ELSEVIER Regional Science and Urban Economics 26 (1996) 77–104

regional SCIENCE & urban ECONOMICS

Simple diagnostic tests for spatial dependence

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Received 1 February 1995; revised version received 5 September 1995

Abstract

In this paper we propose simple diagnostic tests, based on ordinary least-squares (OLS) residuals, for spatial error autocorrelation in the presence of a spatially lagged dependent variable and for spatial lag dependence in the presence of spatial error autocorrelation, applying the modified Lagrange multiplier (LM) test developed by Bera and Yoon (*Econometric Theory*, 1993, 9, 649–658). Our new tests may be viewed as computationally simple and robust alternatives to some existing procedures in spatial econometrics. We provide empirical illustrations to demonstrate the usefulness of the proposed tests. The finite sample size and power performance of the tests are also investigated through a Monte Carlo study. The results indicate that the adjusted LM tests have good finite sample properties. In addition, they prove to be more suitable for the identification of the source of dependence (lag or error) than their unadjusted counterparts.

Keywords: Spatial autocorrelation; Specification tests; Lagrange multiplier tests; Local misspecification; Monte Carlo studies

JEL classification: C12; C21; R10

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SSDI 0166-0462(95)02111-6

1. Introduction

In spatial data analysis, model specification issues have recently become an integral part of spatial econometric modeling (see, for example, Anselin, 1988a,b, 1992a; Blommestein, 1983; Florax and Folmer, 1992; and Kelejian and Robinson, 1992). On the basis of the Lagrange multiplier (LM) principle, Anselin (1988c) proposed several diagnostic tests for spatial econometric models. In particular, the focus was on detecting model misspecification due to spatial dependence (in the form of an omitted spatially lagged dependent variable and spatial error autocorrelation) as well as spatial heterogeneity (in the form of heteroskedasticity). In deriving a joint test for spatial dependence and spatial heterogeneity, Anselin (1988c) observed that the inverse of the information matrix for the joint LM test is block diagonal between the spatially dependent and the heteroskedastic components, and hence the joint test statistic is the sum of the two corresponding component statistics, where the test for the heteroskedastic part is identical to the Breusch and Pagan (1979) statistic. However, the spatially dependent part cannot be decomposed further into two one-directional test statistics that correspond to the spatially lagged dependent variable and spatial error autocorrelation respectively. As emphasized in Anselin (1988c), this is because of the *structural relationship* between spatial autoregressive processes in the dependent variable and the disturbance term, which results in the absence of block diagonality of the information matrix between the corresponding elements (see Anselin, 1988c, p. 8).

Noting this, Anselin (1988c) proposed an LM test for spatial error autocorrelation in the presence of a spatially lagged dependent variable. However, implementation of the suggested test requires nonlinear optimization or the application of a numerical search technique (see Anselin and Hudak, 1992). In this paper we apply the modified LM test developed by Bera and Yoon (1993) to spatial models and propose simple diagnostic tests for spatial dependence that are based on the results of ordinary least-squares (OLS) estimation.

In Section 2 we briefly summarize the main results on the distribution of standard LM tests when the alternative hypothesis is misspecified, and present the modified LM test which is robust under local misspecification. Section 3 develops new diagnostic tests for spatial error autocorrelation and for a spatially lagged dependent variable in the presence of local misspecification in the form of a spatially lagged dependent variable or spatial error autocorrelation, respectively. In Section 4 we provide some evidence on the performance of the robust tests, both in the form of some simple empirical examples as well as on the basis of results of a series of Monte Carlo simulation experiments. We close with some concluding remarks in Section 5.

2. A general approach to testing in the presence of a nuisance parameter

Consider a general statistical model represented by the log-likelihood function $L(\gamma, \psi, \phi)$, where γ is a parameter vector, and for simplicity ψ and ϕ are assumed to be scalars. Suppose an investigator sets $\phi = 0$ and tests H_0: $\psi = 0$ using the log-likelihood function $L_1(\gamma, \psi) = L(\gamma, \psi, 0)$. The LM statistic for testing H_0 in $L_1(\gamma, \psi)$ will be denoted by LM_ψ. Let us also denote $\theta = (\gamma', \psi, \phi)'$ and $\tilde{\theta} = (\tilde{\gamma}', 0, 0)'$, where $\tilde{\gamma}$ is the maximum likelihood (ML) estimator of γ when $\psi = 0$ and $\phi = 0$. The score vector and the information matrix are defined, respectively, as

$$d(\theta) = \frac{\partial L(\theta)}{\partial \theta} = \begin{bmatrix} \dfrac{\partial L(\theta)}{\partial \gamma} \\ \dfrac{\partial L(\theta)}{\partial \psi} \\ \dfrac{\partial L(\theta)}{\partial \phi} \end{bmatrix}$$

and

$$J(\theta) = -E\left[\frac{1}{N}\frac{\partial^2 L(\theta)}{\partial \theta\, \partial \theta'}\right] = \begin{bmatrix} J_\gamma & J_{\gamma\psi} & J_{\gamma\phi} \\ J_{\psi\gamma} & J_\psi & J_{\psi\phi} \\ J_{\phi\gamma} & J_{\phi\psi} & J_\phi \end{bmatrix}.$$

If $L_1(\gamma, \psi)$ were the true model, then it is well known that under H_0: $\psi = 0$, the following certain regularity conditions (see, for example, Serfling, 1980, p. 155),

$$LM_\psi = \frac{1}{N} d_\psi(\tilde{\theta})' J^{-1}_{\psi\cdot\gamma}(\tilde{\theta}) d_\psi(\tilde{\theta}) \xrightarrow{D} \chi^2_1(0),$$

where $J_{\psi\cdot\gamma}(\theta) = J_\psi(\theta) - J_{\psi\gamma}(\theta) J_\gamma^{-1}(\theta) J_{\gamma\psi}(\theta)$. We use $\xrightarrow{D}$ to denote convergence in distribution. Under this set-up, asymptotically the test will have the correct size and will be locally optimal. Now suppose that the true log-likelihood function is $L_2(\gamma, \phi) = L(\gamma, 0, \phi)$, so that the alternative $L_1(\gamma, \psi)$ is misspecified. Using a sequence of local values $\phi = \delta/\sqrt{N}$, Davidson and MacKinnon (1987) and Saikkonen (1989) obtained the asymptotic distribution of LM_ψ under $L_2(\gamma, \phi)$ as

$$LM_\psi \xrightarrow{D} \chi^2_1(\lambda), \tag{1}$$

where the non-centrality parameter λ is given by $\lambda = \delta' J_{\phi\psi\cdot\gamma} J^{-1}_{\psi\cdot\gamma} J_{\psi\phi\cdot\gamma}\delta$, with $J_{\psi\phi\cdot\gamma} = J_{\psi\phi} - J_{\psi\gamma} J_\gamma^{-1} J_{\gamma\phi}$. Owing to the presence of this non-centrality parameter, LM_ψ will reject the null hypothesis H_0: $\psi = 0$ more often than allowed by the size of the test, even when $\psi = 0$. Therefore, the test will have an incorrect size. Note that the crucial quantity is $J_{\psi\phi\cdot\gamma}$ which can be interpreted as the conditional covariance between d_ψ and d_ϕ, given d_γ (see Anderson, 1984, pp. 36–37). If $J_{\psi\phi\cdot\gamma} = 0$, then the local presence of the parameter ϕ has no effect on LM_ψ.

Using (1), Bera and Yoon (1993) suggested a modification to LM_ψ so that the resulting test is robust to the presence of ϕ. The modified statistic is given by

$$\begin{aligned}\mathrm{LM}^*_\psi = \frac{1}{N}&[d_\psi(\tilde\theta) - J_{\psi\phi\cdot\gamma}(\tilde\theta)J^{-1}_{\phi\cdot\gamma}(\tilde\theta)d_\phi(\tilde\theta)]' \\ &\times [J_{\psi\cdot\gamma}(\tilde\theta) - J_{\psi\phi\cdot\gamma}(\tilde\theta)J^{-1}_{\phi\cdot\gamma}(\tilde\theta)J_{\phi\psi\cdot\gamma}(\tilde\theta)]^{-1} \\ &\times [d_\psi(\tilde\theta) - J_{\psi\phi\cdot\gamma}(\tilde\theta)J^{-1}_{\phi\cdot\gamma}(\tilde\theta)d_\phi(\tilde\theta)]\,. \end{aligned} \tag{2}$$

This new test essentially adjusts the asymptotic mean and variance of the standard LM_ψ. Another way to look at LM^*_ψ is to view the quantity $J_{\psi\phi\cdot\gamma}(\tilde\theta)J^{-1}_{\phi\cdot\gamma}(\tilde\theta)d_\phi(\tilde\theta)$ as the prediction of $d_\psi(\tilde\theta)$ by $d_\phi(\tilde\theta)$. Here, $d_\phi(\tilde\theta)$ is the score of the parameter whose effect we want to take into account in constructing the modified test statistic. Therefore, $d_\psi(\tilde\theta) - J_{\psi\phi\cdot\gamma}(\tilde\theta)J^{-1}_{\phi\cdot\gamma}(\tilde\theta)d_\phi(\tilde\theta)$ is the part of $d_\psi(\tilde\theta)$ that remains after eliminating the effect of $d_\phi(\tilde\theta)$. Bera and Yoon (1993) showed that, under $\psi = 0$ and $\phi = \delta/\sqrt{N}$, LM^*_ψ has a central χ^2_1 distribution. Thus, LM^*_ψ has the same asymptotic distribution as LM_ψ with $\psi = 0$ and $\phi = 0$, thereby producing a test with asymptotically the correct size for a locally misspecified model. Two things regarding LM^*_ψ are worth noting. First, LM^*_ψ requires estimation only under the joint null, namely for the constrained model in which both $\psi = 0$ and $\phi = 0$. Given the full specification of the model $L(\gamma, \psi, \phi)$ it is of course possible to derive an LM test for $\psi = 0$ in the presence of ϕ. However, that requires ML estimation of ϕ which could be difficult to obtain in some cases. Secondly, when $J_{\psi\phi\cdot\gamma} = 0$, $\mathrm{LM}^*_\psi = \mathrm{LM}_\psi$. This is a simple condition to check in practice. As mentioned before, if this condition is true, LM_ψ is an asymptotically valid test in the local presence of ϕ.

3. Tests for spatial dependence

As in the treatment of Anselin (1988c), we consider the mixed regressive–spatial autoregressive model with a spatial autoregressive disturbance:

$$\begin{aligned} &y = \phi W_1 y + X\gamma + u\,, \\ &u = \psi W_2 u + \varepsilon\,, \\ &\varepsilon \sim \mathrm{N}(0, \sigma^2 I)\,. \end{aligned} \tag{3}$$

In this model, y is an $(N \times 1)$ vector of observations on a dependent variable recorded at each of N locations, X is an $(N \times k)$ matrix of exogenous variables, and γ is a $(k \times 1)$ vector of parameters. ϕ and ψ are scalar spatial parameters. W_1 and W_2 are $(N \times N)$ observable spatial weights matrices with positive elements, associated with the spatially lagged dependent variable and the spatial autoregressive disturbance, respectively. These spatial weight matrices represent 'degree of potential interaction' between neighboring locations and are scaled such that the sum of the row elements in each matrix is equal to one. After such row standardization, the weights matrix is asymmetric (and positive), with elements less than or equal to one. Typically, the elements of the weights matrix are derived from information on contiguity (i.e. two observations having a common boundary), although more general approaches are possible as well (see Ord, 1975; Cliff and Ord, 1981; Upton and Fingleton, 1985; Anselin 1988a, for discussions of the properties and importance of the W matrix). It is the inclusion of these spatial weights matrices that renders the spatial models to depart from the standard linear model, thereby limiting the applicability of standard econometric procedures based on the OLS method.

Note that for model (3) to be identified, it is necessary that the weights matrices for the spatial autoregressive terms in the dependent variable and the errors be different, $W_1 \neq W_2$, of that the matrix X contain at least one 'exogenous' variable in addition to the constant term. An alternative specification for which there are no such potential problems and which leads to identical results in terms of the tests considered in this paper, is the so-called mixed spatial autoregressive moving average model (Huang, 1984). In such a model the error terms follow a spatial moving average process:

$$u = \psi W_2 \varepsilon + \varepsilon \ , \tag{4}$$

but otherwise the specification is identical to (3).

We are interested in testing H_0: $\psi = 0$ in the presence of the nuisance parameter ϕ. As before, let $\theta = (\gamma', \psi, \phi)'$. Since, under the null of $\psi = 0$ (but not in the general case, see Anselin, 1988a), the information matrix is block diagonal between the θ and σ^2 parameters, we need only consider the scores and the information matrix evaluated at $\theta_0 = (\gamma', 0, 0)'$. On the basis of the results in Anselin (1988a, ch. 6) these follow as

$$d_\gamma = \frac{1}{\sigma^2} X'u \ ,$$

$$d_\psi = \frac{1}{\sigma^2} u'W_2 u \ ,$$

$$d_\phi = \frac{1}{\sigma^2} u'W_1 y \ ,$$

and

$$J = \frac{1}{N\sigma^2}\begin{bmatrix} X'X & 0 & X'(W_1X\gamma) \\ 0 & T_{22}\sigma^2 & T_{21}\sigma^2 \\ (W_1X\gamma)'X & T_{12}\sigma^2 & (W_1X\gamma)'(W_1X\gamma) + T_{11}\sigma^2 \end{bmatrix}, \tag{5}$$

where, as in Anselin (1988a), we use the notation $T_{ij} = \mathrm{tr}[W_iW_j + W_i'W_j]$, $i, j = 1, 2$, with tr denoting the trace of a matrix. From (5) it follows that

$$J_{\psi\phi\cdot\gamma} = \frac{1}{N} T_{21} ,$$

$$J_{\psi\cdot\gamma} = \frac{1}{N} T_{22} ,$$

and

$$J_{\phi\cdot\gamma} = \frac{1}{N\sigma^2}[(W_1X\gamma)'M(W_1X\gamma) + T_{11}\sigma^2] , \tag{6}$$

where $M = I - X(X'X)^{-1}X'$. Note that $J_{\psi\phi\cdot\gamma} \neq 0$, since $T_{21} > 0$ (the elements of the spatial weights matrices are always positive). A modified LM test for the null hypothesis H_0: $\psi = 0$ can easily be obtained as

$$\mathrm{LM}_\psi^* = \frac{[\tilde{u}'M_2\tilde{u}/\tilde{\sigma}^2 - T_{21}(N\tilde{J}_{\phi\cdot\gamma})^{-1}\tilde{u}'W_1y/\tilde{\sigma}^2]^2}{T_{22} - (T_{21})^2(N\tilde{J}_{\phi\cdot\gamma})^{-1}} , \tag{7}$$

where $\tilde{u} = y - X\tilde{\gamma}$ are the OLS residuals, with $\tilde{\sigma}^2 = \tilde{u}'\tilde{u}/N$, and from (6) it follows that

$$(N\tilde{J}_{\phi\cdot\gamma})^{-1} = \tilde{\sigma}^2[(W_1X\tilde{\gamma})'M(W_1X\tilde{\gamma}) + T_{11}\tilde{\sigma}^2]^{-1} .$$

As pointed out in Anselin (1988a), one can interpret $(W_1X\tilde{\gamma})$ as the spatially lagged OLS predicted values.

We can also consider the case where the spatial weights matrices W_1 and W_2 are the same. This is often more realistic in practice, since there may be good reasons to expect the structure of spatial dependence to be the same for both the dependent autoregressive (AR) variable and the error term. Setting $W_1 = W_2 = W$ is always possible when the alternative of interest is a spatial moving average (MA) error term, as in (4). However, for model (3), there may be identification problems. It can easily be shown that the test statistics are the same for an AR and an MA error, since, under the null, the resulting score and information matrix elements are identical; this is a typical characteristic of LM tests (see, for example, Bera and Ullah, 1991).

When $W_1 = W_2 = W$, the following simplifying results hold for the matrix trace expressions:

$$T_{11} = T_{21} = T_{22} = T = \mathrm{tr}[(W' + W)W] ,$$

and the statistic LM_ψ^* becomes

$$LM_\psi^* = \frac{[\tilde{u}'W\tilde{u}/\tilde{\sigma}^2 - T(N\tilde{J}_{\phi\cdot\gamma})^{-1}\tilde{u}'Wy/\tilde{\sigma}^2]^2}{T[1 - T(N\tilde{J}_{\phi\cdot\gamma})]^{-1}} . \tag{8}$$

The conventional one-directional test LM_ψ given in Burridge (1980) is obtained by setting $\phi = 0$ to yield

$$LM_\psi = \frac{[\tilde{u}'W\tilde{u}/\tilde{\sigma}^2]^2}{T} . \tag{9}$$

A comparison of (8) with (9) clearly reveals that LM_ψ^* modifies the standard LM_ψ by correcting the asymptotic mean and variance of the score for the asymptotic correlation between d_ψ and d_ϕ.

Let us now consider the LM test for H_0: $\psi = 0$ in the presence of the ϕ parameter derived in Anselin (1988c). We denote this statistic by LM_ψ^A:

$$LM_\psi^A = \frac{[\hat{u}'W_2\hat{u}/\hat{\sigma}^2]^2}{T_{22} - (T_{21A})^2 \hat{\text{var}}(\hat{\phi})} , \tag{10}$$

where $\hat{u}$ is a vector of ML residuals under the null model, $y = \phi W_1 y + X\gamma + u$, obtained by means of non-linear optimization or a search technique (see Anselin and Hudak, 1992, for practical details). T_{21A} in (10) denotes $\text{tr}[W_2W_1A^{-1} + W_2'W_1A^{-1}]$, with $A = I - \hat{\phi}W_1$. Comparing LM_ψ^A with LM_ψ^* in (8), it is readily seen that LM_ψ^A does not have the mean correction factor in LM_ψ^*. This is because LM_ψ^A uses the restricted ML estimator of ϕ, for which $d_\phi = 0$. We may view LM_ψ^A as the spatial version of the Durbin h statistic, which can also be derived from the general LM principle. Unlike Durbin's h, however, LM_ψ^A cannot be computed using the OLS residuals (this is not a problem for LM_ψ^*), since in the spatial case the model requires nonlinear optimization even under H_0: $\psi = 0$.

We can also obtain LM_ϕ^* easily to test H_0: $\phi = 0$ in the presence of local misspecification involving a spatial dependent error process with parameter ψ, say $\psi = \delta/\sqrt{N}$, which yields

$$LM_\phi^* = \frac{[\tilde{u}'W_1y/\tilde{\sigma}^2 - T_{12}T_{22}^{-1}\tilde{u}'W_2\tilde{u}/\tilde{\sigma}^2]^2}{N\tilde{J}_{\phi\cdot\gamma} - (T_{21})^2T_{22}^{-1}} . \tag{11}$$

For local misspecification in the form of a spatial MA error process (or a properly identified AR error process), assuming $W_1 = W_2 = W$, the above expression simplifies to

$$LM_\phi^* = \frac{[\tilde{u}'Wy/\tilde{\sigma}^2 - \tilde{u}'W\tilde{u}/\tilde{\sigma}^2]^2}{N\tilde{J}_{\phi\cdot\gamma} - T} . \tag{12}$$

It is straightforward to see that the standard and one-directional Lagrangian test statistic, LM_ϕ, given $\psi = 0$, is obtained as

$$LM_\phi = \frac{[\tilde{u}'W_1 y/\tilde{\sigma}^2]^2}{N\tilde{J}_{\phi\cdot\gamma}}. \tag{13}$$

Note that this statistic is identical to the one shown in Eq. (32) in Anselin (1988c).

Similar to the approach taken for LM_ψ^A, we can also formulate a LM test for H_0: $\phi = 0$ in the presence of the ψ parameter. This is another special case of the general framework outlined in Anselin (1988a, ch. 6). We denote such a statistic by LM_ϕ^A:

$$LM_\phi^A = \frac{[\hat{u}'B'BW_1 y]^2}{H_\phi - H_{\theta\phi}\,\hat{\mathrm{var}}(\hat{\theta})H'_{\theta\phi}}, \tag{14}$$

where $\hat{u}$ is a vector of residuals in the ML estimation of the null model with spatial AR errors, $y = X\gamma + (I - \psi W_2)^{-1}\varepsilon$, with $\theta' = [\gamma' \; \psi \; \sigma^2]$ and $B = I - \hat{\psi}W_2$. The terms in the denominator of (14) are

$$H_\phi = \mathrm{tr}\, W_1^2 + \mathrm{tr}(BW_1B^{-1})'(BW_1B^{-1}) + \frac{1}{\sigma^2}(BW_1X\gamma)'(BW_1X\gamma),$$

$$H'_{\theta\phi} = \begin{bmatrix} \frac{1}{\sigma^2}(BX)'BW_1X\gamma \\ \mathrm{tr}(W_2B^{-1})'BW_1B^{-1} + \mathrm{tr}\, W_2W_1B^{-1} \\ 0 \end{bmatrix},$$

and $\hat{\mathrm{var}}(\hat{\theta})$ is the estimated variance matrix for the parameter vector θ in the null model.

As given in Eq. (31) of Anselin (1988c), a test for both ϕ and ψ, based on OLS estimation, takes the form (assuming $W_1 = W_2 = W$):

$$LM_{\phi\psi} = \frac{[\tilde{u}'Wy/\tilde{\sigma}^2 - \tilde{u}'W\tilde{u}/\tilde{\sigma}^2]^2}{N\tilde{J}_{\phi\cdot\gamma} - T} + \frac{[\tilde{u}'W\tilde{u}/\tilde{\sigma}^2]^2}{T}. \tag{15}$$

The statistic is distributed as $\chi_2^2(0)$ and will of course result in a loss of power compared with the proper one-directional test when only one of the two forms of misspecification is present. Note that this statistic is not the sum of (9) and (13), but interestingly the sum of respectively (9) and (12) or (8) and (13) instead:

$$LM_{\phi\psi} = LM_{\psi} + LM_{\phi}^{*} = LM_{\phi} + LM_{\psi}^{*} . \tag{16}$$

The first equality in (16) follows directly from (9) and (12), and the second follows after some straightforward rearrangements of the terms in (8) and (13). In other words, the two-directional LM test for ϕ and ψ can be decomposed into the sum of the uncorrected one-directional test for one type of alternative and the adjusted form for the other alternative. Table 1 summarizes the different forms of the tests for the respective null hypotheses and sources of local misspecification.

Anselin (1988c) also derived an LM test for spatial residual autocorrelation in the presence of heteroskedasticity, assuming no spatially lagged dependent variable. The statistic is given by

$$\frac{[\hat{u}'\hat{\Omega}^{-1}W_2\hat{u}]^2}{T} , \tag{17}$$

where $\hat{u}$ is a vector of residuals in the ML estimation of the null model with a diagonal error covariance matrix Ω incorporating heteroskedasticity. Using the information matrix given in Anselin (1988c) it is easy to check that $J_{\psi\phi\cdot\gamma} = 0$ in this model. This implies that our modified LM* would revert to the conventional LM test given in (9). In other words, the simple standard LM statistic in (9) would give asymptotically the same inference as (17) in the presence of *local* heteroskedasticity without the computational difficulties associated with (17).

Table 1
Overview of tests

Null hypothesis	Parameter		Test statistic
	Spatial error, ψ	Spatial lag, ϕ	
H_0: $\psi = 0$	–	Set to zero	LM_{ψ}
H_0: $\psi = 0$	–	Unrestricted, estimated	LM_{ψ}^{A}
H_0: $\psi = 0$	–	Unrestricted, not estimated	LM_{ψ}^{*}
H_0: $\phi = 0$	Set to zero	–	LM_{ϕ}
H_0: $\phi = 0$	Unrestricted, estimated	–	LM_{ϕ}^{A}
H_0: $\phi = 0$	Unrestricted, not estimated	–	LM_{ϕ}^{*}
H_0: $\psi = 0, \phi = 0$	–	–	$LM_{\phi\psi}$

4. Comparative performance of the tests

4.1. Empirical illustration

To gain more concrete insight into the properties of the new tests in the realistic contexts encountered in empirical work in regional science and urban economics, we first present the results for the various LM tests in three illustrative examples. These examples are chosen specifically to highlight different types of spatial effects. Their substantive interpretation is therefore not considered here. The first regression is a simple relationship between crime and housing value and income in 1980 for 49 neighborhoods in Columbus, OH, and was used extensively to illustrate various spatial regression models in Anselin (1988a).[1] The second model is a neoclassical multiregional investment model from Florax (1992), estimated using 1984 data for 40 COROP regions in The Netherlands.[2] The third model is a simple linear relationship between labor cost (wages) and labor productivity, and an educational variable, degree of unionization and highway investment expenditures (as a proxy for public infrastructure) estimated using 1983 data for the 48 contiguous U.S. states.[3]

The results for the LM tests considered in this paper as well as the ML estimation of the spatial AR coefficients ($\hat{\phi}$ and $\hat{\psi}$) with the associated likelihood ratio (LR) test statistics in their respective alternative models are listed in Table 2. To provide a rough idea of the overall quality of these models, the adjusted R^2 obtained in the OLS regression of the null model is given in the first row of the table as well. All estimates and tests were carried out by means of the SpaceStat software package (Anselin, 1992b, 1994).

The three sets of results reflect two situations often encountered in empirical work that takes spatial effects into account: (a) strong significance for both one-directional tests; and (b) strong significance for one kind combined with weak-or non-significance for the other. For the Columbus model, the two-directional test $\mathrm{LM}_{\phi\psi}$ ($p<0.01$) and both uncorrected one-directional tests LM_{ψ} ($p=0.02$) and LM_{ϕ} ($p<0.01$) are highly

[1] The data are listed in Table 12.1, p. 189 of Anselin (1988a). Estimation results are given in Anselin (198a, ch. 12).

[2] The model used here is the linear version of the model outlined in table 7.1, p. 201 of Florax (1992). It relates investment in buildings by the manufacturing sector to output, investment in equipment, user cost of capital, degree of urbanization, distance to the core region, contagious knowledge diffusion and hierarchical knowledge diffusion. For details and estimation results, see Florax (1992, ch. 8).

[3] Data are based on the U.S. Bureau of Economic Analysis Regional Economic Information System database and selected U.S. census sources. The estimation results are available from the authors.

Table 2
Test results in the empirical examples[a]

	Columbus, Ohio $N=49$	The Netherlands $N=40$	U.S. states $N=48$
R^2	0.533	0.694	0.521
$LM_{\phi\psi}$	9.44	7.97	5.07
LM_{ψ}	5.72	2.43	4.35
LM_{ψ}^{*}	0.08	0.14	3.65
LM_{ψ}^{A}	0.32	0.96	3.36
LM_{ϕ}	9.36	7.83	1.42
LM_{ϕ}^{*}	3.72	5.54	0.72
LM_{ϕ}^{A}	1.76	0.54	1.26
$\hat{\psi}$	0.562	0.459	0.465
	(0.134)	(0.164)	(0.153)
LR_{ψ}	7.99	3.84	5.09
$\hat{\phi}$	0.431	0.349	0.188
	(0.118)	(0.115)	(0.155)
LR_{ϕ}	9.97	7.47	1.38

[a] The one-directional tests are distributed as χ_1^2, with critical levels of 3.84 ($p=0.05$) and 6.63 ($p=0.01$). The two-directional test is distributed as χ_2^2, with critical levels of 5.99 ($p=0.05$) and 9.21 ($p=0.01$). The estimates for ψ and ϕ are based on maximum likelihood estimation in a model with an AR error (setting $\phi=0$) and a model with an AR lag (setting $\psi=0$) respectively. Values in parentheses below the estimates are asymptotic standard errors.

significant. In addition, the spatial AR coefficients $\hat{\phi}$ and $\hat{\psi}$ are positive and strongly significant (at $p<0.01$) in the respective alternative models, as are the associated LR tests. There is a slight edge in favor of the spatial lag model in terms of overall fit (log-likelihood of -182.4 vs. -183.4 for the error model). In other words, this is an instance where the lag model is the likely alternative and the 'impression' of error dependence indicated by the LM_{ψ} test is spurious. In contrast to the uncorrected LM_{ψ} and LM_{ϕ} tests, more convincing and reliable evidence is available from the new LM_{ψ}^{*} and LM_{ϕ}^{*} tests. LM_{ψ}^{*} is not at all significant, while the LM_{ϕ}^{*} test is significant at p slightly greater than 0.05. The LM_{ψ}^{A} test based on the residuals of the ML estimation of the spatial lag model provides an indication similar to LM_{ψ}^{*} ($p=0.57$ vs. $p=0.78$ for LM_{ψ}^{*}), but the LM_{ϕ}^{A} test is not significant ($p=0.18$). For the Dutch data, the two-directional test is significant as well ($p=0.02$), but of the uncorrected one-directional tests, this is only the case for the lag test LM_{ϕ} (at $p<0.01$; the p-value for LM_{ψ} is greater than 0.10). The corrected tests confirm this pattern, though more so for LM_{ψ}^{*} (with $p=0.70$ vs. $p=0.12$ for LM_{ψ}) and again with slightly less power for LM_{ϕ}^{*} ($p=0.02$ vs. $p<0.01$ for LM_{ϕ}). Both autoregressive coefficients are positive and highly significant in the respective alternative models, though here as well the lage model is the correct one, with an edge in terms of fit (log-likelihood of −155.5 vs. −157.3).[4] Again, the LM_{ψ}^{A} and LM_{ψ}^{*} tests are in agreement and fail to reject the null hypothesis, but so does LM_{ϕ}^{A}. In the U.S. model, the two-directional test is not significant ($p = 0.08$), but the uncorrected one-dimensional test LM_{ψ} indicates the presence of error dependence ($p = 0.04$). The uncorrected one-directional test for a spatial lag is not significant ($p = 0.23$). In the alternative models, the spatial autoregressive coefficient in the error model is highly significant ($p < 0.01$), while the one in the lag model is not ($p = 0.23$). Both LM_{ψ}^{*} ($p = 0.06$) and LM_{ψ}^{A} ($p = 0.07$) point in the direction of error dependence, but with lower power than the uncorrected test. As expected, LM_{ϕ}^{A} is not significant. Note that the numerical values of LM_{ψ}^{*} and LM_{ψ}^{A}, both of which taken into account the potential for lage dependence (but in different ways), are very similar, but this is not the case for LM_{ϕ}^{*} and LM_{ϕ}^{A}. Of these two, the former tends to indicate the proper alternative, while the latter has low power in an alternative model with 'significant' error dependence.

Overall, these initial results point to a satisfactory behavior of the new tests. Their power is less than that of the uncorrected tests against the 'correct' alternative. However, they seem less likely to indicate the 'wrong' alternative in the sense that they are not significant against it, while their uncorrected counterparts are. A major computational advantage of the new tests is that they can be calculated from the results of standard OLS regression. In contrast, the LM_{ψ}^{A} and LM_{ϕ}^{A} tests require maximization of the non-linear likelihood function for either the spatial lag or spatial error model. In addition, they may be affected by pre-testing considerations (see Florax and Folmer, 1992).

While these encouraging results are of course very limited in scope, it should be noted that they were obtained for fairly small sample sizes, whereas the properties of the tests are asymptotic in nature.

4.2. Monte Carlo simulation

A more extensive view of the performance of the new tests is provided by the results of a series of Monte Carlo simulations in which the nature of the data generating process, and, in particular, of the "local" misspecification, is under complete control. We focus specifically on the size and power of the new tests relative to their uncorrected one-directional counterparts and to the two-directional test. All these tests are based on estimation by OLS. The LM_{ψ}^{A} and LM_{ϕ}^{A} tests considered in the previous section were not included in

[4] In addition, in the spatial error model the common factor hypothesis is rejected by a Wald test ($p < 0.05$), further indicating the inappropriateness of this alternative. See Anselin (1988a, ch. 13) for further discussion of these specification tests.

the Monte Carlo study, due to the computational burden of estimating the null model by means of ML procedures. Also, the initial evidence provided by the empirical examples suggests they may have inferior power. Anselin and Florax (1994) report the results of a larger set of simulation experiments where other test procedures for spatial dependence are also considered.

4.2.1. Experimental design

The experimental design used in the Monte Carlo simulations is based on a format extensively used in earlier studies (e.g. Anselin and Rey, 1991; Florax and Folmer, 1992). The model under the null hypothesis of no spatial dependence is the classical regression model:

$$y = X\gamma + u \, .$$

The N observations on the dependent variables are generated from a vector of standard normal random variates u. To obtain the explanatory variables X, an $N \times 3$ matrix is generated, consisting of a constant term and two variates drawn from a uniform (0, 10) distribution (consequently, the associate regression coefficients γ equal 1). This matrix of explanatory variables is held fixed in the replications. In addition to a normal error, a lognormal error term is generated as well, with mean and variance equal to that of the normal variates. For each combination of parameter values, 5000 replications were carried out. The tests are evaluated at their theoretical (asymptotic) critical values for $\alpha = 0.05$ and the proportion of rejections (i.e. the proportion of times the computed test statistic exceeded its asymptotic critical value) is reported. For a nominal Type I error of 0.05, the 5000 replications yield a sample standard deviation of 0.0031, which is judged sufficiently precise for our purposes.

The configurations used to generate spatial dependence are formally expressed in four weights matrices. These correspond to sample sizes 40, 81 and 127. The weights matrices of size 40 and 127 are for two actual irregularly shaped regionalizations of The Netherlands.[5] The weights matrices for $N = 81$ correspond to a regular square 9×9 grid, with continuity defined by both the rook criterion (four neighbors, having a side in common; to the north, south, east and west) and the queen criterion (eight neighbors, including the rook neighbors as well as those having a vertex in common). In this series of experiments we included both regular and irregular weights, the irregular ones to reflect the types of economic regions often encountered in empirical work, and the regular ones to focus on the effect of the characteristics of the connectivity structure on the properties of

[5] For $N = 40$, these are the same COROP regions used in the empirical illustration. For $N = 127$, they are the so-called 'economic geographic' regions in The Netherlands, which aggregate into the COROP regions.

the tests. It is important to note that in space the sample size (N) is not the only variable important in achieving convergence to asymptotic properties of tests and estimators. As shown in Anselin (1988a), the degree of interconnectedness between observations (locations) is also an important factor in determining the extent to which the central limit theorems on dependent spatial processes hold (i.e. the various mixing conditions described in Anselin, 1988a, ch. 5). For the weights matrices used here, the average number of connections for an observation are, respectively, 4.2 and 4.7 for the Dutch regions, 3.6 for the rook case and 6.7 for the queen case. The latter two values reflect the influence of boundary conditions (for central observations in the regular lattice, four and eight are the number of connections, respectively). Finally, in our simulation experiments all weights matrices are used in row-standardized form (i.e. such that the row elements sum to one) and the same weights are used in both lag and error specifications (i.e. $W_1 = W_2 = W$, in our notation).

We considered four types of alternative hypothesis of spatial dependence. Three of these are one-directional, i.e. a function of a single spatial parameter, and one is two-directional, i.e. a function of two spatial parameters. The spatially dependent observations are generated by means of an appropriate spatial transformation applied to a vector of errors or 'observations' of uncorrelated values, as follows:

(a) Spatial AR error:

$$u = (I - \psi W)^{-1}\varepsilon ,$$

where ε is a vector of standard normal (log-normal) variates, the other notation is as before. The resulting vector of spatially autocorrelated errors u is added to the $X\gamma$ vector to generate a vector of observations on the dependent variable y.

(b) Spatial MA error:

$$u = (I + \psi W)\varepsilon ,$$

with the spatially autocorrelated errors u added to the explanatory variables in the same way as for (a).

(c) Spatial AR lag:

$$y = (I - \phi W)^{-1}(X\gamma + u) ,$$

where u is a vector of standard normal (log-normal) variates.

(d) Spatial ARMA (SARMA) process:

$$y = (I - \phi W)^{-1}[X\gamma + (I + \psi W)\varepsilon] .$$

For the one-directional alternative hypotheses, the spatial parameters take on values from 0.1 to 0.9. For ease of interpretation, negative parameter values are excluded (see Anselin and Rey, 1991, for a discussion of the complications caused by negative parameter values). The maximum value of 0.9 reflects the constraint on the Jacobian term $|I - \zeta W|$ for the AR processes and $|I + \zeta W|$ for the MA processes, where ζ represents the spatial parameter ϕ or ψ. As is well known the Jacobian term simplifies to an expression in the roots of the weights matrix, as shown in Ord (1975) for AR processes:

$$\ln|I - \zeta W| = \sum_i \ln(1 - \zeta\omega_i) , \tag{18}$$

where the ω_i are the eigenvalues of the weights matrix. Consequently, the restriction on the parameter is of the form $\zeta < 1/\omega_i$, $\forall i$. The resulting acceptable parameter space for AR processes is[6]

$$1/\omega_{\min} < \zeta < 1/\omega_{\max} , \tag{19}$$

where the subscripts indicate the minimum and maximum eigenvalues, respectively, in real terms (see Anselin, 1988a). For MA processes, ζ should be replaced by $-\zeta$ in expression (18). For row-standardized weights, the largest eigenvalue is always 1, and $1/\omega_{\min} \leq -1$, which effectively constrains the positive parameter values to $\zeta < 1$. The combinations of parameter value, spatial configuration and error distribution yield a total of 216 cases for the three one-directional alternatives. For the two-directional SARMA process, constraint (19) holds separately for the AR and MA parameters, yielding 81 parameter combinations (positive values only), for a total of 648 cases.

4.2.2. Results

The empirical size of the tests is given in Table 3 for four spatial weights and for both normal and log normal error terms. Since the specified critical values were for $\alpha = 0.05$, a significant deviation from this rejection proportion would indicate a bias of the tests in finite samples. For 5000 replications and under a normal approximation to the binomial, a 95% confidence interval centered on $p = 0.05$ would include rejection frequencies between 0.044 and 0.056. It is encouraging to note that for $N = 127$, with normal

[6] For a different perspective, see Kelejian and Robinson (1995), where the parameter space is defined over the entire range of real values, with the exception of at most N singularity points.

Table 3
Empirical size of tests[a]

Test	$N=40$	$N=81$ (queen)	$N=81$ (rook)	$N=127$
Normal distribution				
LM_{ψ}	0.046	0.046	0.056	0.049
LM^*_{ψ}	0.046	0.049	0.053	0.051
LM_{ϕ}	0.052	0.054	0.054	0.051
LM^*_{ϕ}	0.055	0.052	0.055	0.054
$LM_{\phi\psi}$	0.051	0.045	0.057	0.048
Log-normal distribution				
LM_{ψ}	0.033	0.034	0.047	0.041
LM^*_{ψ}	0.038	0.041	0.046	0.043
LM_{ϕ}	0.049	0.051	0.052	0.048
LM^*_{ϕ}	0.050	0.052	0.053	0.056
$LM_{\phi\psi}$	0.042	0.043	0.047	0.050

[a] A 95% confidence interval for $p=0.05$ with 5000 replications is $0.044<p<0.056$.

error terms, all tests yield rejection frequencies within this range. Moreover, the four one-directional tests yield rejection frequencies roughly within the 95% confidence interval in all four samples. This indicates a correct size for even moderately sized and small data sets. For LM_{ϕ} and LM_{ψ}, this is in general agreement with the results in Anselin and Rey (1991).

The poorest performance results were for the rook case (relative to the queen configuration for the same sample size), where $LM_{\phi\psi}$ and LM_{ψ} slightly over-reject the null hypothesis, and LM^*_{ϕ} is very close to the upper bound of the confidence interval. A similar result occurred in Anselin and Rey (1991), where differences in empirical size were also found when different weights matrices were used for the same number of observations. It is not clear why the rook case stood out in this respect. The only indication as to how it differs from the other layouts is that it yields the smallest maximum eigenvalue of the four configurations (but its rejection frequencies are always higher). To some extent, this influence of the choice of the weights matrix is counterintuitive, since there is not spatial dependence present. It further highlights the difference between the two-dimensional spatial dependence and serial dependence in time-series analysis, which is one-dimensional (and one-directional). In one dimension, first-order dependence (first-order autocorrelation) is defined unambiguously, while this is not the case in two dimensions. As shown in Anselin and Rey (1991, table 4), this is an issue particularly in small samples and is much less pronounced as the number of observations increases (in the limit, the size of N dominates the effect of the connectedness structure).

A misspecification in the form of a log-normal error term seems to affect the size of the tests more for the error tests than for the lag tests, as was the case in Anselin and Rey (1991). For LM_{ϕ} and LM_{ϕ}^{*}, the rejection frequency remains in the 95% interval for the four cases, while the $LM_{\phi\psi}$ test significantly under-rejects for $N = 40$ and the queen case. Both error tests, LM_{ψ} and LM_{ψ}^{*}, significantly under-reject in three configurations (for $N = 40$ and 127, and for the queen case). In practice, under-rejection of the null hypothesis when no spatial dependence is present does not have any consequences, since the standard estimation results are interpreted as they should be (without taking spatial effects into account).

Overall, these results suggest that the new LM_{ψ}^{*} and LM_{ϕ}^{*} tests are unbiased, even in small to moderately sized data sets, and in this respect perform very similarly to their uncorrected counterparts.

Tables 4, 5 and 6 report the empirical rejection frequencies for the five tests against the three forms of one-directional spatial dependence, for normally distributed error terms.[7] We focus in particular on the extent to which the new tests differ from their uncorrected counterparts. In Table 4, LM_{ψ} clearly achieve the highest power, and in the largest sample results in 90% rejection rates for $\psi \geq 0.4$. The penalty for using LM_{ψ}^{*} is very small, and a small price to pay for the gain in robustness. While its power is inferior to that of LM_{ψ} in the smaller samples, it becomes almost indistinguishable for $N = 127$. As the simulation experiments in Anselin and Rey (1991) showed, LM_{ϕ} has 'good power' against AR error terms, but always less than LM_{ψ}. This is confirmed here. Therefore, the application of LM_{ϕ} alone could lead to the wrong inference. In contrast, the power function for LM_{ϕ}^{*} is always much flatter. This test performs remarkably well, in the sense that it yields low rejection frequencies even for $\psi = 0.9$ (e.g. 25% rejection with $N = 127$). The correction for error dependence in LM_{ϕ}^{*} thus seems to work in the right direction when no lag dependence is present, especially for small values of ψ. The two-directional $LM_{\phi\psi}$ test also has power against AR error dependence, and in this respect is superior to LM_{ϕ}, and very similar to LM_{ψ}^{*}. When comparing these two ways of taking into account potential lag dependence, it turns out that $LM_{\phi\psi}$ is slightly superior for $N = 40$ and the queen case, while there is a slight edge for LM_{ψ}^{*} for the rook case and $N = 127$. However, especially for high values of ψ, the two tests are virtually indistinguishable. In other words, in testing against spatial AR error dependence and relative to LM_{ψ}, a similar loss in power occurs

[7] The results for log-normal errors are very similar. The power of the test is slightly less than for the normal case for small values of the spatial parameter, but not distinguishable for larger values. The relative rankings of these and other tests in terms of power are not affected. Details are given in Anselin and Florax (1994).

Table 4
Power of tests against first-order spatial AR errors – normal distribution[a]

N	ψ	LM_ψ	LM^*_ψ	LM_ϕ	LM^*_ϕ	$LM_{\phi\psi}$
40	0.1	0.064	0.066	0.067	0.067	0.071
	0.2	0.125	0.109	0.089	0.076	0.122
	0.3	0.242	0.207	0.125	0.081	0.222
	0.4	0.401	0.333	0.180	0.096	0.365
	0.5	0.612	0.524	0.253	0.122	0.564
	0.6	0.790	0.689	0.379	0.141	0.753
	0.7	0.910	0.830	0.540	0.154	0.885
	0.8	0.974	0.923	0.724	0.166	0.962
	0.9	0.996	0.972	0.899	0.171	0.994
81 (queen)	0.1	0.066	0.065	0.063	0.063	0.072
	0.2	0.161	0.146	0.091	0.075	0.158
	0.3	0.312	0.285	0.119	0.082	0.276
	0.4	0.533	0.490	0.174	0.105	0.494
	0.5	0.758	0.697	0.279	0.122	0.707
	0.6	0.898	0.866	0.399	0.159	0.871
	0.7	0.973	0.958	0.579	0.199	0.965
	0.8	0.995	0.991	0.767	0.264	0.992
	0.9	1.000	0.999	0.933	0.356	0.999
81 (rook)	0.1	0.072	0.070	0.052	0.048	0.063
	0.2	0.208	0.179	0.079	0.056	0.164
	0.3	0.431	0.389	0.107	0.057	0.350
	0.4	0.691	0.645	0.153	0.063	0.603
	0.5	0.889	0.839	0.271	0.066	0.831
	0.6	0.974	0.952	0.402	0.083	0.953
	0.7	0.997	0.991	0.595	0.093	0.993
	0.8	1.000	0.999	0.811	0.118	0.999
	0.9	1.000	1.000	0.965	0.159	1.000
127	0.1	0.118	0.112	0.068	0.058	0.108
	0.2	0.353	0.318	0.107	0.073	0.300
	0.3	0.683	0.628	0.184	0.084	0.609
	0.4	0.900	0.869	0.301	0.103	0.867
	0.5	0.986	0.975	0.464	0.123	0.978
	0.6	0.998	0.996	0.651	0.157	0.998
	0.7	1.000	1.000	0.844	0.187	1.000
	0.8	1.000	1.000	0.964	0.221	1.000
	0.9	1.000	1.000	0.999	0.247	1.000

[a] The tests are for a one-directional alternative, hence $\phi = 0$.

whether the robust LM^*_ψ or the two-directional $LM_{\phi\psi}$ is used. The former has the advantage that, when compared with LM^*_ϕ, it points to the correct alternative (i.e. LM^*_ψ has higher power than LM^*_ϕ against error

Table 5
Power of tests against first-order spatial MA errors – normal distribution[a]

N	ψ	LM_{ψ}	LM^*_{ψ}	LM_{ϕ}	LM^*_{ϕ}	$LM_{\phi\psi}$
40	0.1	0.061	0.063	0.063	0.058	0.072
	0.2	0.103	0.097	0.086	0.075	0.104
	0.3	0.190	0.163	0.098	0.078	0.178
	0.4	0.307	0.259	0.135	0.085	0.279
	0.5	0.445	0.370	0.178	0.101	0.389
	0.6	0.570	0.479	0.219	0.113	0.522
	0.7	0.682	0.582	0.266	0.118	0.630
	0.8	0.779	0.682	0.313	0.143	0.729
	0.9	0.854	0.766	0.353	0.142	0.816
81	0.1	0.062	0.063	0.061	0.061	0.071
(queen)	0.2	0.134	0.122	0.078	0.067	0.127
	0.3	0.250	0.226	0.106	0.085	0.227
	0.4	0.401	0.370	0.138	0.085	0.360
	0.5	0.550	0.502	0.179	0.096	0.496
	0.6	0.686	0.638	0.223	0.107	0.630
	0.7	0.795	0.749	0.272	0.117	0.749
	0.8	0.875	0.839	0.312	0.131	0.836
	0.9	0.937	0.905	0.379	0.133	0.909
81	0.1	0.071	0.069	0.055	0.050	0.065
(rook)	0.2	0.176	0.161	0.069	0.047	0.140
	0.3	0.400	0.362	0.092	0.045	0.303
	0.4	0.646	0.582	0.131	0.051	0.531
	0.5	0.823	0.771	0.177	0.051	0.730
	0.6	0.924	0.889	0.234	0.053	0.864
	0.7	0.973	0.954	0.290	0.055	0.944
	0.8	0.995	0.984	0.342	0.057	0.984
	0.9	0.997	0.994	0.411	0.058	0.994
127	0.1	0.114	0.106	0.066	0.067	0.103
	0.2	0.322	0.285	0.104	0.069	0.267
	0.3	0.583	0.538	0.154	0.074	0.509
	0.4	0.825	0.781	0.217	0.080	0.771
	0.5	0.937	0.910	0.293	0.103	0.903
	0.6	0.983	0.970	0.385	0.111	0.968
	0.7	0.997	0.993	0.466	0.113	0.992
	0.8	1.000	0.998	0.532	0.124	0.999
	0.9	1.000	1.000	0.618	0.133	1.000

[a] The tests are for a one-directional alternative, hence $\phi = 0$.

dependence), providing an alternative way to carry out the decision rule of Anselin and Rey (1991). In contrast, the results of the $LM_{\phi\psi}$ test do not provide an indication of which alternative may cause the misspecification

Table 6
Power of tests against first-order spatial AR lag – normal distribution[a]

N	ϕ	LM_ψ	LM_ψ^*	LM_ϕ	LM_ϕ^*	$LM_{\phi\psi}$
40	0.1	0.067	0.048	0.193	0.183	0.150
	0.2	0.147	0.040	0.554	0.501	0.458
	0.3	0.331	0.033	0.858	0.797	0.783
	0.4	0.609	0.026	0.978	0.959	0.966
	0.5	0.822	0.018	0.998	0.994	0.996
	0.6	0.956	0.010	1.000	0.999	1.000
	0.7	0.993	0.002	1.000	1.000	1.000
	0.8	1.000	0.001	1.000	1.000	1.000
	0.9	1.000	0.006	1.000	1.000	1.000
81	0.1	0.084	0.052	0.299	0.276	0.234
(queen)	0.2	0.260	0.062	0.810	0.777	0.734
	0.3	0.610	0.118	0.992	0.984	0.980
	0.4	0.904	0.248	1.000	1.000	1.000
	0.5	0.990	0.441	1.000	1.000	1.000
	0.6	1.000	0.632	1.000	1.000	1.000
	0.7	1.000	0.802	1.000	1.000	1.000
	0.8	1.000	0.860	1.000	1.000	1.000
	0.9	1.000	0.351	1.000	1.000	1.000
81	0.1	0.073	0.057	0.463	0.568	0.372
(rook)	0.2	0.162	0.063	0.967	0.955	0.931
	0.3	0.321	0.061	1.000	1.000	1.000
	0.4	0.557	0.061	1.000	1.000	1.000
	0.5	0.806	0.051	1.000	1.000	1.000
	0.6	0.969	0.027	1.000	1.000	1.000
	0.7	0.999	0.010	1.000	1.000	1.000
	0.8	1.000	0.001	1.000	1.000	1.000
	0.9	1.000	0.000	1.000	1.000	1.000
127	0.1	0.142	0.045	0.653	0.607	0.549
	0.2	0.547	0.053	0.996	0.993	0.992
	0.3	0.927	0.092	1.000	1.000	1.000
	0.4	0.998	0.116	1.000	1.000	1.000
	0.5	1.000	0.077	1.000	1.000	1.000
	0.6	1.000	0.027	1.000	1.000	1.000
	0.7	1.000	0.002	1.000	1.000	1.000
	0.8	1.000	0.000	1.000	1.000	1.000
	0.9	1.000	0.000	1.000	1.000	1.000

[a] The tests are for a one-directional alternative, hence $\psi = 0$.

error. These general patterns are confirmed for the spatial MA error dependence in Table 5. There are a few interesting differences, however. First the power of all tests is considerably lower than for comparable values of the AR error parameter.[8] Secondly, the LM_ϕ test no longer has good power against error dependence. For example, for $N = 40$, the null hypothesis is rejected by LM_ϕ in only 35% of the cases for $\psi = 0.9$, compared with 90% in Table 4. To some extent this is to be expected, given the much greater similarity between a lag process and a spatial AR error process (e.g. in the form of a spatial Durbin model), while such a similarity does not exist with MA errors. However, the clear superiority of LM_ψ relative to LM_ϕ in this context, and the same relationship between LM^*_ψ and LM^*_ϕ, would tend to strengthen the decision rule of Anselin and Rey (1991).

In Table 6 the LM_ϕ test is clearly the most powerful test against a spatial AR lag, achieving a 95% rejection level for $\phi > 0.3$ in the smallest sample, and for $\phi > 0.1$ in the rook ($N = 81$) and $N = 127$ configurations. The $LM_{\phi\psi}$ and LM^*_ϕ tests have only slightly less power and are almost indistinguishable in the largest data set. In other words, the penalty in terms of power for the robustness against error dependence in LM^*_ϕ, when none is present, in almost negligible. However, there is hardly any power difference between this test and the two-directional $LM_{\phi\psi}$ test that explicitly takes error dependence into account.

Overall, the power functions of the three lag tests compare favorably with the one for tests against error dependence. This reliability of the lag tests is encouraging, since the consequences of ignoring a spatial lag (as an omitted variable) when one should be included (i.e. inconsistent estimates) are much more serious than the consequences of ignoring spatially correlated errors (less efficient estimates).

The LM_ψ test also has 'power' against a spatial lag, although much less than the lag tests, and therefore LM_ψ by itself cannot be used to identify the dependence structure. The behavior of LM^*_ψ is very interesting. Except somewhat for $\phi = 0.7$ and 0.8 in the queen case ($N = 81$), this test has no power against lag dependence, as it should. Moreover, its power function tends to decrease with increasing values of ϕ. For small values of ϕ, the rejection frequency of LM^*_ψ is very close to its expected value of 0.05, but for large values it becomes almost negligible (except for the queen case). Since the LM^*_ψ test is robust to 'local' misspecification, this is not surprising. However, a clear discrepancy between the indications of LM_ψ and LM^*_ψ, while both LM_ϕ and LM^*_ϕ are significant, would provide strong evidence for lag dependence as opposed to error dependence. The extent to which such a

[8] In a strict sense, the parameter values for an AR error process and an MA error process are not equivalent, since each process implies a different range for the spatial interaction between observations. For an AR process, all observations interact, whereas for a MA process, only the first- and second-order neighbors interact. In other words, the same parameter values imply a stronger interaction for an AR process than for an MA process.

decision rule would hold in a specification search characterized by pre-testing remains to be investigated further.

So far, our results have indicated that there is little loss in power from using the robust tests in situations where the alternative is one-directional. However, it is also important to assess the performance to the tests when a mixture of both forms of dependence is present, e.g. when the alternative is a SARMA process. For this case we present partial results, summarized in four representative figures, which are projections of the power surface for a given value of one parameter onto the axis plane of the other.[9] Not reported are the results for the two-directional $LM_{\phi\psi}$ test, which has excellent power against this set of alternatives, achieving 95% rejection frequencies for $\phi \geq 0.4$, $\forall\psi$, even in the smallest sample. For $N = 127$, a 95% rejection is obtained for all but three parameter combinations ($\phi = 0.1$ and $\psi \leq 0.3$).

The power functions in Figs. 1 and 2 – for $N = 127$ – illustrate the extent to which the LM_ψ^* test is robust to the presence of lag dependence. For small values of ϕ in Fig. 1, the power function mimics that of the tests against first-order MA error dependence, with power slowly increasing with values of ψ. The curves for $\phi = 0.0$ and $\phi = 0.1$, in particular, are very close, confirming the proper behavior of this test against 'local' misspecification. For higher values of ϕ (e.g. $\phi = 0.5$ in Fig. 1), there is a clear loss in power, even for high values of ψ. For the smaller sample sizes, similar patterns are obtained, though overall at a considerable lower level of power. An alternative view is provided by the power functions in Fig. 2. Here, each power curve shows how the rejection frequency changes with ϕ, while holding ψ constant. This illustrates the strange behavior of LM_ψ^* with increasing values of ϕ and provides some insight into the performance of the test under 'non-local' alternatives. As shown in the discussion of one-directional alternatives, the power function is almost horizontal for $\psi = 0.0$ and small values of ϕ, as it should be. However, this is not the case for higher values of ϕ, where power first increases and then decreases to become negligible for high ϕ, even for high values of ψ. Similar erratic patterns are found for the other samples, though less dramatic for $N = 40$ (with a much flatter, decreasing pattern with ϕ). Overall, our results indicate that we will obtain reliable inference from LM_ψ^* in the presence of a modest amount of lag dependence (say $\phi \leq 0.4$). For higher values of ϕ, we can expect LM_ϕ^* to be highly significant (to cast doubts on LM_ψ^*), even in the presence of strong error dependence (see Figs. 3 and 4 and the discussion below).

[9] Complete results, as well as additional figures, are provided in Anselin and Florax (1994).

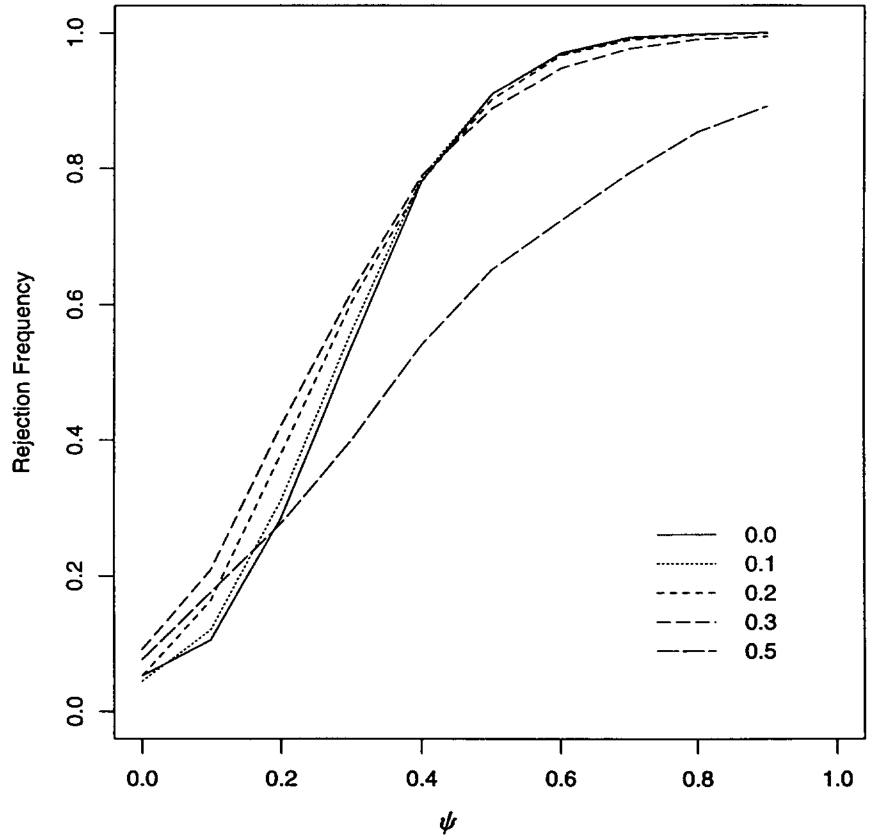

Fig. 1. Power of LM_{ψ}^{*} against SARMA (1, 1), MA dimension, $N = 127$ (dashed lines correspond to $\phi = 0.0$, 0.1, 0.2, 0.3, 0.4 and 0.5).

In contrast to LM_{ψ}^{*}, the power functions of LM_{ϕ}^{*} seem almost unaffected by the value of ψ. In Fig. 3, for $N = 40$, the power curves for the five values of ψ are virtually identical and illustrate the good performance of this test against spatial AR alternatives.[10] Similarly, in Fig. 4, for the same sample size, the power curves are more or less horizontal with ψ, even for high values. The difference in properties between lag and error tests is thus maintained for their robust forms, in the presence of misspecification. Overall, LM_{ϕ}^{*} comes across as much more reliable than LM_{ψ}^{*}, and very

[10] In contrast to the previous case, this good performance is already obvious for $N = 40$. Figures for the other sample sizes are similar, except that still higher power is achieved against $\phi > 0$.

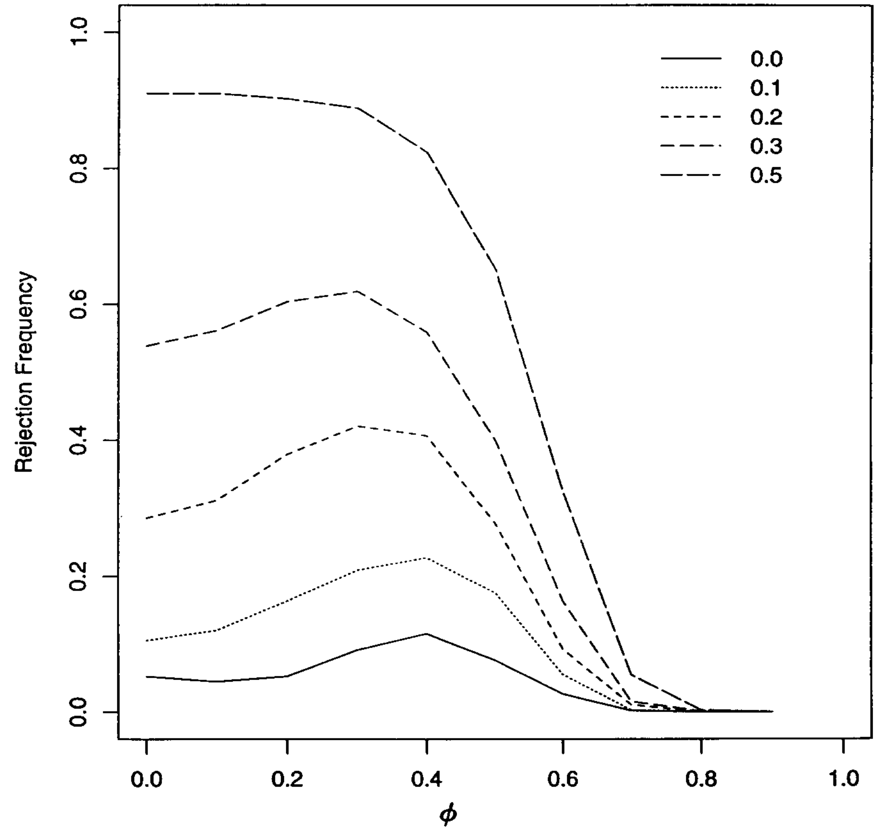

Fig. 2. Power of LM_{ψ}^{*} against SARMA (1, 1), AR dimension, $N = 127$ (dashed lines correspond to $\psi = 0.0$, 0.1, 0.2, 0.3, 0.4 and 0.5).

similar in power to $LM_{\phi\psi}$. The robust tests thus seem more appropriate to test for lag dependence in the presence of error correlation than for the reverse case. Again, this is not unimportant, since the consequences of ignoring lag dependence are more severe.

5. Concluding remarks

In this paper we have proposed simple diagnostic tests for spatial dependence. The proposed tests can be implemented using OLS residuals and are robust to the local presence of a nuisance parameter. Our empirical examples demonstrate the usefulness of the tests. In addition, the results from Monte Carlo experiments show that the proposed tests have very good

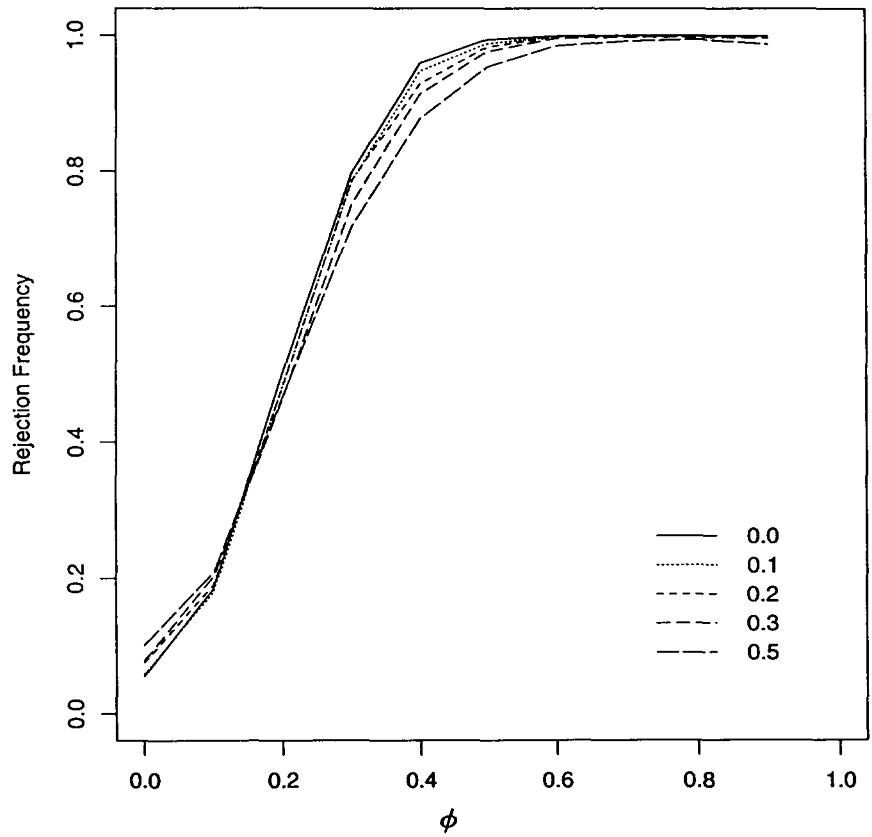

Fig. 3. Power of LM^*_ϕ against SARMA (1, 1), AR dimension, $N = 40$ (dashed lines correspond to $\psi = 0.0$, 0.1, 0.2, 0.3, 0.4 and 0.5).

finite sample properties. In practice, they should be useful in identifying the dependence structure that may be present in spatial regression models.

Anselin (1990) reviewed some other robust approaches to specification testing in the context of spatial econometric models, focusing on techniques that are robust to the presence of heteroskedasticity of an unknown form. For example, following Davidson and MacKinnon (1985), Anselin (1990) considered heteroskedasticity-robust tests for spatial error autocorrelation as well as spatial lag. Essentially these may be viewed as tests for conditional mean specification robust against misspecification of the conditional variance. It is worth pointing out that in the cases considered in Anselin (1990), the information matrix between the parameters of the conditional mean function and those of the conditional variance will be block diagonal. For example, this occurs when the unknown heteroskedasticity is parameterized,

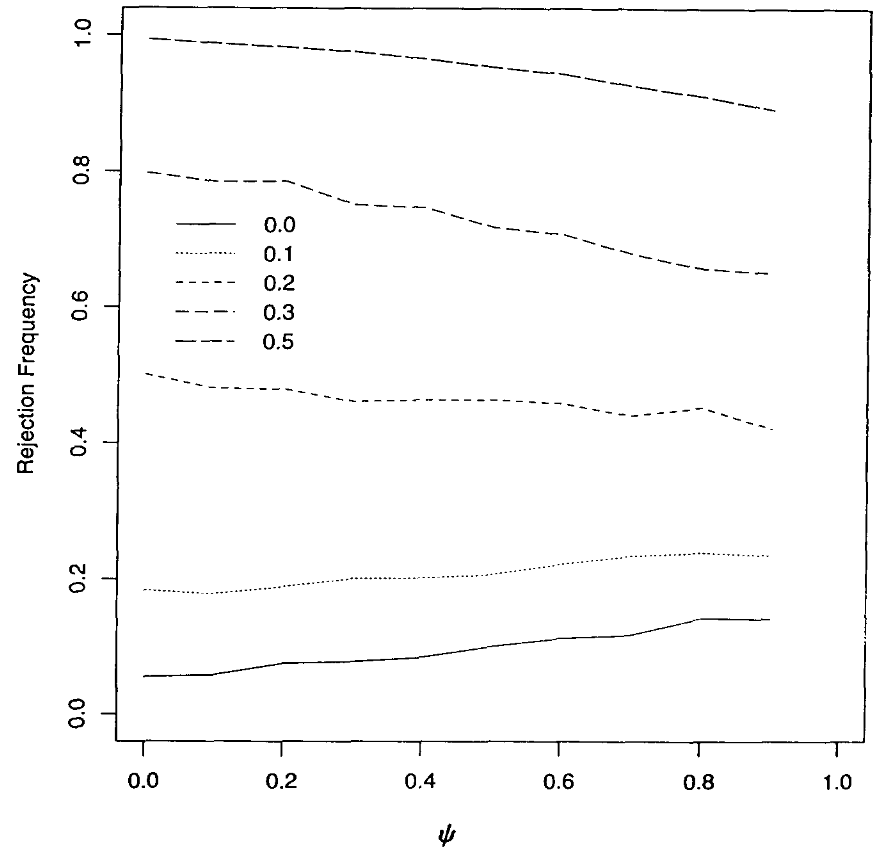

Fig. 4. Power of LM_{ϕ}^{*} against SARMA (1, 1), AR dimension, $N = 40$ (dashed lines correspond to $\phi = 0.0$, 0.1, 0.2, 0.3, 0.4 and 0.5).

as in Breusch and Pagan (1979). In the situations considered here, however, the information matrix is not block diagonal, so that Davidson and MacKinnon's general approach is not applicable. Therefore our proposed tests may be viewed as computationally simple and robust alternatives to some available procedures in spatial econometrics.

Acknowledgements

Anselin's research was supported in part by Grant SES 88-10917 to the National Center for Geographic Information and Analysis (NCGIA) from the U.S. National Science Foundation. Bera gratefully acknowledges financial support from the Department of Economics, the Research Board, and the Bureau of Economic and Business Research of the University of Illinois. The authors wish to thank Joel Horowitz, the referees, and the editor for helpful comments on an earlier version of the paper. However, they retain the possibility for any remaining errors. Programming assistance from Marc Loman is greatly appreciated. An earlier version of the paper was presented at the Midwest Econometric Association Meetings, Iowa City, IA, September 1994.

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