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HW1

Problem 1) Verify:

a) $\delta_{ii} = 3$

$$\delta_{ii} = \sum_{i=1}^{3} \delta_{ii} = \delta_{11} + \delta_{22} + \delta_{33}$$
$$= 1 + 1 + 1 \quad \leftarrow \delta_{ij} = 1 \text{ if } i = j$$
$$= 3 \text{ QED}$$

b) $\delta_{ij}\delta_{ij} = \delta_{ii}$

$$\sum_{i=1}^{3}\sum_{j=1}^{3} \delta_{ij}\delta_{ij} = \delta_{11} + \delta_{12} + \delta_{13} + \delta_{21} + \delta_{22} + \delta_{23} + \delta_{31} + \delta_{32} + \delta_{33}$$
$$= 1 + 0 + 0 + 0 + 1 + 0 + 0 + 0 + 1$$
$$= 3 = \delta_{ii} \text{ QED}$$

c) $\delta_{ij}\delta_{jk} = \delta_{ik}$

$\delta_{ij}\delta_{jk} = 0$ if $i \neq j$. when $i = j$, $(1)\cdot\delta_{ik} = \delta_{ij}\delta_{jk} = \delta_{jk}$ QED

d) $\varepsilon_{ijk}\varepsilon_{ijk} = 6$

$$\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3} \varepsilon_{ijk}\varepsilon_{ijk} = \varepsilon_{111}^2 + \varepsilon_{112}^2 + \varepsilon_{113}^2 + \varepsilon_{121}^2 + \varepsilon_{122}^2 + \cancelto{1\cdot 1}{\varepsilon_{123}^2} + \varepsilon_{131}^2 + \cancelto{-1\cdot -1}{\varepsilon_{132}^2} + \varepsilon_{133}^2$$
$$+ \varepsilon_{211}^2 + \varepsilon_{212}^2 + \cancelto{-1\cdot -1}{\varepsilon_{213}^2} + \varepsilon_{221}^2 + \varepsilon_{222}^2 + \varepsilon_{223}^2 + \cancelto{1\cdot 1}{\varepsilon_{231}^2} + \varepsilon_{232}^2 + \varepsilon_{233}^2$$
$$+ \varepsilon_{311}^2 + \cancelto{+1\cdot +1}{\varepsilon_{312}^2} + \varepsilon_{313}^2 + \cancelto{-1\cdot -1}{\varepsilon_{321}^2} + \varepsilon_{322} + \varepsilon_{323} + \varepsilon_{331} + \varepsilon_{332} + \varepsilon_{333}$$
$$= 1 + 1 + 1 + 1 + 1 + 1 = 6 \text{ QED}$$

e) $a_i a_j \varepsilon_{ijk} = 0$

$$\sum_{i=1}^{3}\sum_{j=1}^{3} a_i a_j \varepsilon_{ijk} = a_1 a_2 \varepsilon_{12k} + a_1 a_3 \varepsilon_{13k}$$
$$+ a_2 a_1 \varepsilon_{21k} + a_2 a_3 \varepsilon_{23k}$$
$$+ a_3 a_1 \varepsilon_{31k} + a_3 a_2 \varepsilon_{32k} = \cancel{A}$$

for k that $k \neq i$ & $k \neq j$,

$\varepsilon_{12k} = -\varepsilon_{21k}$

$\varepsilon_{13k} = -\varepsilon_{31k}$

$\varepsilon_{23k} = -\varepsilon_{32k}$

$\therefore \cancel{A} = 0$ QED

Problem 2) prove $\varepsilon_{ijk}\varepsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}$ using identity for vector triple product

$(\vec{a}\times\vec{b})_i = \varepsilon_{ijk} a_j b_k$

let $\vec{x} = \vec{a}\times(\vec{b}\times\vec{c})$ (1)

$x_j = \varepsilon_{jki}\, a_k (\vec{b}\times\vec{c})_i$

$= \varepsilon_{jki}\, a_k (\varepsilon_{imn} b_m c_n)$

$= \varepsilon_{ijk}\, a_k (\varepsilon_{imn} b_m c_n)$ $\leftarrow$ cyclic permutation: $\varepsilon_{ijk} = \varepsilon_{jki} = \varepsilon_{kij}$

$= \varepsilon_{ijk}\varepsilon_{imn} a_k b_m c_n$

Substitute the ε-δ identity

$x_j = (\underbrace{\delta_{jm}\delta_{kn}}_{\substack{\text{non-zero if}\\ j=m,\ k=n}} - \delta_{jn}\delta_{km})\, a_k b_m c_n$

$x_j = b_j \cdot a_k c_k - c_j b_m a_m$ $\leftarrow$ repeated indices: $\vec{a}\cdot\vec{c} = \sum_{k=1}^{3} a_k c_k = a_k c_k$

$= (\vec{b}(\vec{a}\cdot\vec{c}))_j - (\vec{c}(\vec{a}\cdot\vec{b}))_j$ (2)

Since (1) = (2) by the vector triple product identity, the ε-δ identity used in the proof is proven.

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Problem 3) prove: $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a}\cdot(\vec{c}\times\vec{d})]\vec{b} - [\vec{b}\cdot(\vec{c}\times\vec{d})]\vec{a}$

using vector triple product identity:

$$
\begin{aligned}
(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) &= -(\vec{c}\times\vec{d})\times(\vec{a}\times\vec{b}) \\
&= -\left[\vec{a}\,(\vec{c}\times\vec{d}\cdot\vec{b}) - \vec{b}\,(\vec{c}\times\vec{d}\cdot\vec{a})\right] \\
&= \vec{b}\left[(\vec{c}\times\vec{d})\cdot\vec{a}\right] - \vec{a}\left[(\vec{c}\times\vec{d})\cdot\vec{b}\right] \\
&= [\vec{a}\cdot(\vec{c}\times\vec{d})]\vec{b} - [\vec{b}\cdot(\vec{c}\times\vec{d})]\vec{a} \quad \underline{QED}
\end{aligned}
$$

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Problem 4) $\vec{a} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$, $\vec{r} = \vec{b} - \vec{a} = \underline{\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}}$

find length, direction cosines.

$l = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{9}$

$\boxed{l = 3}$

$$\boxed{\begin{aligned} \cos(\alpha_1) &= \frac{2}{3} \\ \cos(\alpha_2) &= -\frac{2}{3} \\ \cos(\alpha_3) &= \frac{1}{3} \end{aligned}}\begin{aligned} &= \frac{\vec{r} \times \hat{i}}{l} \\ &= \frac{\vec{r} \times \hat{j}}{l} \\ &= \frac{\vec{r} \times \hat{k}}{l} \end{aligned}$$

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Problem 5) $\vec{a} = 3\hat{\imath} - 4\hat{k}$

$\vec{b} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$

a) orthogonal projection of $\vec{a}$ onto $\vec{b}$:

$$\hat{b} = \frac{2}{3}\hat{\imath} - \frac{2}{3}\hat{\jmath} + \frac{1}{3}\hat{k}$$

$$\text{projection} = \vec{a}\cdot\hat{b} = \frac{6}{3} - 0 - \frac{4}{3} = \boxed{\frac{2}{3}}$$

b) angle between the vectors:

$$|\vec{a}| = \sqrt{9+16} = 5$$

$$\vec{a}\cdot\hat{b} = |\vec{a}|\cos(\theta) = \frac{2}{3}$$

$$\Rightarrow \cos\theta = \frac{2}{15}$$

$$\boxed{\begin{gathered}\theta = \cos^{-1}\left(\frac{2}{15}\right) \\ \approx 1.44\,\text{rad}\end{gathered}}$$

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Problem 6: $\vec{e}_1 = \frac{\sqrt{3}}{4}\hat{\imath} + \frac{1}{4}\hat{\jmath}$

$\vec{e}_2 = \frac{1}{2}\hat{\imath} + \frac{3}{2}\hat{\jmath}$

$\vec{e}_3 = \hat{k}$

a. Define the dual/reciprocal basis

first, the scalar triple product: $[\vec{e}_1 \vec{e}_2 \vec{e}_3] = (\vec{e}_1 \times \vec{e}_2) \cdot \vec{e}_3$

$$= \left[(0)\hat{\imath} - (0)\hat{\jmath} + \left(\frac{3\sqrt{3}}{8} - \frac{1}{8}\right)\hat{k}\right] \cdot [1\hat{k}]$$

$$= \frac{3\sqrt{3}-1}{8}$$

$$\vec{e}^{\,1} = \frac{\vec{e}_2 \times \vec{e}_3}{[\vec{e}_1 \vec{e}_2 \vec{e}_3]} = \frac{\frac{3}{2}\hat{\imath} - \frac{1}{2}\hat{\jmath}}{\left(\frac{3\sqrt{3}-1}{8}\right)} = \frac{3 \cdot 4}{3\sqrt{3}-1}\hat{\imath} - \frac{4}{3\sqrt{3}-1}\hat{\jmath} = \boxed{\frac{12}{3\sqrt{3}-1}\hat{\imath} - \frac{4}{3\sqrt{3}-1}\hat{\jmath} = \vec{e}^{\,1}}$$

$$\vec{e}^{\,2} = \frac{\vec{e}_3 \times \vec{e}_1}{[\vec{e}_1 \vec{e}_2 \vec{e}_3]} = \frac{-\frac{1}{4}\hat{\imath} + \frac{\sqrt{3}}{4}\hat{\jmath}}{\frac{3\sqrt{3}-1}{8}} = \boxed{-\frac{2}{3\sqrt{3}-1}\hat{\imath} + \frac{2\sqrt{3}}{3\sqrt{3}-1}\hat{\jmath} = \vec{e}^{\,2}}$$

$$\vec{e}^{\,3} = \frac{\vec{e}_1 \times \vec{e}_2}{\left(\frac{3\sqrt{3}-1}{8}\right)} = \boxed{1\hat{k} = \vec{e}^{\,3}}$$

b. Determine the norms:

$$|\vec{e}_1| = \sqrt{\frac{3}{16} + \frac{1}{16}} = \sqrt{\frac{4}{16}} = \sqrt{\frac{1}{4}} = \boxed{\frac{1}{2} = |\vec{e}_1|}$$

$$|\vec{e}_2| = \sqrt{\frac{1}{4} + \frac{9}{4}} = \sqrt{\frac{10}{4}} = \sqrt{\frac{5}{2}} = \boxed{\frac{\sqrt{10}}{2} = |\vec{e}_2|}$$

$$\boxed{|\vec{e}_3| = 1}$$

$$|\vec{e}^{\,1}| = \sqrt{\frac{144+16}{27-6\sqrt{3}+1}} = \sqrt{\frac{160}{27-6\sqrt{3}+1}} = \boxed{\frac{\sqrt{4480-960\sqrt{3}}}{28-6\sqrt{3}} = |\vec{e}^{\,1}|}$$

$$|\vec{e}^{\,2}| = \sqrt{\frac{4+12}{27-6\sqrt{3}+1}} = \sqrt{\frac{18}{27-6\sqrt{3}+1}} = \boxed{\frac{\sqrt{504-108\sqrt{3}}}{28-6\sqrt{3}} = |\vec{e}^{\,2}|}$$

$$\boxed{|\vec{e}^{\,3}| = 1}$$

Cont. →

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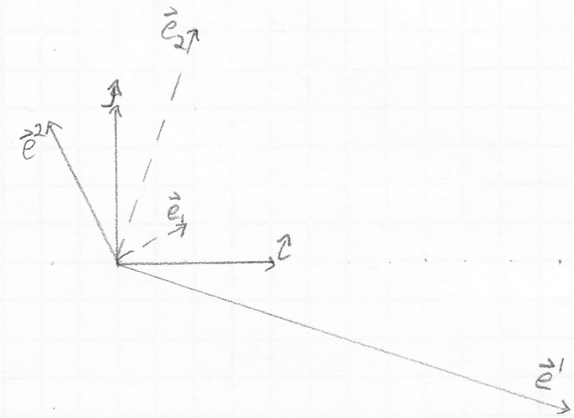
Problem 6c)



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Problem 7) Corresponding to cogredient basis in G, $a^1=1$, $a^2=2$, $a^3=3$

find $\underline{a_1, a_2, a_3}$

$a^i = \vec{a}\cdot\vec{e}^i$
$a_i = \vec{a}\cdot\vec{e}_i$, $\vec{a} = a_i\vec{e}^i$
$\vec{a} = a^i\vec{e}_i$

$\vec{e}_1\cdot\vec{e}_2 = \frac{\sqrt{3}}{8} + \frac{3}{8} = \frac{3+\sqrt{3}}{8}$

$a_i = (a^j\vec{e}_j)\cdot\vec{e}_i = a^j(\vec{e}_j\cdot\vec{e}_i)$

$a_1 = a^1(\vec{e}_1\cdot\vec{e}_1) + a^2(\vec{e}_2\cdot\vec{e}_1) + a^3\cancelto{0}{(\vec{e}_3\cdot\vec{e}_1)}$

$= 1\cdot\frac{1}{4} + 2\cdot\frac{3+\sqrt{3}}{8} + 3\cdot 0$

$\boxed{a_1 = 1 + \frac{\sqrt{3}}{8}}$

$a_2 = a^1(\vec{e}_1\cdot\vec{e}_2) + a^2(\vec{e}_2\cdot\vec{e}_2) + a^3\cancelto{0}{(\vec{e}_3\cdot\vec{e}_2)}$

$= 1\cdot\frac{3+\sqrt{3}}{8} + \cancel{2}\cdot\frac{\cancelto{5}{10}}{4\cancel{2}} + 0$

$\boxed{a_2 = 5 + \frac{3+\sqrt{3}}{8}}$

$a_3 = a^1\cancelto{0}{(\vec{e}_1\cdot\vec{e}_3)} + a^2\cancelto{0}{(\vec{e}_2\cdot\vec{e}_3)} + a^3\cancelto{1}{(\vec{e}_3\cdot\vec{e}_3)}$

$\boxed{a_3 = a^3 = 1}$

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Problem 8: construct orthonormal basis from:

$$\vec{e}_1 = \frac{\sqrt{3}}{4}\hat{\imath} + \frac{1}{4}\hat{\jmath}$$

$$\vec{e}_2 = \frac{1}{2}\hat{\imath} + \frac{3}{2}\hat{\jmath}$$

$$\vec{e}_3 = \hat{k}$$

$$\hat{e}_1 = \frac{\vec{e}_1}{|\vec{e}_1|} = \boxed{\frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath} = \hat{e}_1}$$

$$\vec{e}_2' = \vec{e}_2 - \alpha\hat{e}_1$$

$$\alpha = \hat{e}_1 \cdot \vec{e}_2 = \frac{\sqrt{3}}{4} + \frac{3}{4} = \frac{3+\sqrt{3}}{4}$$

$$\vec{e}_2' = \left(\frac{1}{2} - \frac{(3+\sqrt{3})}{4}\cdot\frac{\sqrt{3}}{2}\right)\hat{\imath} + \left(\frac{3}{2} - \frac{(3+\sqrt{3})}{4}\cdot\frac{1}{2}\right)\hat{\jmath}$$

$$= \frac{4-3\sqrt{3}-3}{8}\hat{\imath} + \frac{12-3-\sqrt{3}}{8}\hat{\jmath} = \frac{1-3\sqrt{3}}{8}\hat{\imath} + \frac{9-\sqrt{3}}{8}\hat{\jmath}$$

$$|\vec{e}_2'| = \sqrt{\frac{1-6\sqrt{3}+27+81-18\sqrt{3}+3}{64}} = \frac{\sqrt{112-24\sqrt{3}}}{8} = \frac{2\sqrt{28-6\sqrt{3}}}{8} = \frac{2}{8}\sqrt{(1+(3\sqrt{3})^2-6\sqrt{3}}$$

$$= \frac{2}{8}\cdot(1-3\sqrt{3})$$

$$\hat{e}_2 = \frac{\vec{e}_2'}{|\vec{e}_2'|} = \frac{1}{2}\hat{\imath} + \frac{9-\sqrt{3}}{2(1-3\sqrt{3})}\cdot\frac{1+3\sqrt{3}}{1+3\sqrt{3}}\hat{\jmath}$$

$$= \frac{1}{2}\hat{\imath} + \frac{\cancel{9}+27\sqrt{3}-\sqrt{3}-\cancel{9}}{2\cdot(1-27)} = \boxed{\frac{1}{2}\hat{\imath} - \frac{\sqrt{3}}{2}\hat{\jmath} = \hat{e}_2}$$

since $\hat{e}_1$ & $\hat{e}_2$ have no $\hat{k}$ component, $\boxed{\hat{e}_3 = \vec{e}_3 = \hat{k}}$

Part B

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Problem 1) $\vec{e}_1 = 3i + 0j$ $\quad \bar{e}_1 = 1i + 0.5j$

$\vec{e}_2 = 1i + 2j$ $\quad \bar{e}_2 = -2i + 1.5j$

$\vec{e}^j$: $\vec{e}_i \cdot \vec{e}^j = \delta_i^j$

$\vec{e}^1$: $3\cdot(\vec{e}^1)_1 + 0(\vec{e}^1)_2 = 1 \Rightarrow (\vec{e}^1)_1 = \frac{1}{3}$

$1\cdot\frac{1}{3} + 2\cdot(\vec{e}^1)_2 = 0 \Rightarrow (\vec{e}^2)_2 = -\frac{1}{6}$ $\Rightarrow \boxed{\vec{e}^1 = \begin{bmatrix}\frac{1}{3}\\ -\frac{1}{6}\end{bmatrix}}$

$\vec{e}^2$: $3\cdot(\vec{e}^2)_1 + 0(\vec{e}^2)_2 = 0 \Rightarrow (\vec{e}^2)_1 = 0$

$0 + 2\cdot(\vec{e}^2)_2 = 1 \Rightarrow (\vec{e}^2)_2 = \frac{1}{2}$ $\Rightarrow \boxed{\vec{e}^2 = \begin{bmatrix}0\\ \frac{1}{2}\end{bmatrix}}$

$\bar{e}^1$: $1(\bar{e}^1)_1 + 0.5(\bar{e}^1)_2 = 1$

$-2(\bar{e}^1)_1 + 1.5(\bar{e}^1)_2 = 0$ $\Rightarrow 2.5(\bar{e}^1)_2 = 2$

$(\bar{e}^1)_2 = \frac{4}{5}$

$(\bar{e}^1)_1 = 1 - \frac{1}{2}\cdot\frac{4}{5} = \frac{3}{5}$ $\Rightarrow \boxed{\bar{e}^1 = \begin{bmatrix}\frac{4}{5}\\ \frac{3}{5}\end{bmatrix}}$

$\bar{e}^2$: $1(\bar{e}^2)_1 + 0.5(\bar{e}^2)_2 = 0$

$-2(\bar{e}^2)_1 + 1.5(\bar{e}^2)_2 = 1$ $\Rightarrow 2.5(\bar{e}^2)_2 = 1 \Rightarrow (\bar{e}^2)_2 = \frac{2}{5}$

$(\bar{e}^2)_1 = 0 - \frac{1}{2}\cdot\frac{2}{5} = \frac{1}{5}$ $\Rightarrow \boxed{\bar{e}^2 = \begin{pmatrix}\frac{2}{5}\\ \frac{1}{5}\end{pmatrix}}$

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Problem 2) $\vec{a} = \frac{1}{3}\vec{e}_1 + \frac{1}{2}\vec{e}_2 = a^i\vec{e}_i$ a) find $a_i, \bar{a}_s, \bar{a}^r$

$a_i = a^j(\vec{e}_j \cdot \vec{e}_i)$

$a_1 = a^1(\vec{e}_1 \cdot \vec{e}_1) + a^2(\vec{e}_2 \cdot \vec{e}_1)$

$= \frac{1}{3}(9+0) + \frac{1}{2}(3+0) = 3 + 3/2 = \underline{4.5}$

$a_2 = a^1(\vec{e}_1 \cdot \vec{e}_2) + a^2(\vec{e}_2 \cdot \vec{e}_2)$

$= \frac{1}{3}(3+0) + \frac{1}{2}(1+4) = 1 + 5/2 = \underline{3.5}$

$$\boxed{a_i = \begin{bmatrix} 4.5 \\ 3.5 \end{bmatrix}}$$

cogredient law for $a_j \to \bar{a}_s$

$\bar{e}_s = (\vec{e}^{\,i} \cdot \bar{e}_s)\vec{e}_i = \alpha_s^i \vec{e}_i$

$$\alpha_s^i = \begin{bmatrix} \vec{e}^{\,1} \cdot \bar{e}_1 & \vec{e}^{\,2} \cdot \bar{e}_1 \\ \vec{e}^{\,1} \cdot \bar{e}_2 & \vec{e}^{\,2} \cdot \bar{e}_2 \end{bmatrix} = \begin{matrix} \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{6} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} & \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} \\ 0.5 \end{bmatrix} \\ \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{6} \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1.5 \end{bmatrix} & \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1.5 \end{bmatrix} \end{matrix} = \begin{matrix} \frac{4}{12} - \frac{1}{12} & \frac{1}{4} \\ -\frac{2}{3} - \frac{3}{2} \cdot \frac{1}{6} & \frac{1}{2} \cdot \frac{3}{2} \\ -\frac{8}{12} \; -\frac{3}{12} & \end{matrix} = \underline{\begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{11}{12} & \frac{3}{4} \end{bmatrix}}$$

$$\bar{a}_s = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{11}{12} & \frac{3}{4} \end{bmatrix} \begin{bmatrix} \frac{9}{2} \\ \frac{7}{2} \end{bmatrix} = \begin{bmatrix} \frac{9}{8} + \frac{7}{8} \\ \frac{33}{8} + \frac{21}{8} \end{bmatrix} = \begin{bmatrix} 2 \\ \frac{54}{8} \end{bmatrix} = \boxed{\begin{bmatrix} 2 \\ \frac{27}{8} \end{bmatrix} = \bar{a}_s}$$

contragredient law for $a^i \to \bar{e}^r$

$\bar{e}^r = (\vec{e}_j \cdot \bar{e}^r)\vec{e}^{\,j} = b_j^r \vec{e}^{\,j}$

$$= \begin{matrix} \vec{e}_1 \cdot \bar{e}^1 & \vec{e}_2 \cdot \bar{e}^1 \\ \vec{e}_1 \cdot \bar{e}^2 & \vec{e}_2 \cdot \bar{e}_2 \end{matrix} = \begin{matrix} \begin{bmatrix} 3 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{4}{5} \\ \frac{3}{5} \end{bmatrix} & \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 4/5 \\ 3/5 \end{bmatrix} \\ \begin{bmatrix} 3 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{5} \\ \frac{1}{5} \end{bmatrix} & \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2/5 \\ 1/5 \end{bmatrix} \end{matrix} = \begin{bmatrix} \frac{12}{5} & \frac{4}{5} + \frac{6}{5} \\ \frac{6}{5} & \frac{2}{5} + \frac{2}{5} \end{bmatrix} = \underline{\begin{bmatrix} \frac{12}{5} & 2 \\ \frac{6}{5} & \frac{4}{5} \end{bmatrix}}$$

$$\bar{a}^r = \begin{bmatrix} \frac{12}{5} & 2 \\ \frac{6}{5} & \frac{4}{5} \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{4}{5} + 1 \\ \frac{2}{5} + \frac{2}{5} \end{bmatrix} = \boxed{\begin{bmatrix} \frac{9}{5} \\ \frac{4}{5} \end{bmatrix} = \bar{a}^r}$$

b) from cogredient law above,

$\bar{e}_1 = \frac{1}{4}\vec{e}_1 + \frac{1}{4}\vec{e}_2$

$\bar{e}_2 = \frac{11}{2}\vec{e}_1 + \frac{3}{4}\vec{e}_2$

c) from contragredient law above,

$\bar{e}^1 = \frac{12}{5}\vec{e}^{\,1} + 2\vec{e}^{\,2}$

$\bar{e}^2 = \frac{6}{5}\vec{e}^{\,1} + \frac{4}{5}\vec{e}^{\,2}$