Thursday, February 28, 2019 6:51 PM

Image restoration

- reduction or elimination of degradations in an image

Ex: - camera out of focus
- camera or object in motion
- noisy acquisition system
- geometric warp due to lens distortion

Applications:
- forensics (blurry picture of getaway car license plate)
- military reconnaissance
- Hubble space telescope (spherical aberration)
- consumer photos

Image observation model

$$v(x,y) = g[w(x,y)] + n(x,y)$$

$g[\cdot]$ is a pointwise nonlinearity

$$w(x,y) = \iint h(x,y;x',y')\,u(x',y')\,dx'dy' \quad \text{(blurring)}$$

$\eta(x,y)$ is noise (possibly image-dependent)

$u(x,y)$ is the original image

For shift-invariant systems, point-spread function (PSF) is constant over image.

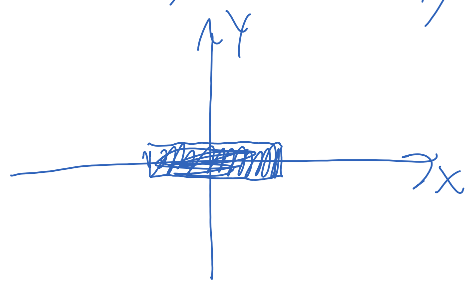
PSF $\equiv$ impulse response

$$h(x,y;x',y') = h(x-x', y-y'; 0,0)$$
$$\triangleq h(x-x', y-y')$$

Then $w(x,y) = h(x,y) * u(x,y)$

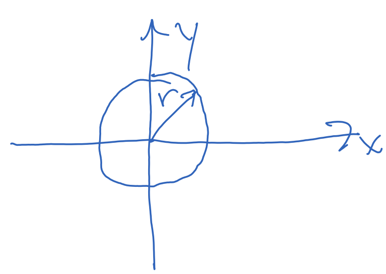
~~blur models~~

horizontal uniform motion



$$\Pi\left(\frac{x}{L}\right)\cdot\delta(y) \xleftrightarrow{\mathcal{F}} \mathrm{sinc}(L\omega_x)$$

out-of-focus



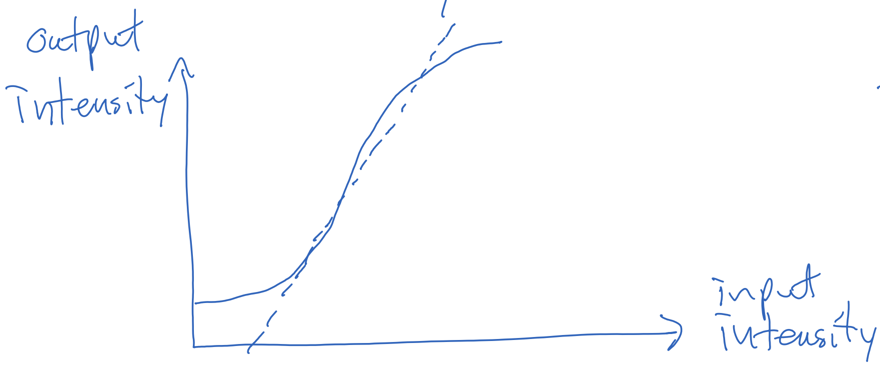
$$\begin{cases} \frac{1}{\pi r^2}, & x^2+y^2 \le r^2 \\ 0, & \text{otherwise} \end{cases}$$

$$\xleftrightarrow{\mathcal{F}} \quad \underline{J_1(2\pi f r)}$$

$f = \sqrt{f_x^2 + f_y^2}$ $\pi f r$ "jinc"

J_1 is Bessel function of first kind of order 1

* sensor nonlinearity



typical sensor nonlinearity

- photographic film
- some CMOS sensors

- either include nonlinear model in restoration or assume image is acquired in linear region

* noise

$$n(x,y) = \sqrt{w(x,y)}\, n_1(x,y) + n_2(x,y)$$

$\sqrt{w(x,y)}\, n_1(x,y)$ is Poisson-distributed — from random photon emissions/detections

$n_2(x,y)$ is Gaussian distributed —

electronic (thermal) noise

* can also model quantization, although this is actually uniformly distributed

From this point:

- shift-invariant blur
- linear sensor
- image-independent noise

~~Inverse filter~~

$$v(x,y) = u(x,y) * h(x,y) + n(x,y)$$

$$v(m,n) = u(m,n) * h(m,n) + n(m,n)$$

$$V(\omega_m,\omega_n) = U(\omega_m,\omega_n) H(\omega_m,\omega_n) + N(\omega_m,\omega_n)$$

⇒ stack up pixels into a tall vector $\quad v = Hu + n$

inverse filter: $\hat{U}(\omega_m,\omega_n) = \frac{1}{H(\omega_m,\omega_n)} V(\omega_m,\omega_n)$

$$= U(\omega_m,\omega_n) + \frac{\cancel{N(\omega_m,\omega_n)}}{H(\omega_m,\omega_n)}$$

two problems:

- noise amplification

– may not be invertible

$\Rightarrow$ pseudoinverse

$$H^{-}(\omega_m, \omega_n) = \begin{cases} \dfrac{1}{H(\omega_m, \omega_n)}, & |H(\omega_m, \omega_n)| > \varepsilon \\ 0, & \text{otherwise} \end{cases}$$

Wiener filter

Minimize $E\{[u(m,n) - \hat{u}(m,n)]^2\}$

for our choice of restoration filter $g(m,n)$

where $\hat{u}(m,n) = g(m,n) * v(m,n)$

– assumes we have a statistical description of $u(m,n)$

$$r_{uv}(m,n) = E[u(m',n')\, v(m'-m, n'-n)]$$

crosscorr of u & v

$$\boxed{r_{uv}(m,n) = g(m,n) * r_{vv}(m,n)}$$

autocorr of u

$\Rightarrow$ In frequency domain,

$$S_{uv}(\omega_m, \omega_n) = G(\omega_m, \omega_n)\, S_{vv}(\omega_m, \omega_n)$$

If $v(m,n) = h(m,n) * u(m,n) + \eta(m,n)$

$$V(\omega_m, \omega_n) = H(\omega_m, \omega_n)\, U(\omega_m, \omega_n) + N(\omega_m, \omega_n)$$

then $S_{vv}(\omega_m, \omega_n) = |H(\omega_m, \omega_n)|^2 S_{uu}(\omega_m, \omega_n) + S_{\eta\eta}(\omega_m, \omega_n)$

$$S_{uv}(\omega_m, \omega_n) = H^*(\omega_m, \omega_n)\, S_{uu}(\omega_m, \omega_n)$$

$\Rightarrow$ assume signal & noise are uncorrelated and noise is zero-mean

$$\Rightarrow G(\omega_m, \omega_n) = \frac{H^*(\omega_m, \omega_n)\cancel{S_{uu}(\omega_m, \omega_n)}}{|H(\omega_m, \omega_n)|^2 S_{uu}(\omega_m, \omega_n) + S_{\eta\eta}(\omega_m, \omega_n)}$$

$$= \frac{H^*(\omega_m, \omega_n)}{|H(\omega_m, \omega_n)|^2 + \dfrac{S_{\eta\eta}(\omega_m, \omega_n)}{S_{uu}(\omega_m, \omega_n)}}$$

* Wiener filter can also be used for smoothing noisy images that are not blurred.

$\Rightarrow$ no blur $H(\omega_m, \omega_n) = 1$

$h(m,n) = \delta(m,n)$

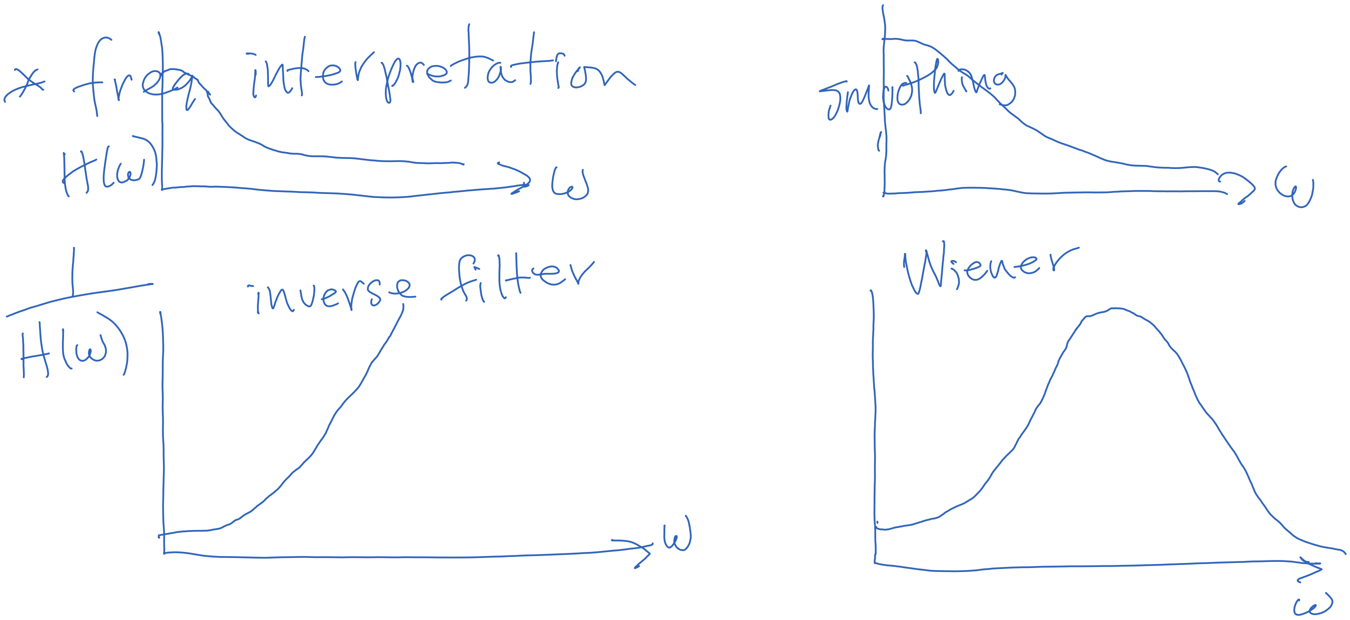
$$G(\omega_m,\omega_n) = \cfrac{1}{1 + \cfrac{S_{\eta\eta}(\omega_m,\omega_n)}{S_{uu}(\omega_m,\omega_n)}}$$

* relation to inverse

$$S_{\eta\eta}(\omega_m,\omega_n) = 0 \Rightarrow G(\omega_m,\omega_n) = H(\omega_m,\omega_n)$$

$$S_{\eta\eta}(\omega_m,\omega_n) \rightarrow 0 \Rightarrow G(\omega_m,\omega_n) = \cancel{H}^{-1}(\omega_m,\omega_n)$$

* freq interpretation



Computing correlation/spectra

* noise
  - assume white noise → constant spectrum
  - estimate variance from a flat region of image

* image
  - use another similar image
  - use model

model for autocorrelation

$$r(m,n) = A\rho^{\sqrt{m^2+n^2}} + (\bar{u})^2$$

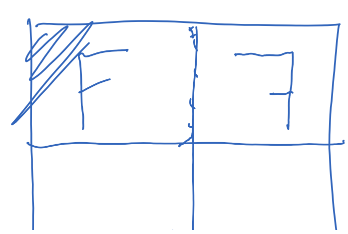
$\bar{u}$ = mean of original

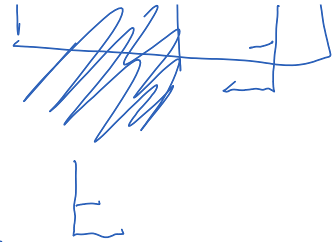
Boundary effects in FFT processing

- assumes image is periodic
- filter is implemented with circular convolution
- boundary artifacts can be significant without compensation (preprocessing)

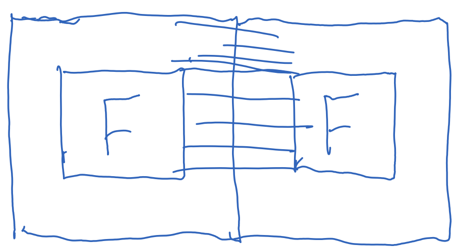
different approaches:

- inverse filter / periodic boundaries
- apply a rolloff window / restore / undo window
- symmetric

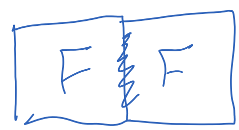




- edgetaper
- my method
  - create larger image – size of original that explains blurry image
- fill unknown parts of larger image with a smooth periodic extrapolation



not this



- replicate boundaries
- apply a progressive blur as we approach boundaries (edgetaper)

Notes:

- smoothing / edgetapering boundaries & Wiener-like solution works well

- padding first & then smoothing / edgetapering works better

<u>Constrained least squares</u>

N-1

Let $\|x\|^2 = \sum_{i=0} x_i^2$

- can represent an image as a vector of pixels stacked up
- can write blur process in vector-matrix notation!

$$g = Df + n$$

blurred image

noise

original image

convolution that implements blur process

$$\min_f \|Lf\|^2 \text{ subject to } \|g - Df\|^2 \le \varepsilon^2$$

L is a highpass filter (often a discrete laplacian)

$\|Lf\|^2$ measures roughness of f

$\Rightarrow$ minimize roughness without allowing solution ~~after~~ being blurred to be very different from blurry image data

$$E\{\|g - Df\|^2\} = E\{\|n\|^2\} = E\{\sum n_i^2\}$$
$$= MN\sigma_n^2$$

original

$\Rightarrow$ choose $\varepsilon^2 = MN\sigma_n^2$

ead 5.11, 7.1

olor overheads

- Lagrange multiplier version

$$\min_f \{ \|Lf\|^2 + \lambda [\|g - Df\|^2 - \varepsilon^2] \}$$

with λ such that $\|g - Df\|^2 = \varepsilon^2$

$$\equiv \min_f \{ \|Lf\|^2 + \lambda \|g - Df\|^2 \}$$

$$\alpha = \frac{1}{\lambda} \quad \equiv \min_f \{ \|g - Df\|^2 + \alpha \|Lf\|^2 \}$$

$\Rightarrow$ regularized restoration

Regularized restoration

- method for making solution "regular" or well-behaved

* minimizes a combination of data error and roughness penalty

$\Rightarrow$ yields a smooth restoration

α controls the tradeoff between smoothing and deblurring
(larger α $\Rightarrow$ more smoothing)

Σ L controls the manner of smoothing

$$[g(m,n) - d(m,n) * f(m,n)]^2 + \alpha \sum [l(m,n) * f(m,n)]^2$$

'arseval's

$$\Rightarrow \frac{1}{4\pi^2} \iint |G(\omega_m,\omega_n) - D(\omega_m,\omega_n) F(\omega_m,\omega_n)|^2 \, d\omega_m \, d\omega_n$$

$$+ \frac{\alpha}{4\pi^2} \iint |L(\omega_m,\omega_n) F(\omega_m,\omega_n)|^2 \, d\omega_m \, d\omega_n$$

$$\min_F |G - DF|^2 + \alpha |LF|^2$$

et $\Rightarrow \hat{F}(\omega_m,\omega_n) = \dfrac{D^*(\omega_m,\omega_n)}{|D(\omega_m,\omega_n)|^2 + \alpha |L(\omega_m,\omega_n)|^2} G(\omega_m,\omega_n)$

$$|L(\omega_m,\omega_n)|^2 = \frac{1}{S_{ff}(\omega_m,\omega_n)}$$

equivalent to Wiener filter

In matrix notation,

$$\hat{f} = (D^T D + \alpha L^T L)^{-1} D^T g$$

Sample to get DFT solution:

$$\hat{F}(k,l) = \frac{D^*(k,l)}{|D(k,l)|^2 + \alpha\,|L(k,l)|^2}\, G(k,l)$$