

# CarND-Controls-MPC

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Self-Driving Car Engineer Nanodegree Program

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This README can also be found in this repo as a [PDF](#) (in case the equations don't show properly).

## Dependencies

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- cmake >= 3.5
- All OSes: [click here for installation instructions](#)
- make >= 4.1
  - Linux: make is installed by default on most Linux distros
  - Mac: [install Xcode command line tools to get make](#)
  - Windows: [Click here for installation instructions](#)
- gcc/g++ >= 5.4
  - Linux: gcc / g++ is installed by default on most Linux distros
  - Mac: same deal as make - [install Xcode command line tools] (<https://developer.apple.com/xcode/features/>)
  - Windows: recommend using [MinGW](#)
- [uWebSockets](#) == 0.14, but the master branch will probably work just fine
  - Follow the instructions in the [uWebSockets README](#) to get setup for your platform. You can download the zip of the appropriate version from the [releases page](#). Here's a link to the [v0.14 zip](#).
  - If you have MacOS and have [Homebrew](#) installed you can just run the ./install-mac.sh script to install this.
- [Ipopt](#)
  - Mac: `brew install ipopt --with-openblas`
  - Linux
    - You will need a version of Ipopt 3.12.1 or higher. The version available through `apt-get` is 3.11.x. If you can get that version to work great but if not there's a script `install_ipopt.sh` that will install Ipopt. You just need to download the source from [here](#).
    - Then call `install_ipopt.sh` with the source directory as the first argument, ex: `bash install_ipopt.sh Ipopt-3.12.1`.
  - Windows: TODO. If you can use the Linux subsystem and follow the Linux instructions.
- [CppAD](#)
  - Mac: `brew install cppad`
  - Linux `sudo apt-get install cppad` or equivalent.
  - Windows: TODO. If you can use the Linux subsystem and follow the Linux instructions.
- [Eigen](#). This is already part of the repo so you shouldn't have to worry about it.
- Simulator. You can download these from the [releases tab](#).

## Basic Build Instructions

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1. Clone this repo.
2. Make a build directory: `mkdir build && cd build`
3. Compile: `cmake .. && make`
4. Run it: `./mpc`.

## Building using Docker

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1. `docker build -t mpcp .`
2. `docker run -p 127.0.0.1:4567:4567 mpcp ./mpc`

## Discussion

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### Pre-processing of Waypoints

All the waypoints are transformed from the global reference frame to the vehicle reference frame. The following transform is used for each of the waypoints:

$$\begin{aligned} x_{trans} &= [x_{waypoint} - x_p] * \cos(\psi) + [y_{waypoint} - y_p] * \sin(\psi) \\ y_{trans} &= -[x_{waypoint} - x_p] * \sin(\psi) + [y_{waypoint} - y_p] * \cos(\psi) \end{aligned} \quad (1)$$

Where  $\psi$  is the angle of the vehicle with respect to the x-axis and  $x_p$  and  $y_p$  are the location of the vehicle in the global reference frame.  $x_{waypoint}$  and  $y_{waypoint}$  are the x, y coordinates of a given waypoint in the global reference frame.

### Polynomial Fitting

The transformed waypoints are fit to a 3rd order polynomial  $f(x_t)$ .

$$f(x_t) = c_0 + c_1 x_t + c_2 x_t^2 + c_3 x_t^3 \quad (2)$$

With  $c_i$   $i \in \{0, 1, 2, 3\}$  being the coefficients.

Given that the waypoints are now in the reference frame of the vehicle, the current cross track error  $cte_0$  is simply the value of the polynomial at point  $x_0 = 0.0$ , so we have  $cte_0 = f(x_0) = f(0.0) = c_0$ .

The current orientation error  $e\psi$  is determined by the arc-tangent of the derivative of the polynomial.

$$e\psi = \arctan(f'(x_t)) = \arctan(c_1 + 2c_2 x_t + 3c_3 x_t^2) \quad (3)$$

The result  $e\psi_0$  is also at point  $x_0 = 0.0$  and hence  $e\psi_0 = \arctan f'(x_0) = \arctan f'(0.0) = \arctan(c_1)$ .

## The Model

The **state** is a 6 element vector with the following elements:

- $x$  position of vehicle
- $y$  position of vehicle
- $\psi$  - the orientation of the vehicle
- $v$  - speed of the vehicle

- $cte$  - cross track error
- $e\psi$  - orientation error

The **actuators** are contained in a 2 element vector:

- $\delta$  - steering angle
- $a$  - acceleration

The following **update equations** were used:

$$\begin{aligned}
 x_{t+1} &= x_t + v_t * \cos(\psi_t) * dt \\
 y_{t+1} &= y_t + v_t * \sin(\psi_t) * dt \\
 \psi_{t+1} &= \psi_t + \frac{v_t}{L_f} * \delta * dt \\
 v_{t+1} &= v_t + a_t * dt \\
 cte_{t+1} &= f(x_t) - y_t + v_t * \sin(e\psi_t) * dt \\
 e\psi_{t+1} &= \psi_t - \psi_{des_t} + \frac{v_t}{L_f} * \delta_t * dt
 \end{aligned} \tag{4}$$

With the current state being  $[x_0, y_0, \psi_0, v_0, cte_0, e\psi_0] = [0.0, 0.0, 0.0, v, f(0.0), \arctan(f'(0.0))]$ .

## Timestep Length, Frequency and MPC Latency

Given that MPC has a latency of 100ms, I decided to choose a  $dt$  slightly higher than that, namely  $dt = 0.15$ . I then experimented with different values for  $N$ . Here the best suited value turned out to be  $N = 11$ . The prediction horizon  $T$  then becomes  $T = (N - 1) * dt = 10 * 0.15 = 1.5$ . With higher values of  $N$ , the vehicle would drive erratically and quickly drive off the track. Lower values, would make the prediction very limited.

## Demo Video

A demo video of this project can be found on Youtube [here](#).