Problem Set 1

1. Find $\exp\{-ax - by\}u(x, y) * \delta(y)$.

2. Find the Fourier transform of $f(x, y) = \delta(x - ay)$.

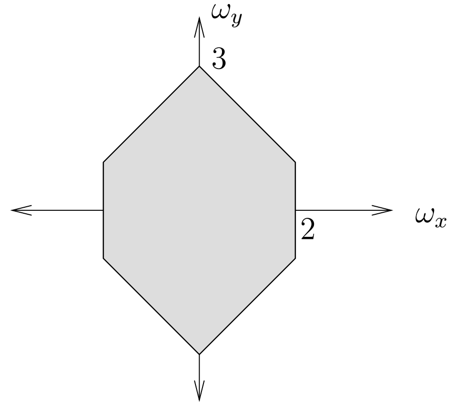
3. Let $\tilde{f}(x, y) = \int_{-\frac{\Delta y}{2}}^{\frac{\Delta y}{2}} \int_{-\frac{\Delta x}{2}}^{\frac{\Delta x}{2}} f(x - x', y - y')dx'dy'$. Determine whether the system is shift-invariant. Demonstrate your answer.

4. For the system above, determine whether the system is linear. Demonstrate your answer.

5. Let $f(x, y) = \begin{cases} 1, & -\frac{\Delta x}{2} < x \leq \frac{\Delta x}{2}, -\frac{\Delta y}{2} < y \leq \frac{\Delta y}{2}. \\ 0, & \text{otherwise.} \end{cases}$ Is this signal separable? Explain.

Problem Set 2

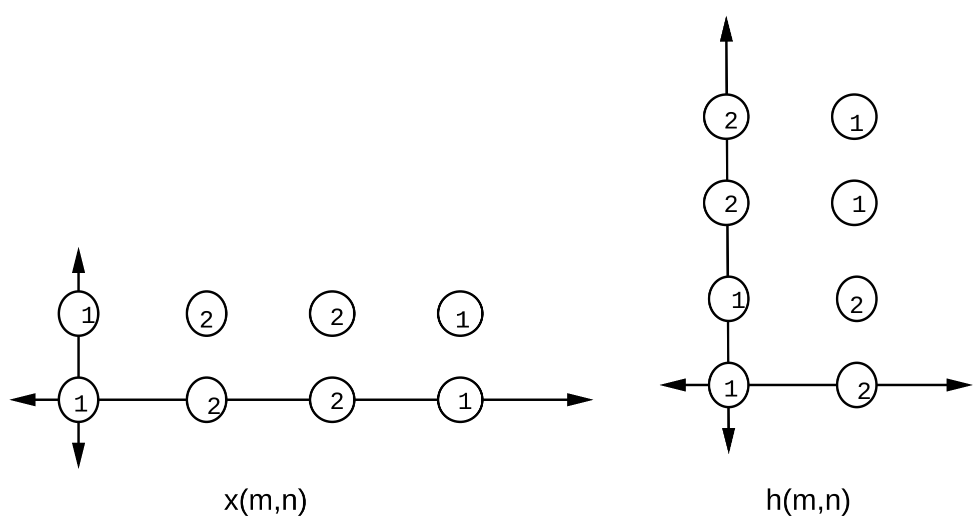
1. Find the maximum sample spacings for a 2-D signal whose Fourier transform has the following region of support (consider the shape to be a square rotated 45 degrees with two corners removed):



   Find the maximum sample spacings if the signal is sampled diagonally.

2. Determine which systems are i) linear, and/or ii) shift-invariant.
   1. $T\{x(m,n)\} = (x(m,n))^2$
   2. $T\{x(m,n)\} = x(m,n) + x(m,n-1)$
   3. $T\{x(m,n)\} = x(m,n) + e^n x(m,n-1)$

3. Given the two two-dimensional signals shown below, determine $x(m,n) * h(m,n)$.



Problem Set 3

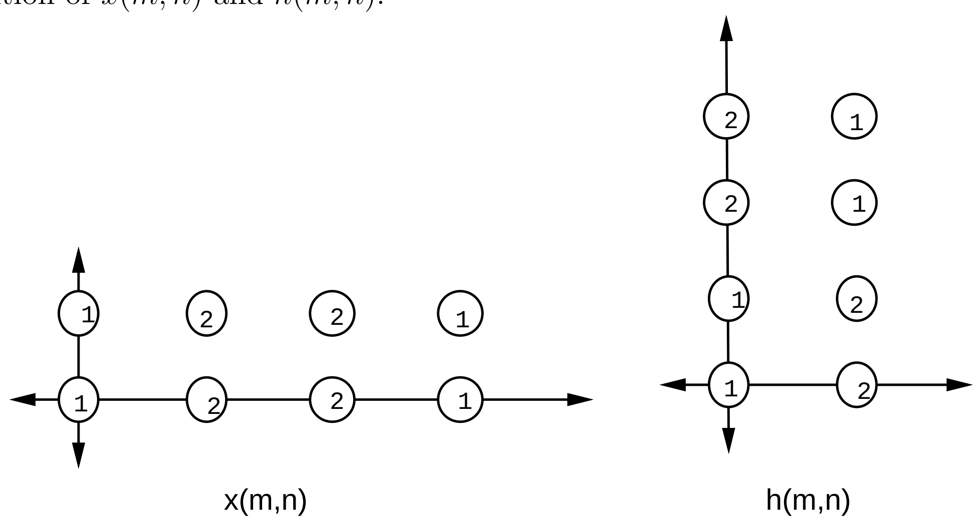
1. Find the Fourier transform of each of the following:

   (a) $x(n) = \delta(n) + \delta(n-2)$

   (b) $x(m,n) = \delta(m,n) + \delta(m-2,n-1)$

2. Compute the 4×4 DFT of 1(b) above.

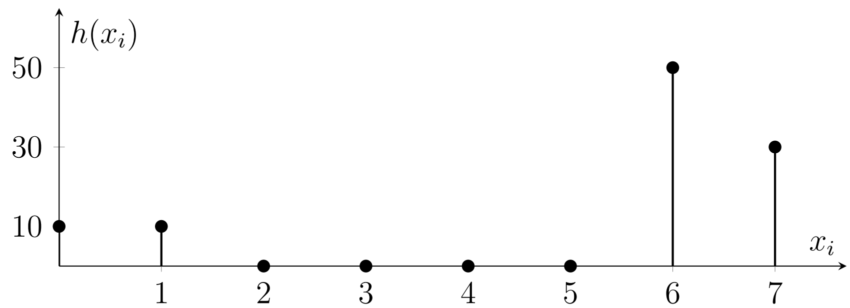
3. Given the two two-dimensional signals shown below, determine the 4×4 circular convolution of $x(m,n)$ and $h(m,n)$.



4. Suppose that we want to use an FFT to implement linear convolution of the two signals above. What size should the zeropadded sequences be to guarantee that the result is a linear convolution? Explain.

Problem Set 4

1. For the histogram below, determine the mapping to equalize the histogram, and sketch the resulting histogram.



2. If a film positive is made by exposing film to a light passing through a previously exposed negative, what is the relationship between the object intensity and the intensity measured by passing a light through the film positive?

3. Let $f(x) = \begin{cases} 2x, & 0 \leq x \leq 0.5 \\ 1, & \text{otherwise} \end{cases}$ represent the response of an image sensor to the incident light. Discuss the options and pitfalls of attempting to compensate for this sensor nonlinearity.

4. If an image has intensities in the range $[50, 120]$, find a mapping that will stretch the contrast to a range of $[0, 255]$.

Problem Set

1. A video inspection system operating at 30 frames/s is used to inspect widgets for uneven texture. The noise variance in each frame is determined to be a factor of 10 too high. Image averaging can be used to average frames and bring down the noise variance. Assuming that the noise is uncorrelated from frame to frame and has zero mean, how long will each widget need to remain stationary under the camera to make the noise level acceptable?

2. Show that subtracting a fraction of the Laplacian boosts high frequencies.

Problem Set

1. Consider the following corresponding points:

input	output
(0,0)	(5,5)
(1,6)	(7,10)
(4,4)	(10,9)
(8,3)	(14,7)
(9,5)	(15,9)
(3,5)	(9,10)

   Find the least-squares fit for a) translation and b) affine mapping for these point pairs.

2. Find the continuous impulse response for bilinear interpolation. (Assume a single unit sample value at the origin and zero elsewhere, and find the response.)