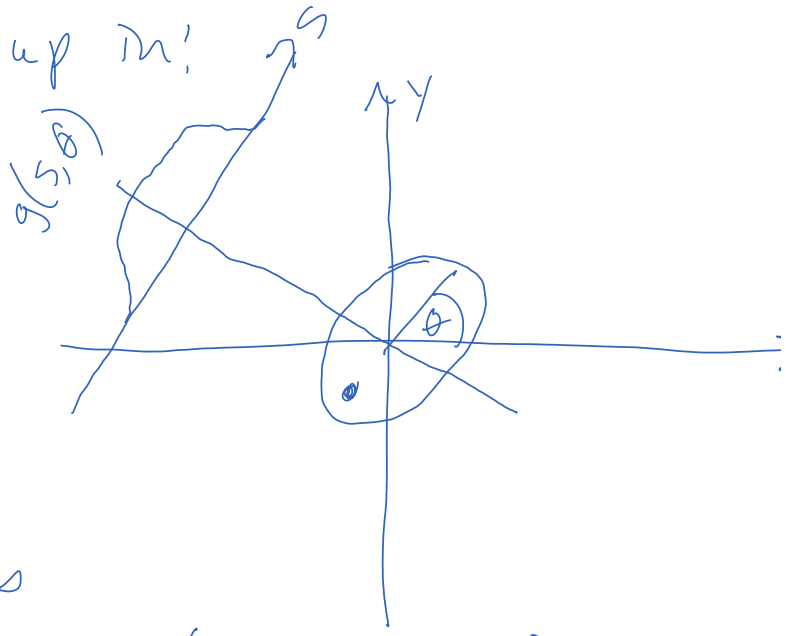
Monday, March 25, 2019 8:30 AM

# Tomography

- tomo - "slice"
- graphy - "picture"
- image inside an object (cross-section)
- typically shows up in:
  - X-ray CT
  - SPECT
  - PET




$$g(s,\theta) \triangleq \mathcal{R}f = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta - s)\,dx\,dy$$

$-\infty < s < \infty$

$0 \le \theta < \pi$

- Radon transform

$s = x\cos\theta + y\sin\theta$

$u = -x\sin\theta + y\cos\theta$

$x = s\cos\theta - u\sin\theta$

$y = s\sin\theta + u\cos\theta$

$g(s,\theta) =$

$$\int_{-\infty}^{\infty} f(s\cos\theta - u\sin\theta,\ s\sin\theta + u\cos\theta)\,du$$

Back-projection operator:

$$b(x,y) = \mathcal{B}g = \int_0^{\pi} g(x\cos\theta + y\sin\theta, \theta)\,d\theta$$

- this is the accumulation of ray-sums of all rays that pass through $(x,y)$

$$\tilde{f}(x,y) = \mathcal{B}g = \mathcal{B}\mathcal{R}f = f(x,y) * (x^2+y^2)^{-1/2}$$

$\Rightarrow \mathcal{B}$ not inverse of $\mathcal{R}$

HW: 5.31, 5.33

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Read 7.2, 7.4-7.5

HW: Find the Radon transform of $f(x,y) = e^{-\pi(x^2+y^2)}$

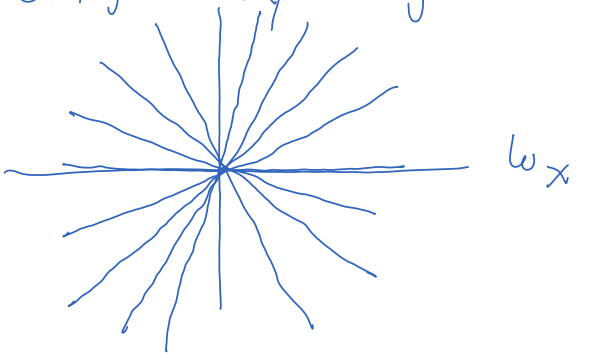
## Projection-slice theorem

If $g(s,\theta) \xrightarrow{\mathcal{F}} G(w,\theta)$

then $G(w,\theta) = F(w\cos\theta, w\sin\theta)$

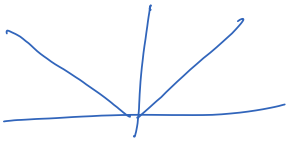
The FT of the projection at angle $\theta$ is the FT of the object evaluated along a

line through origin at angle $\theta$.




## Reconstruction

- can show that



$$f(x,y) = \mathcal{B}\,\hat{g}$$

where $\hat{g}(s,\theta) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\omega_s|\, G(\omega_s,\theta)\, e^{j\omega_s s}\, d\omega_s$

* inverse FT of a frequency-weighted Radon transform (projections)

Radon transform ≡ sinogram

- filtered backprojection (FBP) – frequency-domain
- convolution backprojection (CBP) – spatial-domain
- restoration approach

Read 8.1, 8.2, 8.4, 8.8

HW: 7.5, 7.8, 7.23, 7.25

Project 6 due week from Mon.

HSI is often advantageous for image processing.

- add or subtract constant to/from hue
  (adds or subtracts red, green, or blue tint)
  * makes image warmer or cooler

- can map ~~Saturation~~ to larger values


"postcard effect"

- process intensity (enhancement methods)
  * if you filter RGB components independently, it may create color artifacts

$RGB \rightarrow HSI \xrightarrow{IP} \hat{H}\hat{S}\hat{I} \rightarrow \hat{R}\hat{G}\hat{B}$