DeepBayes Summer School 2018
Theoretical Assignment

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1 Problem 1

In problem 1, we have a random variable ξ distributed according to $\textbf{Poisson}(\lambda)$, and parameterized by λ, where λ is an average number of events per interval. Additionally, random variable η is distributed according to $\textbf{Binomial}(k, p)$ parameterized by probability of successful Bernoulli trial p, and number of trials k, where parameter k is a realization of random variable ξ. Therefore, we are have a model of the form

$$p(\eta|k, p)\ p(k|\lambda)$$

In order to show that random variable η is Poisson distributed with rate parameter $p\lambda$ we can marginalize over the the Binomial parameter k i.e. all possible realizations of ξ to obtain $p(\eta|p, \lambda)$:

$$\begin{aligned}
p(\eta|p, \lambda) &= \sum_{k=\eta}^{\infty} p(\eta|k, p)\ p(k|\lambda) && (1)\\
&= \sum_{k=\eta}^{\infty} \frac{p^{\eta}(1-p)^{k-\eta} \times \exp(-\lambda)\lambda^{k}}{\eta!(k-\eta)!} && (2)\\
&= \frac{p^{\eta}\exp(-\lambda)}{\eta!} \sum_{k=\eta}^{\infty} \frac{1}{(k-\eta)!}(1-p)^{k-\eta}\lambda^{k-\eta}\lambda^{\eta} && (3)\\
&= \frac{p^{\eta}\lambda^{\eta}\exp(-\lambda)}{\eta!} \sum_{i=0}^{\infty} \frac{((1-p)\lambda)^{i}}{i!} && (4)\\
&= \frac{p^{\eta}\lambda^{\eta}\exp(-\lambda)}{\eta!} \times \exp(\lambda - \lambda p) && (5)
\end{aligned}$$

$$p(\eta|p, \lambda) = \frac{\exp(-p\lambda)(p\lambda)^{\eta}}{\eta!} \tag{6}$$

By marginalizing out all possible realizations k of ξ in (1) to (6), we show that η is distributed according to $\textbf{Poisson}(p\lambda)$.

c	$p(t=10\|\mu_c,\sigma_c^2,c=k)$	p(c=k)	$p(t=10\|\mu_c,\sigma_c^2,c=k)\times p(c=k)$	prob.
1 (strict)	0.005	0.5	0.0025	0.33
2 (kind)	0.01	0.5	0.005	0.67

Table 1: Evaluated at $t = 10$ class-conditional likelihood, prior, and the resulting class-conditional probabilities.

2 Problem 2

In problem 2 we are given two Gaussian class conditional distributions for a random variable t. We may treat the problem as a Naive Bayes classification, except that we are already given the value of the parameters of the class conditional distributions, and the class prior is uniform $p(c=k) = \frac{1}{C}$, where C is the number of classes. In our case we have two reviewers hence $C = 2$ and $p(c=k) = \frac{1}{2}$.

Let $c = 2$ denote kind reviewer, and $c = 1$ denote strict reviewer. To obtain class-conditional probability we can multiply the class-conditional likelihood $p(t|\mu_c,\sigma_c^2,c=k)$ with class prior $p(c=k)$ and normalize with respect to marginal distribution $p(t)$. We evaluate the above-mentioned in Table 1, and the conditional probability that the $t = 10$ has been reviewed by a kind reviewer $(c = 2)$ is 0.67.

3 Links to Practical assignment solutions and research

Please find below links to the solution code for practical assignment, as well as the research part:

- Practical assig. solution: link to raw code on github
- Optional research assig. solution: link to jupyter notebook on github

The source code for practical assignment has also been submitted as part of the application form as required.