

Problem 1  
*Red and Blue Hats*

Years ago, to an exceptionally noisy class, Ms. Flick was at her wits end with how to get this class to learn math and so she offered her class a deal. She showed them she had two red hats and three blue hats.

She then set out upon describing the deal:

Three student volunteers would close their eyes, and while their eyes were closed, the teacher would put a hat on each of their heads (and hide the other two hats). Then, one at a time, the students would open their eyes, look at the other two students' heads, and try to guess which color hat was on their own head. Any students that guessed correctly would have no homework to do the rest of the semester. But any students that guessed wrong would not only have to do their own homework, but they would have to help grade everyone else's work also.

The students drew numbers to see who would guess first. Then they closed their eyes and Ms. Flick put a hat on each one's head. Anthony, who was to go first, opened his eyes, looked at the others' heads, and said he didn't really want to play. He couldn't tell for sure and he didn't want to guess in case he was wrong. Next, Belicia opened her eyes and looked at the others' heads. She also thought about the fact that Anthony had said he couldn't tell. Then she said she didn't want to risk it either. She couldn't tell for sure.

Carletta was third. She just stood there with her eyes still closed tightly and a big grin on her face. "I know what color hat I have on," she said.

And she gave the right answer.

Your problem is to figure out what color hat Carletta had on and how she knew for sure. Remember: Carletta didn't even look.

Hint: Try writing out a table of all the possibilities. Think about when any of the students wouldn't know the hat color. Eliminate possibilities.

## Solution:

Suppose we enumerate all the possible arrangements:

Anthony	Belicia	Carletta	
-----	-----	-----	
b	b	b	No red hats
r	b	b	One red hat
b	r	b	One red hat
b	b	r	One red hat
r	r	b	Two red hats
r	b	r	Two red hats
b	r	r	Two red hats

The only way for Anthony to guess what color his own hat would be is if he can see two red hats. That would let him know that he must be wearing a blue hat. Since he doesn't make that guess, we can eliminate that possibility:

Anthony	Belicia	Carletta	
-----	-----	-----	
b	b	b	No red hats
r	b	b	One red hat
b	r	b	One red hat
b	b	r	One red hat
r	r	b	Two red hats
r	b	r	Two red hats

Similarly, if Belicia sees two red hats, she knows she's wearing a blue hat. But she doesn't make that guess, so we can eliminate that possibility:

Anthony	Belicia	Carletta	
-----	-----	-----	
b	b	b	No red hats
r	b	b	One red hat
b	r	b	One red hat
b	b	r	One red hat
r	r	b	Two red hats

Now, note that as soon as Anthony refuses to guess, Belicia and Carletta both know something: at least one of them is wearing a blue hat! (Possibly both of them are.)

So imagine you're Belicia for a moment. You know that there is at least one blue hat between you and Carletta. So if you see Carletta in a red hat, you know your hat is blue, and you can win the game. But you don't draw that conclusion, which means that Carletta doesn't have a red hat. So we can eliminate that possibility.

Anthony	Belicia	Carletta	
-----	-----	-----	
b	b	b	No red hats
r	b	b	One red hat
b	r	b	One red hat
r	r	b	Two red hats

Now if you're Carletta, and you've been following along, you know that you must have a blue hat! And you don't even have to look to know that.

This seems like a lot of work for Carletta, to keep a table like this in her head. Fortunately, she doesn't really have to do that. Here's how she can reason her way to the answer without constructing a table:

- 1) If Anthony had seen two red hats, he would have guessed that his own hat was blue. So he saw at least one blue hat.
- 2) If Belicia had seen Carletta wearing a red hat, she would have guessed that her own hat was blue (since at least one of them had to have the blue hat that Anthony had seen).
- 3) So Carletta already knows that she isn't wearing a red hat. Which means she must have a blue one.