Problem Set 7
Dynamic Programming
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Specification of the economic environment

- State variable: A oil company stores w tons oil in the warehouse.

- Control variable: The company can sell s tons of oil

The more the firm sells, the lower the price will be. Therefore, the price is conditional on the amount of oil sales. If the company sells s tons, the price will be

$$price = \frac{100}{e^S} \tag{1}$$

Then, he can pledge the rest w' tons of oil to fund investment, and the return is inv dollars per ton. Each year, they can produce q tons of oil, and we assume this system is infinite. This information are known at the moment of the transaction.

Therefore, the profit function will be

$$\pi(s) = s \times p + inv \times w' \tag{2}$$

where

- $w' = (w - s) \times (1 + R)$

- R and inv are known parameters

The value function is

$$V(w) = \max_{s} \pi(s, w') + \beta V(w'), \forall w \in [0, \hat{w}] \tag{3}$$

substitute in the law of motion: w' = w - s

$$V(w) = \max_{0 \leq w' \leq w} \pi(w - w', w') + \beta V(w'), \forall w \tag{4}$$

where

- β is the known discount factor

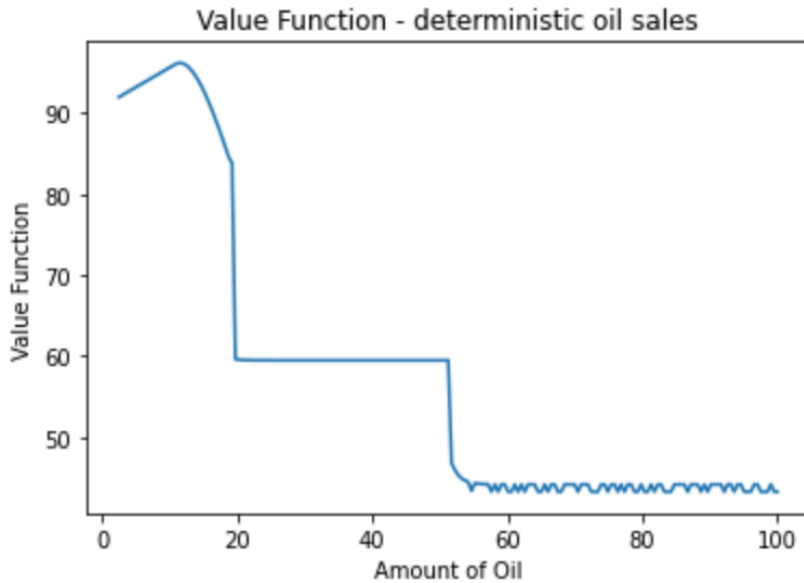- The value function, V(w), and the policy function $s = \phi(s)$ are unknown

Bellman Equation

$$V(w) = \max_{0 \leq w' \leq w} \pi(w, w - w') + \beta V(w'), \forall w \tag{5}$$
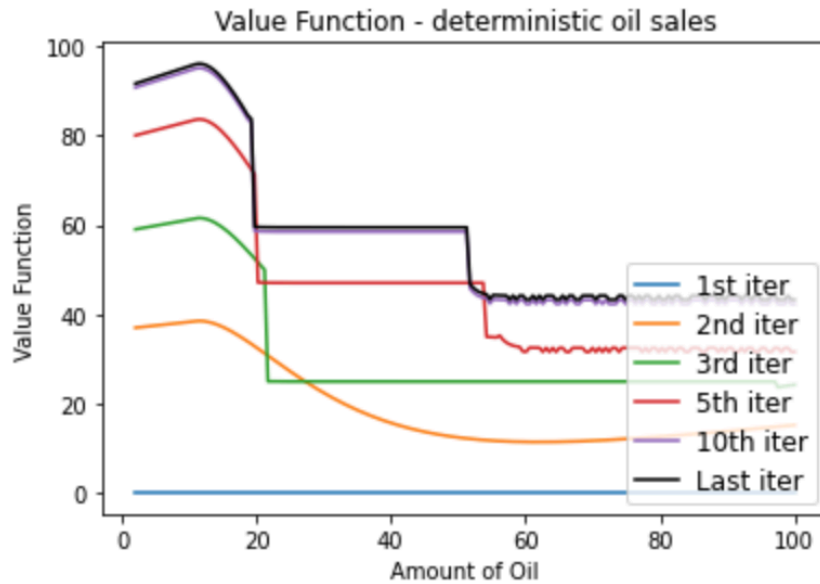
FOCs in this infinite horizon problem should be:
$$\frac{dV}{dw'} = -\pi'(w, w - w') + \beta V'(w') = 0 \tag{6}$$
$$\frac{dV}{dw} = -\pi'(w, w - w') \tag{7}$$

Output


This figure shows the solution for the value function. This demonstrates the maximum profit that can be generated for different amount of oil.

Value Function - deterministic oil sales

This figure shows the value function in different iterations. The blue line is the value function for the initial guess. As the purple line is very close the black one, this means the iteration almost converges since 10th iteration.

Policy Function, sales - deterministic oil sales
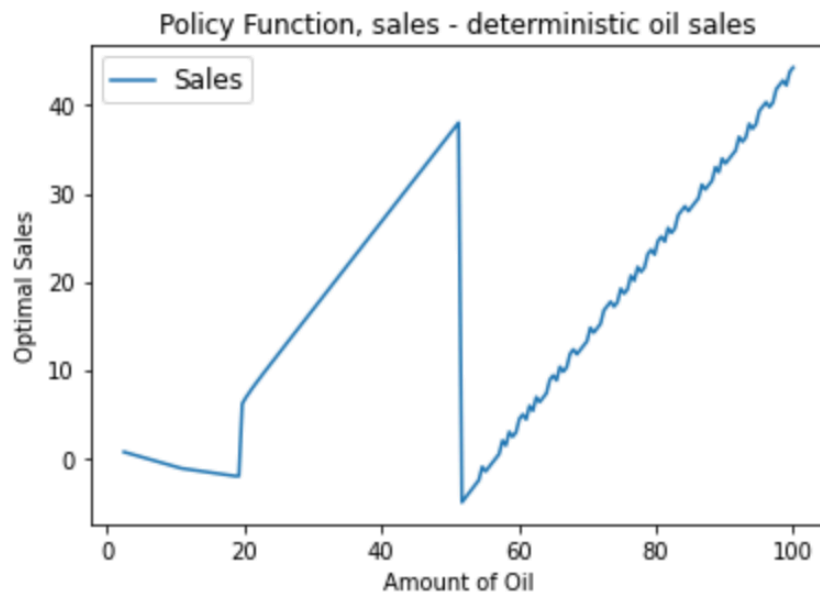
This figure is the policy function, which shows the optimal amount to sell at different level of oil

Policy Function, storage - deterministic oil sales
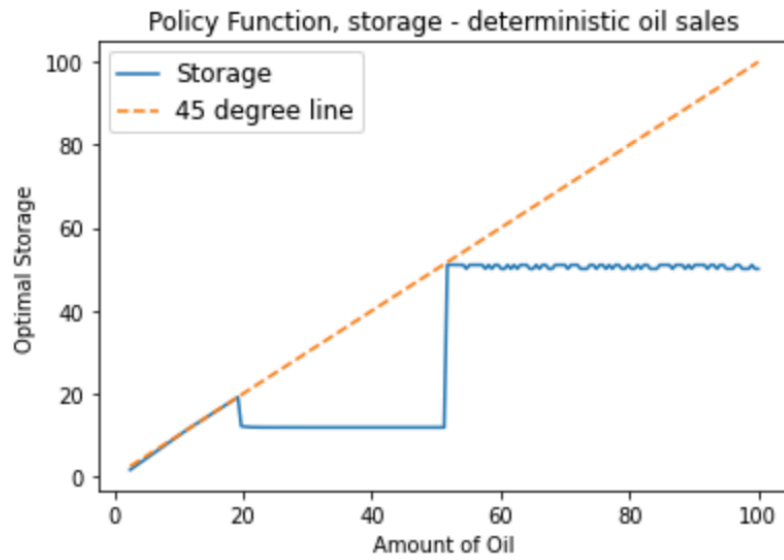
This figure is the policy function for storage, which shows the optimal amount to store at different level of oil