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# The market for bitcoin transactions

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ABSTRACT

Transaction fees in the bitcoin system work differently from those in conventional payment systems due to the design of the bitcoin mining algorithm. In particular, transaction fees and transaction volume in the bitcoin system increase whenever the network is congested, and our VAR results confirm that is indeed the case. To account for the empirical findings, we build a model where users and miners together determine transaction fees and transaction volume. Even though the mechanism of fluctuating transaction fees in bitcoin introduces an extra cost of uncertainty to users, a back-of-envelope calculation shows that the cost of using the bitcoin network for transactions is still smaller than the cost of using the current conventional payment system with a fixed transaction fee rate. However, this calculation may underestimate the cost due to the crowding-out effect on small transactions during the congested period.



## 1. Introduction

As a new currency initially created in the computer science community, bitcoin was born with some unique characteristics. Bitcoin is a currency that exists only on the internet.[1] It is accessible publicly, and anyone who is connected to the internet can create a bitcoin wallet and begin to use bitcoin. In this paper, we take some of these technical aspects of bitcoin into consideration and study how its transaction fees are determined in the bitcoin network.

In the early days, most bitcoin studies focus on technical or legal issues. However, recently bitcoin is getting more attention from economists, especially after the bullish period in 2017. Numerous studies are done focusing on the price discovery mechanism.[2] However, not much research is done on bitcoin as a payment system, which is the original impetus of the creation of bitcoin.

In this paper, we first use the VAR method to examine the reactions to a congested network in terms of transaction volume and transaction fees. Then we try to build a simple structural model to capture what we observe in our VAR results. Our model focuses on the short-run dynamics inside the bitcoin system in response to a demand shock from the users' side. We

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1 Some researchers view the bitcoin as the internet of money (Antonopoulos, 2016), but in this paper, we focus on the original intention of bitcoin: a digital currency.

2 Brandvold et al. (2015) is the first to study the bitcoin price discovery mechanism from the exchange level. By using the method of De Jong et al. (1999) on seven exchanges, they find that MtGox and Btce were price leaders in the early days. Kraaijeveld and De Smedt (2020) find out Twitter sentiment can be used to predict the price return of bitcoin. Corbet et al. (2018) also provide a comprehensive review of literature treating bitcoin as a financial asset.

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allow the service rate from miners to be responsive to financial incentives, and, together with users' decisions, we will have a simple demand-supply model in which transaction fees and transaction volume are endogenously determined. Furthermore, we summarize three different costs in the bitcoin system and do a back-of-envelope calculation on the cost of using a payment system with a fluctuating transaction fee rate.

There are a number of related studies on the bitcoin market. Dimitri (2019) models "the transaction fee as a Nash equilibrium outcome of an auction game with complete information," and the conclusion is that the optimal block size for miners depends on the distribution of users' willingness to pay. Basu et al. (2019) argue that the transaction fee mechanism in the bitcoin system acts as a generalized first-price auction on multiple and identical items. Noda et al. (2019) point out a potential vulnerability in the bitcoin payment system: the difficulty adjustment algorithm in the bitcoin network may fail to adjust the block arrival rate timely when a large group of miners suddenly leave the network. Auer (2019) argues that the bitcoin payment system cannot be functional by solely relying on transaction fees. Once the payoff from block rewards becomes zero, it may take months for transactions to be confirmed by miners. A counter argument is proposed by Garratt and van Oordt (2019), the authors argue that because the mining flexibility is low in the bitcoin market, the fixed cost can keep miners staying in the bitcoin market even when the block reward declines. Most of these studies focus on the normative questions about the bitcoin system, like what the optimal block size should be, or how the vulnerability of this system should be dealt with. In this study, we instead take the current bitcoin transaction system as a given and study why the reactions to a congested network in terms of transaction volume and transaction fees may vary in different periods.

There are also studies using queueing theory to look into the mechanism of the Bitcoin system. Easley et al. (2019) use queuing theory to study the performance of bitcoin as a transaction medium, and find out how transaction fees arise from the bitcoin network. Huberman et al. (2019) embed queuing theory into their analysis of users' optimal strategies and establish a closed-form formula of users' transaction fees and waiting time given their different delaying costs. Again using queuing theory, Li et al. (2018) examine the conditions under which different Nash equilibria exist in the bitcoin payment system, assuming that transaction fees preassigned to each user. Kasahara and Kawahara (2016) and Kawase and Kasahara (2017, 2018) use a signal-server priority queue with batch service rate to model the transaction process in the bitcoin payment system and study the effect of block size on transaction-confirmation time. Kasahara and Kawahara (2016) find that increased block size cannot effectively reduce transaction-confirmation time. By excluding the newly arrival transactions from the current mining block, Kawase and Kasahara (2017, 2018) show that increased block size can reduce the transaction-confirmation time of the high-priority group but not the low-priority group.

While queuing theory is clearly a suitable tool to understand the nature of the bitcoin system, to the best of our knowledge, the arrival rate of users and the service rate of miners in these bitcoin-related studies are assumed to be exogenous and constant for technical reasons.[3] The stable service rate from the miners' side can properly capture the supply side of the bitcoin system in the long run, given the existence of the difficulty adjustment algorithm embedded in the bitcoin blockchain protocol. However, the difficulty adjustment takes roughly two weeks, during which the supply rate is not constant. Another restriction of queuing theory is that the results only apply to the steady-state of the system, and not much is said on the adjustments of the bitcoin system. Our study focuses on the short-run dynamics inside the bitcoin system. We adopt a demand-supply model to allow the arrival rate from the users' side and the service rate from the miners' side to be both flexible, which can better capture the characteristics of the bitcoin system in the short-run.

We are also interested in the costs of employing the bitcoin transaction system in our society. Chiu and Koeppl (2017) adopt the Lagos and Wright (2005) model to study the welfare loss incurred by double-spending in the bitcoin system. They argue that the current bitcoin system is "generating a welfare loss of 1.4% relative to an efficient cash system," and the mining cost encouraged by the double-spending incentive is at least one of the main sources. In this study, we calculate the other costs due to occasional congestion of the bitcoin system, costs that are present even without the double-spending issue.

## 2. Background

In 2008, a paper entitled "Bitcoin: A Peer-to-Peer Electronic Cash System" began to circulate on the internet under the pseudonym of Satoshi Nakamoto (Nakamoto, 2008). In this paper, Nakamoto proposes a decentralized electronic currency system, and its most innovative part is a mechanism called "Proof-of-Work" that solves the double-spend problem without a central clearinghouse. Under this system, every miner keeps an individual ledger of all the transactions, and miners reach a consensus on the state of transactions roughly every 10 min and then update their ledgers. Such a decentralized ledger system, compared with a central ledger system, is supposed to be more resilient to data loss and manipulation.

Based on this paper, Nakamoto created the bitcoin network in 2009. The network becomes a new way to distribute information and eliminates the need for a trusted third-party to facilitate the distribution process, and the trading parties do not need to reveal their identities to others. The anonymity helped bitcoin gain popularity beyond the computer science community, and people began to use it for transactions without revealing their identities.

In this section, we explain the technical details about bitcoin that are necessary for understanding the rest of the paper. We discuss three unique aspects of the bitcoin network that motivate this paper.

[3] Arrival rate or service rate can be time-dependent in the queuing theory, but the results are often intractable.

### 2.1. Mining

An important feature of mining is that, based on the computing power currently in the network, the bitcoin system automatically adjusts the difficulty of the Proof-of-Work algorithm to make sure that blocks are generated, on average, every 10 min. However, the adjustment is not immediate and takes 2016 blocks. After every 2016 blocks, all nodes will recalibrate the difficulty of the Proof-of-Work algorithm based on the average mining time in the past 2016 blocks.

Due to the difficulty of the adjustment algorithm, we can reasonably assume that, when looking at the weekly or monthly horizon, the mining speed or service rate from the miners' side is relatively stable. However, when looking at a shorter period of time, we do see the mining speed changes based on how often the difficulty adjusts in a certain period (see Fig. 1).

Another important feature of mining relates to the miners in the system. In the beginning, mining bitcoin is more or less a "hobby" to people in the computer science community. When bitcoin price took off in 2017, the competition of mining bitcoin became more intensive, as seen in the exponentially growing hash rate in Fig. 1. It became no longer profitable to mine bitcoin using the CPU or GPU on a personal computer, and the miner community shifted to dedicated ASIC bitcoin mining equipment. Unlike a CPU or GPU, which can be used for other purposes, the mining ASIC equipment is specially designed to be used on mining bitcoins. The technology change implies that more professional miners join the bitcoin network recently, and mining bitcoin may become unprofitable for amateur miners. While we do not have direct evidence to prove the demographic change of bitcoin miners, Google trend data in Fig. 2 on "antminer" and "bitmain" (the two biggest ASIC mining equipment producers) is indirect evidence on this mining technology change on the supply side.

### 2.2. Transaction volume

In the bitcoin system, there are two kinds of transactions: on-chain and off-chain transactions. On-chain transactions refer to the confirmed transactions stored in the blockchain, and anyone in the bitcoin network can verify them. Off-chain transactions are not stored in the blockchain, and their most common applications are Lighting Network transactions and exchange transactions. Off-chain transactions usually incur no transaction fees (e.g. transactions in the Lighting Network), or the transaction fee rate is preset by a third party (e.g. exchange transactions). However, the transaction fees in on-chain transactions fluctuate and are determined by demand (users) and supply (miners). Since the unique transaction fee mechanism is the main focus in our paper, it is natural that we only look at on-chain transactions.

Unfortunately, as the bitcoin system operates more like cash than bank accounts, it is challenging to measure precisely the volume of on-chain transactions. Suppose wallet A wants to send wallet B one bitcoin, and wallet A holds two bitcoins. The bitcoin ledger records the transaction as wallet A sending one bitcoin to wallet B and one bitcoin to wallet A itself. However, if the person owns both wallet A and C, then the ledger record may look like wallet A sends one bitcoin to wallet B and one bitcoin to wallet C, misleadingly looking like a transaction of two bitcoins.[4]

Hence, there is no foolproof way to filter out all of the "spurious" transactions and calculate the actual on-chain transaction volume. Some companies use different heuristics to parse on-chain transactions (see CoinMetrics for an example), and try to recover the accurate on-chain transaction volume. In this paper, we use the parsed on-chain transaction trade volume data compiled by CoinMetrics, but we want to remind our readers that the transaction volume used in this paper may still contain some measurement errors.

### 2.3. Transaction fees

In the bitcoin network, transaction fees have two essential roles in the transaction confirmation process:

(1) Security: transaction fees make it economically unattractive to attack the network with fake transactions.
(2) Motivation: transaction fees motivate miners to stay in the network, preventing the network from being controlled by a small group of people and becoming centralized.[5]

Before 2015, transaction fees were never a concern to bitcoin users. In Fig. 3, we can see that before June 2015, the average block size was always below 0.5 megabyte, which is half of the block size limit. During that period, people could even submit a transaction without offering any transaction fees, and some miners would still put the transaction into a half-empty block and get it confirmed. However, starting around 2016, a growing number of transactions needed to be confirmed, and the block size limit began to create competition among those unconfirmed transactions. From then on, people need to attach transaction fees in their transactions to motivate miners to promptly confirm their transactions.

Fig. 4 shows that when the number of transactions waiting to be confirmed in the mempool is high, we also observe a higher average transaction fee recorded in the bitcoin network. Fig. 5 shows that when average transaction fees are high, so is the average transaction volume. One plausible explanation for these phenomena is that users do not make a transaction

[4] A more detailed discussion on "spurious" transactions can be found at Yang (2020).

[5] Nowadays, miners' rewards still mostly come from Coinbase reward, which means the miner who successfully adds a block into the blockchain will be rewarded 12.5 BTC. However, Coinbase reward halves every 210,000 blocks. At this speed, approximately, the reward will become 0 in 2140, and then the only source of reward comes from the transaction fee.

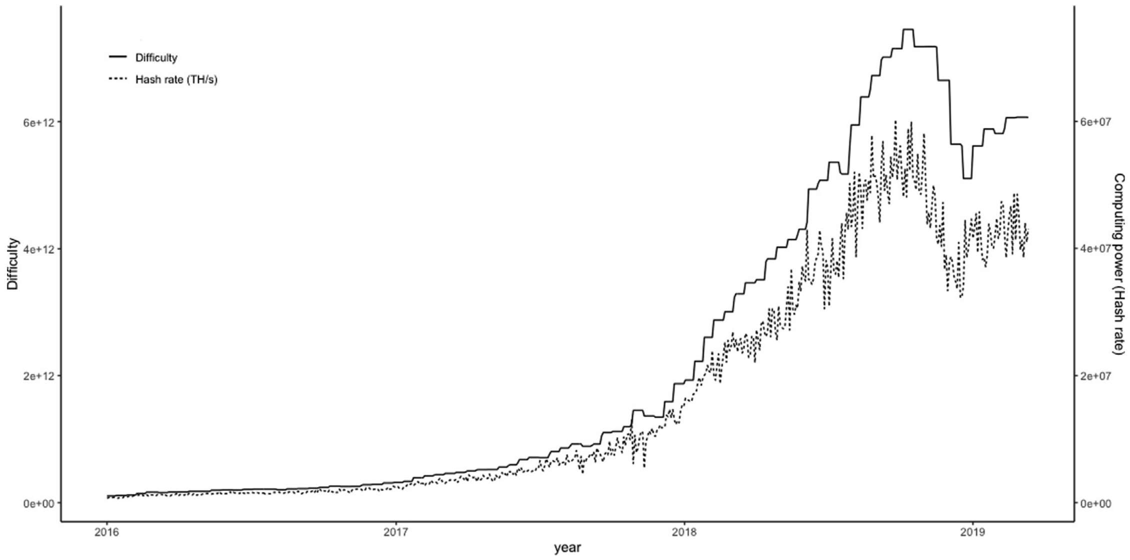



**Fig. 1.** Difficulty and computing power (hash rate) since 2016. The data comes from blockchain.com. This figure shows the change of difficulty and computing power in the bitcoin network since 2016. The difficulty will be re-calibrated by the Proof-of-Work algorithm after mining every 2016 blocks to match with the computing power in the network. The computing power measures how much computing resource has been devoted into the network to mine blocks.

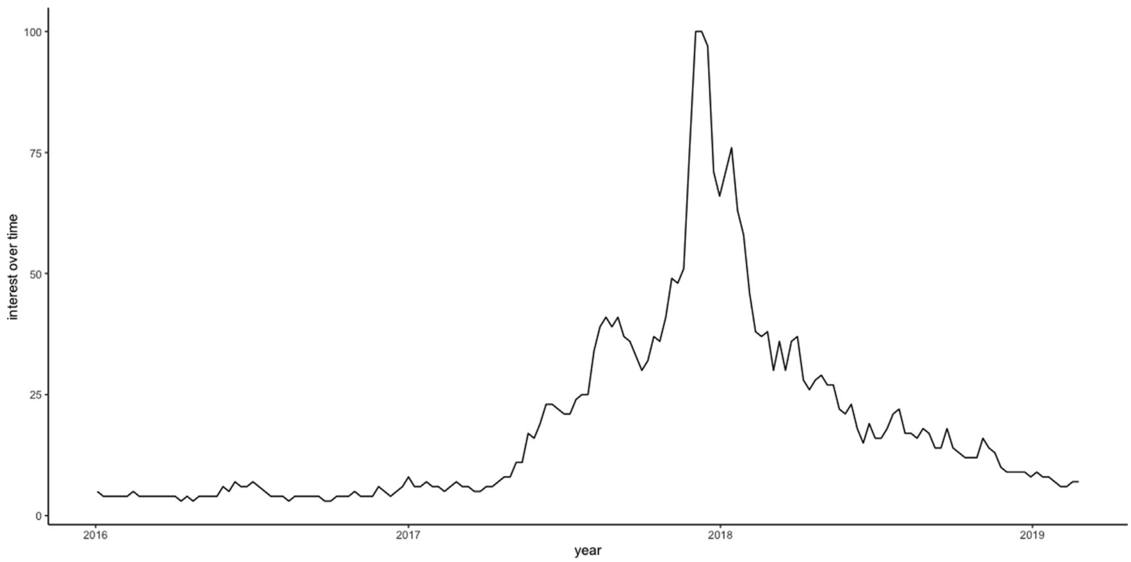



**Fig. 2.** Searching popularity of ‘‘antminer” and ‘‘bitmain”. The data comes from the Google Trend. The Google trend data shows how many inquires to Google are related to ‘‘antminer” and ‘‘bitmain”, which are the brands of the two biggest ASIC mining equipment producers. This figure shows the dynamic change of the popularity of ‘‘antminer” and ‘‘bitmain” since 2016.

decision based on the absolute level of transaction fees but the transaction fees as a percentage of the transaction amount (TFR henceforth). Users find smaller transactions more expensive, and their number decreases in the bitcoin payment system when the network is busy or when transaction fees are high. The positive correlation between average transaction fee and average transaction volume becomes not so obvious after 2018, and it becomes almost unobservable since the end of 2018. We think one plausible explanation is the gradual adoption of SegWit in the miner community. SegWit is a proposal to lift the block size limit. It enables miners to process more transactions in one block.

Due to the mining mechanism and transaction fee system, the relationship between transaction sizes and transaction fees is quite different from what we usually see in conventional payment systems. In the credit card system, the credit card company acts as a centralized clearinghouse, and sets the transaction fee unilaterally. In the bitcoin network, however, transac-

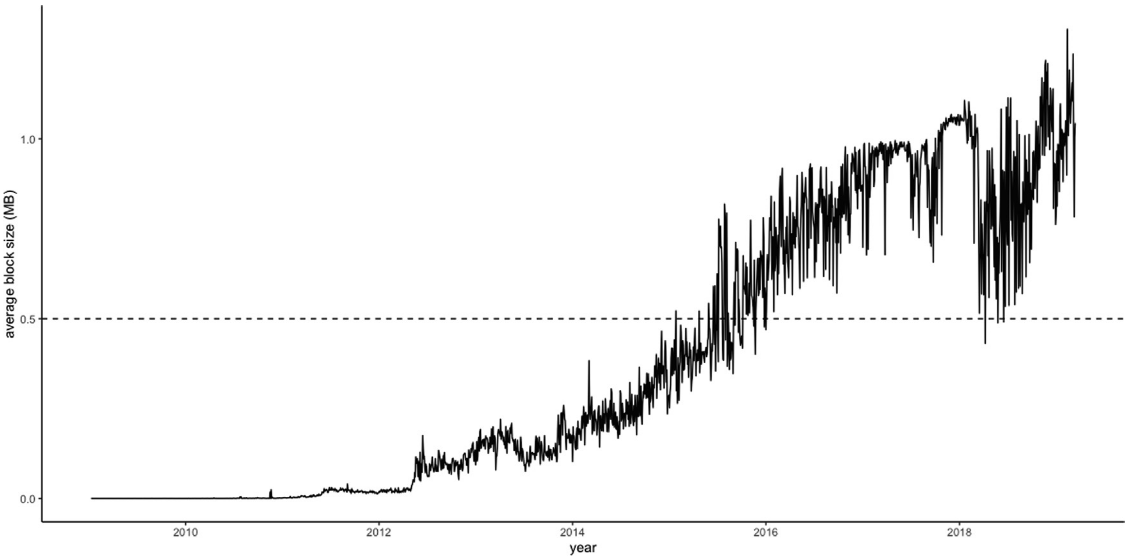



**Fig. 3.** Average daily block size. The data comes from blockchain.com. This graph shows the average block size on daily basis since the beginning the bitcoin network. Originally, the block size limit is set to be 1 MB by the bitcoin algorithm. We can see that, before June 2015, the blocks are always less than half full in the bitcoin network.

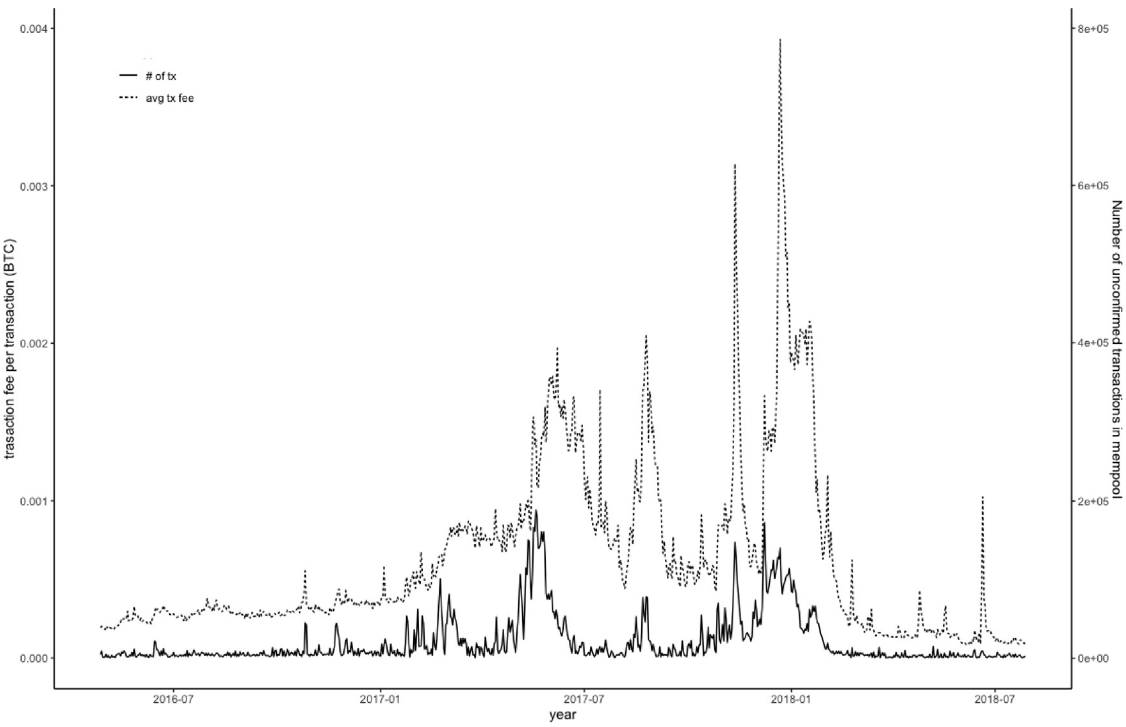



**Fig. 4.** Number of transactions in *mempool* and average transaction fees. The data comes from blockchain.com. This graph shows the relationship between the number of transactions in mempool (unconfirmed transactions) and the average transaction fees paid in those confirmed transactions in the bitcoin network. When the number of unconfirmed transactions is high, we can observe an increase in transaction fees in those confirmed transactions.

tion sizes and transaction fees depend on how congested the network is. In the next section, we use VAR model to exam the reactions to a congested network in terms of transaction sizes and transaction fees.

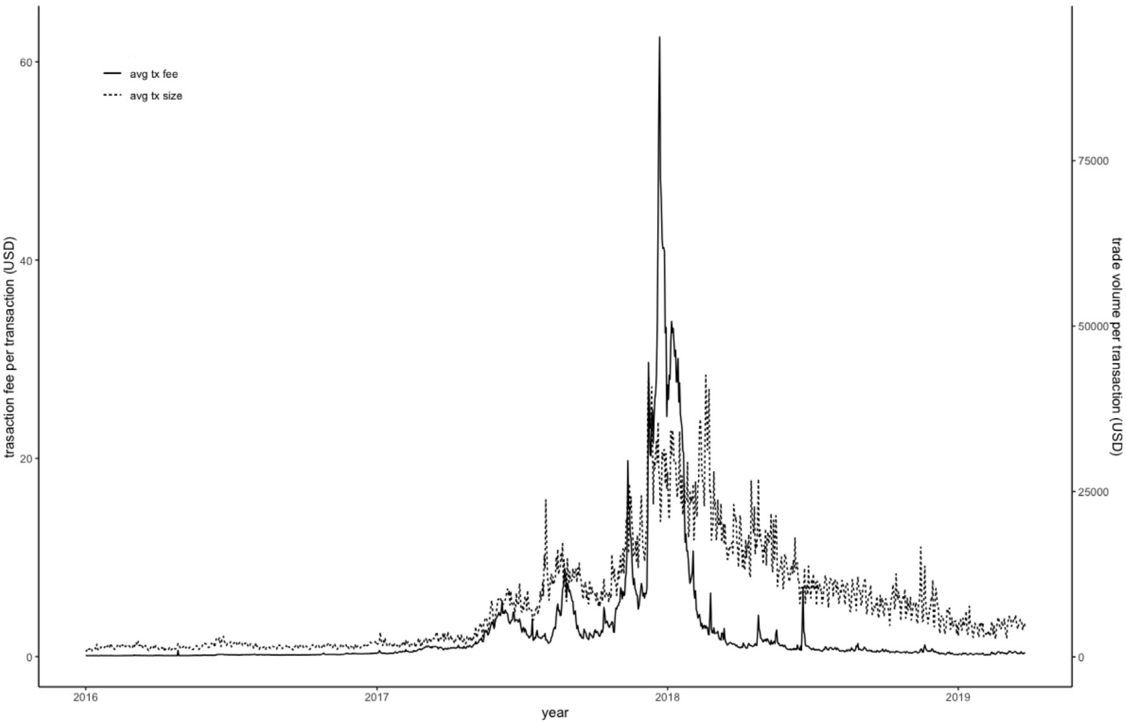



**Fig. 5.** Average transaction volume and average transaction fees. The data comes from blockchain.com. This graph shows the relationship between the average transaction volume and average transaction fees. We can see that when transaction fees are high, so is the average transaction volume. This relationship becomes not so obvious since 2018, and it becomes almost unobservable since the end of 2018. One plausible explanation is the gradually adoption of SegWit inside the miner community.

## 3. Empirical results

### 3.1. VAR with a particular ordering

In this section, we use a VAR model to study the effect of a demand shock on transaction fees and transaction volume in different periods.

To study the effect of demand shock in the bitcoin network, this VAR model consists of four variables: $D_t, F_t, V_t$ and $P_t$.[6] $D_t$ represents the transaction demand in the bitcoin network. Unfortunately, we cannot directly observe the exact demand for bitcoin transactions. Google trend and tweet data have been used as the proxy variables for bitcoin transaction demand in some studies (Liu and Tsyvinski, 2018; Yelowitz and Wilson, 2015). In this study, we choose to use the number of unconfirmed transactions in mempool (mempool size) to represent bitcoin transaction demand based on two reasons. The first reason is mempool size measures the materialized demand. Google trend and Twitter data may add more noise into the measure of demand because mass media sentiment may not eventually become real demand in the network. The second reason is that the Google trend adjusts their based line from time to time, which may bring more noise into our analysis. $F_t$ represents average transaction fees per confirmed transaction in BTC. $V_t$ represents the average transaction volume per confirmed transaction in BTC. $P_t$ represents the bitcoin price in USD. Hence, the model can be written as:

$$\begin{bmatrix} a_{11}^0 & 0 & 0 & 0 \\ a_{21}^0 & a_{22}^0 & 0 & 0 \\ a_{31}^0 & a_{32}^0 & a_{33}^0 & 0 \\ a_{41}^0 & a_{42}^0 & a_{43}^0 & a_{44}^0 \end{bmatrix} \begin{bmatrix} D_t \\ F_t \\ V_t \\ P_t \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 & a_{14}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 & a_{24}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 & a_{34}^1 \\ a_{41}^1 & a_{42}^1 & a_{43}^1 & a_{44}^1 \end{bmatrix} \begin{bmatrix} D_{t-1} \\ F_{t-1} \\ V_{t-1} \\ P_{t-1} \end{bmatrix} + \cdots + \begin{bmatrix} \epsilon_{D,t} \\ \epsilon_{F,t} \\ \epsilon_{V,t} \\ \epsilon_{P,t} \end{bmatrix}$$

The data comes from Coinmetrics.io and Blockchain.com, and the $D\_t, F\_t, V\_t$, and $P\_t$ are all in log. Seasonal effects have been removed by adding the days of a week as exogenous variables, and the lags are chosen in different periods based on AIC.[7] We choose three lags for the early period and two lags for the rest two periods.

[6] Thanks to our referee's suggestion, we add bitcoin price into our model and it helps us better capture the effect of $D_t$ on $F_t$ and $V_t$.

[7] Other information criterion rules give similar results, and do not change the final conclusions.

We split the data into three periods: the early period (from 2016-05-01 to 2017-01-01), the volatile period (from 2017-06-01 to 2018-02-01), and the recent period (from 2018-03-01 to 2018-10-01). The statistical description of data is shown in the following Table 1. These three periods are chosen based on two characteristics embodied in the bitcoin network: (1) From the miner side: the mining technology is evolving from general-purpose equipment (i.e. CPU, GPU) to dedicated mining equipment (i.e. ASIC). The latest expensive mining equipment makes mining bitcoin no longer a cheap hobby for most people. We think the evolution of mining technology induces the demographic change of bitcoin miners: more professional miners joined the network recently. (2) From the user side: the uncertainty faced by users in the bitcoin network is changing, and the uncertainty is measured by bitcoin price volatility.

Based on the discussion in Section 2.1, the whole industry begins to shift to relay on dedicated ASIC bitcoin mining equipment after 2017. By ending the early period before 2017, we can safely assume that the proportion of miners who mine bitcoins as a hobby is higher in the early period. Meanwhile, The volatile period is chosen by the standard deviation of the bitcoin price in USD. Based on the bitcoin historical price data on Blockchain.com, the standard deviation of the bitcoin price is \$4752.05 in the volatile period, which is much larger than that in the early period and the recent (\$108.48 and \$1209.4 respectively). Hence the early period represents a network with more amateur miners and low uncertainty; the volatile periods represents a network with more professional miners and high uncertainty; the recent period represents a network with more professional miners and low uncertainty, and is served as a benchmark to the other two periods. We also intentionally leave a gap on the time horizon to isolate these characteristics into different periods.

Due to the mempool data limitation, we do not have data earlier than 2016-05. We did not include the most recent data into our analysis due to the implementation of SegWit. SegWit is a proposal to lift the block size limit (1 MB) on the blockchain by removing the signature part out of the input section. Miners gradually begin to adopt SegWit since the 2017-07-21. However, the effect of SegWit is not so obvious before the end of 2018 because, most of the time, the average block size is still smaller than 1 MB (see Fig. 3), which could be an indicator of a low adoption rate of SigWit among miners. However, from the end of 2018, we can observe that the average block size has become consistently larger than 1 MB. To minimize the effect of SigWit, we do not consider the data after 2018-10.

In the bitcoin network, miners are profit-oriented and tech-savvy. They monitor the bitcoin network 24/7 and respond immediately to the network changes, which means miners will respond to the change of mempool size immediately by choosing to process more profitable transactions first. The mempool size, on the other hand, represents users' demand for transactions. When the transaction fee changes, users still need time to search for trade opportunities first. Meanwhile, if the transactions are already in the mempool, users cannot withdraw the transactions from the mempool, so the reaction from the mempool side to other shocks will be sluggish. For these reasons, we think it is reasonable to assume that transaction fees and transaction volume can be affected by a current shock to the mempool size, but the mempool size may not immediately respond to current shocks from transaction fees or transaction volume. Also, we think bitcoin price should be most sensitive to all the other shocks, so $P\_t$ can immediately react to all current shocks from other variables. Hence, we order the mempool size ($D\_t$) first and the bitcoin price ($P\_t$) last in our VAR model. The positions for $F\_t$ and $V\_t$ are arbitrary, and the final ordering in this main result is chosen as $D\_t, F\_t, V\_t$, and $P\_t$. In the following part, we will also show the ordering-free result. The level values of variables $D\_t$ and $V\_t$ are stationary in all three periods, $F\_t$ is stationary in the early and recent periods, but not in the volatile period, $P\_t$ is not stationary. To avoid the issue of spurious regressions, we use the first difference value of $F\_t$ in the volatile period and the first difference value of $P\_t$ across all three periods.[8]

To make the results comparable, all the shocks in the VAR results have been normalized to one-unit shocks from the number of unconfirmed transactions in mempool ($D\_t$). First of all, we compare the responses across different periods. The impulse responses from the three periods are shown in Figs. 6 and 7. In Fig. 6, we can see that, compared with the response in the recent period, the average transaction volume ($V\_t$) experiences a period of dropping below the steady-state value 0 after the shock in the early period. Also the magnitude of the response from the average transaction fee ($F\_t$) in the recent period is slightly larger. In Fig. 7, the responses from $V\_t$ is slightly larger in the recent period. Also, the magnitude of the response from $F\_t$ is larger in the volatile period. In both graphs, we can see that demand shocks do not necessarily increase bitcoin price. This is understandable because the demand ($D\_t$) we measured here is for bitcoin transactions, not for bitcoin itself. Some transaction demand shocks are caused by negative news about bitcoin, which may drive down bitcoin prices. In the following event study section, we will have further discussion on this topic. From what we observe in these two graphs, we can reach two conclusions: (1) a demand shock can induce a positive jump on the average transaction volume, and the reaction can turn into negative in the earlier period; (2) a demand shock can trigger a larger positive response from the average transaction fee in the volatile period.

Then we compare the magnitudes of the responses from $F\_t$ and $V\_t$ within the same period in Fig. 8. We can see only in the volatile period the magnitude of the response from $F\_t$ is observably larger than that from $V\_t$. Meanwhile, as mentioned above, the negative response from $V\_t$ is much more observable in the early period.

[8] To make sure our results are not driven by how we treat $F\_t$ differently in the volatile period, we also use the first difference values and the level values of $F\_t$ across all three periods, the results are similar and do not change the conclusions we draw in this paper.

**Table 1**
Summary statistics. This table reports the descriptive statistics of the data used in the three periods: the early period (from 2016-05-01 to 2017-01-01), the volatile period (from 2017-06-01 to 2018-02-01), and the recent period (from 2018-03-01 to 2018-10-01). *D_t* represents the unconfirmed transactions in the mempool, *f_t* represents the average transaction fee, *V_t* is the average transaction volume, *P_t* is the bitcoin price in USD. The data comes from Coinmetrics.io and Blockchain.com, and the variables are all in log.

| | Early period | | | | Volatile period | | | | Recent period | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *D_t* | *F_t* | *V_t* | *P_t* | *D_t* | *F_t* | *V_t* | *P_t* | *D_t* | *F_t* | *V_t* | *P_t* |
| Mean | 8.38 | −8.15 | 1.89 | 6.45 | 9.89 | −6.88 | 1.85 | 8.61 | 7.53 | −8.96 | 1.13 | 8.92 |
| Med | 8.31 | −8.15 | 1.88 | 6.46 | 10.08 | -6.93 | 1.87 | 8.42 | 7.58 | −8.97 | 1.12 | 8.89 |
| Max | 11.00 | −7.50 | 3.16 | 6.88 | 12.11 | -5.54 | 3.79 | 9.88 | 9.97 | −6.89 | 1.72 | 9.35 |
| Min | 5.27 | −8.63 | 1.13 | 6.08 | 5.08 | -7.77 | 1.17 | 7.57 | 4.16 | −9.64 | 0.55 | 8.68 |
| Std.dev | 0.79 | 0.17 | 0.36 | 0.17 | 1.28 | 0.51 | 0.36 | 0.17 | 1.04 | 0.38 | 0.23 | 0.66 |
| Obs. | 246 | 246 | 246 | 246 | 246 | 246 | 246 | 246 | 215 | 215 | 215 | 215 |

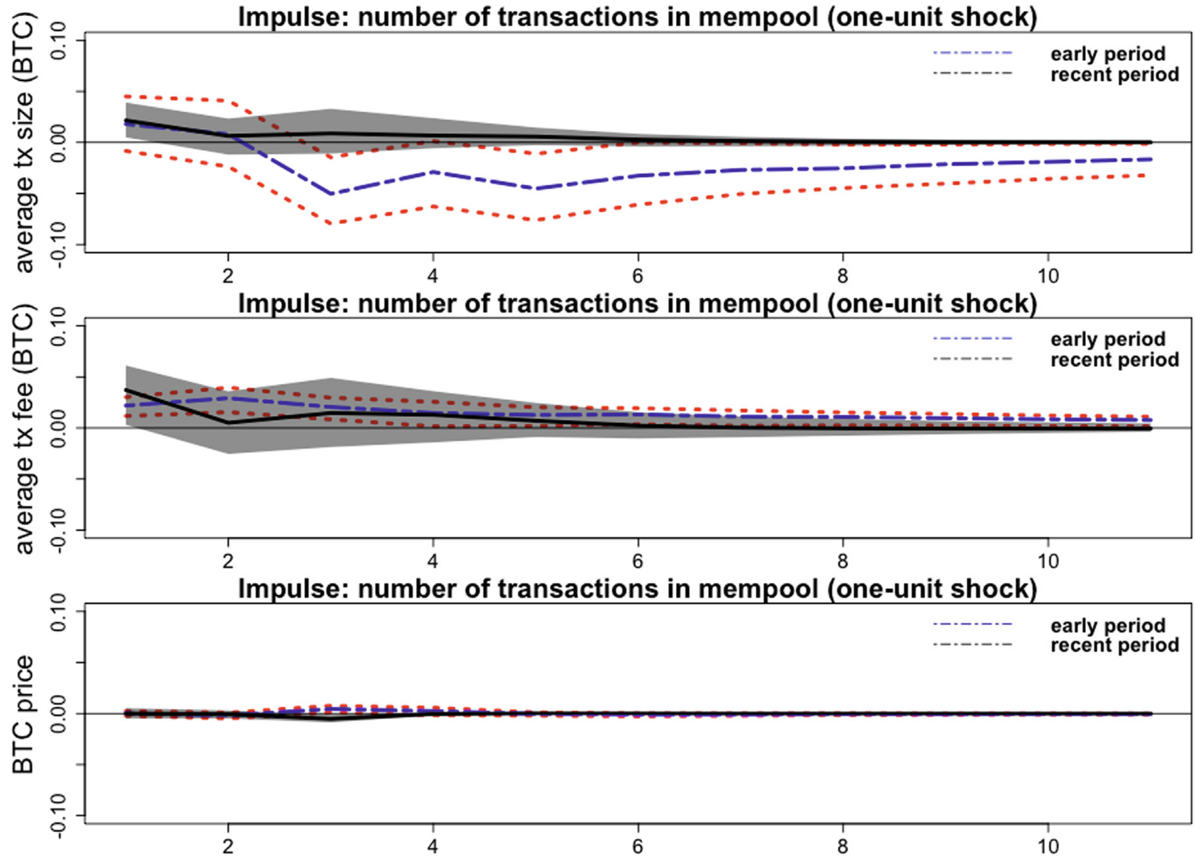



**Fig. 6.** Impulse responses in the early period and the recent period. This graph compares the responses to a one-unit shock from demand in the early period and the recent period. Compared with the responses in the recent period, we cans see that *V_t* experiences a period of dropping below the steady-state value 0 after the shock in the early period. The magnitude of the response from *F_t* is slightly larger in the recent period.

### *3.2. VAR without a particular ordering*

One concern about VAR results is that our impulse response results may be driven by the particular ordering chosen in our VAR. As mentioned in Primiceri (2005), and following Diebold and Yilmaz (2009) and Klößner and Wagner (2014), now we try to be agnostic on the ordering and present average results obtained from all 24 possible orderings. The impulse response results are shown in Figs. 9–11. Comparing the early period and the recent period, we can see that the results are almost identical in Figs. 9 and 6. Figs. 10 and 7 show that the magnitude difference of the demand effect on average transaction fee (*F_t*) in the volatile period, and the recent period becomes slightly smaller in the average VAR results, even though the effect is still larger in the volatile period. Also, similar to the early period, the average transaction volume (*V_t*) experiences a period of dropping below the steady-state value in the volatile period. We also do not observe any noticeable

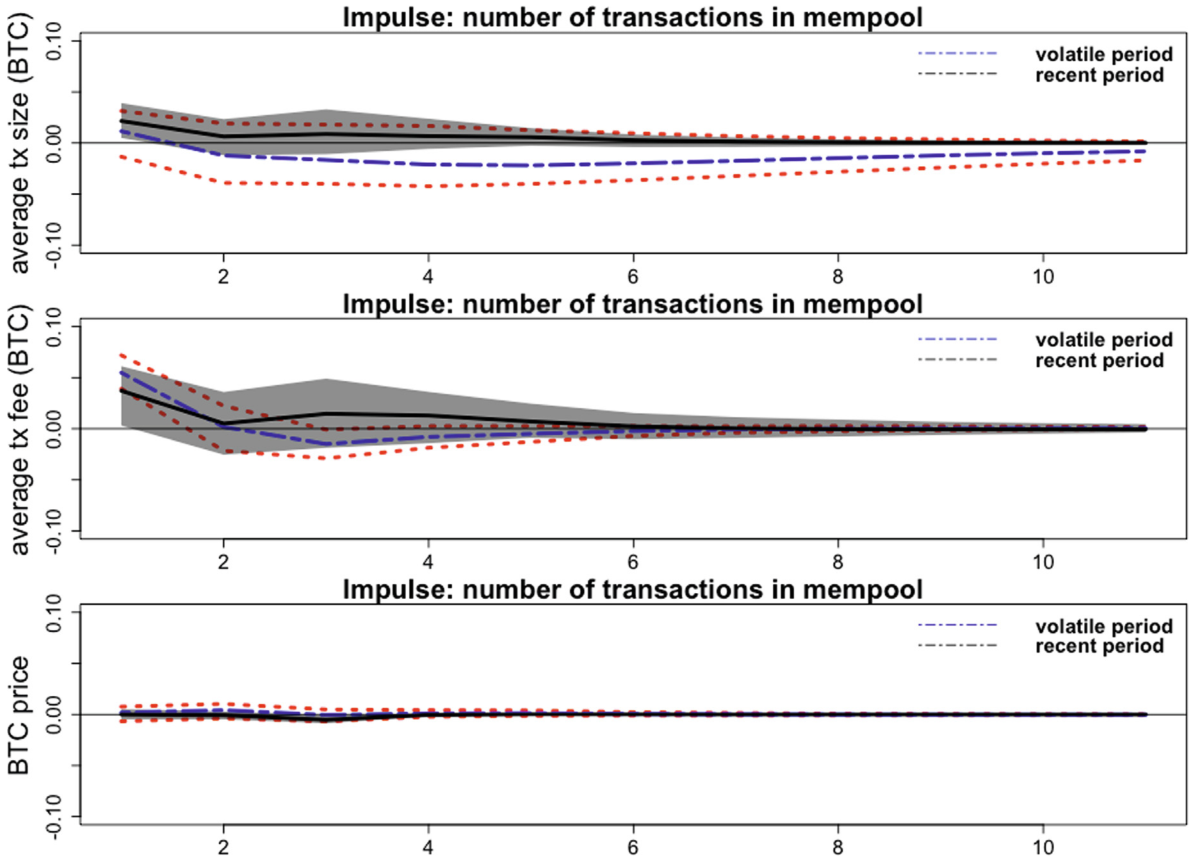



**Fig. 7.** Impulse responses in the volatile period and the recent period. This graph compares the responses to a one-unit shock from demand in the volatile period and the recent period. The responses from the $V\_t$ is slightly larger in the recent period. However, the magnitude of the response from $F\_t$ is larger in the volatile period.

demand effect on bitcoin price in the average VAR results. By comparing the magnitudes of the responses from $F\_t$ and $V\_t$ in the same period (see Fig. 11), we still observe that, only in the volatile period, the magnitude of the responses from $F\_t$ is observably larger than that from $V\_t$. All these average VAR results service as the robustness check showing that our main VAR results are not merely driven by a particular ordering.

### 3.3. Event study

Beside the VAR results, we also examine events happened in the real world to see if the bitcoin market behaves as we observed in our VAR results. From the three periods mentioned in subsection 3.1, we pick out events that satisfied the following two criteria:

(1) The bitcoin transaction demand change should be larger than 50% on the event day.
(2) We can connect the event to a specific incident reported by media in $\pm1$ days.

Eventually, we find four events that roughly happened around the early period, six events for the volatile period, and six events for the recent period.[9] In these event studies, we define the inter-day movement as the log difference of a variable from its previous day. We find that the event study results are consistent with our VAR results.

Figs. 12–14 show the average event study results in the early period, volatile period and recent period respectively. The red line indicates the date when the event happened. The demand shocks in these three periods have been normalized to be a unit shock for easier comparison. Across these three periods, we do see a consistent pattern that a demand shock on bitcoin transactions ($D\_t$) is associated with an increase in average transaction volume ($V\_t$) and average transaction fees ($F\_t$). Also,

[9] We also observe two events that show a negative demand shock on 2017-10-25, Bitcoin Gold hard fork launch day (volatile period), and on 2018-02-15, Lunar New Year's Eve, a holiday celebrated in many Asian countries (recent period). The negative shock is associated with a drop on average transaction fees and average transaction volume. However, the data points are too sparse to draw a reliable conclusion.

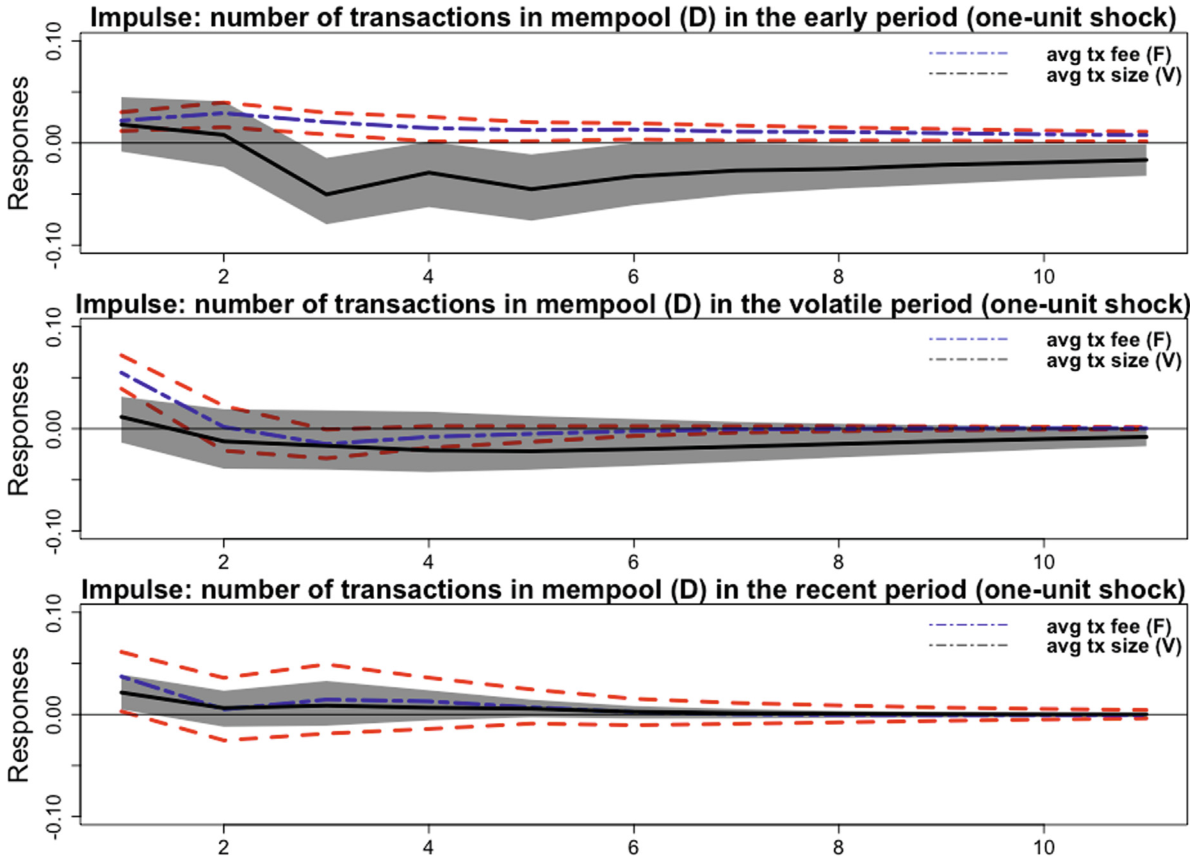



**Fig. 8.** Impulse responses in the three periods. This graph compares the magnitudes of the responses from $F\_t$ and $V\_t$ within the same period. We can see only in the volatile period that the magnitude of the response from $F\_t$ is obviously larger than that from $V\_t$. Meanwhile, the response from $V\_t$ turns into negative after the initial positive response is much more observable in the early period.

one unit demand shock is associated with a higher average transaction fee change in the volatile period than the other two periods. However, there is no clear pattern of how demand shock for bitcoin transactions affects bitcoin prices. If we dive into the individual event study figures, we can see that demand shock for bitcoin transactions can be associated with bitcoin price goes up (e.g. 2016-06-14, 2017-10-13 or 2018-07-18) or goes down (e.g. 2016-09-16, 2017-09-15 or 2018-06-27). This observation also explains why there is no noticeable effect of demand shock to bitcoin price in our VAR results. The inter-day movements in $F\_t, V\_t$ and $P\_t$ for each event date and the specific incident that trigged the demand change can be found in Appendix.

Based on all the empirical results listed above, we have observed that, in response to a demand shock of bitcoin transactions ($D\_t$), the responses from $V\_t$ and $F\_t$ behave differently in different periods. In the next section, we try to account for these differences by using a supply-demand model.

## 4. A simple model

As mentioned above, several studies use queuing theory to model the bitcoin transaction network (Huberman et al., 2019; Easley et al., 2019). In such models, users usually arrive in the bitcoin network as a Poisson process with a rate $\lambda$, and the service time provided by miners is exponentially distributed with mean $1/\mu$. The models can tell us how the bitcoin network looks like when $T \to \infty$, which is the steady-state. In this paper, we leave the steady-state aside and propose a model to capture the short-run dynamics of the bitcoin network, in particular when the network is hit by a shock from the demand side. We hope this model can help us further understand the VAR results we observed in Section 3.

### 4.1. Miners

Due to the bitcoin Proof-of-Work algorithm, most models assume that the service rate of miners is a Poisson process with an exogenous rate $\lambda$, which indeed is an accurate description when looking at a horizon longer than two weeks. Hence, if the

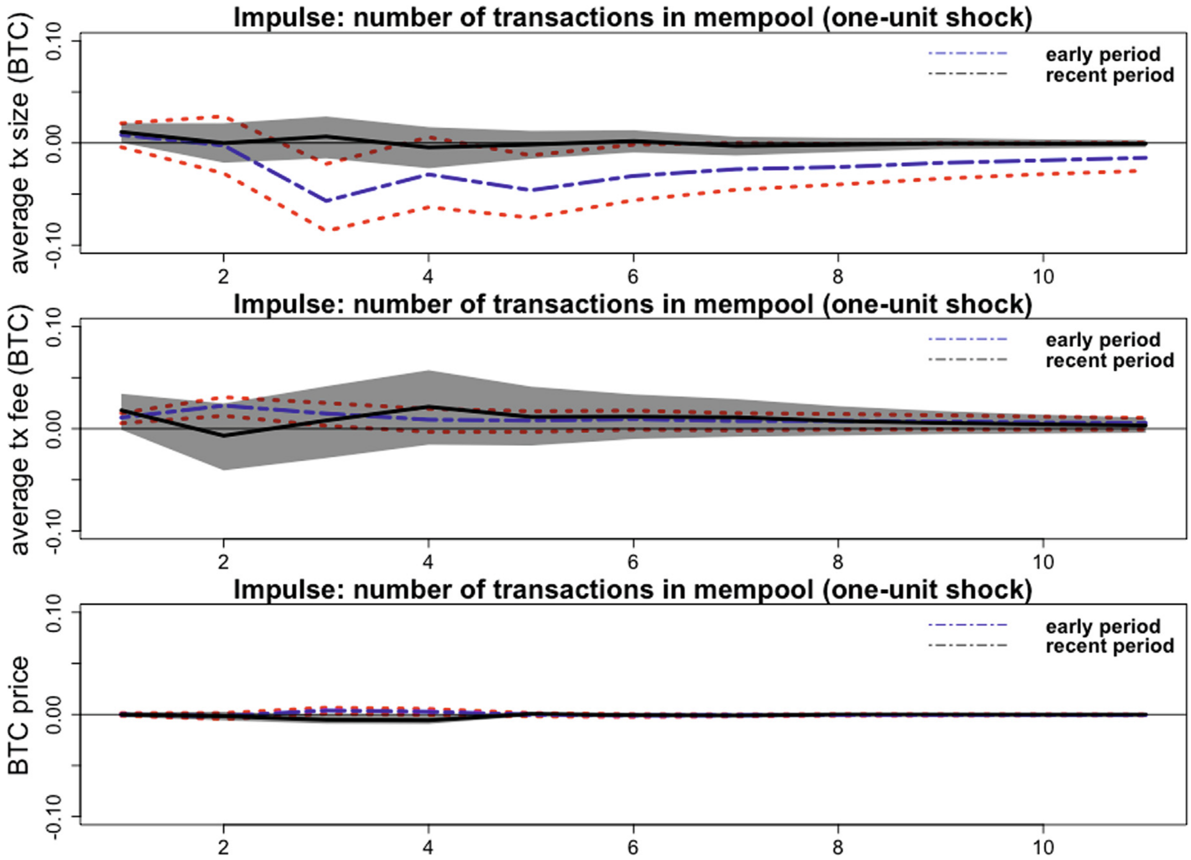



**Fig. 9.** Average impulse responses in the early period and the recent period. This graph compares the responses to a one-unit shock from demand in the early period and the recent period. Compared with the responses in the recent period, we can see that $V\_t$ experiences a period of dropping below the steady-state value 0 after the shock in the early period. The magnitude of the response from $F\_t$ is slightly larger in the recent period.

time horizon we are interested in is shorter than two weeks, it is more appropriate to assume that the service rate as not determined by a fixed parameter $\mu$. In this model, we allow the service rate to respond to transaction fees in any period.

Without loss of generality, we introduce a new concept called "effective service rate" into this model. We consider a case in which all miners join a mining pool, where the total reward is decided by the service rate provided by the mining pool. The reward is then divided based on how much computation power each miner puts into the mining pool, and by joining a mining pool, a miner can receive rewards based on how much effective service rate this miner provides, not on whether this miner successfully mined a block. The effective service rate $(q\_i, t)$ of miner $i$ in any period $t$ is defined as:

$$q\_i, t \equiv \frac{\text{Computation power from miner } i \text{ in period } t}{\text{Total computation power in the network in period } t} \times \text{Number of confirmed transactions in period } t$$

We assume no miner has market power, which means each miner's effective service supply is negligible compared with total demand and miners are price-takers. We can write down the individual supply function of miner $i$ as:

$$q\_i, t = f\_t^{\alpha\_i}$$

where $q\_i, t$ is the effective service rate of miner $i$ in period $t, f\_t$ is the transaction fees in period $t$, and $\alpha\_i$ describes supply elasticity.

In Section 2.1, we point out that miners in the bitcoin network have changed from treating mining as a "hobby" to participating as professionals. Likewise, there are two kinds of miners in the model: professional miners and amateurs. For professional miners, their individual supply function is $q\_1, t = f\_t^{\alpha\_1}$, and for amateurs, their individual supply function is $q\_2, t = f\_t^{\alpha\_2}$. We assume $\alpha\_1 > \alpha\_2$, meaning that supply from professional miners is more flexible or price elastic.

If there are $A\_1$ professional miners and $A\_2$ amateurs, then market supply can be written as:

$$S\_t' = A\_1 f\_t^{\alpha\_1} + A\_2 f\_t^{\alpha\_2}$$

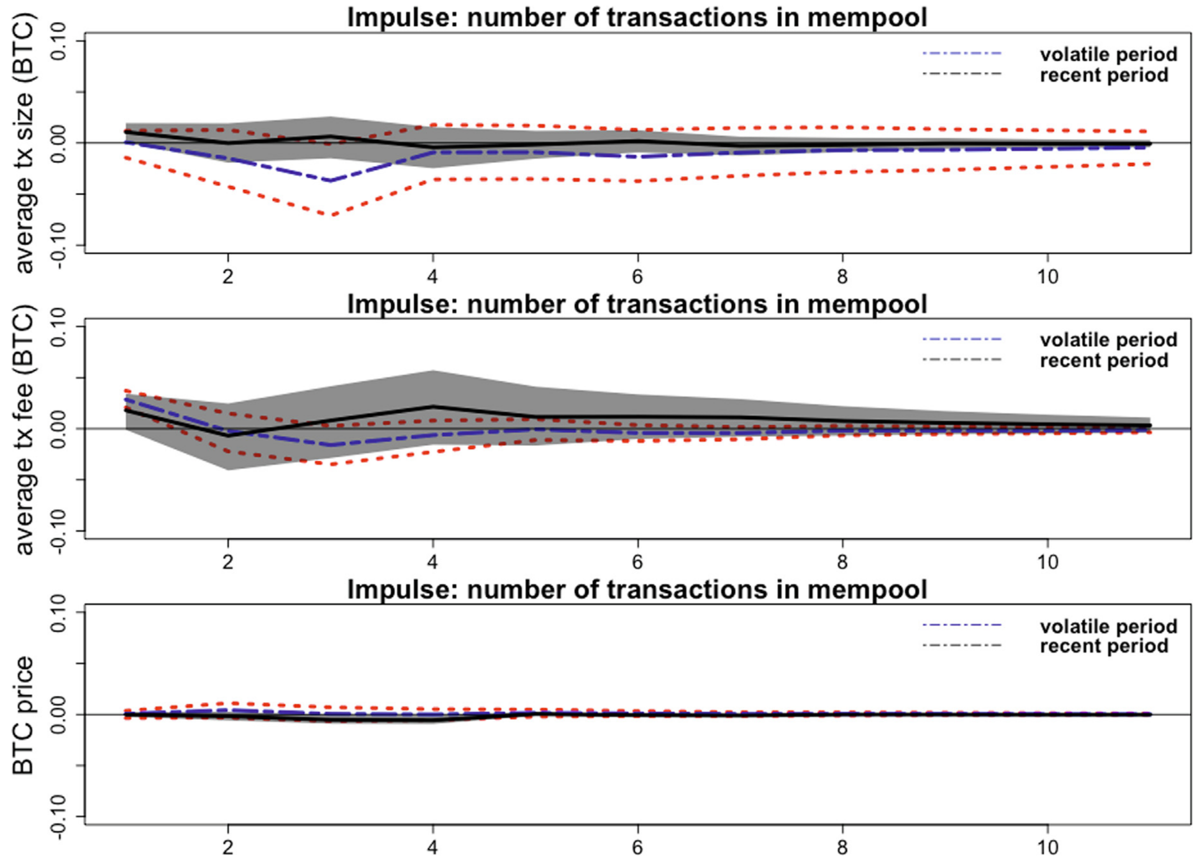



**Fig. 10.** Average impulse responses in the volatile period and the recent period. This graph compares the responses to a one-unit shock from demand in the volatile period and the recent period. Compared with the responses in the recent period, we can see that $V_t$ experiences a period of dropping below the steady-state value 0 after the shock in the volatile period. However, the magnitude of the response from $F_t$ is larger in the volatile period.

However, $S_t'$ only captures the financial incentive through transaction fees. The bitcoin system also rewards miners for providing their service, and this block reward is fixed in a short period.[10] Even without transaction fees from users, miners are still willing to provide a certain level of service for the rewards from the system. A more realistic way to write down market supply is:

$$S_t = S + A_1 f_t^{\alpha_1} + A_2 f_t^{\alpha_2} \tag{1}$$

where $S$ is the fixed service rate provided by miners when there is no financial incentive from users, $S_t$ is the total service rate provided by miners.

The elasticity of market supply is:

$$E_t = \frac{\partial S_t}{\partial f_t} \cdot \frac{f_t}{S_t} = \frac{\alpha_1 A_1 f_t^{\alpha_1} + \alpha_2 A_2 f_t^{\alpha_2}}{S + A_1 f_t^{\alpha_1} + A_2 f_t^{\alpha_2}}$$

The elasticity of the market supply will be in between $\alpha_2$ and $\alpha_1$, and its value is determined by $A_1$ and $A_2$, meaning that the ratio of the two kinds of miners in the network indirectly affects the elasticity of the market supply curve.

### 4.2. Users

Another feature that is missing in queuing theory concerns the users (the demand side). In those models, a user may choose to stay in a queue indefinitely until she is served. However, when a user submits a transaction into the bitcoin network, it will first stay in a mempool. Every miner maintains his own mempool, which is costly to him, and as a result, a trans-

[10] The block reward in the bitcoin network halves every 210,000 blocks. A countdown for bitcoin halving can be found at https://www.bitcoinclock.com/.

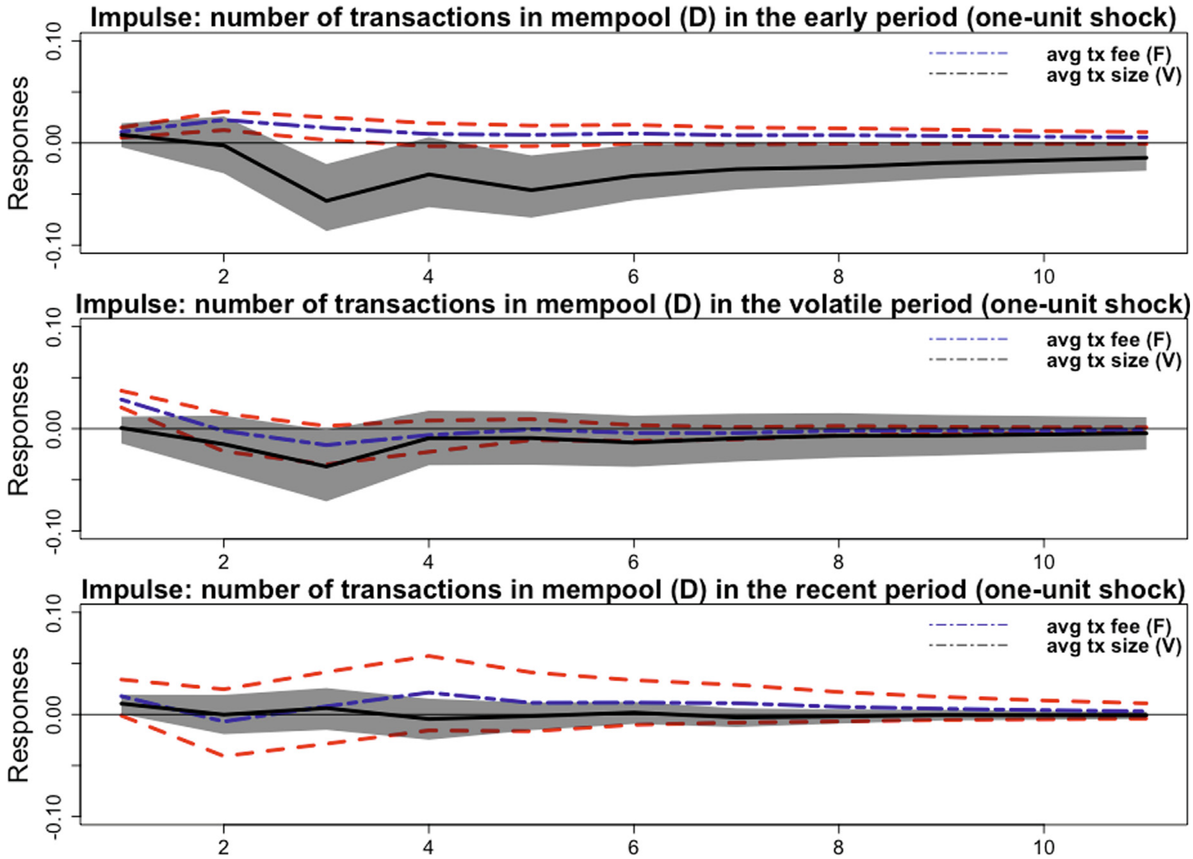



**Fig. 11.** Average impulse responses in the three periods. This graph compares the magnitudes of the responses from $F\_t$ and $V\_t$ within the same period. We can see only in the volatile period that the magnitude of the response from $F\_t$ is observably larger than that from $V\_t$. Meanwhile, the initial positive response from $V\_t$ turns negative in the following up periods becomes observable in the volatile period, even though it is still more obvious in the early period.

action may be cleared out of the mempool if it has stayed for too long. Typically, an unconfirmed transaction is allowed to stay in a mempool for less than a week.[11]

In our model, the bitcoin network exists for a total of $T$ periods, and everybody knows that. To capture the mempool feature, we impose an "overlapping generations" structure that each transaction has to be confirmed either in the current or the next period. After that, it will be rejected from the bitcoin network. Users in the first $T-1$ periods can choose to finish a transaction in the current period, next period, or just leave the network. Users in the last $T^{th}$ period can only process a transaction in the current period or leave the network.

To simplify the model, in each period, the transaction sizes of all the transactions satisfy a uniform distribution with the range $\overline{R} = (0, 1]$, and there is a continuum of users of mass $Q\_t$, which is exogenous. We assume users can choose to do their transactions in their current period and gain utility $R\_i - f\_t$, postpone their transactions into next period and gain utility $\beta(R\_i - f\_t + 1)$, or leave the network and gain zero utility. Here $R\_i$ is user $i$'s transaction volume, the $f\_t$ is the equilibrium transaction fee at time $t$, and $\beta$ is the discount rate. Suppose in each period $t$, the user who has a transaction volume $R\_t$ is indifferent between doing the transaction in the current period and postponing the transaction into the next period, and the transaction fee $f\_t$ paid by this user becomes the equilibrium transaction fee in the bitcoin network at time $t$.

In the $T^{th}$ period, users behave as follow:

$$R\_T = f\_T \tag{2}$$

$$S + A\_1 f\_T^{\alpha\_1} + A\_2 f\_T^{\alpha\_2} = \begin{cases} (R\_T - 1 - f\_T)Q\_T - 1 + (1 - R\_T)Q\_T & if \quad R\_T - 1 > f\_T \\ (1 - R\_T)Q\_T & if \quad R\_T - 1 \leqslant f\_T \end{cases} \tag{3}$$

[11] The bitcoin protocol does not specify how long an unconfirmed transaction can stay in a mempool, and the decision is made by miners themselves. The time limit is approximately 2 days (Beigel, 2020).

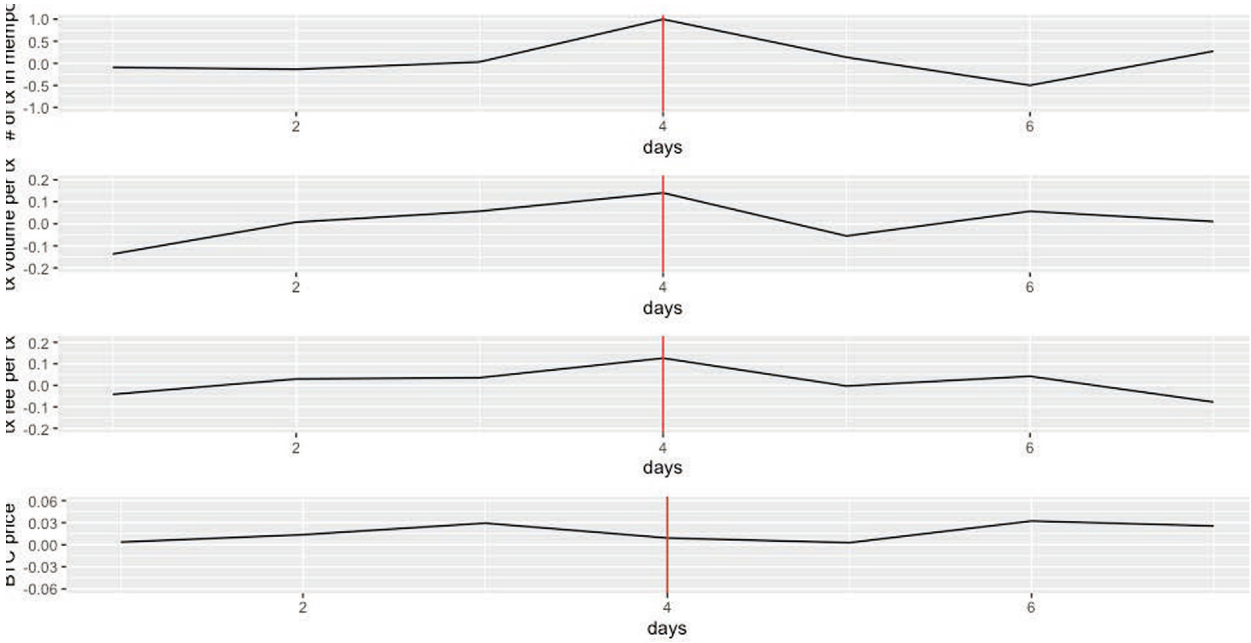



**Fig. 12.** Event study in the early period. This graph shows the average event study result in the early period, the event dates includes in the early period are 2016-05-27, 2016-06-14, 2016-09-16, and 2016-10-09. The specific incidents happened in these dates can be found in Appendix.

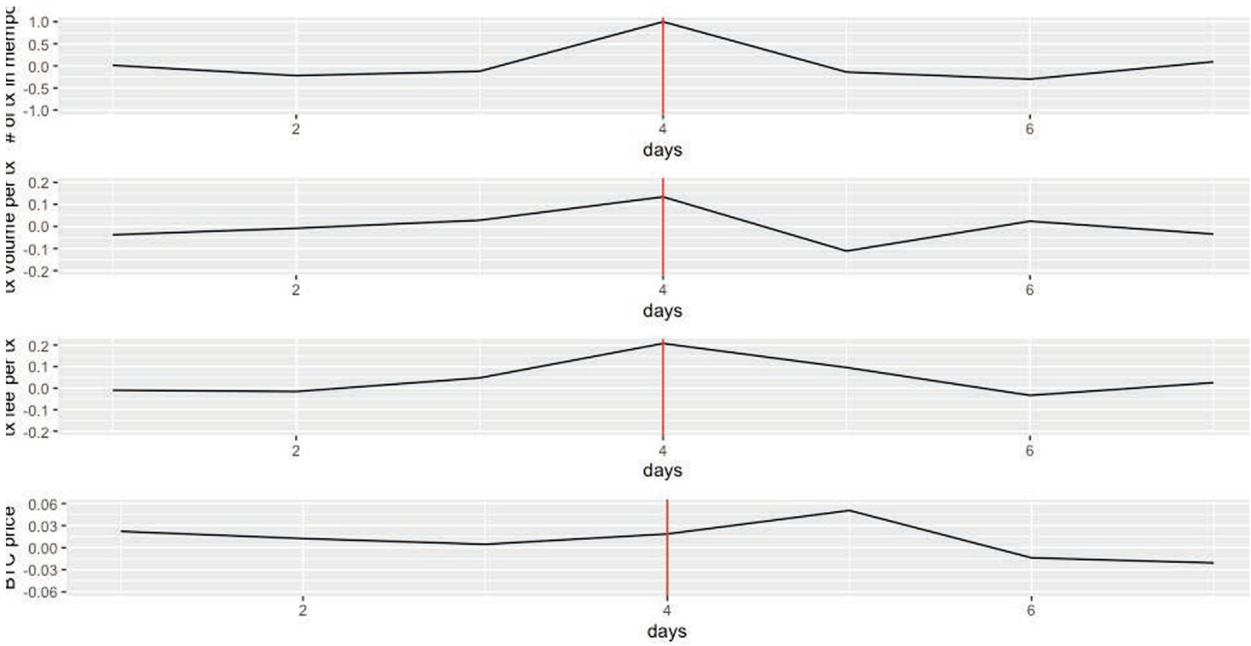



**Fig. 13.** Event study in the volatile period. This graph shows the average event study result in the early period, the event dates includes in the early period are 2017-05-09, 2017-08-14, 2017-09-15, 2017-10-12, 2017-12-07, and 2017-12-20. The specific incidents happened in these dates can be found in Appendix.

Eq. (2) says that, in the last period, no one can postpone transactions into the next period. The fees $f\_T$ directly determine that only the users whose transaction volume is larger than $f\_T$ will be willing and able to do transactions in the period $T$. Eq. (3) says that if $R\_T-1>f\_T$, the service provided by miners will satisfy the demand from users who postponed their transactions from period $T-1$ and also the demand from users in period $T$ who are willing and able to afford $f\_T$. Moreover,

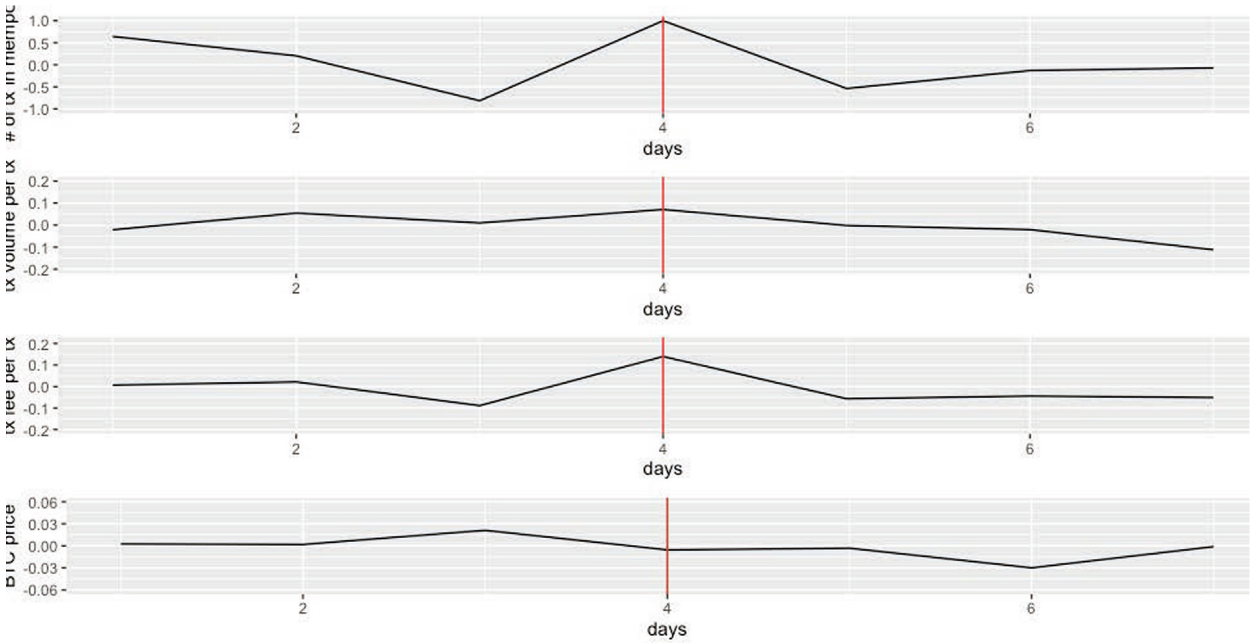



**Fig. 14.** Event study in the recent period. This graph shows the average event study result in the early period, the event dates includes in the early period are 2018-03-13, 2018-04-24, 2018-05-23, 2018-06-13, 2018-06-27, and 2018-09-04. The specific incidents happened in these dates can be found in Appendix.

only $(R\_T-1-f\_N)Q\_T-1$ users from period $T-1$ will get their transactions confirmed in period $T$, the rest will be cleared out from the system. If $R\_T-1 \leqslant f\_T$, all postponed transactions from period $T-1$ will be removed from the system.

For the second to the $T-1^{th}$ periods, users make decisions based on the following two rules:

$$R\_t - f\_t = \begin{cases} \beta(R\_t - f\_t+1) & \textit{if} \quad R\_t > f\_t+1 \\ 0 & \textit{if} \quad R\_t \leqslant f\_t+1 \end{cases} \tag{4}$$

$$S + A\_1 f\_t^{\alpha\_1} + A\_2 f\_t^{\alpha\_2} = \begin{cases} (R\_t-1-f\_t)Q\_t-1+(1-R\_t)Q\_t & \textit{if} \quad R\_t-1 > f\_t \\ (1-R\_t)Q\_t & \textit{if} \quad R\_t-1 \leqslant f\_t \end{cases} \tag{5}$$

$\beta$ in Eq. (4) represents the uncertainty of postponing transactions into the next period. In the bitcoin network, the value of $\beta$ is primarily decided by how unpredictable the purchasing power of bitcoin is. This unpredictability can be measured by the volatility of bitcoin price in USD. When the volatility is low, then the purchasing power of bitcoin is more predictable, which means $\beta$ is large, and people feel more comfortable to postpone their transaction into the next period.

Eq. (4) says that if $R\_t > f\_t+1$, some users will choose to postpone their current transaction to the next period. The number of users who will postpone depends on how large the $\beta$ is. When $\beta$ is larger, users are more patient, and more users will postpone their transactions. If $R\_t \leqslant f\_t+1$, users anticipate that transaction fees in the next period will be higher, and all transactions smaller than $f\_t$ will eventually be removed from the bitcoin network.

In the first period, users follow:

$$R\_1 - f\_1 = \begin{cases} \beta(R\_1 - f\_2) & \textit{if} \quad R\_1 > f\_2 \\ 0 & \textit{if} \quad R\_1 \leqslant f\_2 \end{cases} \tag{6}$$

$$S + A\_1 f\_1^{\alpha\_1} + A\_2 f\_1^{\alpha\_2} = (1-R\_1)Q\_1 \tag{7}$$

Eq. (7) shows that, in the first period, miners only serve users who intend to do transactions in the current period. Eventually, the average transaction volume per transaction $(V\_t)$ in each period can be written as in Eq. (8).

$$V\_t = \begin{cases} \frac{\frac{R\_t-1+f}{2}(R\_t-1-f\_n)Q\_t-1+\frac{1+R\_t}{2}(1-R\_t)Q\_t}{(R\_t-1-f\_t)Q\_t-1+(1-R\_t)Q\_t} & \textit{if} \quad R\_t-1 > f\_t \\ (1+R\_t)/2 & \textit{if} \quad R\_t-1 \leqslant f\_t \end{cases} t = 1,2,\dots T \tag{8}$$

Given Eqs. (2)–(8), we can calculate the equilibrium average transaction volume $V\_t$ and the transaction fee $f\_t$ for any period $t$. However, to solve how the two variables change when the system is hit by demand shock to $Q\_t$, closed-form solutions are hard to come by. To understand how the model works, we instead do some simple simulations in the following section.

# 5. Simulation results

## 5.1. The evolution of the bitcoin network

We choose $T = 10$, corresponding to a period shorter than 2 weeks. In addition, we choose $S = 25$ and $Q\_t = 25$ for $t \neq 2$, so that in the steady-state, transaction fees are zero and all transactions will not be postponed to the next period. The average transaction volume per transaction ($V\_t$) is calculated as in Eq. (8), and since the steady-state $V\_t$ is 0.5, to bring it down to 0, we use $V\_t - 0.5$ instead. The $V\_t$ curve shows the responses from the average transaction volume to the shock from demand $Q\_t$, and the $f\_t$ curve shows the responses from the transaction fees to the shock from demand $Q\_t$. Lastly, we choose $\alpha\_1 = 2$ and $\alpha\_2 = 0.5$ to differentiate the professional miners and amateurs.

Now suppose there is a demand shock in period 2 of $Q\_2 = 50$. By choosing different values of $A\_1, A\_2$, and $\beta$, we try to describe the bitcoin network under different situations. As mentioned in Section 3, we intentionally choose three discrete periods to ensure each period can preserve different characteristics. Compared with the rest two periods, a larger portion of miners mine bitcoin as a hobby in the early period, so the ratio of professional miners to amateur miners will be low in this period. Hence, we choose $A\_1 = 3, A\_2 = 15$ to simulate the early period, and $A\_1 = 15, A\_2 = 3$ for the rest two periods. In the volatile period, people face higher uncertainty for postponing transactions into the next period, so we choose $\beta = 0.2$ to simulate the volatile period, and $\beta = 0.7$ for the other two periods. The numeric values of these parameters are arbitrary. We are not trying to match the absolute magnitudes of the VAR results. What we aim at are the changing directions and the relative magnitude changes among different variables. In particular, we focus on the ratio of $A\_1$ to $A\_2$ and $\beta$, and see how they contribute to the different responses we observed in the VAR results in different periods.

**Comparing the responses across different periods:** The simulation results corresponding to the VAR results can be found in Figs. 15 and 16. The simulation results for the early period and the recent period match the corresponding VAR results (Fig. 6): the response from $V\_t$ switches to negative in the early period. This means many users choose to postponed transactions into their next period in the model, which drives down the average transaction volume. Also, in the simulation results, the respond from $f\_t$ is slightly larger in the recent period, which matches what we observe in the VAR results. The simulation results for the volatile period and the recent period also match the corresponding VAR results (Fig. 7): the magnitude of the responses from $V\_t$ is slightly larger in the recent periods, but the response from $f\_t$ is much stronger in the volatile period.

**Comparing the responses within each period:** The simulation results corresponding to Fig. 8 can be found in Fig. 17. We can see the magnitudes of the responses from $V\_t$ and $f\_t$ are almost the same in the early period and the recent period. However, the response from $f\_t$ in the volatile period is much larger than the response from $V\_t$. These phenomena are also observed in the VAR results.

After matching the simulation results with the VAR results, the following question is what causes the different VAR results in different periods. In the following part, we will distinguish the roles of the composition of miners (ratio of $A\_1$ to $A\_2$) and users' patience ($\beta$) in our simulation model.

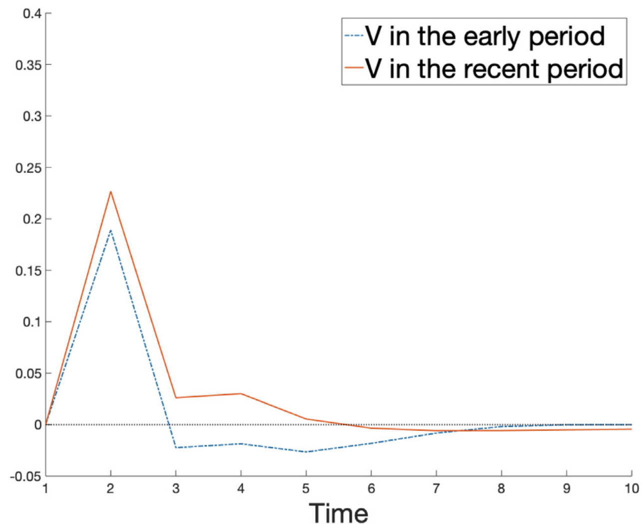
## 5.2. Comparative statics

The results above can be traced back to the changes in two parts of our model, one relates to the change in users' patience, and one relates to the change in the composition of miners.

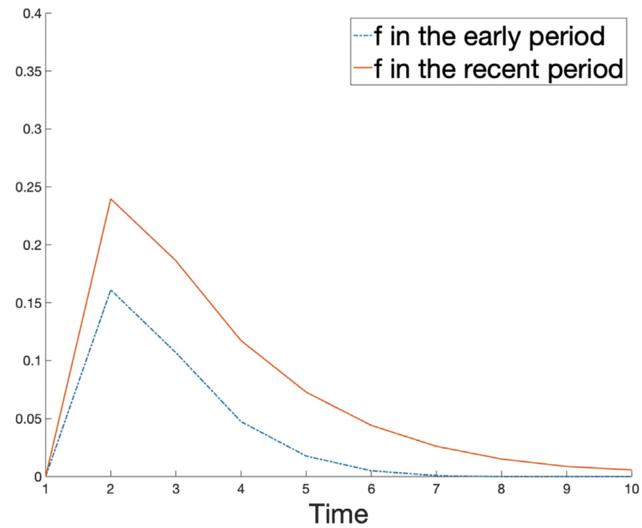
**Effect of $\beta$:** The most obvious effect we can observe is that low $\beta$ significantly increases the magnitude of the response from transaction fees. When people are more impatient, or when the future is more uncertain, the utility of postponing a transaction into the next period is almost 0. So users who have large transactions to make are willing to pay more to finish their transactions now.

**Effect of the ratio of $A\_1$ to $A\_2$:** When the ratio of $A\_1$ to $A\_2$ decreases, the most obvious effect is that $V\_t$ can drop significantly below 0 in the early period. This means more users will choose to postpone their transactions when there are more amateur miners and less professional miners in the bitcoin network. We think this is due to it is less costly to scale up service rates when there are more professional miners because they are more responsive to financial incentives. So it becomes reasonable to attract some users who initially will postpone their transactions into the next period to process their transactions in the current period by increasing the transaction fee a little bit. An increase in transaction fee will further deter users from postponing their transactions into the next period because they know the users in the next period are also willing to pay more transaction fee to boost up the service rate, so we do not observe a big drop on $V\_t$ when there are more professional miners ($A\_1$) than armature miners ($A\_2$). The unwillingness of postponing transactions drives up the fee and makes it only profitable for fewer transactions, that is why we can see the average transaction volume and transaction fee are higher in the recent period compared with that in the early period.

Based on the above analysis, we can see that: (1) the willingness of postponing transactions into the next period, which can be observed as a drop of $V\_t$ in the early period, is mainly caused by the low ratio of $A\_1$ to $A\_2$; (2) even though more professional miners in the network will deter users from postponing their transactions and drive up the transaction fee to some extent, the overreaction from $f\_t$ in the volatile period is mainly driven by a low $\beta$.




(a) Average transaction volume



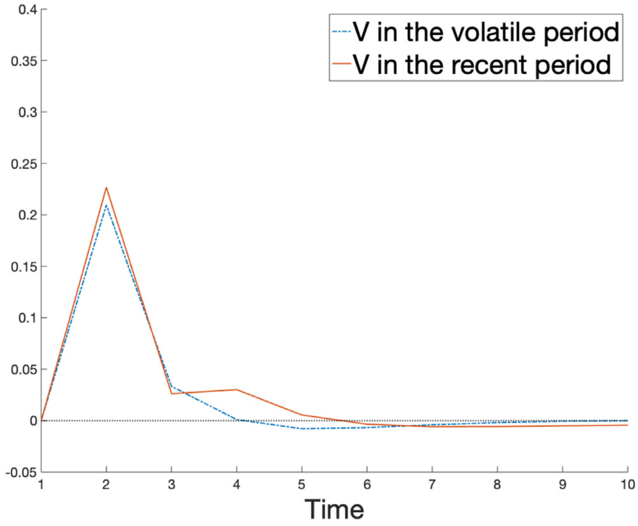

(b) Average transaction fee

**Fig. 15.** Responses in the early period and the recent period. The graph shows the simulation results from the early period and the recent period. The response from $V\_t$ switches to negative in the early period. Meanwhile, in the simulation results, the respond from $f\_t$ is slightly larger in the recent period.

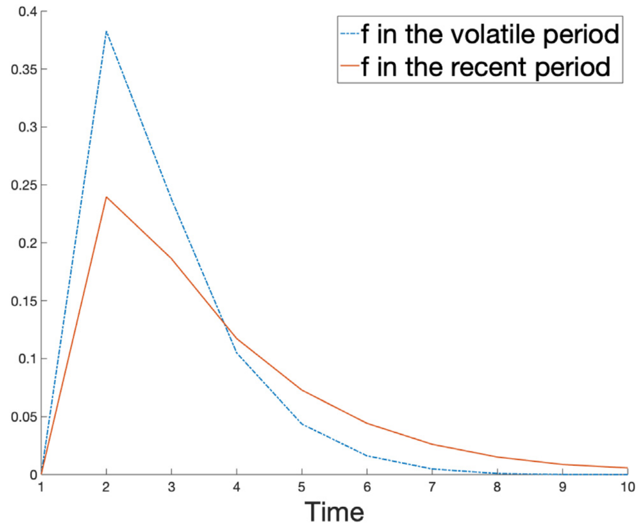
Till now, we have discussed the demand-supply mechanism behind the fluctuating transaction fee in the bitcoin system and explored the factors that can affect the transaction fee. In the next section, we try to measure the effect of this transaction fee system on bitcoin users.

## 6. Three costs of using the bitcoin system

In the bitcoin network, every transaction has to be confirmed by miners. The price of confirming a transaction (transaction fee) is decided by both the demand side (users) and the supply side (miners). When the bitcoin network is congested, a demand shock can drive up the transaction fee. This unique transaction-fee-decision mechanism incurs three different kinds of costs when bitcoin is used as a payment system.




(a) Average transaction volume

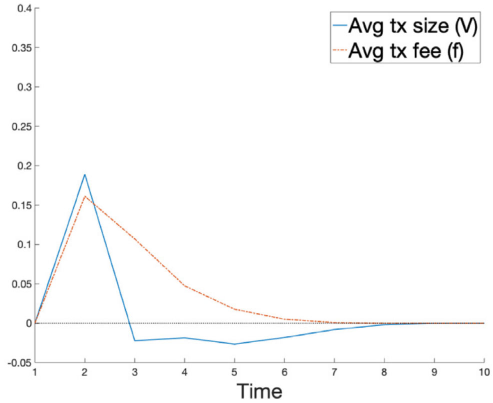



(b) Average transaction fee

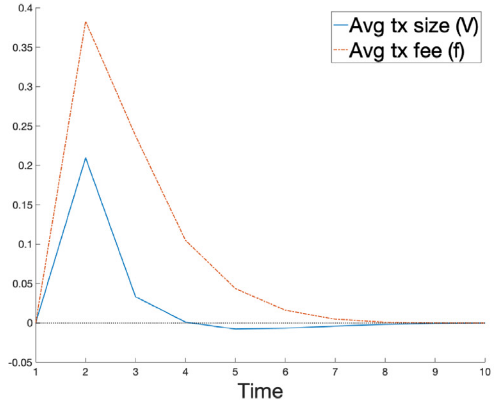
**Fig. 16.** Responses in the volatile period and the recent period. The graph shows the simulation results from the volatile period and the recent period. The magnitude of the responses from $V\_t$ are very similar in these two periods, but the response from $f\_t$ is much stronger in the volatile period.

## *6.1. The transaction fee rate*

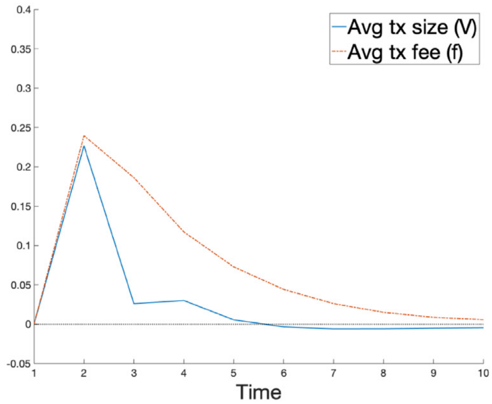
At the end of 2017, the high transaction fee becomes a major concern inside the bitcoin community (Browne, 2017). However, we argue that the transaction fee rate rather than transaction fee itself is a better index to measure how expensive a transaction network is. In Fig. 18, we plot the daily transaction fee rate from 2016-01-01 to 2019-03-01 in the bitcoin network. We can see the transaction fee rates are actually relatively low. Even the highest rate at the end of 2017 is still lower than 0.1%. As a comparison, in 2019, the average credit card transaction fee rates in the U.S. range from 1.7% to 3.5% (Prakash, 2019). Hence, a congested bitcoin network may drive up the transaction fee rate. However, given the current historical data, the bitcoin network is still much cheaper than those conventional payment networks, like Visa and MasterCard.




(a) Early period




(b) Volatile period




(c) Recent period

**Fig. 17.** Simulation responses in each period. This figure shows the simulation results inside each period. The magnitudes of the responses from $V\_t$ and $f\_t$ are almost the same in the early period and the recent period. However, the response from $f\_t$ in the volatile period is much larger than the response from $V\_t$. These phenomena are also can be observed in the VAR results.

### 6.2. The uncertainty of the transaction fee rate

Unlike a centralized transaction network, where transaction fee rate is pre-decided, the transaction fee rate in the bitcoin network is decided by the supply-demand mechanism, which makes the transaction fee rate fluctuating. The uncertainty of the transaction fee rate imposes another cost on the bitcoin payment system.

By borrowing the approach from Lucas (2003), we try to evaluate the potential cost of having a fluctuating transaction fee rate to a payment system. Consider a representative consumer is living in a world using bitcoin. Suppose the consumer's real consumption ($C\_t$) satisfies lognormal distribution, with $log(C\_t) \sim N(\mu, \sigma^2)$ and $E(C\_t) = e^{\mu + \frac{\sigma^2}{2}}$. As in Lucas, the consumer's utility function of consumption is of CRRA form:

$$U(C\_t) = \frac{C\_t^{1-\gamma}}{1-\gamma} \qquad \text{with} \gamma \neq 1$$

where $\gamma$ is the coefficient of risk aversion.

The expected utility of the consumer's real consumption is

$$E(U(C\_t)) = \frac{e^{\mu(1-\gamma)+\frac{\sigma^2}{2}(1-\gamma)^2}}{1-\gamma} = U\left(e^{\mu+\frac{\sigma^2}{2}(1-\gamma)}\right).$$

Hence, the utility of the real consumption $C\_t$ in the bitcoin network equals to a certain consumption of $C\_eq = e^{\mu+\frac{\sigma^2}{2}(1-\gamma)}$. The latter can be interpreted as the consumption level in an ideal payment system without transaction fees. Thus, the ratio of the consumption ($\lambda$) that the consumer is willing to give up to switch from this bitcoin system to this ideal transaction system is:

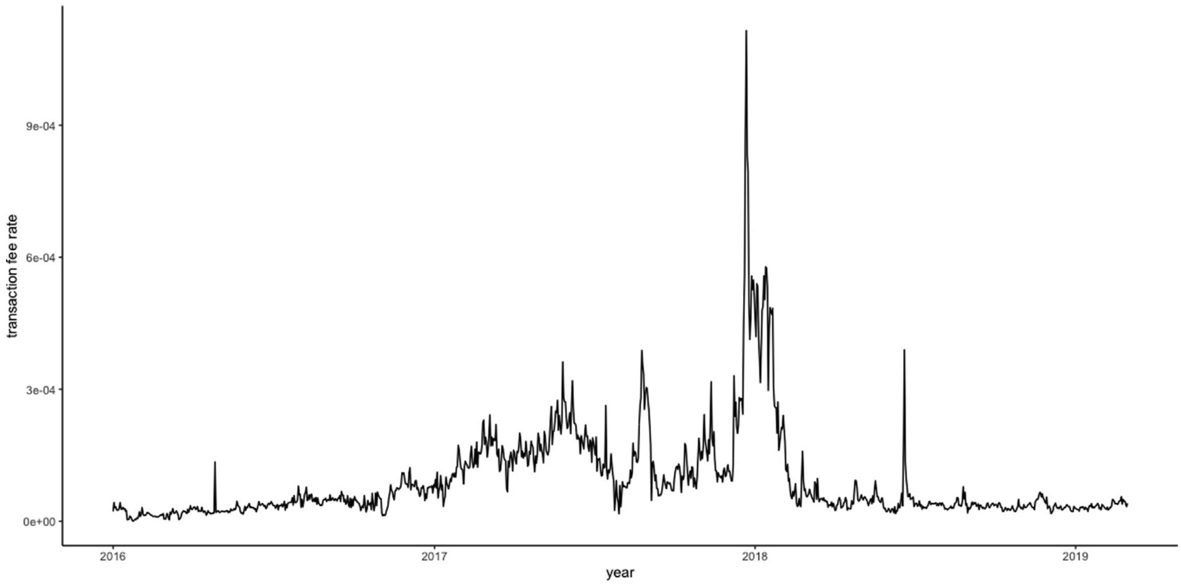



**Fig. 18.** Transaction fee rate. The data comes from blockchain.com. This figure shows the daily transaction fee rate in the bitcoin network during the period 2016-01-01 ~ 2019-03-01. We can see the transaction fee rate is actually relatively low. Even the highest rate is still lower than 0.1%.

$$\lambda = \frac{E(C\_t) - C\_eq}{E(C\_t)} = 1 - e^{-\frac{\sigma^2\gamma}{2}}$$

when $\frac{\sigma^2\gamma}{2}$ is very small, then $\lambda \approx \frac{1}{2}\gamma\sigma^2$.

To estimate how large the $\lambda$ can be, we need to find out the variance of $C\_t$. Suppose the transaction fee rate is a random variable $r\_f,t$. The representative consumer's income is $W$, and the consumer spends all the income, then: $C\_t = (1 - r\_f,t)W$. Then the variance of consumption is

$$\sigma^2 = Var(log(C\_t)) = Var(log(1 - r\_f,t) + log(W)) = Var(log(1 - r\_f,t)). \tag{9}$$

Eq. (9) shows that the variance of $C\_t$ is only related to the variance of transaction fee rate, $r\_f,t$. This conclusion also reconciles with the TFR rule discussed in subsection 2.3: people do not care about the absolute value of the transaction fee itself. What people care about is the transaction fee rate.

Using the whole bitcoin historical data, we can calculate that $\sigma^2 = 1.096e^{-8}$ in the bitcoin payment system. Even when we shrink the time window to only include the most fluctuating period, like from 2017-08-01 to 2018-04-01, $\sigma^2 = 2.815e^{-8}$.

If the consumer is extreme risk-averse, like $\gamma = 4$, in the utility function, and given $\sigma^2 = 2.815e^{-8}$, we can calculate that $\lambda = 5.630e^{-8}$. This means the cost of enduring the uncertain transaction fee rate in the bitcoin system only counts as $5.63e^{-6}\%$ of the consumer's consumption, which is negligible.

Now compare the bitcoin transaction system with the credit card system. If the consumer chooses to use credit card system, then the processing fee rate is certain, so the effective consumption in credit card system can be written as $C\_effective = (1 - r\_cc)e^{\mu}$, where $r\_cc$ is the flat processing fee rate charged by credit card companies and $\mu = log(W)$. In the bitcoin system, if the expected transaction fee rate is $r\_btc$, then we can get $\mu' = log((1 - r\_btc)W) = log(1 - r\_btc) + \mu$ and $C\_eq' = e^{\mu' + \frac{\sigma^2}{2}(1-\gamma)}$.

$$C\_eq' = C\_effective \Rightarrow \sigma\_eq^2 = \frac{2ln\left(\frac{1-r\_cc}{1-r\_btc}\right)}{1-\gamma} \tag{10}$$

Eq. (10) shows how fluctuating the transaction fee rate in the bitcoin payment system could be that the representative consumer would still be indifferent between the bitcoin payment system and the credit card system. Based on previous discuss about the average credit card processing fees in the US, we assume the fee rate is 1% in the credit card system ($r\_cc = 0.01$). And the representative consumer is extreme risk-averse ($\gamma = 4$). To make the argument strong, we consider the worst scenario in bitcoin system by choosing $r\_btc = 0.001$, then we can get the condition for this consumer becomes indifferent between these two payment systems is $\sigma\_eq^2 = 6.03e^{-3}$. However, from above, we can see that, given the current transaction fee in the bitcoin system, the variance of consumption is $\sigma^2 = 2.815e^{-8}$, which is much smaller than $6.03e^{-3}$.

Hence, consumers may need to deal with a fluctuating transaction fee rate if they choose to use the bitcoin payment system. However, given the current historical data, the cost of the uncertainty from fluctuating transaction fee rates is still much smaller than the cost of using the conventional transaction networks, like Visa and MasterCard.

### *6.3. The cost of the crowding-out effect*

In subsection 2.3, we talked about the TFR rule, which says people make transaction decisions based on how large the percentage of the current transaction fee will be given the sizes of their potential transactions. This rule implies that, when the transaction fee is high, only those big-size transactions can still maintain a reasonable transaction fee rate. The preference for big-size transactions during high-transaction-fee periods can be called the crowding-out effect. This crowding-out effect is caused by both users and miners. When the bitcoin network is congested, miners prefer to process the transactions which pay the highest fee. Meanwhile, the users realize that only when the sizes of their transactions are big enough can a high transaction fee be justified, which leads to the crowding out of those small-size transactions. The crowding-out effect may be a plausible explanation for what we observed in our VAR results that transaction fee and transaction volume become higher during the high demand period, and the bitcoin network is congested.[12] If crowding-out effect exists, it may cause two parts of costs that we failed to capture: (1) the crowding-out effect can block out a group of users who are willing to pay low transaction fees. This is the direct cost of the crowding-out effect. (2) We may underestimate the standard deviation of the transaction fees users are willing to pay. The crowding-out effect may stabilize the transaction fee rate observed in the historical data and cause us to underestimate the cost of having a fluctuating transaction fee system. Unfortunately, due to the lack of data, we cannot directly measure the cost of the crowding-out effect.

In this section, we talk about three different kinds of costs associated with the bitcoin system. Using historical data, we find these costs are not unbearable compared with the conventional payment networks, like Visa and MasterCard. However, our back-of-envelope calculation may underestimate the costs because of the crowding-out effect.

## 7. Conclusion

In this paper, we study bitcoin as a payment system with fluctuating transaction fees. We find that a demand shock to bitcoin transactions is associated with higher average transaction fees, and the phenomenon is more obvious in the volatile period. We propose a simple model to capture the behavior of users and miners in the bitcoin network, and we find that the bigger effect of demand shock on average transaction fees in the volatile period is due to users' impatience in that period. More professional miners in the network can also discourage users from postponing their transactions and drive up the average transaction volume. Furthermore, using historical data, we estimate the disadvantage of using a payment system with fluctuating transaction fee rates. We find that the cost is negligible. However, our calculation does not consider the crowding-out effect on transactions of small volume, and it may impose an extra cost to a certain group of users during the congested period.

This paper has several limitations. First, the trade volume data used in the VAR model are probably measured with error, as discussed in the paper. Second, if the bitcoin network can solve the scalability problem and significantly increase the number of transactions that can be processed in a given period, our current calculation based on the historical data may underestimate the variance of the transaction fee rate and also the cost of using the future bitcoin system. Third, in our model, the number of miners and the ratio of professional miners to amateur miners are exogenous, ignoring entry and exit of miners.

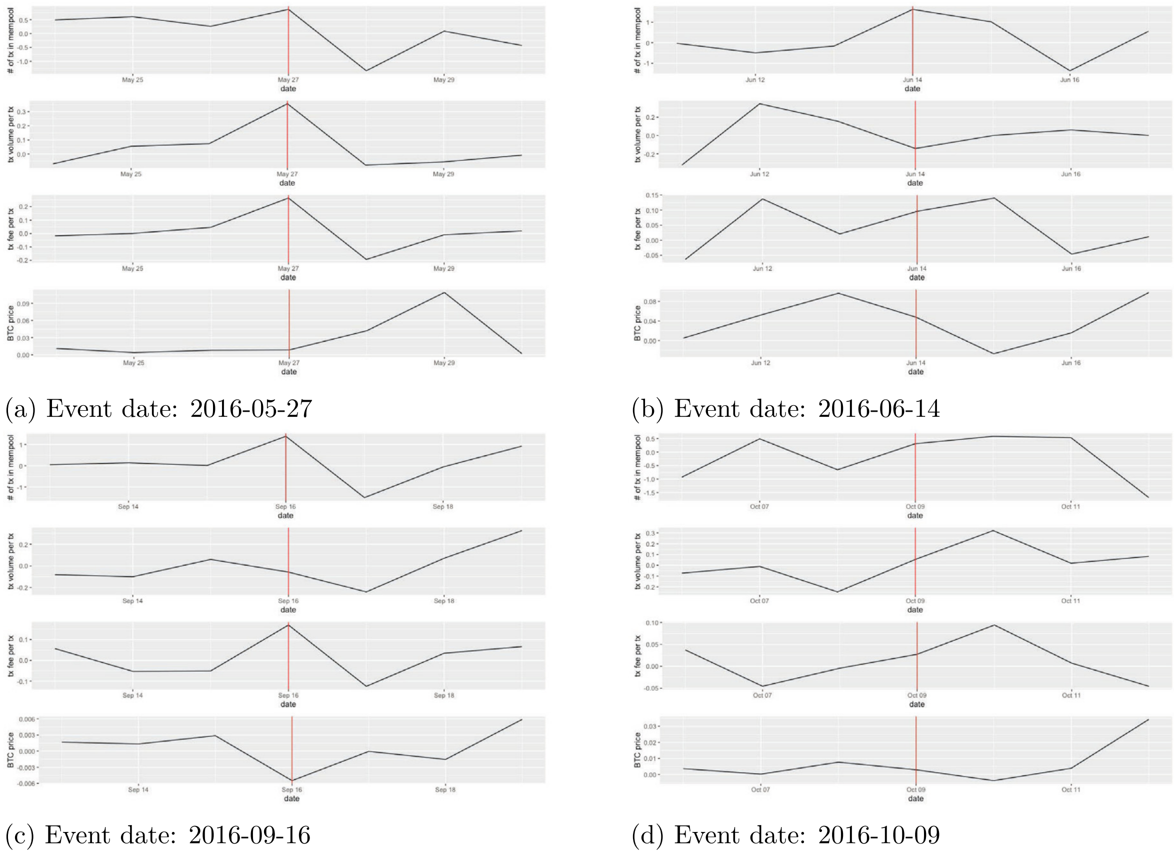
## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Appendix A. Event study

Figs. 19–21

[12] Thanks to the referee for pointing out that there are other plausible explanations for the higher transaction fee and transaction volume during a high demand period. For example, there may be more trading and less payments activity during the high demand periods and all the trading involves higher transaction fee and transaction volume.



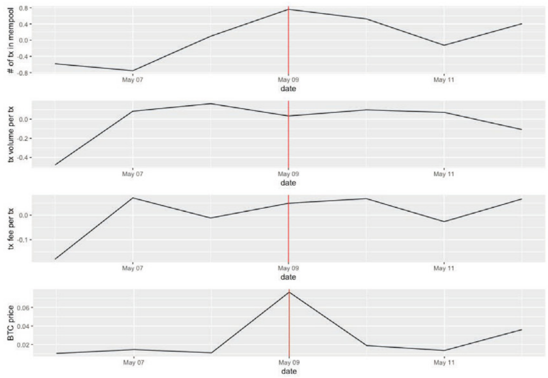

(a) Event date: 2016-05-27

(b) Event date: 2016-06-14

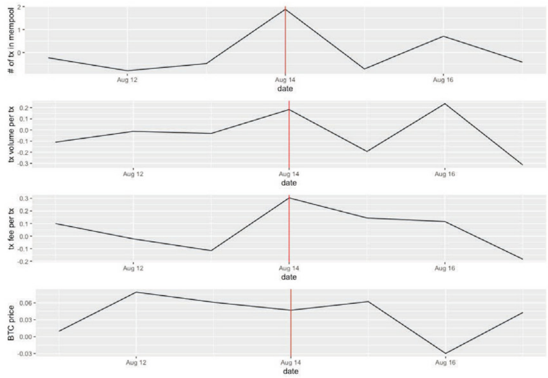
(c) Event date: 2016-09-16

(d) Event date: 2016-10-09

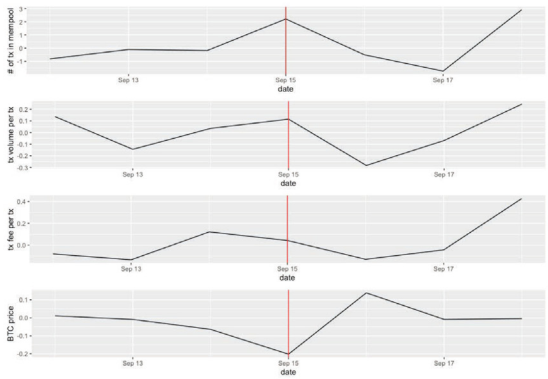
**Fig. 19.** Event studies in the early period (a) 2016-05-27, bitcoin price reached 6-month high. (b) 2016-06-14, bitcoin price reached 2-years high, $700. (c) 2016-09-16, Russia government blocked finnish exchange. (d) 2016-10-09, China yuan fell to its lowest level in 6 years.



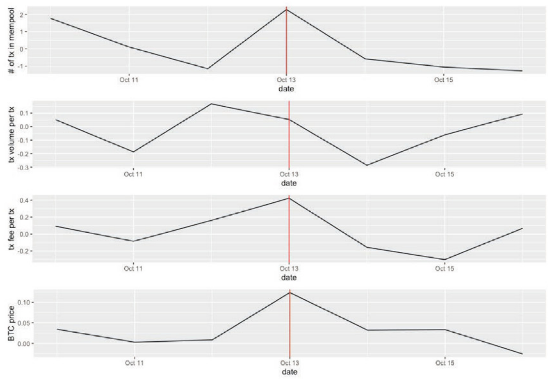

(a) Event date: 2017-05-09



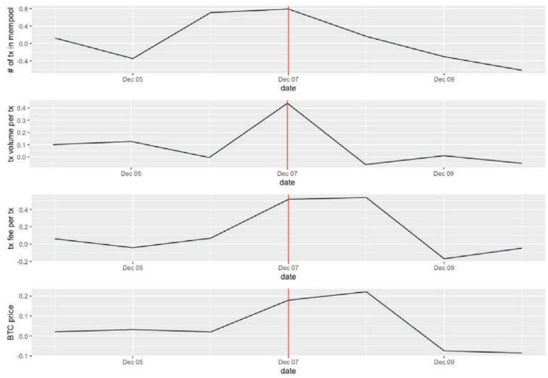

(b) Event date: 2017-08-14



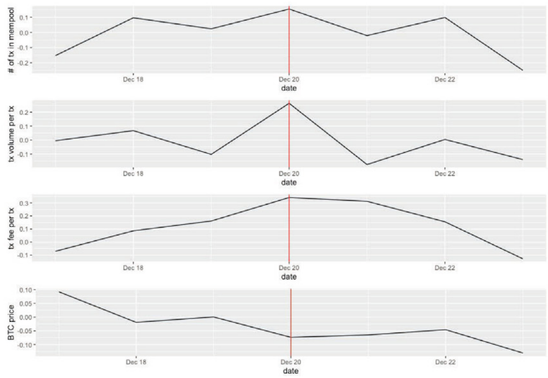

(c) Event date: 2017-09-15




(d) Event date: 2017-10-13

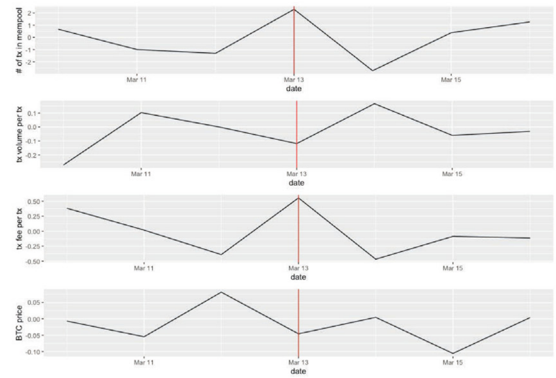



(e) Event date: 2017-12-07

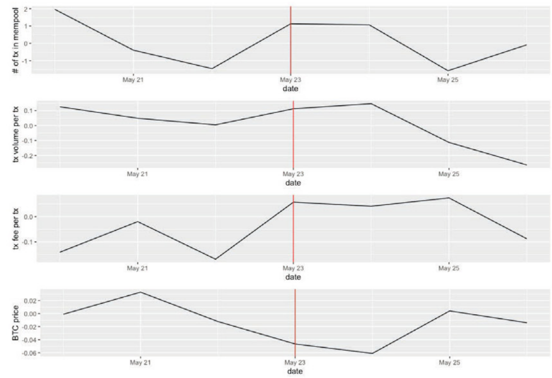



(f) Event date: 2017-12-20

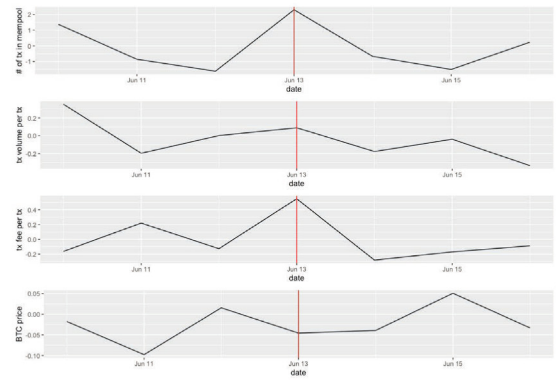
**Fig. 20.** Event studies in the volatile period (a) 2017-05-09, bitcoin price jumped from $1600 to $1700 in one day and investors think the price can reach $4000 soon. (b) 2017-08-14, bitcoin price reached $4000. (c) 2017-09-15, China's second-largest digital-currency exchange was shut down by government. (d) 2017-10-13, bitcoin price reached $5000. (e) 2017-12-07, bitcoin price passed $15000. (f) 2017-12-20, bitcoin price dropped from historical high $19000.



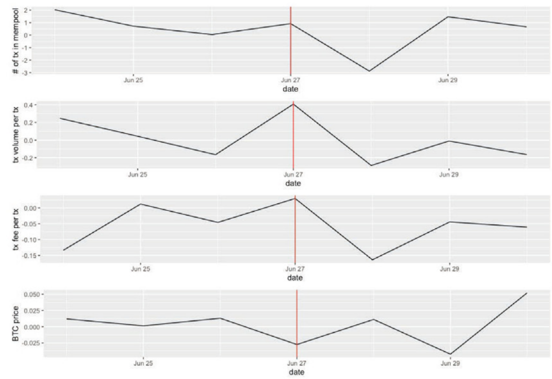

(a) Event date: 2018-03-13



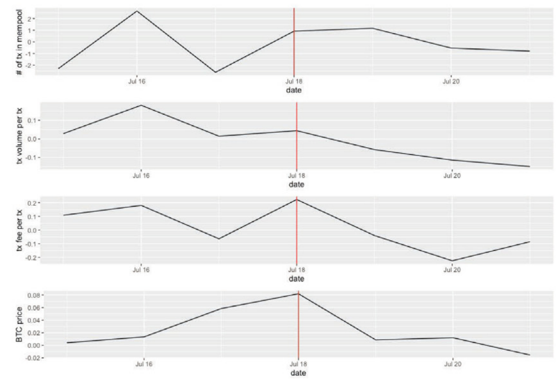

(b) Event date: 2018-05-23



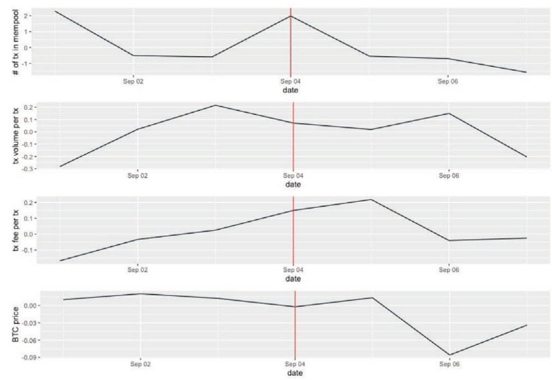

(c) Event date: 2018-06-13




(d) Event date: 2018-06-27




(e) Event date: 2018-07-18




(f) Event date: 2018-09-04

**Fig. 21.** Event studies in the recent period (a) 2018-03-13, IMF Managing Director Christine Lagarde calls for bitcoin crackdown. (b) 2018-05-23, U.S. Department of Justice probed bitcoin price manipulation. (c) 2018-06-13, South Korean exchange CoinRail got hacked. (d) 2018-06-27, U.S. authorities seized over $20 M in crypto in massive Darknet crackdown. (e) 2018-07-18, bitcoin price surged 10% in 30 min. (f) 2018-09-04, Iran officially recognized cryptocurrency mining.

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