AUBURN UNIVERSITY
Department of Electrical and Computer Engineering

ELEC 7450

Midterm Exam

Wednesday, March 6, 2019
50 Minutes

General Instructions

1. Put your name on the line below.
2. Please put all of your work on the exam itself. You may use the backs of the pages if necessary, but if you do, please indicate that clearly. Otherwise, you will not receive credit for the work done on the backs of pages.
3. This is a *closed book, closed notes* exam. You may, however, use two handwritten 3"×5" cards.
4. Please note the value of each problem. Do not spend too much time on any one problem.
5. Please be as neat and well-organized as possible.

Name: Solution

Problem	*Value*	*Score*
1	15	
2	20	
3	20	
4	15	
5	30	
Total	100	

Problem 1:

(a) Let $F(\omega_x, \omega_y) = 3\exp(-2\omega_x - \omega_y)u(\omega_x, \omega_y) + \delta(\omega_x - 2\pi, \omega_y - \pi)$. Determine the inverse Fourier transform of $F(\omega_x, \omega_y)$.

$$f(x,y) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(\omega_x,\omega_y)\exp\{j(\omega_x x + \omega_y y)\}\,d\omega_x\,d\omega_y$$

$$= \frac{1}{4\pi^2}\int_0^{\infty}\int_0^{\infty} 3\exp\{\omega_x(-2+jx)\}\exp\{\omega_y(-1+jy)\}\,d\omega_x\,d\omega_y + \underbrace{\frac{1}{4\pi^2}\exp\{j(2\pi x + \pi y)\}}_{\text{sifting property}}$$

$$= \frac{1}{4\pi^2}\frac{3}{-2+jx}\exp\{\omega_x(-2+jx)\}\Big|_0^{\infty} \times \frac{1}{-1+jy}\exp\{\omega_y(1+jy)\}\Big|_0^{\infty} + \text{''}$$

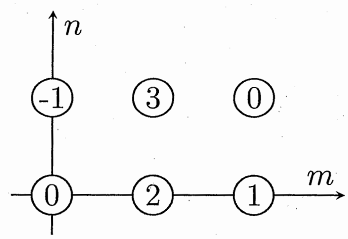
$$= \frac{3}{4\pi^2}\frac{1}{(-2+jx)(-1+jy)} + \frac{1}{4\pi^2}\exp\{j(2\pi x + \pi y)\}$$

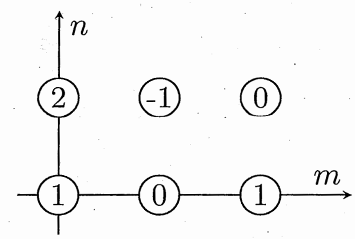
(b) An image has a maximum frequency in the x direction of 4π radians/mm and a maximum frequency in the y direction of 3π radians/mm. Determine the sampling requirements on the image that will avoid aliasing.

$$\frac{2\pi}{\Delta x} > 2(4\pi) \qquad \frac{2\pi}{\Delta y} > 2(3\pi)$$

$$\Delta x < \frac{2\pi}{8\pi} = \frac{1}{4}\text{ mm} \qquad \Delta y < \frac{2\pi}{6\pi} = \frac{1}{3}\text{ mm}$$

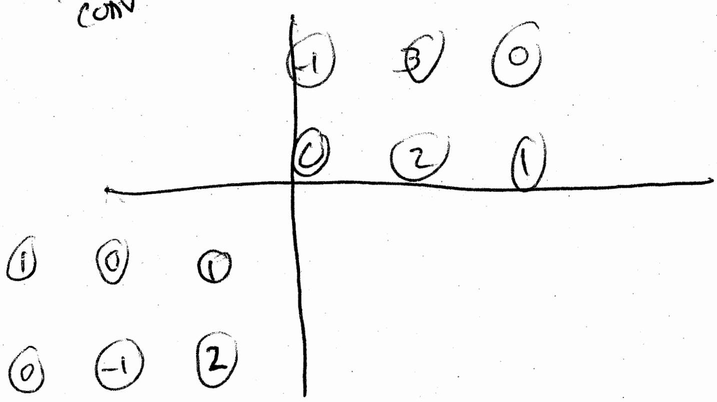
Problem 2:



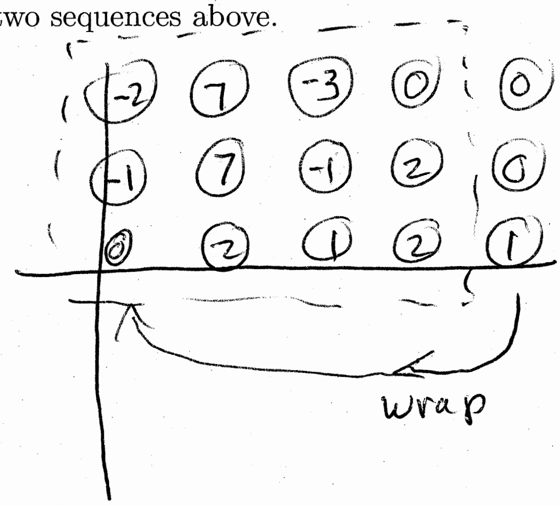


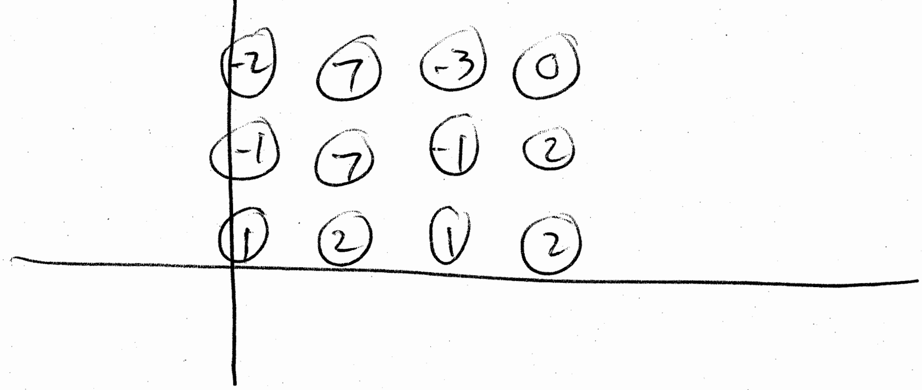
linear conv

(a) Find the 4×3 circular convolution of the two sequences above.



$\Rightarrow$





(b) Find the DFT of the sequence on the right.

$$F(k,l) = \sum_{m=0}^{2} \sum_{n=0}^{1} f(m,n) \exp\left\{-j2\pi\left(\frac{km}{3} + \frac{ln}{2}\right)\right\}$$

$$= 1 + 2e^{-j\pi l} - e^{-j\pi\left(\frac{2}{3}k + l\right)} + e^{-j\pi\frac{4}{3}k}$$

Problem 3:

(a) Show using MATLAB commands how to zoom an image by a factor of N using FFTs.

```
X = fft2(x);
[MM,NN] = size(X);
Xpad = zeros(MM*N, NN*N);
Xpad(1:M,1:N) = X;
Xs = circshift(Xpad,[-MM/2,-NN/2]);
xzoom = ifft2(Xs);
```

(b) The following point pairs are identified:

input	output
(2,1)	(6,2)
(4,4)	(10,8)
(5,3)	(12,6)
(1,5)	(4,10)

Set up the matrix-vector equations to solve for the unknown parameters of a backward mapping that includes a global scaling and a global shift in both dimensions.

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} \begin{bmatrix} x_o \\ y_o \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 4 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 & 1 & 1 \\ 10 & 1 & 1 \\ 12 & 1 & 1 \\ 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} S \\ a \\ b \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 4 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 8 & 1 & 1 \\ 6 & 1 & 1 \\ 10 & 1 & 1 \end{bmatrix} \begin{bmatrix} S \\ a \\ b \end{bmatrix}$$

Problem 4:

Given the following histogram, find a) the pointwise histogram-equalizing mapping and b) the equalized histogram for a target grayscale range of $[0, 255]$.

index	50	100	220	230
pixel count	100	350	50	500

$L_d = 256$

$L_d - 1 = 255$

* don't use left half of left pulse or right half of right pulse

$$tot = \frac{1}{2}(100) + 350 + 50 + \frac{1}{2}(500) = 700$$

$$K = \text{round}\left[\frac{255}{700}\left[\sum_{i=1}^{j} \frac{1}{2}(h(x_i) + h(x_{i-1}))\right]\right]$$

j	K formula	K
50	$\frac{255}{700}(0)$	0
100	$\frac{255}{700}\left[\frac{1}{2}(100+350)\right]$	82
220	$\frac{25\cancel{6}}{700}\left[\frac{1}{2}(450) + \frac{1}{2}(350+50)\right]$	155
230	$\frac{25\cancel{6}}{700}\left[\frac{1}{2}(450) + \frac{1}{2}(400) + \frac{1}{2}(50+500)\right]$	255

Problem 5:

Circle the best answer:

(a) The FFT is best described as:

i. the Fourier transform of a discrete signal.
ii. a fast DFT.
iii. a circular convolution.
iv. none of the above

(b) Thresholding is an example of:

i. a spatial operation.
ii. an algebraic operation.
iii. a pointwise operation.
iv. a combination of operations.

(c) Zooming in can be accomplished by:

i. zeropadding and taking an FFT.
ii. taking an FFT, zeropadding, and taking an inverse FFT.
iii. cropping the image and taking an FFT.
iv. taking an FFT, cropping the result, and taking an inverse FFT.

(d) Aliasing can be reduced by

i. smoothing the image after it is digitized.
ii. defocusing slightly before digitizing.
iii. unsharp masking.
iv. none of the above

(e) (True | False) The vector-radix FFT requires fewer multiplies than the row-column FFT.

(f) If $y(m,n)$ is the output and $x(m,n)$ the input, a system defined by $y(m,n) = \sqrt{|x(m,n)|+1}$ is

i. shift-invariant.
ii. linear.
iii. both (a) and (b).
iv. none of the above

(g) Nearest-neighbor interpolation yields images that may be

i. blurry.
ii. blocky.
iii. none of the above

(h) The Mach-band effect (viewing four vertical bars of constant intensity) demonstrates that

i. aliasing occurs in sampling non-bandlimited images.
ii. contouring is generated by images with too few bits.
iii. sampled images have periodic copies of the baseband signal in frequency.
iv. none of the above

(i) To create light patterns, LCD displays

i. use polarization filters.
ii. use an electron gun and a phosphorescent screen.
iii. use randomly accessible miniature fluorescent lights.

(j) Median filtering is good for

i. sharpening images.
ii. removing impulsive noise.
iii. smoothing step edges.