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Graph theory-based radial load flow analysis to solve the dynamic network reconfiguration problem

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SUMMARY

Radial load flow (RLF) methods are frequently used in solving distribution system problems. The conventional Jacobian-based load flow methods (e.g., Newton–Raphson) may fail to converge in solving distribution system due to high resistance to reactance ratio of distribution lines. RLF methods are based on sweep-based mechanism and require that the line data must be arranged in accordance with network topology. This problem needs special attention particularly in case of solving dynamic power system problems, where topology of the network changes with tie switches position (network reconfiguration problem). This paper has presented an intelligent graph theory-based RLF analysis to solve "dynamic" problems. The proposed algorithm will help in arranging line data for any combination of tie switch positions, to check the radiality of the system and to ensure that all nodes are connected with the source node. The effectiveness of the proposed method is also validated by solving 16-bus and 33-bus network reconfiguration problem using graph theory-based RLF method. Copyright © 2015 John Wiley & Sons, Ltd.

KEY WORDS: graph theory; radial load flow analysis; network reconfiguration; distribution system

1. INTRODUCTION

Network reconfiguration is one of the key tools in planning and operation of medium voltage distribution system. Network reconfiguration is defined as altering the topological structures of the distribution feeders, by changing the position of tie and sectionalizing switches; however, under normal operation, medium voltage distribution networks operate in radial manner [1–4]. Authors have utilized this tool for minimization of power losses [5–12]. Some of the authors have also used network reconfiguration as a tool for load balancing [13–17], power supply restoration [18–20], optimum power flow [21], voltage stability improvement [22–24], and operation cost reduction [25].

Network reconfiguration problem differs from other power system problems including distributed generator placement and shunt capacitor bank placement and can be referred as "dynamic" problem. In network reconfiguration, the topology of the network changes with the tie switch position. Further, it is also needed that the radiality of the system must be maintained and all the buses must remain in contact with the source node (main substation) during reconfiguration process. Therefore, an efficient and robust load flow method is required to solve the network reconfiguration problem. In literature, the conventional Newton–Raphson and fast-decoupled load flow methods have also been utilized in solving the distribution system [26,27]. Some of the load flow methods are an approximate form of

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Newton–Raphson methods and are generally used for solving transmission systems. These methods are based on solving Jacobian matrix or *Z*-matrix method. In order to solve the load flow methods based on Jacobian matrix, the inverse of Jacobian matrix must exist. However, in some cases, the Jacobian matrix becomes singular because of high resistance to reactance (*R*/*X*) ratio of distribution lines [28]. The Z-matrix-based load flow methods need the set of equations proportional to the number of buses, which utilizes more computational resources and time [29]. This necessity has resulted into the development of radial load flow (RLF) methods.

Radial load flow methods utilize "backward/forward sweep" approach in solving the distribution network [30–35]. Such approaches are quite commonly used in solving radial or weakly meshed networks. However, "sweep"-based methods require the appropriate line-data arrangement in accordance with the network topology for load flow analysis. Inappropriate line-data input may result in wrong load flow results output. In literature, bus renumbering, branch renumbering, and parent–child node approaches have been proposed to arrange the line data [31,34]. Such techniques work well for "static" system where the topology of the network does not change (e.g., optimum distributed generator placement or shunt capacitor bank placement in radial distribution system). However, in case of solving "dynamic" problems (e.g., network reconfiguration problem), the aforementioned approaches need some intelligent mechanism to arrange the line data for different tie switch combination, maintaining the radiality of the system and to ensure that all nodes are connected with source node (main substation). In this paper, MATLAB-based graph theory tool box (MathWorks Inc., Natick, MA, USA) will be utilized to solve the network reconfiguration problem using RLF analysis. The proposed method has also been tested on 12.66-kV, 16-bus, and 33-bus radial distribution test systems having three and five tie switches, respectively.

This paper is organized as follows. Section 2 will briefly cover the introduction of existing RLF methods. The limitation of RLF methods will be discussed in Section 3. In Section 4, necessity of graph theory-based load flow analysis will be explained in solving network reconfiguration problem. The basic introduction of graph theory will be presented in Section 5. The proposed algorithm of graph theory-based RLF analysis to solve network reconfiguration problem will be discussed in Section 6. In the last section, the proposed algorithm will be tested on standard 16-bus distribution test system, and the results will also be discussed in detail in the same section.

2. RADIAL LOAD FLOW METHODS—A BRIEF INTRODUCTION

Distribution systems are majorly considered as radial or weakly mesh network. Different RLF methods have been proposed to solve such networks by updating system parameters (e.g., bus voltages and line current) in backward–forward manner. Six most welll-known RLF methods and their convergence criteria are presented as follows [29,31–35].

2.1. D. Shirmohammedi radial load flow method [31]

Shirmohammedi theorem is one of the oldest methods in solving radial distribution system. This theorem is also equally validated for weakly meshed networks. The backward sweep helps in determining the branch current *I*, calculated from the far end to the root node. The load current at bus *i* is calculated using Equation (1).

$$I_i = conj\ \ (S_i/V_i) \qquad \text{where} \quad i = 1, 2, 3, \ldots n \tag{1}$$

The Kirchhoff current law is utilized to compute the current (*I*) in each branch of the network, using Equation (2).

$$I = -\text{current injected at node } 'n' + \sum \text{current in branches extended from node } 'n' \tag{2}$$

The forward sweep helps in estimating receiving end voltages (V_s) from sending bus voltage (V_r) and line voltage drop, using Equation (3).

$$V_r = V_s - I\ \ (R + jX) \tag{3}$$

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Apply the same approach to all buses/branches in the radial system. The convergence criteria of Shirmohammedi theorem are given in Equations (4) and (5).

$$Max\ \left[\Delta P_i^k \quad \Delta Q_i^k\right] < \text{tolerance} \qquad \text{where} \quad i = 1, 2, 3, \dots n \tag{4}$$

where,

$$\Delta P_i^k = \mathrm{Re}\left[S_i^k - S_i\right] \qquad \text{where} \quad i = 1, 2, 3, \dots n \tag{5a}$$

$$\Delta Q_i^k = \mathrm{Im}\left[S_i^k - S_i\right] \qquad \text{where} \quad i = 1, 2, 3, \dots n \tag{5b}$$

$$S_i = V_i \ \times \ conj(I_i) \qquad \text{where} \quad i = 1, 2, 3, \dots n \tag{5c}$$

where k is the number of iteration and n is the total number of buses.

2.2. G. X. Luo radial load flow method [32]

Luo has presented an RLF method based on backward–forward sweep. The backward sweep helps in determining the total sum of load and power losses. However, in forward sweep, updated bus voltages are obtained. In the backward sweep, the sending end power is calculated using Equation (6).

$$P_s = P_r + R\frac{P_r^2 + \ Q_r^2}{V_r^2} \tag{6a}$$

$$Q_s = Q_r + X\frac{P_r^2 + Q_r^2}{V_r^2} \tag{6b}$$

In the forward sweep, the receiving end voltage is calculated, from the following relations given, in Equations (7) and (8).

$$V_r = \sqrt{(V_s - \Delta V')^2 + \Delta V''^2} \tag{7}$$

where

$$\Delta V' = \frac{RP_s + XQ_s}{V_s} \tag{8a}$$

$$\Delta V'' = \frac{XP_s - RQ_s}{V_s} \tag{8c}$$

The receiving bus angle will be calculated from Equation (9a) and will help in finding the receiving end voltage and can be calculated using Equation (9b).

$$\delta_r = \delta_s - \tan^{-1}\left[\frac{\Delta V''}{V_s - \Delta V'}\right] \tag{9a}$$

$$\overline{V_r} = |V_r|e^{j\delta_r} \tag{9b}$$

Apply the same approach to all buses in the radial system. The convergence criteria of Luo method are given in Equation (10).

$$Max\left[V_i^k - V_i^{k-1}\right] < \text{tolerance} \qquad \text{where} \quad i = 1, 2, 3, \dots n \tag{10}$$

where k is the number of iteration and n is the total number of buses.

2.3. M. H. Haque radial load flow method [33]

Haque has also presented an RLF method based on backward–forward sweep. The backward sweep is identical with Luo method and helps in determining the total sum of load and power losses. In the backward sweep, the sending end power is calculated using Equation (11).

$$P_s = P_r + R\frac{P_r^2 + Q_r^2}{V_r^2} \tag{11a}$$

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$$Q_s = Q_r + X\frac{P_r^2 + Q_r^2}{V_r^2} \quad (11b)$$

In the forward sweep, the receiving end voltage is calculated using Equations (12) and (13).

$$\overline{V_r} = \sqrt{V_s^2 - 2(P_sR + Q_sX) + \left(P_s^2 + Q_s^2\right)\left(R^2 + X^2\right)/V_s^2} \quad (12a)$$

$$\delta_r = \delta_s - \tan^{-1}\left[\frac{a_1}{a_2{}'}\right] \quad (12b)$$

where

$$a_1 = \frac{XP_s - RQ_s}{V_s} \quad (13a)$$

$$a_2 = V_s - \frac{RP_s + XQ_s}{V_s} \quad (13b)$$

Apply the same approach to n buses in the radial system. The convergence criteria of Haque method are given in Equation (14).

$$Max\left[V_i^k - V_i^{k-1}\right] < \text{tolerance} \qquad \text{where} \quad i = 1, 2, 3, \ldots n \quad (14)$$

where k is the number of iteration and n is the total number of buses.

2.4. *D. Thukaram radial load flow method [34]*

Thukaram load flow differs from the foregoing two methods. In backward sweep, the load current is computed for nth loads in the network, using Equation (15).

$$I_i = conj(S_i/V)_i \qquad \text{where} \quad i = 1, 2, 3, \ldots n \quad (15)$$

However, in forward sweep, the receiving end voltage is calculated using Equation (16).

$$V_r = V_s - I(R + jX) \quad (16)$$

Apply the same approach to nth buses in the radial system. The convergence criteria of Thukaram method are given in Equation (17).

$$Max\left[V_i^k - V_i^{k-1}\right] < \text{tolerance} \qquad \text{where} \quad i = 1, 2, 3, \ldots n \quad (17)$$

where k is the number of iteration and n is the total number of buses.

2.5. *D. Rajicic radial load flow method [35]*

Rajicic method is also based on backward–forward sweep. The backward sweep is identical with Luo method and helps in determining the total sum of load and power losses. In the backward sweep, the sending end power is calculated using Equation (18).

$$P_s = P_r + R\frac{P_r^2 + Q_r^2}{V_r^2} \quad (18a)$$

$$Q_s = Q_r + X\frac{P_r^2 + Q_r^2}{V_r^2} \quad (18b)$$

In the forward sweep, the receiving end voltage V_r in complex form is calculated using Equation (19).

$$\overline{V_r} = \overline{V_s}\left(1 - \frac{P_sR + Q_sX}{V_s^2} + j\frac{Q_sR - P_sX}{V_s^2}\right) \quad (19)$$

Apply the same approach to n buses in the radial system. The convergence criteria of Rajicic method are given in Equation (20).

$$Max\ \left[\left|V_i^k\right| - \left|V_i^{k-1}\right|\right] < \text{tolerance} \qquad \text{where} \quad i = 1, 2, 3, \ldots n \quad (20)$$

where k is the number of iteration and n is the total number of buses.

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2.6. J. H. Teng radial load flow method [29]

Teng method is considered as a direct approach for solving radial and weakly meshed system. This method is based on constructing two derived matrices, branch injection to branch current (BIBC) and branch current to bus voltage (BCBV) matrices. The formation of BIBC and BCBV matrix is discussed in [29]. In each iteration k, the load current is updated using Equation (21).

$$I_i^k = \left(\frac{P_i + Q_i}{V_i^k}\right)^* \tag{21}$$

The receiving end voltage is calculated using Equation (21):

$$\left[V^{k+1}\right] = \left[V^0\right] + \left[\Delta V^{k+1}\right] \tag{22}$$

where

$$\left[\Delta V^{k+1}\right] = [DLF]\ \left[I^k\right] \tag{23a}$$

$$[DLF] = [BCBV]\ [BIBC] \tag{23b}$$

The change in voltage w.r.t. to the root node V1 is calculated using Equation (24).

$$[\Delta V] = [BCBV]\ [BIBC]\ [I] = [DLF]\ [I] \tag{24}$$

The convergence criteria of Teng method are given in Equation (25).

$$Max\ \left[\left|V_i^k\right| - \left|V_i^{k-1}\right|\right] < \text{tolerance} \qquad \text{where} \quad i = 1, 2, 3, \ldots n \tag{25}$$

where k is the number of iteration and n is the total number of buses.

Radial load flow methods (discussed previously) based on backward–forward sweep have some limitations in terms of line-data arrangement. Such limitations and the existing methods to correct the line data are discussed in Section 3.

3. LIMITATIONS OF BACKWARD–FORWARD LOAD FLOW METHODS

Radial load flow methods are basically based on backward–forward techniques and are highly sensitive to line-data arrangement. In RLF methods, the buses present in the same lateral are modeled in such a manner that the successive bus and the corresponding line must come in the same lateral in the line data. This statement is better illustrated using six-bus radial distribution test system. Six-bus system is a reduced form of 12-bus radial distribution system [36]. Figure 1 is representing the single line diagram of six-bus radial distribution test system.

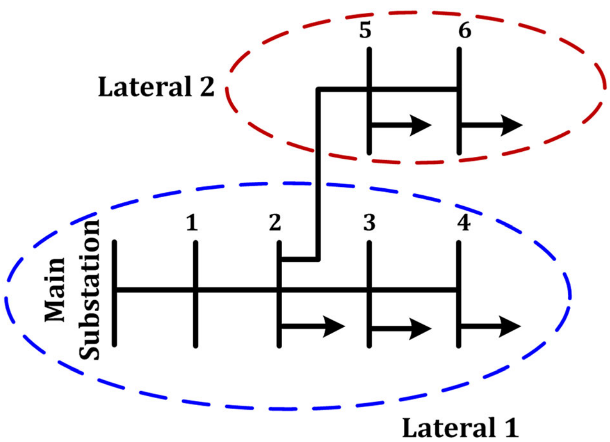

Figure 1. Single line diagram of six-bus radial distribution system.

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The necessary details of the system are given in Table I.

Thukaram RLF [34] is used to carry out the load flow analysis of the six-bus system, and the results are given in Table II.

In the first case, when the line-data position is changed and the branch(s) present in the same lateral move to the other position within the same lateral, wrong load flow results are obtained, as shown in Table IIIa.

In the second case, when the line-data position is changed and the branch(s) present in the same lateral is shifted to other laterals in line data, wrong load flow results are obtained, as shown in Table IIIb.

Similar is the case when the From–To position in the line data is changed, wrong load flow results are obtained, as shown in Table IIIc.

Table I. Line data and bus data of six-bus system [36].*

Lateral	From	To	*R* (pu)	*X* (pu)	*P* (MW)	*Q* (MW)
1	1	2	9.00E−05	3.80E−05	0.06	0.06
	2	3	9.80E−05	4.10E−05	0.04	0.03
	3	4	0.000173	7.20E−05	0.055	0.055
2	2	5	0.000263	0.00011	0.03	0.03
	5	6	9.00E−05	3.80E−05	0.02	0.015

*Base MVA = 0.01; base kV = 11; accuracy = 1E−004.

Table II. Load flow analysis results.

Bus number	Voltage magnitude (V)		Angle (degree)	
1	V1	1.0000	δ_1	0.0000
2	V2	0.9974	δ_2	0.0535
3	V3	0.9961	δ_3	0.0791
4	V4	0.9948	δ_4	0.1112
5	V5	0.9956	δ_5	0.0900
6	V6	0.9954	δ_6	0.0934
	Active power losses (kW)			1.1010
	Reactive power losses (kVAR)			0.4631

Table IIIa. Changing line-data position (when the branch moves to the same lateral).

	Input line data				
Lateral	From	To	*R*	*X*	Remarks
1	1	2	9.00E−05	3.80E−05	The position of line data has been changed within the same lateral.
	3	4	0.000173	7.20E−05	
	2	3	9.80E−05	4.10E−05	
2	2	5	0.000263	0.00011	
	5	6	9.00E−05	3.80E−05	
	Load flow analysis results*				
Bus number	Voltage magnitude (V)		Angle (degree)		Remarks
1	V1	1.0000	δ_1	0.0000	Changing the line-data position in the same lateral has resulted in wrong load flow results.
2	V2	0.9981	δ_2	0.0370	
3	V3	0.9976	δ_3	0.0445	
4	V4	0.9963	δ_4	0.0765	
5	V5	0.9963	δ_5	0.0735	
6	V6	0.9961	δ_6	0.0769	
	Active power losses (kW)			0.6241	
	Reactive power losses (kVAR)			0.2623	

*D. Thukaram load flow.

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Table IIIb. Changing line-data position (when the branch moves to other lateral).

Lateral	Input line data				Remarks
	From	To	R	X	
1	1	2	9.00E−05	3.80E−05	The position of line data has been changed into the other lateral.
	5	6	9.00E−05	3.80E−05	
	3	4	0.000173	7.20E−05	
2	2	5	0.000263	0.00011	
	2	3	9.80E−05	4.10E−05	

Load flow analysis results*					
Bus number	Voltage magnitude (V)		Angle (degree)		Remarks
1	V1	1.0000	δ_1	0.0000	Changing the line-data position from one lateral to other lateral has resulted in wrong load flow results.
2	V2	0.9984	δ_2	0.0336	
3	V3	0.9979	δ_3	0.0411	
4	V4	0.9965	δ_4	0.0731	
5	V5	0.9972	δ_5	0.0600	
6	V6	0.9970	δ_6	0.0634	
	Active power losses (kW)			0.4661	
	Reactive power losses (kVAR)			0.1959	

*D. Thukaram load flow.

Table IIIc. Changing From–To position in line data.

Lateral	Input line data				Remarks
	From	To	R	X	
1	1	2	9.00E−05	3.80E−05	The position of line data has been changed into the other lateral.
	2	3	9.80E−05	4.10E−05	
	4	3	0.000173	7.20E−05	
2	2	5	0.000263	0.00011	
	5	6	9.00E−05	3.80E−05	

Load flow analysis results*					
Bus number	Voltage magnitude (V)		Angle (degree)		Remarks
1	V1	1.0000	δ_1	0.0000	Changing the To–From position inline data has resulted in wrong load flow results.
2	V2	0.9981	δ_2	0.0370	
3	V3	0.9991	δ_3	0.0132	
4	V4	1.0000	δ_4	0.0000	
5	V5	0.9963	δ_5	0.0735	
6	V6	0.9961	δ_6	0.0769	
	Active power losses (kW)			0.5616	
	Reactive power losses (kVAR)			0.2363	

*D. Thukaram load flow.

From the foregoing three cases, it can be recognized that it is highly important that the line-data arrangement must match with the network topology. This problem must be highlighted because of the following reasons:

(1) In case of large radial distribution test system, the arrangement of line data with the network topology is a difficult job. The addition of graph theory to existing RLF methods helps the user in terms of entering line data, and the user does not need to think about the line-data arrangement. Graph theory will also help in determining the radiality of the system and to ensure that all nodes are connected with source node (main substation). This is particularly useful in graphical user interface-based application. The addition of graph theory with graphical user interface-based application such as voltage stability and optimization programming [37] tool and distribution system power flow analysis package [30] will improve the robustness and efficiency of such programs to solve the distribution system problems.

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(2) In case of solving "dynamic" problem such as finding the optimum tie switch position in network reconfiguration problem. This problem will be discussed in detail in the next section.

In literature, bus renumbering, branch renumbering, and parent–child node have also been utilized to correct the line-data format [31,34]. For smaller system, visual inspection can also be carried out to arrange the line data. Such approaches work well in case of "static" system where the network topology does not change. However, for dynamic power system problem, for example, network reconfiguration problem, it is necessary to utilize some intelligent technique. This statement is better presented in the next section.

4. NECESSITY OF GRAPH THEORY-BASED RADIAL LOAD FLOW ANALYSIS IN SOLVING NETWORK RECONFIGURATION PROBLEM

In the aforementioned analysis, it has been seen that the arrangement of line data is necessary to carry out the RLF analysis. However, for smaller bus system, it can be carried out by visual inspection or node branch renumbering approach, proposed in [29,31–35]. However, such approaches cannot be utilized in such cases where the system configuration is dynamic, for example, in case of solving network reconfiguration. Network reconfiguration is defined as altering the topological structures of the distribution feeders, by changing the position of tie and sectionalizing switches; however, under normal operation, medium voltage distribution networks operate in radial manner, as shown in Figure 2 [1,2]. Tie lines' circuit breakers under normal condition remain open; however, in case of necessity, for example, fast restoration of power, it will be closed.

In present case, there can be many different options of tie switch positions, as shown in Figure 3.

If the foregoing system configurations (shown in Figure 3) are solved using RLF analysis, searching mechanism is required to arrange the line data for each system configuration. Such small system can also be solved visually because of limited number of combinations. However, in case of large system, for example, 33-bus, 69-bus, or 127-bus distribution system having five tie switches, there must be some intelligent mechanism to rearrange the line data and carry out the RLF analysis for different system configurations. In this paper, graph theory-based RLF methods have been proposed to solve the limitation of existing load flow problem as well as to solve the network reconfiguration problem using MATLAB tools. Here, it is also important to note that graph theory also utilizes parent–child node approach to find the connectivity among different nodes; however, MATLAB® graph theory tool box has some standard searching mechanism (e.g., breadth-first search (BFS) and depth-first search (DFS)), which can be utilized to arrange the line data more effectively.

5. GRAPH THEORY

Graph theory began in 1736, and one of the first people to experiment with graph theory was Leonhard Euler [38]. He attempted to solve the problem of crossing seven bridges onto an island without using

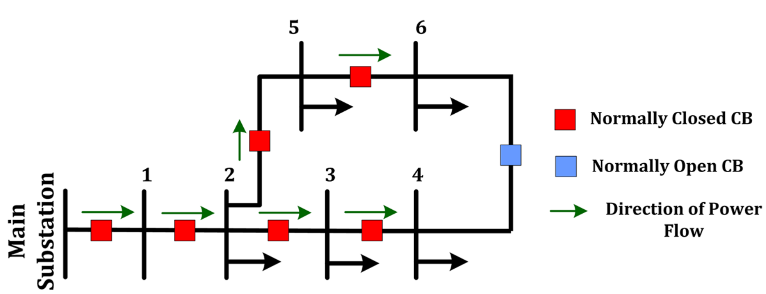

Figure 2. Single line diagram of six-bus radial distribution system with 01 normally open circuit breaker (CB).

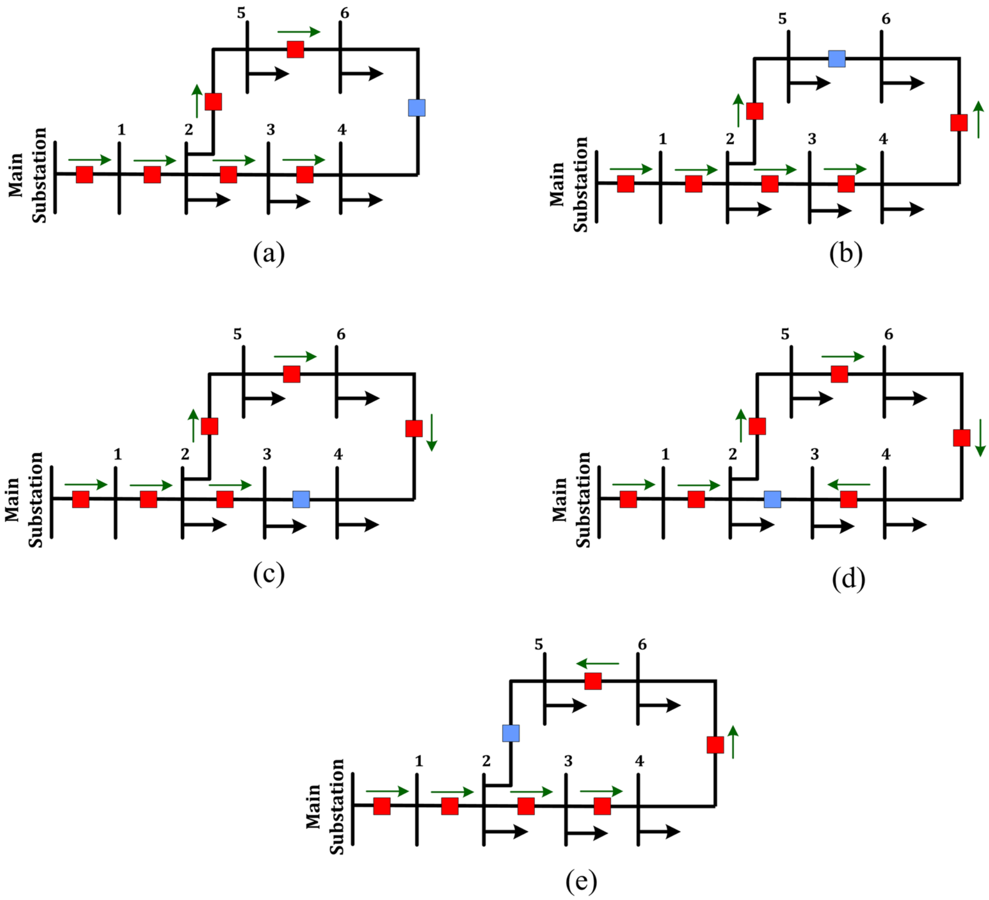

Figure 3. Different system configurations in case of six-bus network.

any of them more than once. Graph theory models and represents the interconnection between objects (nodes) in a certain set. In general, the graph G (V, E) is a pair of sets mapped by the pair of vertices or nodes (V) through the edges (E). The vertices V are usually labeled as $V = \{v_1, v_2 \cdots \cdots, v_n\}$. V is the vertex set of G, and E is the edge set. The vertices are said to be adjacent to each other if they are interconnected through an edge [39,40]. The example of nodes and edges is shown in Figure 4 using six-bus test system (shown previously in Figure 1).

In Figure 4, there are six nodes interconnected with five lines. In the present case of distribution system, the vertices (V) will represent the system buses, and the edges (E) will represent the distribution lines (connecting two buses). Therefore, in distribution system, RLF problem can be formulated as a problem of identifying the desired graph topology.

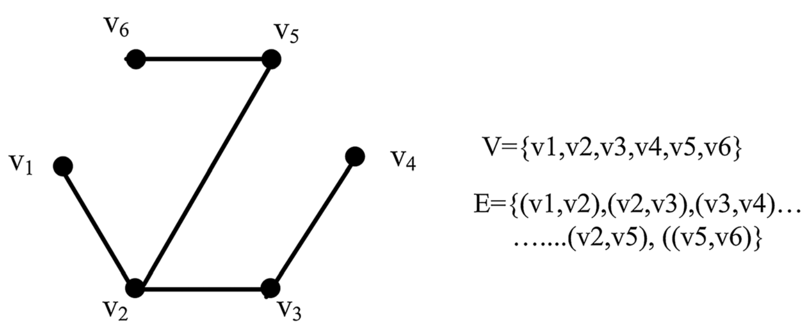

Figure 4. Example of nodes and edges using six-bus test system.

5.1. Directed undirected graph

In graph theory, two different types of graphs are normally used, that is, directed graph and undirected graph. A directed graph is formed by vertices connected with directed edges, that is, from vertices v_1 to v_2. While in undirected graph, the nodes are connected to each other in a bidirectional manner. Or in another words, an undirected graph is a graph in which edges have no orientations. One of the examples of directed and undirected graph is shown in Figure 5.

5.2. Sparse form and adjacency matrix

Sparse form represents the line connection between two nodes in the system and removes all zero elements in the matrix. From the sparse matrix, the adjacency matrix (A) is created. The adjacency matrix (A) represents the complete connection of each node in the graph [41].

Equation (26a) represents the adjacency matrix for the directed graph according to Figure 5a. Here, it can be seen that each element in adjacency matrix $a_{ij} = 1$ if $(i,k) \in E$ and $a_{ij} = 0$ if $(i,k) \notin E$, where E is a set of directed edges {(1,2), (2,3), (3,4), (2,5), (5,6)}.

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{26a}$$

Equation (26b) represents the adjacency matrix for the undirected graph, according to Figure 5b. Here, it can be seen that each element in adjacency matrix $a_{ij} = 1$ if $(i,k) \in E$ and $a_{ij} = 0$ if $(i,k) \notin E$, where E is a set of undirected edges {(1,2), (2,1), (2,3), (2,5), (3,2), (3,4), (4,3), (5,2), (5,6), (6,5)}. For undirected graph, the adjacency matrix is symmetric because the edges are bidirectional, that is, $a_{ij} = a_{ji}$.

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \tag{26b}$$

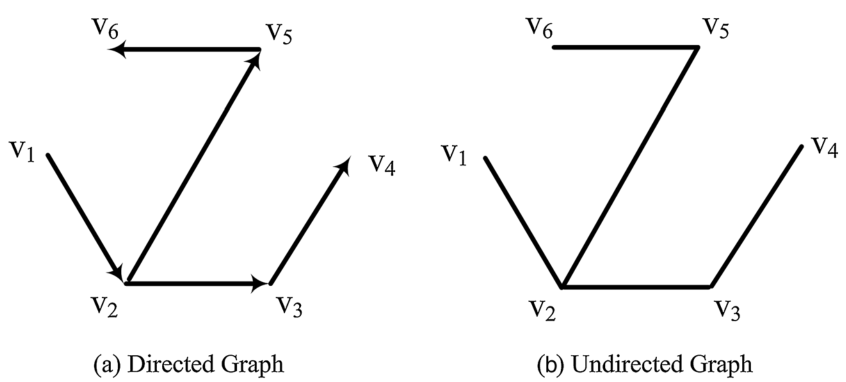

Figure 5. Example of (a) directed and (b) undirected graphs.

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In graph theory, adjacency matrix is also used to calculate Kirchoff's matrix or Laplacian matrix. Using the adjacency matrix, the number of spanning trees in the graph can be determined based on the Laplacian matrix. The spanning tree is defined as a selection of edges that will result in a tree-like structure, which includes every vertex in the graph [42,43].

5.3. Search algorithm for finding the connectivity among nodes

In order to measure the connectivity among all nodes or sequence of nodes interconnection, two searching methods are commonly used, BFS and DFS methods [44].

In BFS, all the nodes present in the same level will be identified before proceeding to the next level. This process will continue till the last vertex is reached. For example, in Figure 6, using BFS method, the traveled path starting from source node A will be A → B → C → D → E → F → G.

However, in DFS method, the nodes are searched in depth-first order where the search algorithm will trace the path from the search point (root) to the extended node (descendants) [45]. For example, in Figure 7, using DFS method, the traveled path starting from source node A will be A → B → D → G → E → C → F.

In current application of RLF analysis, DFS method is utilized in order to determine all nodes present in the same lateral. DFS method returns the order of the bus in the system, which will be used to rearrange the line data according to the flow that is traced out.

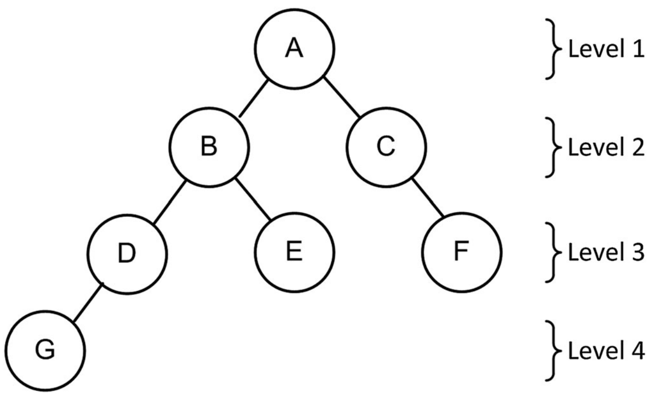

Figure 6. Breadth-first search-order technique.

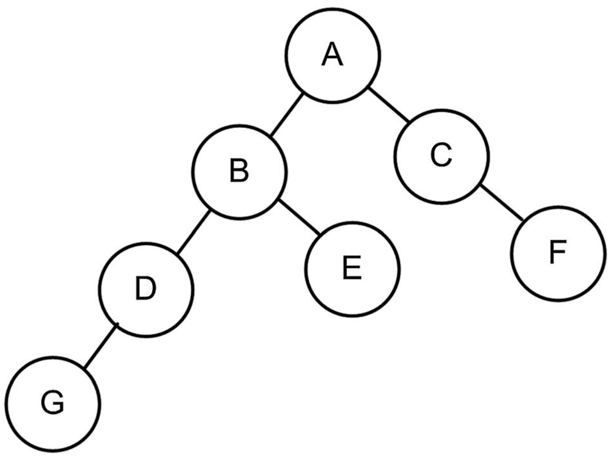

Figure 7. Depth-first search-order technique.

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6. PROPOSED ALGORITHM—GRAPH THEORY-BASED RADIAL LOAD FLOW ANALYSIS FOR SOLVING NETWORK RECONFIGURATION PROBLEM

In the aforementioned Sections 3 and 4, it can be seen that the arrangement of line data in correspondence with the network topology is mandatory to carry out the RLF analysis. Graph theory-based RLF will help in solving dynamic network reconfiguration problem where the topology of the system changes with tie position. In this case, graph theory will help in following ways:

(1) To arrange the line data according to the network topology.
(2) To determine the radiality of the system.
(3) To ensure that all nodes are connected with source node (main substation).

The detailed steps of solving network reconfiguration problem using graph theory-based RLF analysis are given as follows:

Step 1 Initialization: Enter the necessary bus data and line data, as shown in Equation (27).

$$\text{Bus Data} = [\text{Bus No.} \quad \text{BusType} \quad \text{Active Load} \quad \text{Reactive Load}] \tag{27a}$$

$$\text{Line Data} = [\text{Line No} \quad \text{From } (i) \quad \text{To } (j) \quad \text{Resistance} \quad \text{Reactance}] \tag{27b}$$

To explain the proposed algorithm, a simple six-bus system with a single tie line switch is considered, as shown in Figure 8.The necessary bus data and line data of the foregoing network are given in Equations (28) and (29), respectively.

$$\text{bus\_data} = \begin{bmatrix} 1 & 1 & P_1 & Q_1 \\ 2 & 0 & P_2 & Q_2 \\ 3 & 0 & P_3 & Q_3 \\ 4 & 0 & P_4 & Q_4 \\ 5 & 0 & P_5 & Q_5 \\ 6 & 0 & P_6 & Q_6 \end{bmatrix} \tag{28}$$

$$\text{line\_data} = \begin{bmatrix} 1 & 1 & 2 & r_1 & x_1 \\ 2 & 2 & 3 & r_2 & x_2 \\ 3 & 3 & 4 & r_3 & x_3 \\ 4 & 4 & 6 & r_4 & x_4 \\ 5 & 5 & 6 & r_5 & x_5 \\ 6 & 2 & 5 & r_6 & x_6 \end{bmatrix} \tag{29}$$

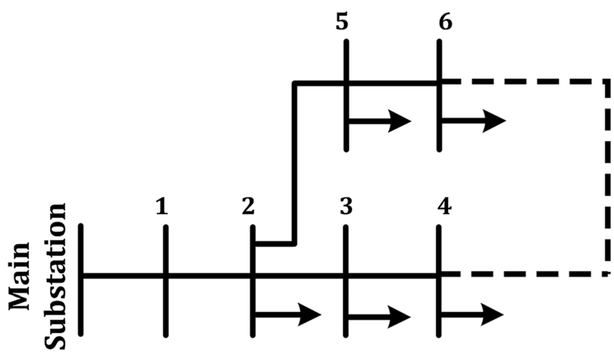

Figure 8. Six-bus radial distribution system with a single tie switch.

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where r and x are resistance and reactance of the line, respectively. P and Q are active and reactive loads at the bus, respectively.

Step 2: Generation of sparse and adjacency matrix: Once the bus system data have been selected, the sparse form of the system is created on the basis of the line data. Sparse form represents the line connection between two nodes in the system and removes all zero elements in the matrix. Equation (4) is used to generate sparse matrix of undirected graph. Undirected graph will help in generation of adjacency matrix and switching of the selected tie line positions.

$$S = \text{sparse}\,(a, b) \tag{30}$$

where

$$a = [\,\text{line} - \text{data (from)} \quad \text{line} - \text{data (to)}\,]$$
$$b = [\,\text{line} - \text{data (to)} \quad \text{line} - \text{data (from)}\,]$$

The adjacency matrix (A) of the system is also formed using the sparse form of the system to represent the network topology in matrix form. Equation (31) is used to generate adjacency matrix (A) of the system.

$$A = \text{full}(S)) \tag{31}$$

The adjacency matrix (A) is an $n \times n$ matrix (where n represents the total number of buses in the system). The adjacency matrix (A) of six-bus system, with all tie line switches that are also closed, is given by Equation (32).

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \tag{32}$$

Step 3: Generation of random tie switch positions and updating adjacency matrix (A): Random tie switch positions are generated (for example, $r = [r1\ r2\ r3\ r4\ r5]$, where r is an array representing tie switch position), and the adjacency matrix (A) of the system is updated to represent the tie switch positions. For example, in six-bus test system, when the tie line circuit breaker between four-bus and six-bus systems is open ($r = [4]$), the new adjacency matrix (A') is given by Equation (33):

$$A' = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \tag{33}$$

Here, it can be observed that by using undirected graph, it is easy to switch off the connection between four-bus and six-bus systems. Further, it is also noted that the dimension of new adjacency matrix (A') will remain the same, that is, $n \times n$.

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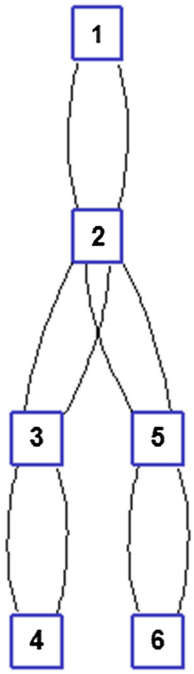

Figure 9. Biograph of six-bus test system.

Using the sparse form S of the new system (when the tie line positions have opened), Equation (34) is used to generate the biograph of the new system, as shown in Figure 9.

$$\text{biograph} = (S, \text{label}, \text{`ShowArrows'}, \text{`off'}) \tag{34}$$

Step 4: Checking radiality of the system. The reconfigured graph based on A′ is checked for radiality to ensure that there are no loops or mesh present in the network. The graph is traversed in DFS order, starting from node n. Equation (35) is used to traverse the graph:

$$[D, P] = \text{graphtraverse}\ \ (\text{sparse}\ \ \text{of}\ \ A',\ n) \tag{35}$$

where D is a vector of node indices in the order in which they are discovered. P is a vector of predecessor node indices (listed in the order of the node indices) of the resulting spanning tree. The path is mapped out between all nodes using the graphpred2path() command, given in Equation (36).

$$\text{path} = \text{graphpred2path}\ (P, D) \tag{36}$$

If the length of the graphpred2path() is found equal to 2, then the circuit is believed to be a radial system (status = 1), otherwise non-radial. In case of non-radiality in the system (status = 0), the program exits, and step 3 is repeated.

Step 5: Check connectivity. Even though step 4 ensures that the graph is radial, however, there might be instances that the graph is disconnected, that is, some nodes are not connected to the source node (bus number 1). In order to ensure the connectivity among all nodes in the system, the number of buses in the traversed path should be equal to the total number of buses in the system, that is, $n_{\text{bus (traverse)}} = n_{\text{bus(system)}}$. The following Equation (37) is used to calculate the number of buses in traversed path.

$$\text{bus\_order} = \text{graphtraverse}(\text{sparse}\ \ \text{of}\ \ A',\ \ 1, \text{`Method'}, \text{`}DFS\text{'}) \tag{37a}$$

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$$n_{\mathrm{bus(traverse)}} = \mathrm{length(bus\_order)} \tag{37b}$$

Equation (37a) is used to obtain the traverse path, and the detail of traverse path is stored in variable DFS_order. The length of DFS_order will represent the total number buses that come across during traversing. For example, in Figure 9, the following DFS_order, given in Equation (38), is obtained, and the numbers of buses in traversed path are given in Equation (39).

$$\mathrm{DFS\_order} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{pmatrix} \tag{38}$$

$$n_{\mathrm{bus(traverse)}} = 6 \tag{39}$$

Step 6: Line data arrangement. In order to carry out the RLF analysis, the line data of the reconfigured network have to be arranged according to the load flow requirement. The following substeps are necessary:

(1) Arranging the From–To position: The edge that represents the line-data $E_{(\mathrm{From–To})}$ is sorted out to ensure that From < To in each element of line data.
(2) Determining the number of descendants (NoD) of each node: The NoD of each node is determined based on the DFS order using Equation (40).

$$\mathrm{NoD} = \mathrm{size(getdescendants())} \tag{40}$$

The NoD will determine the characteristic of each node. Three different node characteristics can be found, as shown in Figure 10. The NoD = 2 represents an ending node, and the NoD = 3 represents a transitional node. Any number higher than two (NoD > 3) represents a branch node.The DFS order and the corresponding NoD values are stored in an array form in variable DFS_Node_Order, as given by Equation (41). The NoD values are used to separate the branch nodes (NoD > 3) and stored in variable branch, as given by Equation (42). The DFS order and NoD information are used to arrange the line data according to the RLF requirement.

$$\mathrm{DFS\_order\_NoD} = \begin{array}{c} \begin{matrix} \text{DFS Order} & \text{NoD} \end{matrix} \\ \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 3 \\ 4 & 2 \\ 5 & 3 \\ 6 & 2 \end{bmatrix} \end{array} \tag{41}$$

$$\mathrm{branch} = [2] \tag{42}$$

(3) Constructing line-data variable for RLF analysis: Using the DFS_order variable, the line data are constructed, as given by Equation (43).

$$\mathrm{DFS\_order\_NoD} = \begin{array}{c} \begin{matrix} \text{From} & \text{To} \end{matrix} \\ \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 4 & 5 \\ 5 & 6 \end{bmatrix} \end{array} \tag{43}$$

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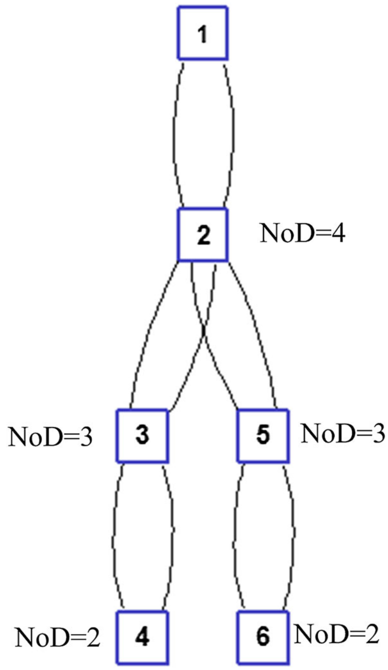

Figure 10. Number of descendants (NoD) identifications.

The line-data variable will be compared with original line data in order to find out non-existing connection between the lines. For example, in given line data, there is no connection between four-bus and five-bus systems. Thus, the correct "from" position will be found out from branch variable. In the present case, from position 4 will be replaced with 2, as shown in Equation (44).

$$line\_data = \begin{array}{c} \begin{array}{cc} \text{From} & \text{To} \end{array} \\ \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 2 & 5 \\ 5 & 6 \end{bmatrix} \end{array} \tag{44}$$

Step 7: Load flow analysis: After rearranging the line data, the RLF analysis is carried out.

The flow chart of proposed graph theory-based RLF analysis to solve network reconfiguration problem is shown in Figure 11.

7. RESULTS AND DISCUSSION

The proposed algorithm has been applied on 16-bus and 33-bus distribution test systems on the basis of minimization of power losses. Thukaram RLF method is used with graph theory to carry out the load flow.

7.1. Test result on 16-bus system

Sixteen-bus system is a standard 12.66-kV network, having three feeders and three tie switches [11]. The system details are given in Appendix Table A-I. In order to solve the three-feeder system, the system has been transformed to a single-feeder radial distribution test system, as shown in Figure 12.

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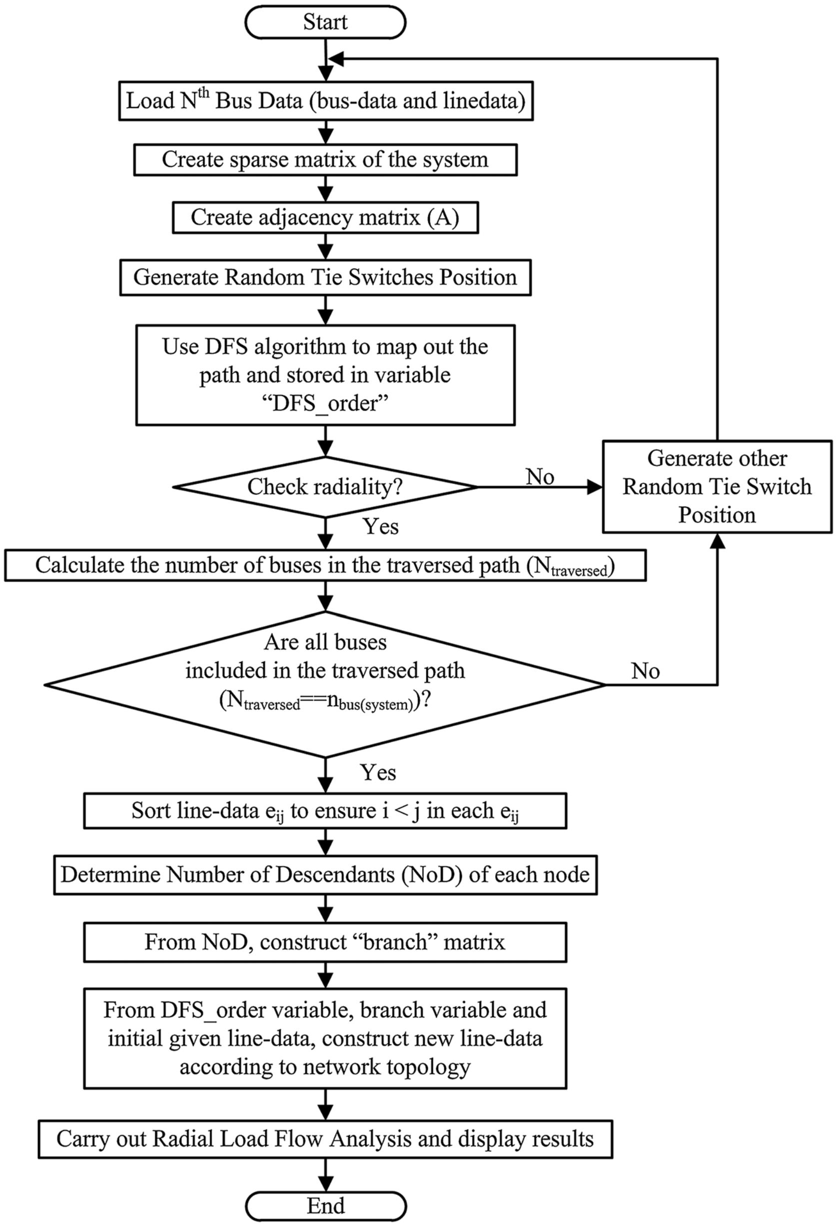

Figure 11. Flow chart of proposed algorithm. DFS, depth-first search.

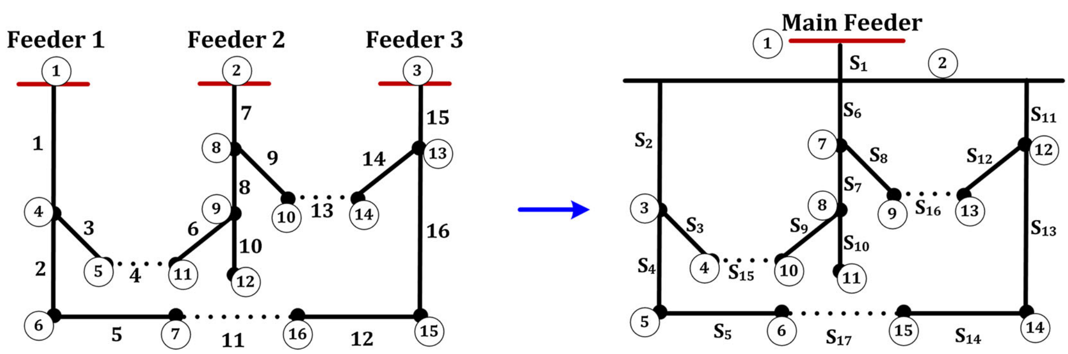

Figure 12. 16-Bus distribution network.

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The sparse of the system, when the initial tie switches S_{15}, S_{16}, and S_{17} are open, is given as follows:

Sparse of 16-bus network = {(2,1), (1,2), (3,2), (7,2), (12,2), (2,3), (4,3), (5,3), (3,4), (10,4), (3,5), (6,5), (5,6), (15,6), (2,7), (8,7), (9,7), (7,8), (10,8), (11,8), (7,9), (13,9), (4,10), (8,10), (8,11), (2,12), (13,12), (14,12), (9,13), (12,13), (12,14), (15,14), (6,15), (14,15)}.

The adjacency matrix of the base case system, when the tie switches are present at S_{15}, S_{16}, and S_{17}, is given in Table IV.

In case of network topology, when the tie switches position is changed to S_2, S_4, and S_{11}, the adjacency matrix of the system is given in Table V. The other necessary steps to rearrange the line data from graph theory-based RLF analysis are summarized in Table VIa.

Similarly, when the tie switches position is changed to S_6, S_9, and S_{13}, the adjacency matrix of the system is given in Table VIb. The other necessary steps to rearrange the line data from graph theory-based RLF analysis are summarized in Table VII.

Table IV. Adjacency matrix of the system when tie line switches S_{15}, S_{16}, and S_{17} are open.

A	Columns														
Rows	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	1	0	0	0	1	0	0	0	0	1	0	0	0
3	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0
4	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
5	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0
6	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
7	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0
8	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0
9	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0
10	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
12	0	1	0	0	0	0	0	0	0	0	0	0	1	1	0
13	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
15	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0

Table V. Adjacency matrix of the system when tie line switches S_2, S_4, and S_{11} are open.

A'	Columns														
Rows	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
4	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
5	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
6	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
7	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0
8	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0
9	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0
10	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
13	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
15	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0

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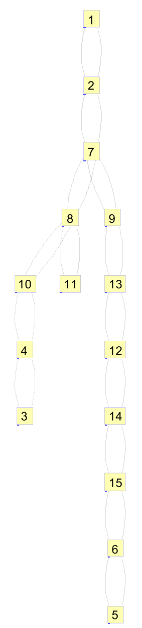
Table VIa. Line-data arrangement when the tie switch positions are S_2, S_4, and S_{11}.

Initial line data	*Step 1: For r=[2 4 11] replace position 2,4,11 in Linedata0 with ‘0’	**Step 2: Remove tie lines and reconstruct Line data (Linedata2)	***Step 3: Using DFS algorithm, generate search order		Step 4: Find DFS order and NoD. Also construct branch matrix.	Step 5: Reconstruct line data from DFS_order	Step 6: Reconstruct Line data (Linedata4)
Linedata0= $\begin{bmatrix}1&2\\2&3\\3&4\\3&5\\5&6\\2&7\\7&8\\7&9\\8&10\\8&11\\2&12\\12&13\\12&14\\14&15\\4&10\\9&13\\6&15\end{bmatrix}$	Linedata1= $\begin{bmatrix}1&2\\0&0\\3&4\\0&0\\5&6\\2&7\\7&8\\7&9\\8&10\\8&11\\0&0\\12&13\\12&14\\14&15\\4&10\\9&13\\6&15\end{bmatrix}$	Linedata2= $\begin{bmatrix}1&2\\3&4\\5&6\\2&7\\7&8\\7&9\\8&10\\8&11\\12&13\\12&14\\14&15\\4&10\\9&13\\6&15\end{bmatrix}$ With this line-data (Linedata2), wrong load flow results are obtained. e.g. Ploss=451.2		DFS_order= $\begin{bmatrix}1\\2\\7\\8\\10\\4\\3\\11\\9\\13\\12\\14\\15\\6\\5\end{bmatrix}$	DFS_oder_NoD= $\begin{bmatrix}1&2\\2&3\\7&4\\8&4\\10&3\\4&3\\3&2\\11&2\\9&3\\13&3\\12&3\\14&3\\15&3\\6&3\\5&2\end{bmatrix}$ branch= $\begin{bmatrix}7\\8\end{bmatrix}$	Linedata3= $\begin{bmatrix}1&2\\2&7\\7&8\\8&10\\10&4\\4&3\\3&11\\11&9\\9&13\\13&12\\12&14\\14&15\\15&6\\6&5\end{bmatrix}$ This is incorrect line-data (Linedata3). Use branch variable to correct the line-data	Linedata4= $\begin{bmatrix}1&2\\2&7\\7&8\\8&10\\10&4\\4&3\\8&11\\7&9\\9&13\\13&12\\12&14\\14&15\\15&6\\6&5\end{bmatrix}$ This is the correct line data and matches with the network topology. Correct load flow results are obtained e.g. Ploss= 1666.1

This methodology will apply for all tie switch combinations present in the system. In this paper, the process of switch combination is repeated for 100 good solution (radial system only), and the switch position $r = [9\ 8\ 17]$ was found the best tie switch position having minimum power losses of 466.10 kW, which matches with the results of [11]. Optimization technique can also be used to reach

Table VIb. Adjacency matrix of the system when tie line switches S_6, S_9, and S_{13} are open.

A’		Columns														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Rows	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	2	1	0	1	0	0	0	0	0	0	0	0	1	0	0	0
	3	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0
	4	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
	5	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0
	6	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
	7	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0
	8	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0
	9	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0
	10	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
	12	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0
	13	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0
	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	15	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0

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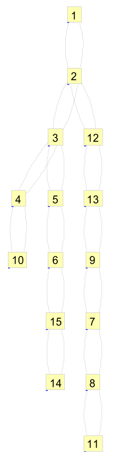
Table VII. Line-data arrangement when the tie switch positions are S_6, S_9, and S_{13}.

Initial line data	*Step 1: For r=[6, 9, 13] replace position 6,9,13 in Linedata0 with '0'.	**Step 2: Reconstruct Line data (Linedata2)	***Step 3: Using DFS algorithm, generate search order		Step 4: DFS order and Degree	Step 5: Reconstruct line data from DFS_order	****Step 6: Reconstruct Line data (Linedata4)
Linedata0= $\begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 3 & 5 \\ 5 & 6 \\ 2 & 7 \\ 7 & 8 \\ 7 & 9 \\ 8 & 10 \\ 8 & 11 \\ 2 & 12 \\ 12 & 13 \\ 12 & 14 \\ 14 & 15 \\ 4 & 10 \\ 9 & 13 \\ 6 & 15 \end{bmatrix}$	Linedata1= $\begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 3 & 5 \\ 5 & 6 \\ 0 & 0 \\ 7 & 8 \\ 7 & 9 \\ 0 & 0 \\ 8 & 11 \\ 2 & 12 \\ 12 & 13 \\ 0 & 0 \\ 14 & 15 \\ 4 & 10 \\ 9 & 13 \\ 6 & 15 \end{bmatrix}$	Linedata2= $\begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 3 & 5 \\ 5 & 6 \\ 7 & 8 \\ 7 & 9 \\ 8 & 11 \\ 2 & 12 \\ 12 & 13 \\ 14 & 15 \\ 4 & 10 \\ 9 & 13 \\ 6 & 15 \end{bmatrix}$ With this line-data (Linedata2), wrong load flow results are obtained. e.g. Ploss= 255.5		DFS_oder= $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 10 \\ 5 \\ 6 \\ 15 \\ 14 \\ 12 \\ 13 \\ 9 \\ 7 \\ 8 \\ 11 \end{bmatrix}$	DFS_oder_NoD= $\begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 4 \\ 4 & 3 \\ 10 & 2 \\ 5 & 3 \\ 6 & 3 \\ 15 & 3 \\ 14 & 2 \\ 12 & 3 \\ 13 & 3 \\ 9 & 3 \\ 7 & 3 \\ 8 & 3 \\ 11 & 2 \end{bmatrix}$ branch= $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$	Linedata3= $\begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 4 & 10 \\ 10 & 5 \\ 5 & 6 \\ 6 & 15 \\ 15 & 14 \\ 14 & 12 \\ 12 & 13 \\ 13 & 9 \\ 9 & 7 \\ 7 & 8 \\ 8 & 11 \end{bmatrix}$ This is incorrect line-data (Linedata3). Use branch variable to correct the data	Linedata4= $\begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 4 & 10 \\ 3 & 5 \\ 5 & 6 \\ 6 & 15 \\ 15 & 14 \\ 2 & 12 \\ 12 & 13 \\ 13 & 9 \\ 9 & 7 \\ 7 & 8 \\ 8 & 11 \end{bmatrix}$ This is the correct line data and matches with the network topology. Correct load flow results are obtained e.g. Ploss= 1277.6

*For random generated switches(r), if the system is found radial, start from step 1, else generate another good solution of switches.
**Line data will also be sort out to ensure From(i) < To(j) position.
***Here bio-graph is shown here to visualize the DFS order only.
****Carried out RLF analysis after step 6.

to the final solution. However, optimum network reconfiguration is out of scope of this paper; thus, the discussion is limited up to the graph theory-based RLF analysis in solving network reconfiguration problem.

Sixteen-bus system is a small distribution system in case of solving network reconfiguration problem. The proposed method is also applied on 33-bus distribution systems, and it was found that the proposed method works efficiently for large systems also.

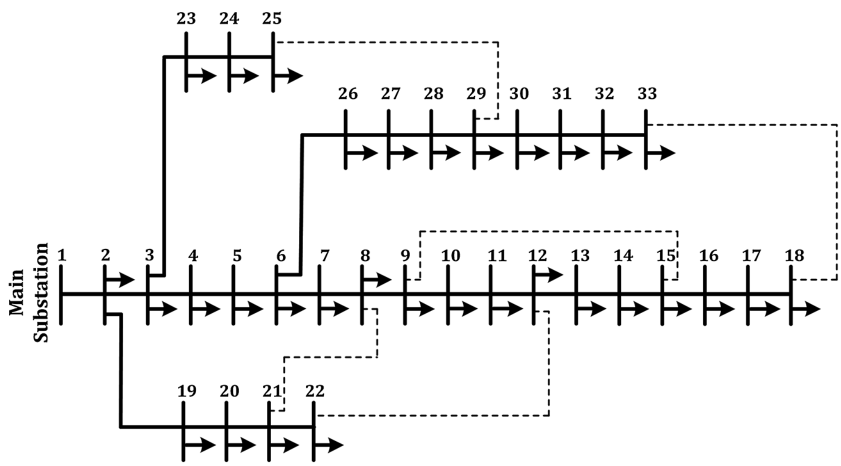


Figure 13. Single line diagram of 33-bus distribution test system.

7.2. Test results on 33-bus test system

In the second case, the proposed algorithm is applied on 12.66-kV, 33-bus distribution test system having five tie switches present in the system. The system details are given in Appendix Table A-II, and the single line diagram of the system is shown in Figure 13. The adjacency matrix of the base case system, when the tie switches are present at S_{33}, S_{34}, S_{35}, S_{36}, and S_{37}, is given in Table VIII.

When the tie switches positions are changed, the necessary steps to rearrange the line data from graph theory-based RLF analysis are summarized in Table VIa and VII. The RLF analysis is carried

Table VIII. Adjacency matrix of the system when tie line switches S_{33}, S_{34}, S_{35}, S_{36}, and S_{37} are open.

A		Columns																																
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
Row	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	2	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	4	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	6	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
	7	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	12	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	13	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	14	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	15	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	19	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
	22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	23	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0
	25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	26	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0
	28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
	29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0
	30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
	31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
	33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0

Table IX. Adjacency matrix of the system when tie line switches S_7, S_9, S_{14}, S_{32}, and S_{37} are open.

A		Columns																																
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
Row	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	2	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	4	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	6	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
	7	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	12	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	13	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	14	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	15	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	19	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	21	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
	22	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	23	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0
	25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	26	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0
	28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
	29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0
	30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
	31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
	33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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out once the line data have been rearranged. The adjacency matrix of the system when the tie switches position is changed to S_7, S_9, S_{14}, S_{32}, and S_{37} is given in Table IX. Figure 14 represents the biograph for 33-bus distribution system for different tie switches positions.

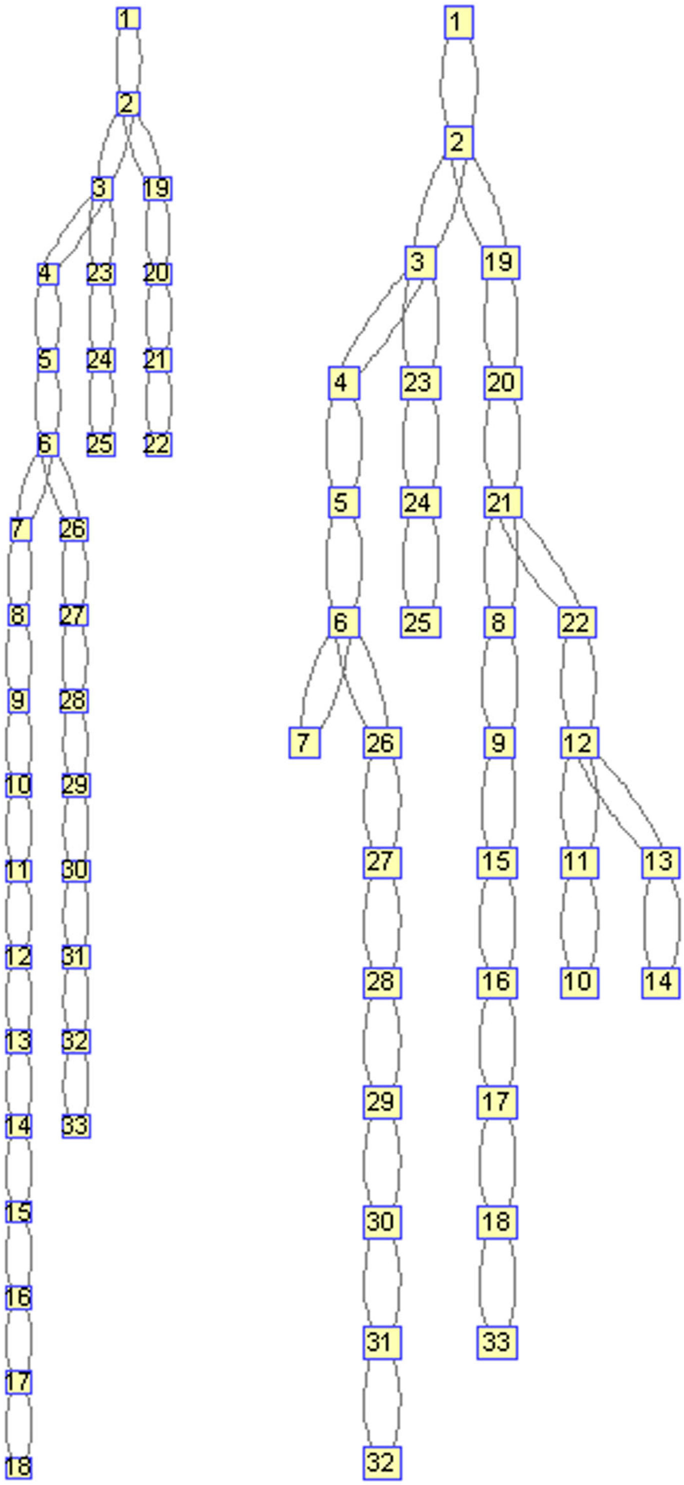

Figure 14. Biograph of 33-bus distribution system for different tie switches positions. (a) When tie switches positions are S33, S34, S35, S36, and S37. (b) When tie switches positions are S7, S9, S14, S32, and S37.

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The proposed methodology is applied for all tie switches combination present in the system. In this paper, the process of switch combination is repeated for 100 times for 20 good solution (radial system only), and the switch position $r = [7,9,14,32,37]$ was found the best tie switches position having minimum power losses of 139.55 kW.

8. CONCLUSION

This paper has presented a graph theory-based RLF analysis to solve "dynamic" power system problems in which the configuration of the system does not remain static, for example, network reconfiguration problem. The proposed algorithm will help in arranging the line data for any combination of tie switch positions, to check the radiality of the system and to ensure that all nodes are connected with the source node. The proposed algorithm has been tested on standard 16-bus and 33-bus distribution test systems having three and five tie switches, respectively, for minimization of power losses. From the obtained results and discussion, it can be concluded that the graph theory approach has increased the robustness of existing RLF methods and can be utilized to solve the dynamic power system problems.

9. LIST OF SYMBOLS AND ABBREVIATIONS

RLF	radial load flow analysis
R	resistance of the line
X	reactance of the line
V_r	receiving end bus voltage
V_s	sending end bus voltage
δ_r	receiving end bus angle
δ_s	sending end bus angle
k	number of iteration
n	number of buses
BCBV	branch current to bus voltage matrix
BIBC	branch injection to branch current matrix
DLF	distribution load flow matrix
I	load current
P	active power
Q	reactive power
S	apparent power
V	vertices of graph "G"
E	edges of graph "G"
S	sparse form
A	adjacency matrix
BFS	breadth-first search
DFS	depth-first search

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APPENDIX A

Table A-I: Modified form of 16-bus distribution system data [11].

Line number	From	To	P (kW)	Q (kVAR)	R (Ω)	X (Ω)	$I_{max}(A)$
1	1	2	0	0	1.00E−10	1.00E−10	2500
2	2	3	2000	1600	0.12021	0.16028	1400
3	3	4	3000	400	0.12822	0.1763	500
4	3	5	2000	−400	0.14425	0.2885	500
5	5	6	1500	1200	0.06411	0.06411	500
6	2	7	4000	2700	0.1763	0.1763	1400
7	7	8	5000	1800	0.12822	0.1763	1000
8	7	9	1000	900	0.1763	0.1763	500
9	8	10	600	−500	0.1763	0.1763	500
10	8	11	4500	−1700	0.12822	0.1763	500
11	2	12	1000	900	0.1763	0.1763	1400
12	12	13	1000	−1100	0.14425	0.19233	500
13	12	14	1000	900	0.12822	0.1763	500
14	14	15	2100	−800	0.06411	0.06411	500
15	4	10	—	—	0.06411	0.06411	500
16	9	13	—	—	0.06411	0.06411	500
17	6	15	—	—	0.144248	0.192331	500

P and Q are representing the receiving end load.

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Table A-II: 33-Bus distribution system data [13].

Line number	From	To	R (Ω)	X (Ω)	P (kW)	Q (kVAR)
1	1	2	100	60	0.0922	0.0477
2	2	3	90	40	0.493	0.2511
3	3	4	120	80	0.366	0.1864
4	4	5	60	30	0.3811	0.1941
5	5	6	60	20	0.819	0.707
6	6	7	200	100	0.1872	0.6188
7	7	8	200	100	1.7114	1.2351
8	8	9	60	20	1.03	0.74
9	9	10	60	20	1.04	0.74
10	10	11	45	30	0.1966	0.065
11	11	12	60	35	0.3744	0.1238
12	12	13	60	35	1.468	1.155
13	13	14	120	80	0.5416	0.7129
14	14	15	60	10	0.591	0.526
15	15	16	60	20	0.7463	0.545
16	16	17	60	20	1.289	1.721
17	17	18	90	40	0.732	0.574
18	2	19	90	40	0.164	0.1565
19	19	20	90	40	1.5042	1.3554
20	20	21	90	40	0.4095	0.4784
21	21	22	90	40	0.7089	0.9373
22	3	23	90	50	0.4512	0.3083
23	23	24	420	200	0.898	0.7091
24	24	25	420	200	0.896	0.7011
25	6	26	60	25	0.203	0.1034
26	26	27	60	25	0.2842	0.1447
27	27	28	60	20	1.059	0.9337
28	28	29	120	70	0.8042	0.7006
29	29	30	200	600	0.5075	0.2585
30	30	31	150	70	0.9744	0.963
31	31	32	210	100	0.3105	0.3619
32	32	33	60	40	0.341	0.5302
33	21	8	—	—	2	2
34	9	14	—	—	2	2
35	12	22	—	—	2	2
36	18	33	—	—	0.5	0.5
37	25	29	—	—	0.5	0.5

P and Q are representing the receiving end load.

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