PROBLEM SET # 2

Astro 512 – Spring 2017
Extragalactic Astronomy

Problem 1: The Annoying Surface Density μ

Assume that a galaxy disk has an exponential surface brightness profile $I(r) = I_0 e^{-r/h}$.

a) Derive the conversion from I_0 in solar luminosities per square parsec to μ_0 in magnitudes per square arcsecond. (Hint: It's helpful to think of putting the galaxy at 10 pc.)

b) What is an approximate luminosity surface density that would correspond to the characteristic Freeman (1970) central surface brightness of disks? Cite the sources of any conversions you used.

Problem 2: Surface Brightness Biases

a) Given the disk in Problem 1, what is the luminosity of the disk within r, expressed in terms of I_0 and the exponential scale length h? Please try to put as many terms into dimensionless ratios as possible (i.e. $h(1 + r/h)$ is easier to think about than $(h + r)$).

b) What is the radius r_{50} that contains half the light, expressed in terms of the exponential disk scale length h?

Now consider what type of disks might be targets for the Sloan Digital Sky Survey (SDSS). SDSS measured spectra for sources that had a limiting magnitude of $r_{lim} = 17.77$, above a limiting isophote of $\mu_{r,lim} \approx 26.2$ r-magnitudes per arcsec^2 (Kniazev et al 2004). Note that there is also an explicit surface brightness selection criteria as well — that the surface brightness at the half-light radius is brighter than $\mu_{50} = 24.5$ r-magnitudes per arcsec^2 — but you can ignore it for the sake of this exercise.

c) For a galaxy with a Freeman central surface brightness, out to how many scale lengths would the disk be detected in the r band? Don't forgot to make an appropriate transformation from the B band to the r band.

d) For the galaxy in (c), what fraction of the galaxy's light is detected?

e) What is the absolute r-band magnitude of the galaxy in (c), assuming it has a scale length comparable to the Milky Way's ($h \approx 3$ kpc)? And how many magnitudes of the r-band flux have been "lost"?

e) Considering only the *detectable* flux, out to what distance (in Mpc) would the galaxy from (c) be included in SDSS's spectroscopic sample? Assume Euclidean geometry, rather than worrying about luminosity distance, though the prudent thing to do would be to check that that assumption holds. Is there significant $(1+z)^4$ surface brightness dimming out to this redshift that you should have taken to into account, if you wanted to do this calculation scrupulously?

f) Now generalize through part (e) for galaxies that have the same absolute magnitude as the galaxy from (c), but for central surface brightnesses from one magnitude brighter than the Freeman value, down to 0.25 magnitudes per arcsec^2 brighter than the limiting isophote. As a function of disk central surface brightness, plot (i) the number of scale lengths out to which the galaxy is detected; (ii) the fraction of light detected; (iii) the distance out to which the galaxy would be detected, divided by the distance out to which it would be detected if all the light were visible.

g) Now consider if you were doing a survey over some fixed area of the sky. Assuming that galaxies of some luminosity are equally likely to have any surface brightness in the range you plotted in (f), plot the relative number of the galaxies of that intrinsic luminosity that you would detect as a function of their central surface brightness, normalized to detecting 1,000 Freeman central surface brightness galaxies over the survey area.

We know that the assumptions in (g) are not quite correct. Galaxies of a fixed luminosity do not have uniformly distributed surface brightnesses. Instead, there is a correlation between luminosity and surface brightness, with a (broad) Gaussian distribution of surface brightness at each luminosity.

h) Compared to a galaxy with an absolute magnitude of $M_r \approx -20$, by what factor is the effective survey volume smaller for a dwarf galaxy with $M_r \approx -15$, ignoring any surface brightness effects?

i) By what factor is it smaller when you adjust for the difference in their intrinsic surface brightness (assuming the most likely central surface brightness for each type?

Problem 3: The Vertical Structure of Disks

In class we derived the vertical structure for an isothermal, self-gravitating slab. We will now test some of our assumptions in the derivation, estimate some scales to the problem, and expand it into a different physical situation.

a) In the standard derivation of vertical structure, we assumed that we could neglect any radial accelerations. Show that this is a valid assumption, using the definition of $\nabla \cdot \vec{g}$ in cylindrical coordinates, and assuming that the only radial acceleration is centripital. You may also assume that one is considering a disk in a galaxy with a rotation curve like the Milky Way's.

b) We derived that $\rho(z) = \rho_0 \, \mathrm{sech}^2(\frac{z}{2z_0})$ is the form for the vertical structure of an isothermal self-gravitating disk. Solve for the value of z_0 in terms of the vertical velocity dispersion σ_z and the surface density Σ of the disk. (No need to substitute in actual numbers for various physical constants).

c) Given typical values for σ_z and z_0 for old stars in the thin disk, what is your predicted value for the surface density of the Milky Way's disk near the solar circle? Does this agree with currently accepted values that you find in the literature?

d) If spiral galaxy disks were truly self-gravitating isothermal disks, with a single value of σ_z that applies throughout the disk, what radial dependence would you expect to observe for the scale height z_0?

e) Edge-on late-type spiral disks appear to have a relatively constant disk scale height with radius. Can you speculate as to the source of any discrepancies with your answer for (d)?

f) Can you find observational evidence in the literature for your suggested solution to the discrepancy in (e)?

g) In the outskirts of spiral disks, the mass density within the stellar disk is probably dominated by the dark matter halo, not the stars themselves. Show that the resulting vertical profile of the stars is not sech^2, but is instead gaussian with a scale height $z_0^2 = \sigma_z^2 / 4\pi G \rho_{halo}$, where ρ_{halo} is the local density of the dark matter halo. You may assume that the density of the halo varies little with height within the disk, and thus that ρ_{halo} is approximately constant with z. You may assume that $\rho_{stars}(z)$ is everywhere negligible compared with ρ_{halo}. (Note that the above derivation probably also applies to young stellar populations or the gas layer, which are fully embedded within a thicker, more massive old thin disk.)

h) What is the timescale for vertical oscillation within a disk (assuming a constant background density as in (g), with $\rho \sim 0.1\,\mathrm{M_\odot / pc^3}$), and how many times might a star like the sun be expected to oscillate up and down in a single trip around the center of the galaxy?