27 \times 75}{100}$
$c = 20.25 \uparrow 21$

The 21st value is 43, so the 75th percentile corresponds to 43.

$c = \dfrac{27 \times 90}{100}$
$c = 24.3 \uparrow 25$

The 25th value is 97, so the 90th percentile corresponds to 97.

Percentile Rank - Q_1 and Q_3

The 25th percentile is known as the *first quartile*, and is denoted by Q_1.

Percentile Rank - Q_1 and Q_3

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The 25th percentile is known as the *first quartile*, and is denoted by Q_1.

The 75th percentile is known as the *third quartile*, and is denoted by Q_3.

Note: Graphing calculators will find these values for you (see last week's videos).

Percentile Rank - Q_1 and Q_3

The 25th percentile is known as the *first quartile*, and is denoted by Q_1.

The 75th percentile is known as the *third quartile*, and is denoted by Q_3.

Note: Graphing calculators will find these values for you (see last week's videos).

To find Q_1 and Q_3 by hand:

1) Arrange data from lowest to highest, and find the median as we did before.
2) Find the median of values below the median; this is Q_1
3) Find the median of values above the median; this is Q_3

Percentile Rank - Q_1 and Q_3

The 25th percentile is known as the *first quartile*, and is denoted by Q_1.

The 75th percentile is known as the *third quartile*, and is denoted by Q_3.

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The *interquartile range* is the difference between the third and first quartiles.
$IQR = Q_3 - Q_1$

Percentile Rank - Q_1 and Q_3

Find Q_1, Q_3, and the interquartile range for the data below.

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Percentile Rank - Q_1 and Q_3

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$10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 29, 35, 37, 40, 43, 58, 66, 67, 97, 120, 129$

First, find the median (25)

Percentile Rank - Q_1 and Q_3

Find Q_1, Q_3, and the interquartile range for the data below.

$10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 29, 35, 37, 40, 43, 58, 66, 67, 97, 120, 129$

First, find the median (25)

Q_1 is the median of all data values below 25 (17)

Percentile Rank - Q_1 and Q_3

Find Q_1, Q_3, and the interquartile range for the data below.

$10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 29, 35, 37, 40, 43, 58, 66, 67, 97, 120, 129$

First, find the median (25)

Q_1 is the median of all data values below 25 (17)

Q_3 is the median of all data values above 25 (43)

Percentile Rank - Q_1 and Q_3

Find Q_1, Q_3, and the interquartile range for the data below.

$10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 29, 35, 37, 40, 43, 58, 66, 67, 97, 120, 129$

First, find the median (25)

Q_1 is the median of all data values below 25 (17)

Q_3 is the median of all data values above 25 (43)

$IQR = 43 - 17 = 26$

Outliers

An *outlier* is an extremely high or an extremely low value when compared with the rest of the data.

Procedure Table

Procedure for Identifying Outliers

Step 1 Arrange the data in order from lowest to highest and find Q_1 and Q_3.

Step 2 Find the interquartile range: $\text{IQR} = Q_3 - Q_1$.

Step 3 Multiply the IQR by 1.5.

Step 4 Subtract the value obtained in step 3 from Q_1 and add the value obtained in step 3 to Q_3.

Step 5 Check the data set for any data value that is smaller than $Q_1 - 1.5(\text{IQR})$ or larger than $Q_3 + 1.5(\text{IQR})$.

Outliers

Check the following data set for outliers.

$5, 6, 12, 13, 15, 18, 22, 50$

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Check the following data set for outliers.

$5, 6, 12, 13, 15, 18, 22, 50$

First, find the median (14), Q_1 (9), and Q_3 (20)

Outliers

Check the following data set for outliers.

$5, 6, 12, 13, 15, 18, 22, 50$

First, find the median (14), Q_1 (9), and Q_3 (20)

So the interquartile range is $20 - 9 = 11$; $1.5 \times 11 = 16.5$

Outliers

Check the following data set for outliers.

$5, 6, 12, 13, 15, 18, 22, 50$

First, find the median (14), Q_1 (9), and Q_3 (20)

So the interquartile range is $20 - 9 = 11$; $1.5 \times 11 = 16.5$

Lower boundary: $9 - 16.5 = -7.5$; Upper boundary: $20 + 16.5 = 36.5$

Outliers

Check the following data set for outliers.

$5, 6, 12, 13, 15, 18, 22, 50$

First, find the median (14), Q_1 (9), and Q_3 (20)

So the interquartile range is $20 - 9 = 11$; $1.5 \times 11 = 16.5$

Lower boundary: $9 - 16.5 = -7.5$; Upper boundary: $20 + 16.5 = 36.5$

50 is larger than the upper boundary, so it is an outlier.

Outliers

Check the following data for outliers.

$19, 21, 25, 28, 29, 32, 34, 46$

Outliers

Check the following data for outliers.

$19, 21, 25, 28, 29, 32, 34, 46$

First, find the median (28.5), Q_1 (23), and Q_3 (33)

Outliers

Check the following data for outliers.

$19, 21, 25, 28, 29, 32, 34, 46$

First, find the median (28.5), Q_1 (23), and Q_3 (33)

So the IQR is $33 - 23 = 10$; $1.5 \times 10 = 15$

Outliers

Check the following data for outliers.

$19, 21, 25, 28, 29, 32, 34, 46$

First, find the median (28.5), Q_1 (23), and Q_3 (33)

So the IQR is $33 - 23 = 10$; $1.5 \times 10 = 15$

Lower boundary: $23 - 15 = 8$; Upper boundary: $33 + 15 = 48$

Outliers

Check the following data for outliers.

$19, 21, 25, 28, 29, 32, 34, 46$

First, find the median (28.5), Q_1 (23), and Q_3 (33)

So the IQR is $33 - 23 = 10$; $1.5 \times 10 = 15$

Lower boundary: $23 - 15 = 8$; Upper boundary: $33 + 15 = 48$

There are no outliers.

Next Steps

- Read 3-4
- Watch Video Lesson #8
- Begin preparing for Midterm 1 (Study Guide is available on Moodle)

Thanks for watching!