Financial Frictions and the Wealth Distribution

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July 7, 2019

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Motivation

Our goal

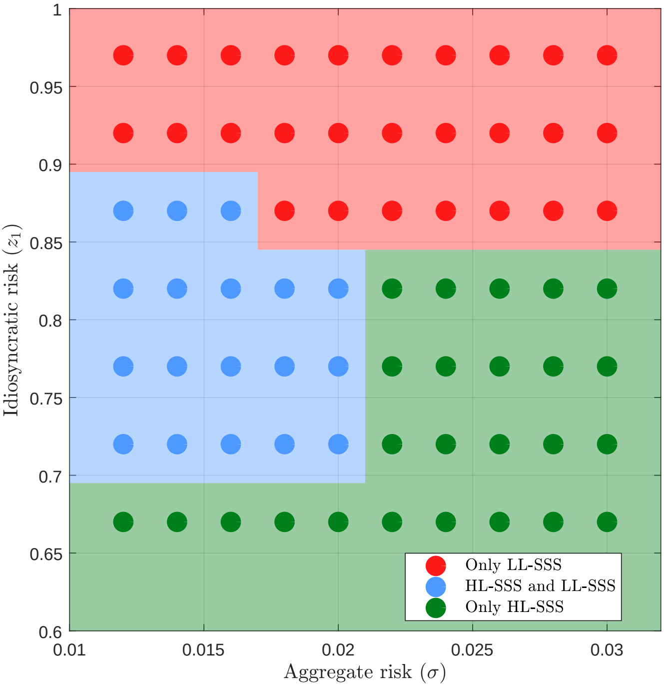
We investigate how, in a HA-model with financial frictions, idiosyncratic individual shocks interact with exogenous aggregate shocks to generate:

1. highly nonlinear behavior,
2. endogenously time-varying volatility and levels of leverage, and
3. endogenous aggregate risk.

- To do so, we postulate, compute, and estimate a continuous-time model *à la* Basak and Cuoco (1998) and Brunnermeier and Sannikov (2014) with a financial expert and a non-trivial distribution of wealth among households.

Four main results

- Multiple stochastic steady states or SSS(s):
  - Depending on the the volatility of the idiosyncratic and aggregate shocks, we can have one high-leverage SSS, one low-leverage SSS, or both.
  - Why? Interaction of precautionary behavior by households with desire to issue debt by the financial expert.
  - Higher micro turbulence leads to higher macro volatility, more inequality, and more leverage.
- Strong state-dependence on the responses of endogenous variables (GIRFs and DIRFs) to aggregate shocks.
- Long spells at different basins of attraction.
  - Multimodal and skewed ergodic distributions of endogenous variables, with endogenous time-varying volatility and aggregate risk.
- Thus, key importance of heterogeneity and breakdown of "quasi-aggregation."



Idiosyncratic risk (z_1)
Aggregate risk (σ)
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.01
0.015
0.02
0.025
0.03
Only LL-SSS
HL-SSS and LL-SSS
Only HL-SSS

Methodological contribution

- New approach to (globally) compute and estimate with the likelihood approach HA models:
  1. Computation: we use tools from machine learning.
  2. Estimation: we use tools from inference with diffusions.
- Strong theoretical foundations and many practical advantages.
  1. Deal with a large class of arbitrary operators efficiently.
  2. Algorithm that is easy to code, stable, and massively parallel.

The firm

- Representative firm with technology:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}$$

- Competitive input markets:

$$w_t = (1-\alpha) K_t^{\alpha} L_t^{-\alpha}$$
$$rc_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha}$$

- Aggregate capital evolves:

$$\frac{dK_t}{K_t} = (\iota_t - \delta)\, dt + \sigma dZ_t$$

- Instantaneous return rate on capital dr_t^k:

$$dr_t^k = (rc_t - \delta)\, dt + \sigma dZ_t$$

The expert

- Financial expert holds capital $\widehat{K}_t$ and issues risk-free debt $\widehat{B}_t$ at rate r_t to households.
- Financial friction: expert cannot issue state-contingent claims (i.e., outside equity) and must absorb all risk from capital.
- Expert's net wealth: $\widehat{N}_t = \widehat{K}_t - \widehat{B}_t$ with leverage ratio $\widehat{\omega}_t \equiv \frac{\widehat{K}_t}{\widehat{N}_t}$.
- The law of motion for expert's net wealth $\widehat{N}_t$:

$$d\widehat{N}_t = \left[\left(r_t + \widehat{\omega}_t \left(rc_t - \delta - r_t \right) \right) \widehat{N}_t - \widehat{C}_t \right] dt + \sigma \widehat{\omega}_t \widehat{N}_t dZ_t$$

- The expert decides her consumption levels and capital holdings to solve:

$$\max_{\left\{ \widehat{C}_t, \widehat{\omega}_t \right\}_{t \geq 0}} \mathbb{E}_0 \left[\int_0^\infty e^{-\widehat{\rho} t} \log(\widehat{C}_t) dt \right]$$

given initial conditions and a NPG condition.

Households

- Continuum of infinitely-lived households heterogeneous in wealth a_m and labor supply z_m for $m \in [0,1]$ with distribution $G_t(a,z)$.
- Households save $a_t \geq 0$ in the riskless debt issued by experts:

$$da_t = (w_t z_t + r_t a_t - c_t)\,dt = s(a_t, z_t, K_t, G_t)\,dt$$

- z_t units of labor valued at wage w_t that evolves following a Markov chain:
  1. $z_t \in \{z_1, z_2\}$, with $z_1 < z_2$.
  2. Ergodic mean of z_t is 1.
  3. Jump intensity from state 1 to state 2: λ_1 (reverse intensity is λ_2).
- Preferences:

$$\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\frac{c_t^{1-\gamma}-1}{1-\gamma}dt\right]$$

- Total consumption by households:

$$C_t \equiv \int c(a_t, z_t, K_t, G_t)\,dG_t(da, dz)$$

Characterizing the equilibrium

- Using the equilibrium conditions of the model, we find:

$$\widehat{C}_t = \widehat{\rho} N_t$$

$$w_t = (1-\alpha) K_t^{\alpha}$$

$$rc_t = \alpha K_t^{\alpha-1}$$

$$r_t = \alpha K_t^{\alpha-1} - \delta - \sigma^2 \frac{K_t}{N_t}$$

$$dN_t = \underbrace{\left(\alpha K_t^{\alpha-1} - \delta - \widehat{\rho} - \sigma^2 \left(1 - \frac{K_t}{N_t}\right) \frac{K_t}{N_t}\right) N_t}_{\mu_t^N(B_t, N_t)} dt + \underbrace{\sigma K_t}_{\sigma_t^N(B_t, N_t)} dZ_t$$

- Thus, if we know the states of the economy, we only need to compute:

$$dB_t = \left((1-\alpha) K_t^{\alpha} + \left(\alpha K_t^{\alpha-1} - \delta - \sigma^2 \frac{K_t}{N_t}\right) B_t - C_t\right) dt$$

and we can compute the HJB of the households and KFE characterizing the household's distribution.

Solution

- We approximate

$$dB_t = \left(\left(1 - \alpha\right) K_t^{\alpha} + \left(\alpha K_t^{\alpha - 1} - \delta - \sigma^2 \frac{K_t}{N_t} \right) B_t - C_t \right) dt$$

with $dB_t = h\left(B_t, N_t\right) dt$ using a neural network.

- Simulation-based algorithm with outstanding properties:

1. It is a universal nonlinear approximator.
2. It breaks the curse of dimensionality (less important in this application).
3. Easy to code, stable, and with good extrapolation properties.
4. You can flush the algorithm to a graphics processing unit (GPU) or a tensor processing unit (TPU) instead of a standard CPU.

Estimation with aggregate variables

- $D+1$ observations at fixed time intervals $Y_0^D = \{Y_0, Y_\Delta, Y_{2\Delta}, ..., Y_D\}$.
- Likelihood:

$$\mathcal{L}_D\left(Y_0^D|\Psi\right) = \prod_{d=1}^{D} p_Y\left(Y_{d\Delta}|Y_{(d-1)\Delta};\Psi\right),$$

  where

$$p_Y\left(Y_{d\Delta}|Y_{(d-1)\Delta};\Psi\right) = \int f_{d\Delta}(Y_{d\Delta}, B)dB.$$

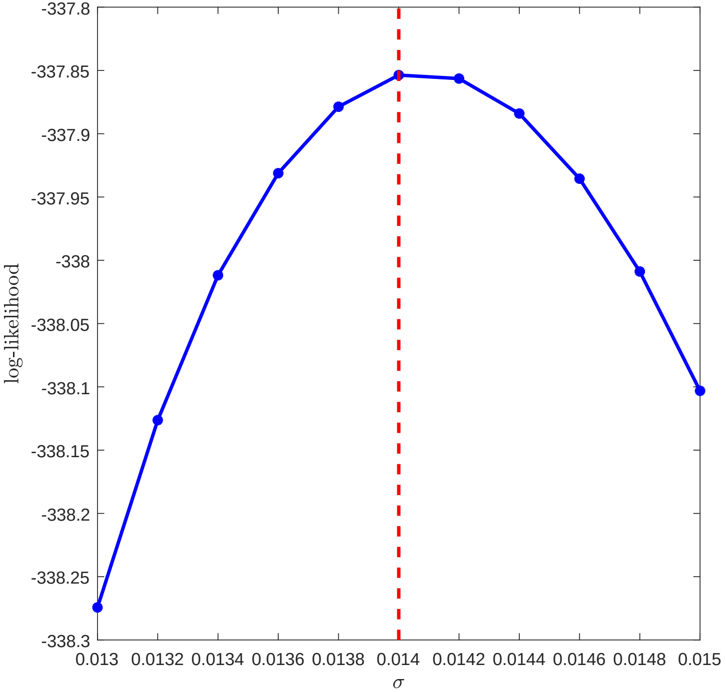
- $f_t^d(Y,B)$ follows the KFE:

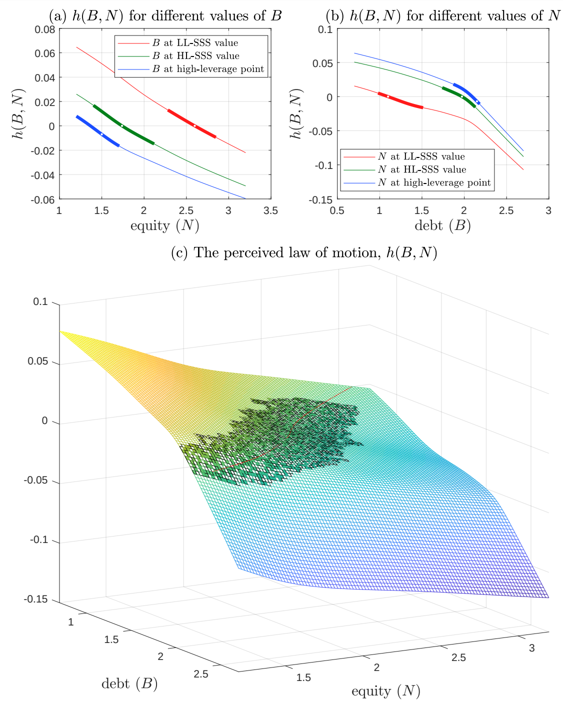
$$\begin{aligned}\frac{\partial f_t}{\partial t} &= -\frac{\partial}{\partial Y}\left[\mu^Y(Y,B)f_t(Y,B)\right] - \frac{\partial}{\partial B}\left[h(B, Y^{\frac{1}{\alpha}} - B)f_t^d(Y,B)\right] \\ &\quad +\frac{1}{2}\frac{\partial^2}{\partial Y^2}\left[\left(\sigma^Y(Y)\right)^2 f_t(Y,B)\right]\end{aligned}$$

- The operator in the KFE equation is the adjoint of the infinitesimal generator generated by the HJB of the household we solved above.

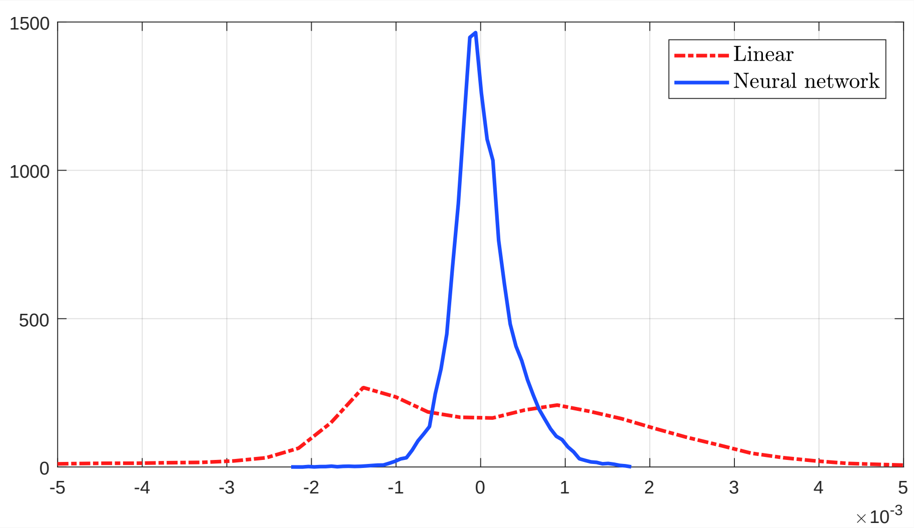
Parametrization

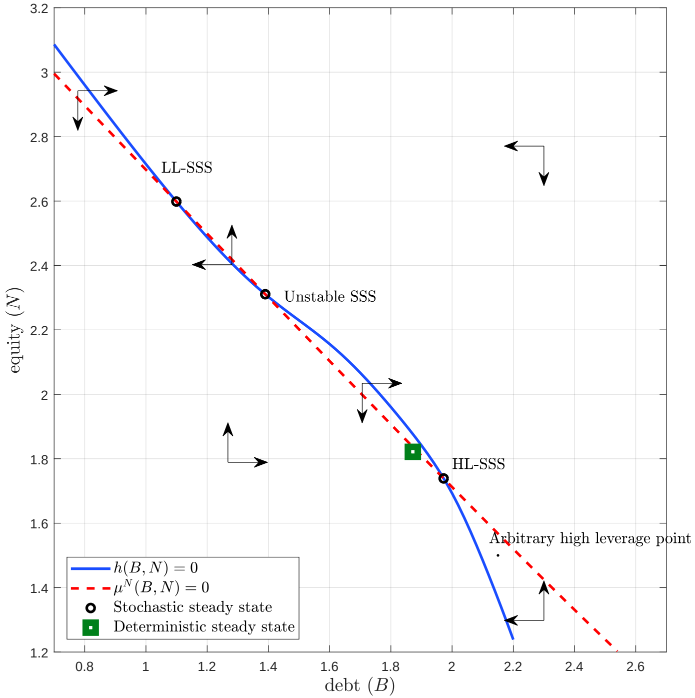
Parameter	Value	Description	Source/Target
α	0.35	capital share	standard
δ	0.1	yearly capital depreciation	standard
γ	2	risk aversion	standard
ρ	0.05	households' discount rate	standard
λ_1	0.986	transition rate u.-to-e.	monthly job finding rate of 0.3
λ_2	0.052	transition rate e.-to-u.	unemployment rate 5 percent
y_1	0.72	income in unemployment state	Hall and Milgrom (2008)
y_2	1.015	income in employment state	$\mathbb{E}(y) = 1$
$\widehat{\rho}$	0.0497	experts' discount rate	$K/N = 2$


-337.8
-337.85
-337.9
-337.95
-338
-338.05
-338.1
-338.15
-338.2
-338.25
-338.3
log-likelihood
0.013
0.0132
0.0134
0.0136
0.0138
0.014
0.0142
0.0144
0.0146
0.0148
0.015
σ

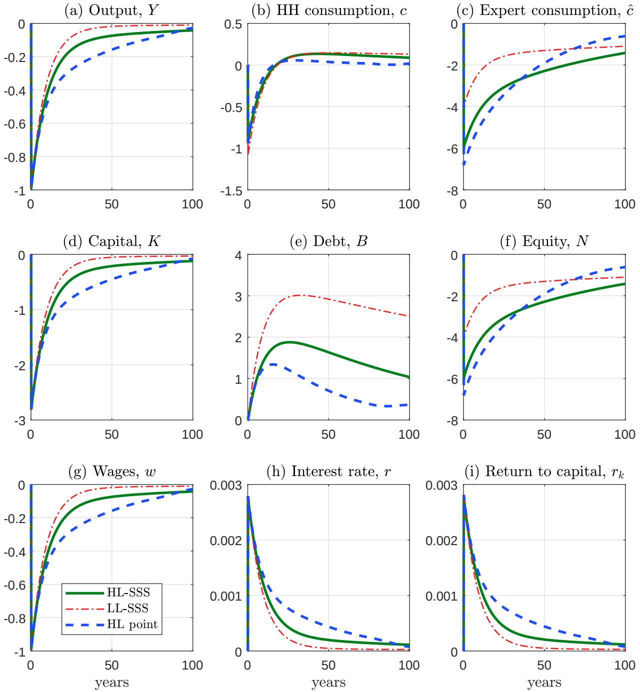


(a) $h(B,N)$ for different values of B
B at LL-SSS value
B at HL-SSS value
B at high-leverage point
$h(B,N)$
equity (N)
(b) $h(B,N)$ for different values of N
N at LL-SSS value
N at HL-SSS value
N at high-leverage point
$h(B,N)$
debt (B)
(c) The perceived law of motion, $h(B,N)$
debt (B)
equity (N)

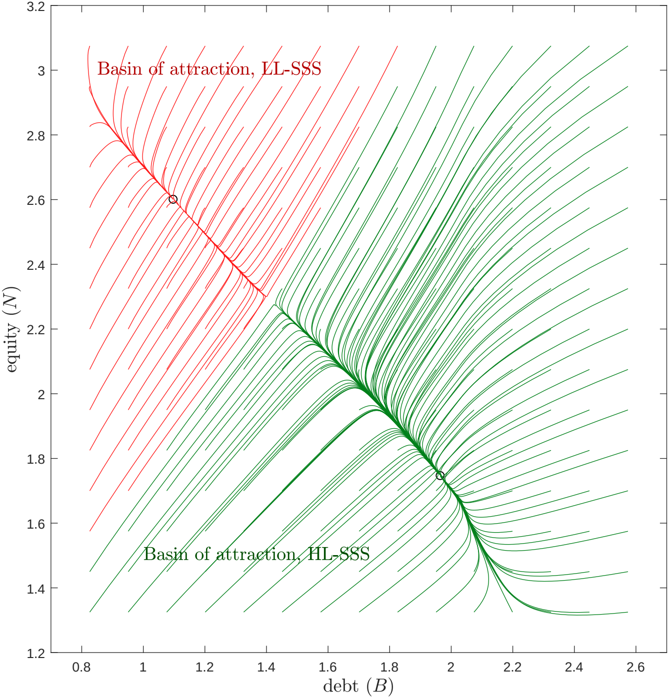

1500
1000
500
0
-5
-4
-3
-2
-1
0
1
2
3
4
5
×10^-3
Linear
Neural network

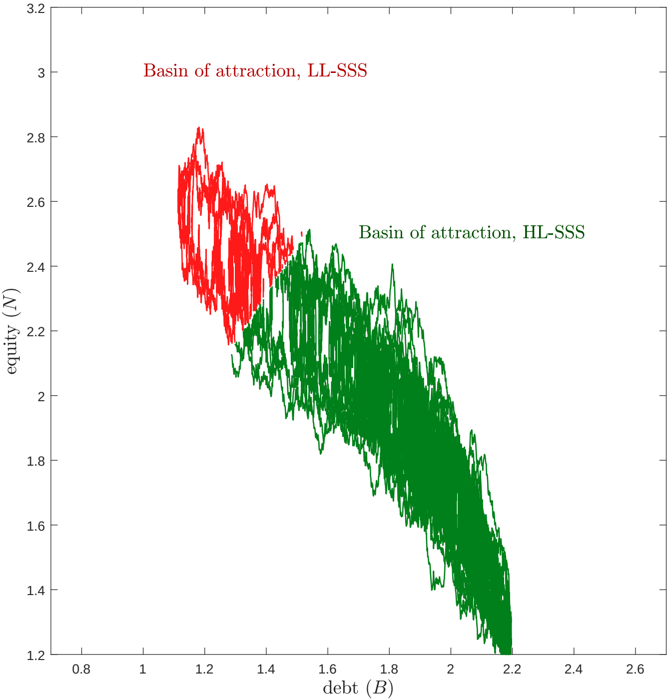


LL-SSS
Unstable SSS
HL-SSS
Arbitrary high leverage point
$h(B,N)=0$
$\mu^N(B,N)=0$
Stochastic steady state
Deterministic steady state
equity (N)
debt (B)



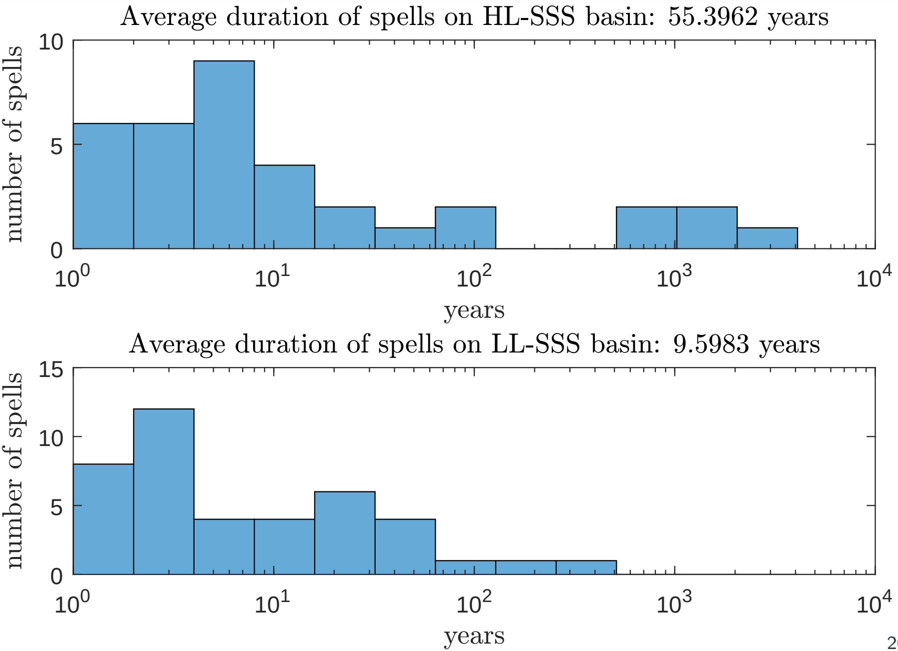
(a) Output, Y
(b) HH consumption, c
(c) Expert consumption, $\hat{c}$
(d) Capital, K
(e) Debt, B
(f) Equity, N
(g) Wages, w
(h) Interest rate, r
(i) Return to capital, r_k
HL-SSS
LL-SSS
HL point
years


Basin of attraction, LL-SSS
Basin of attraction, HL-SSS
equity (N)
debt (B)
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6

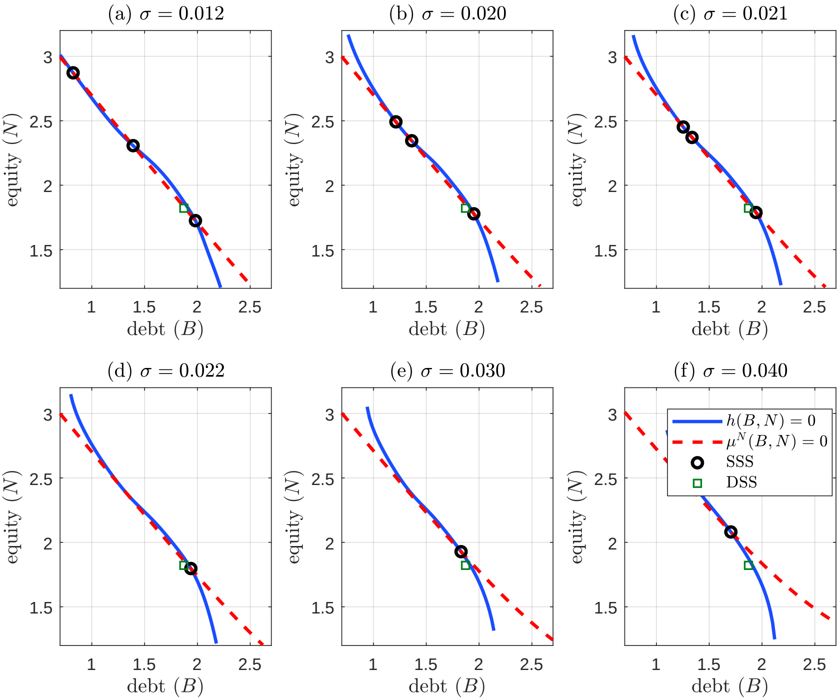

Basin of attraction, LL-SSS
Basin of attraction, HL-SSS
equity (N)
debt (B)
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6

	Mean	Standard deviation	Skewness	Kurtosis
$Y^{\text{basin } HL}$	1.5807	0.0193	-0.0831	2.8750
$Y^{\text{basin } LL}$	1.5835	0.0166	0.16417	3.1228
$r^{\text{basin } HL}$	4.92	0.3360	0.1725	2.8967
$r^{\text{basin } LL}$	4.88	0.2896	-0.0730	3.0905
$w^{\text{basin } HL}$	1.0274	0.0125	-0.0831	2.875
$w^{\text{basin } LL}$	1.0293	0.0108	0.1642	3.1228

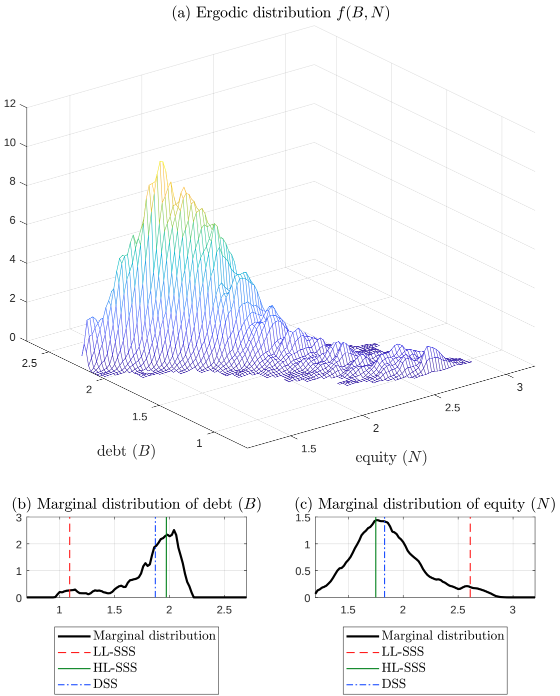
Table 1: Moments conditional on basin of attraction.

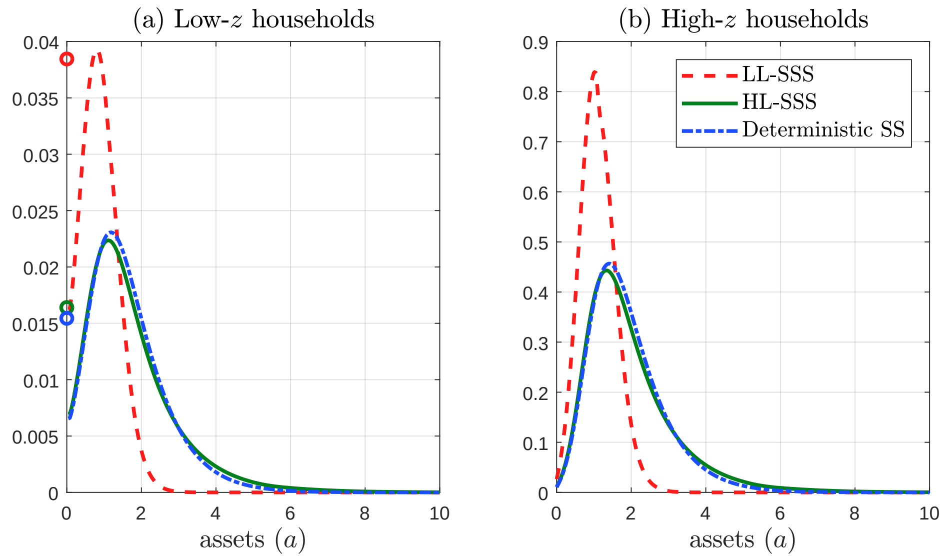


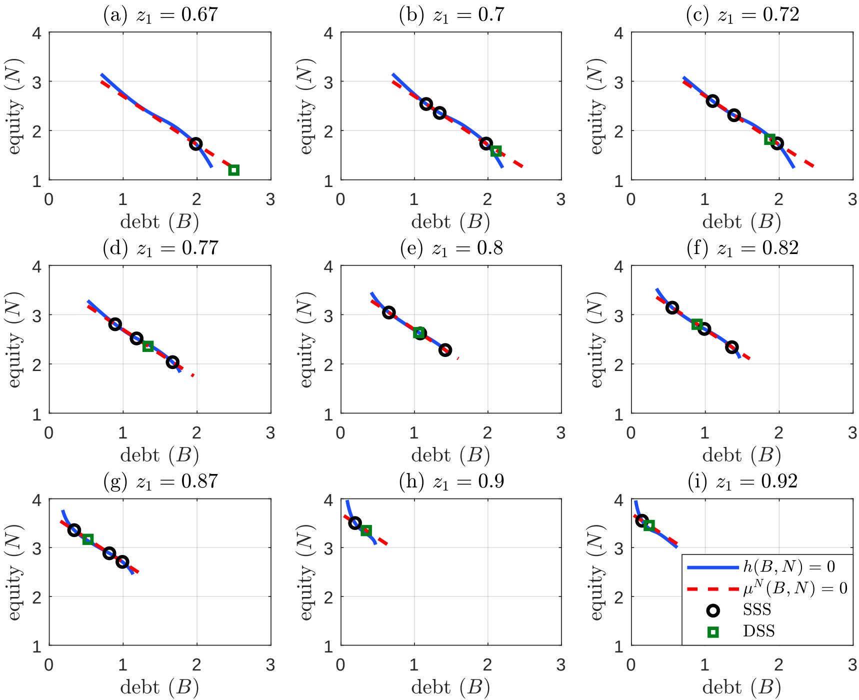
Average duration of spells on HL-SSS basin: 55.3962 years
number of spells
10
5
0
10^0
10^1
10^2
10^3
10^4
years
Average duration of spells on LL-SSS basin: 9.5983 years
number of spells
15
10
5
0
10^0
10^1
10^2
10^3
10^4
years

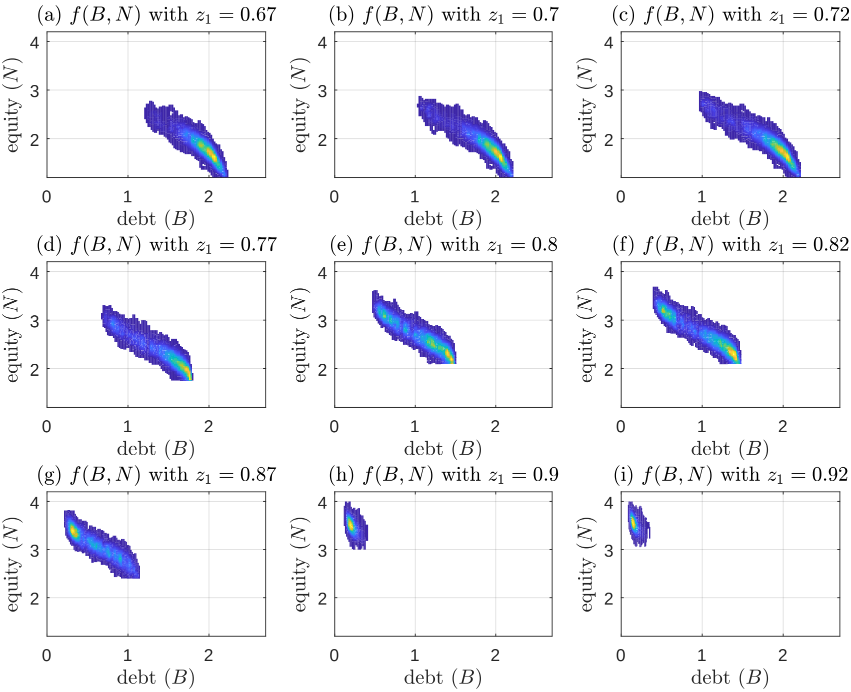


(a) $\sigma = 0.012$
(b) $\sigma = 0.020$
(c) $\sigma = 0.021$
(d) $\sigma = 0.022$
(e) $\sigma = 0.030$
(f) $\sigma = 0.040$
equity (N)
debt (B)
$h(B,N)=0$
$\mu^N(B,N)=0$
SSS
DSS

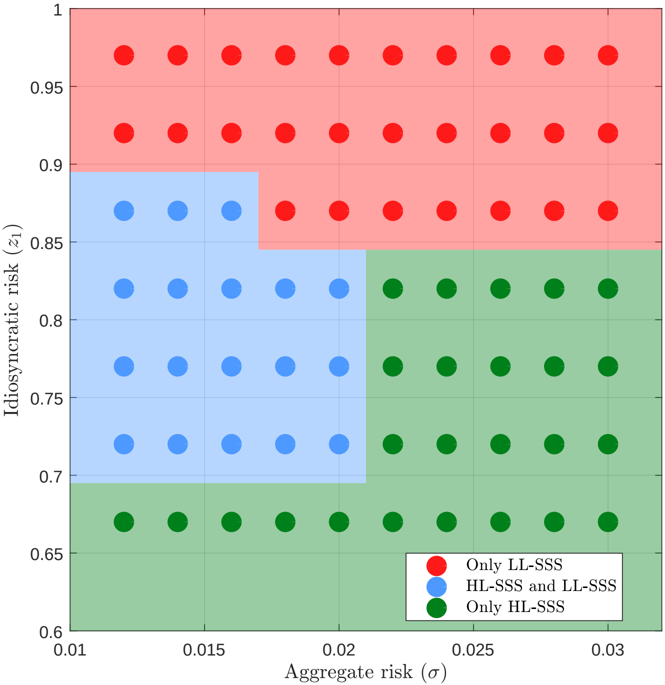

(a) Ergodic distribution $f(B, N)$
12
10
8
6
4
2
0
2.5
2
1.5
1
debt (B)
1.5
2
2.5
3
equity (N)
(b) Marginal distribution of debt (B)
3
2
1
0
1
1.5
2
2.5
Marginal distribution
LL-SSS
HL-SSS
DSS
(c) Marginal distribution of equity (N)
1.5
1
0.5
0
1.5
2
2.5
3
Marginal distribution
LL-SSS
HL-SSS
DSS


(a) Low-z households
(b) High-z households
LL-SSS
HL-SSS
Deterministic SS
assets (a)
assets (a)
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
2
4
6
8
10

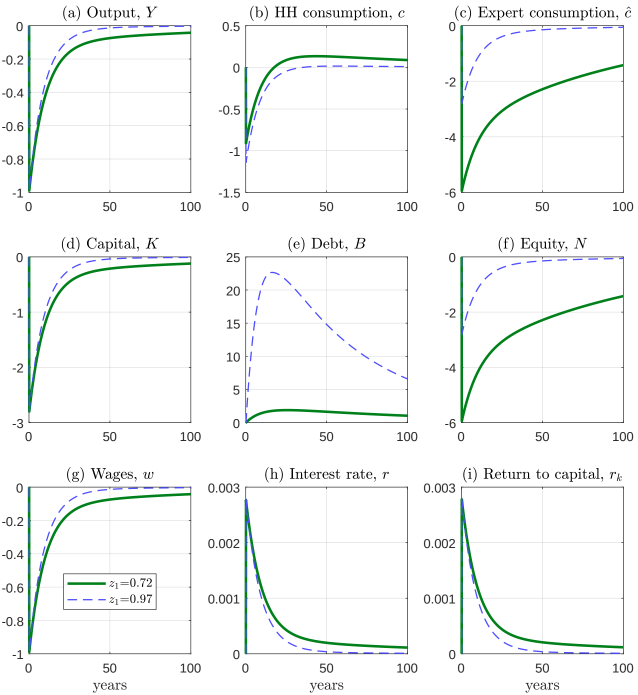

(a) $z_1 = 0.67$
(b) $z_1 = 0.7$
(c) $z_1 = 0.72$
(d) $z_1 = 0.77$
(e) $z_1 = 0.8$
(f) $z_1 = 0.82$
(g) $z_1 = 0.87$
(h) $z_1 = 0.9$
(i) $z_1 = 0.92$
equity (N)
debt (B)
$h(B, N) = 0$
$\mu^N(B, N) = 0$
SSS
DSS



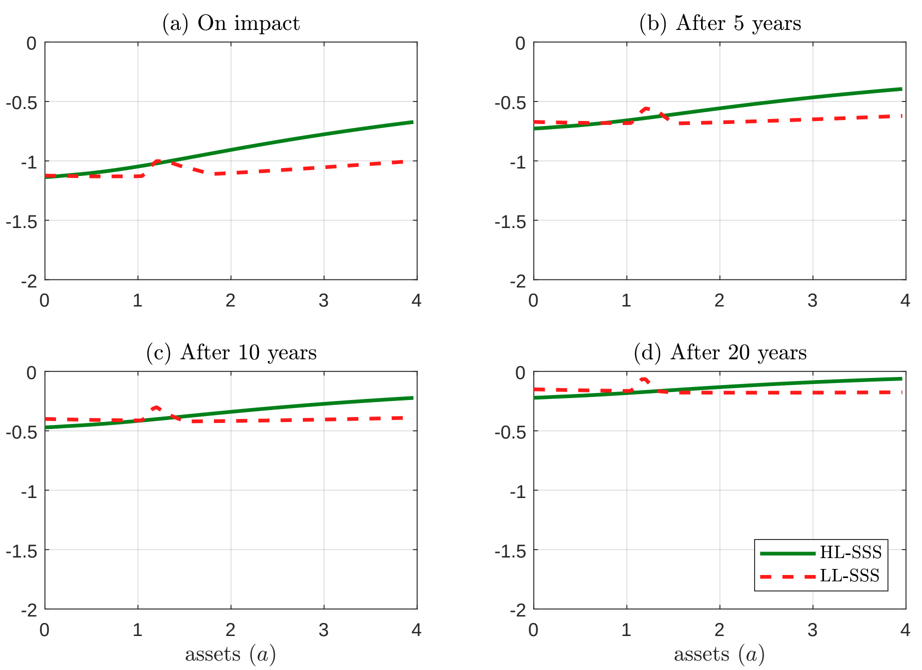
(a) $f(B,N)$ with $z_1 = 0.67$
(b) $f(B,N)$ with $z_1 = 0.7$
(c) $f(B,N)$ with $z_1 = 0.72$
(d) $f(B,N)$ with $z_1 = 0.77$
(e) $f(B,N)$ with $z_1 = 0.8$
(f) $f(B,N)$ with $z_1 = 0.82$
(g) $f(B,N)$ with $z_1 = 0.87$
(h) $f(B,N)$ with $z_1 = 0.9$
(i) $f(B,N)$ with $z_1 = 0.92$
equity (N)
debt (B)

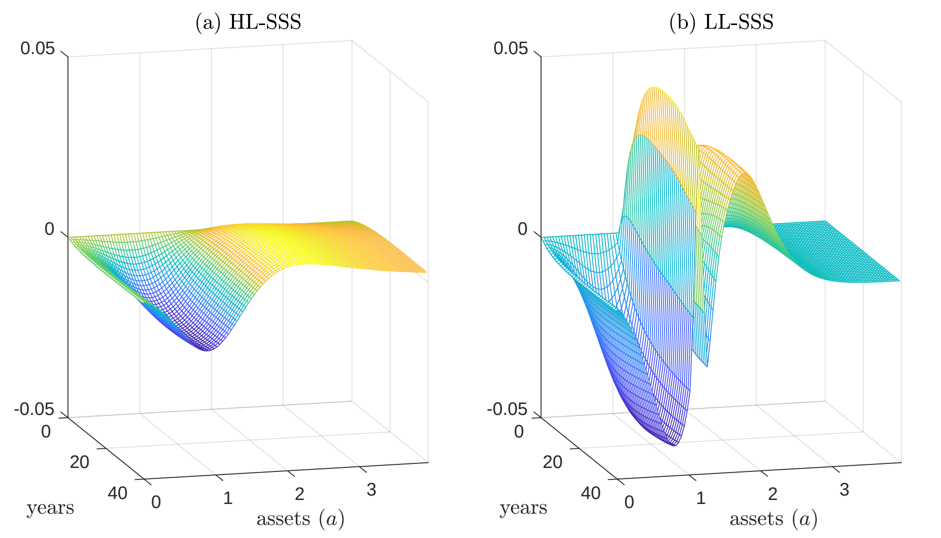


Idiosyncratic risk (z_1)
Aggregate risk (σ)
Only LL-SSS
HL-SSS and LL-SSS
Only HL-SSS
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.01
0.015
0.02
0.025
0.03

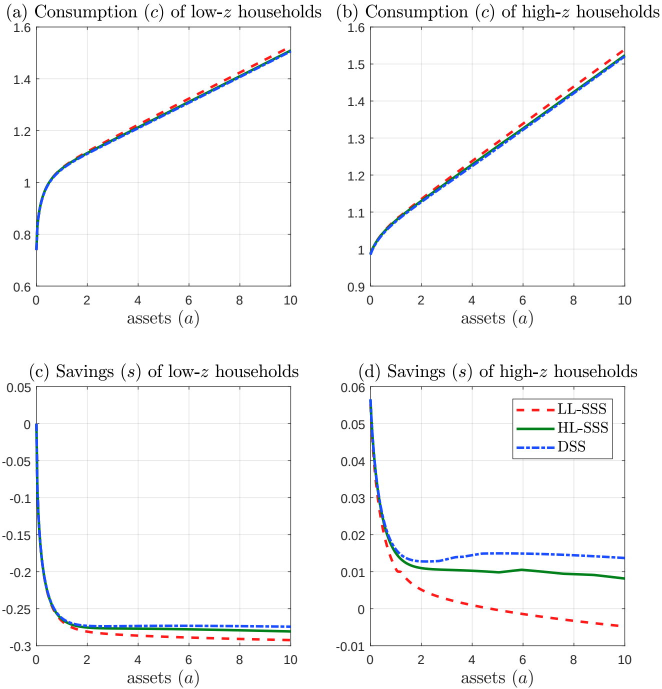


(a) Output, Y
(b) HH consumption, c
(c) Expert consumption, $\hat{c}$
(d) Capital, K
(e) Debt, B
(f) Equity, N
(g) Wages, w
(h) Interest rate, r
(i) Return to capital, r_k
z_1=0.72
z_1=0.97
years
years
years


(a) On impact
(b) After 5 years
(c) After 10 years
(d) After 20 years
0
-0.5
-1
-1.5
-2
0
1
2
3
4
assets (a)
HL-SSS
LL-SSS



(a) HL-SSS
0.05
0
-0.05
0
20
40
years
0
1
2
3
assets (a)
(b) LL-SSS
0.05
0
-0.05
0
20
40
years
0
1
2
3
assets (a)


(a) Consumption (c) of low-z households
(b) Consumption (c) of high-z households
(c) Savings (s) of low-z households
(d) Savings (s) of high-z households
assets (a)
LL-SSS
HL-SSS
DSS

Concluding remarks

- We have shown how a continuous-time model with a non-trivial distribution of wealth among households and financial frictions can be built, computed, and estimated.
- Four important economic lessons:
  1. Multiplicity of SSS(s).
  2. State-dependence of GIRFs and DIRFs.
  3. Long spells at different basins of attraction.
  4. Importance of household heterogeneity.
- Many avenues for extension.