Problem 9: S&J 3.18

Create B', B'', and B''*B' DCMs

In[661]:=
```
BpDCM = {{B0'^2 + B1'^2 - B2'^2 - B3'^2,
     2*(B1'*B2' + B0'*B3'),  2*(B1'*B3' - B0'*B2')},
    {2*(B1'*B2' - B0'*B3'), B0'^2 - B1'^2 + B2'^2 - B3'^2,
     2*(B2'*B3' + B0'*B1')}, {2*(B1'*B3' + B0'*B2'),
     2*(B2'*B3' - B0'*B1'), B0'^2 - B1'^2 - B2'^2 + B3'^2}};

BppDCM = {{B0''^2 + B1''^2 - B2''^2 - B3''^2,
     2*(B1''*B2'' + B0''*B3''),  2*(B1''*B3'' - B0''*B2'')},
    {2*(B1''*B2'' - B0''*B3''), B0''^2 - B1''^2 + B2''^2 - B3''^2,
     2*(B2''*B3'' + B0''*B1'')}, {2*(B1''*B3'' + B0''*B2''),
     2*(B2''*B3'' - B0''*B1''), B0''^2 - B1''^2 - B2''^2 + B3''^2}};
BpDCM // MatrixForm
BppDCM // MatrixForm
compositeMat = FullSimplify[BppDCM.BpDCM]  // MatrixForm;
```

Out[663]//MatrixForm=

$$\begin{pmatrix} (B0')^2 + (B1')^2 - (B2')^2 - (B3')^2 & 2\,(B1'\,B2' + B0'\,B3') & 2\,(-B0'\,B2' + B1'\,B3') \\ 2\,(B1'\,B2' - B0'\,B3') & (B0')^2 - (B1')^2 + (B2')^2 - (B3')^2 & 2\,(B0'\,B1' + B2'\,B3') \\ 2\,(B0'\,B2' + B1'\,B3') & 2\,(-B0'\,B1' + B2'\,B3') & (B0')^2 - (B1')^2 - (B2')^2 + (B3')^2 \end{pmatrix}$$

Out[664]//MatrixForm=

$$\begin{pmatrix} (B0'')^2 + (B1'')^2 - (B2'')^2 - (B3'')^2 & 2\,(B1''\,B2'' + B0''\,B3'') & 2\,(-B0''\,B2'' + B1''\,B3'') \\ 2\,(B1''\,B2'' - B0''\,B3'') & (B0'')^2 - (B1'')^2 + (B2'')^2 - (B3'')^2 & 2\,(B0''\,B1'' + B2''\,B3'') \\ 2\,(B0''\,B2'' + B1''\,B3'') & 2\,(-B0''\,B1'' + B2''\,B3'') & (B0'')^2 - (B1'')^2 - (B2'')^2 + (\end{pmatrix}$$

Find B from S&J Eqn 3.98

In[666]:=
```
BppMat = {{B0'', -B1'', -B2'', -B3''}, {B1'', B0'', B3'', -B2''},
    {B2'', -B3'', B0'', B1''}, {B3'', B2'', -B1'', B0''}} ;
B = BppMat.{{B0'}, {B1'}, {B2'}, {B3'}};
BppMat // MatrixForm
B // MatrixForm
```

Out[668]//MatrixForm=

$$\begin{pmatrix} B0'' & -B1'' & -B2'' & -B3'' \\ B1'' & B0'' & B3'' & -B2'' \\ B2'' & -B3'' & B0'' & B1'' \\ B3'' & B2'' & -B1'' & B0'' \end{pmatrix}$$

Out[669]//MatrixForm=

$$\begin{pmatrix} B0'\,B0'' - B1'\,B1'' - B2'\,B2'' - B3'\,B3'' \\ B1'\,B0'' + B0'\,B1'' - B3'\,B2'' + B2'\,B3'' \\ B2'\,B0'' + B3'\,B1'' + B0'\,B2'' - B1'\,B3'' \\ B3'\,B0'' - B2'\,B1'' + B1'\,B2'' + B0'\,B3'' \end{pmatrix}$$

Verify DCM B''*B' has terms that fit the Shepard's Method Bi*Bj equations

For some reason, I had to have the "1" in the front when I tried to access the composite DCM. Not sure why.

In[670]:=
```
FullSimplify[(compositeMat[[1, 2, 3]] - compositeMat[[1, 3, 2]]) / 4 - B[[1]] * B[[2]]]
FullSimplify[(compositeMat[[1, 3, 1]] - compositeMat[[1, 1, 3]]) / 4 - B[[1]] * B[[3]]]
FullSimplify[(compositeMat[[1, 1, 2]] - compositeMat[[1, 2, 1]]) / 4 - B[[1]] * B[[4]]]
FullSimplify[(compositeMat[[1, 2, 3]] + compositeMat[[1, 3, 2]]) / 4 - B[[3]] * B[[4]]]
FullSimplify[(compositeMat[[1, 3, 1]] + compositeMat[[1, 1, 3]]) / 4 - B[[4]] * B[[2]]]
FullSimplify[(compositeMat[[1, 1, 2]] + compositeMat[[1, 2, 1]]) / 4 - B[[2]] * B[[3]]]
```

Out[670]= {0}

Out[671]= {0}

Out[672]= {0}

Out[673]= {0}

Out[674]= {0}

Out[675]= {0}

Create B'*B'' DCM

In[676]:=
```
compositeMat = FullSimplify[BpDCM.BppDCM]  // MatrixForm;
```

Find B from S&J Eqn 3.99

In[677]:=
```
BppMat = {{B0'', -B1'', -B2'', -B3''}, {B1'', B0'', -B3'', B2''},
    {B2'', B3'', B0'', -B1''}, {B3'', -B2'', B1'', B0''}} ;
B = BppMat.{{B0'}, {B1'}, {B2'}, {B3'}};
BppMat // MatrixForm
B // MatrixForm
```

Out[679]//MatrixForm=

$$\begin{pmatrix} B0'' & -B1'' & -B2'' & -B3'' \\ B1'' & B0'' & -B3'' & B2'' \\ B2'' & B3'' & B0'' & -B1'' \\ B3'' & -B2'' & B1'' & B0'' \end{pmatrix}$$

Out[680]//MatrixForm=

$$\begin{pmatrix} B0'\,B0'' - B1'\,B1'' - B2'\,B2'' - B3'\,B3'' \\ B1'\,B0'' + B0'\,B1'' + B3'\,B2'' - B2'\,B3'' \\ B2'\,B0'' - B3'\,B1'' + B0'\,B2'' + B1'\,B3'' \\ B3'\,B0'' + B2'\,B1'' - B1'\,B2'' + B0'\,B3'' \end{pmatrix}$$

Verify DCM B'*B'' has terms that fit the Shepard's Method Bi*Bj equations

In[681]:=
```
FullSimplify[(compositeMat[[1, 2, 3]] - compositeMat[[1, 3, 2]]) / 4 - B[[1]] * B[[2]]]
FullSimplify[(compositeMat[[1, 3, 1]] - compositeMat[[1, 1, 3]]) / 4 - B[[1]] * B[[3]]]
FullSimplify[(compositeMat[[1, 1, 2]] - compositeMat[[1, 2, 1]]) / 4 - B[[1]] * B[[4]]]
FullSimplify[(compositeMat[[1, 2, 3]] + compositeMat[[1, 3, 2]]) / 4 - B[[3]] * B[[4]]]
FullSimplify[(compositeMat[[1, 3, 1]] + compositeMat[[1, 1, 3]]) / 4 - B[[4]] * B[[2]]]
FullSimplify[(compositeMat[[1, 1, 2]] + compositeMat[[1, 2, 1]]) / 4 - B[[2]] * B[[3]]]
```

Out[681]= {0}

Out[682]= {0}

Out[683]= {0}

Out[684]= {0}

Out[685]= {0}

Out[686]= {0}