BASIC STRUCTURAL MODELING PROJECT

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CONCEPTUAL SOLUTION REPORT APPENDICES

Version 1.00

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Appendix A – Structuring Process Example

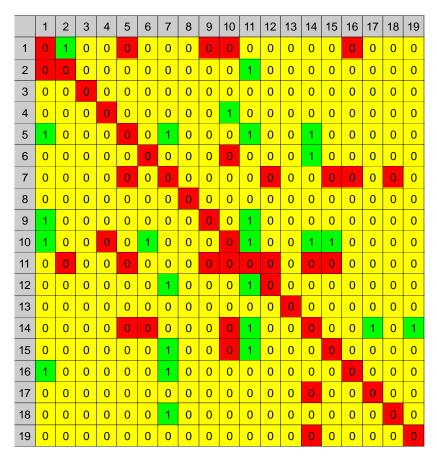


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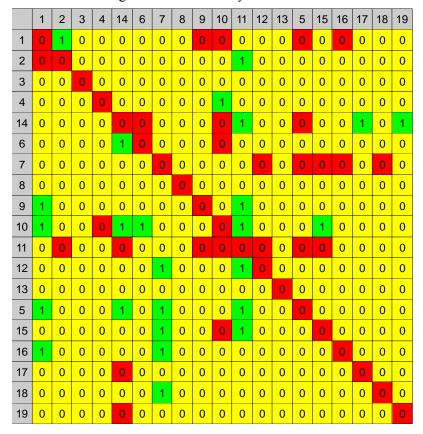


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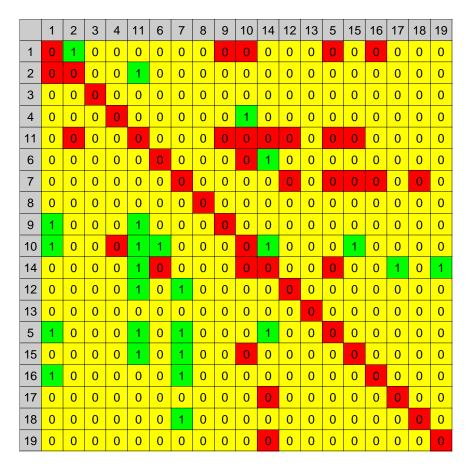


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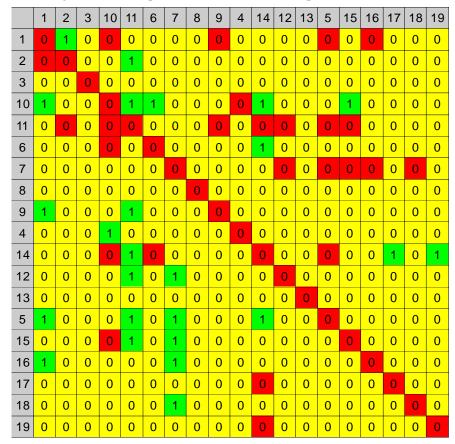


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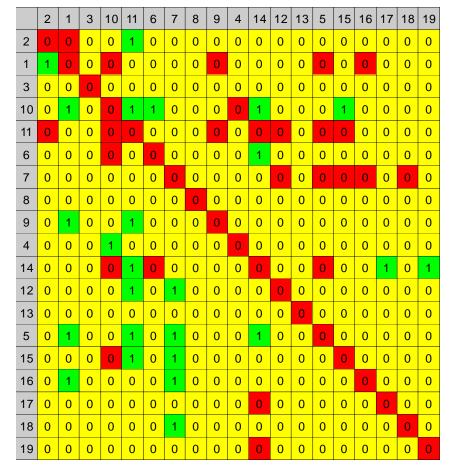


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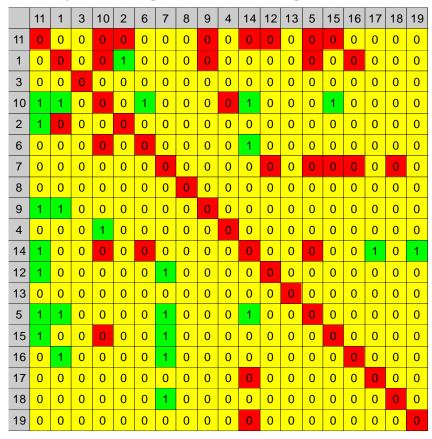


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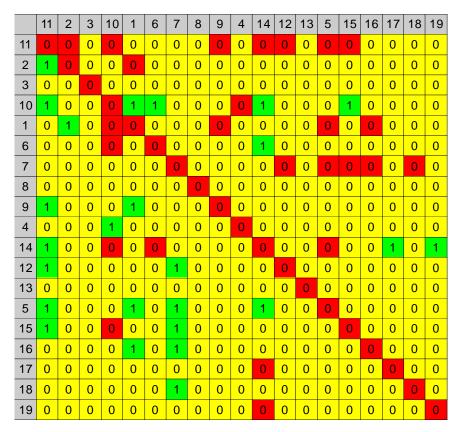


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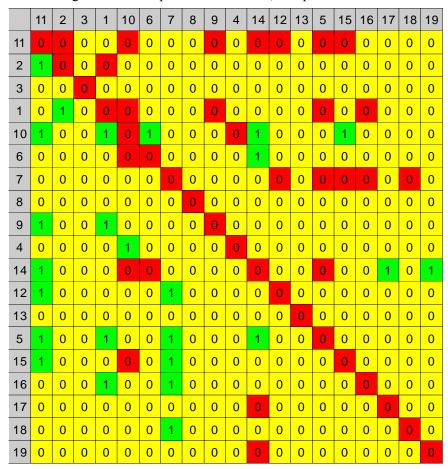


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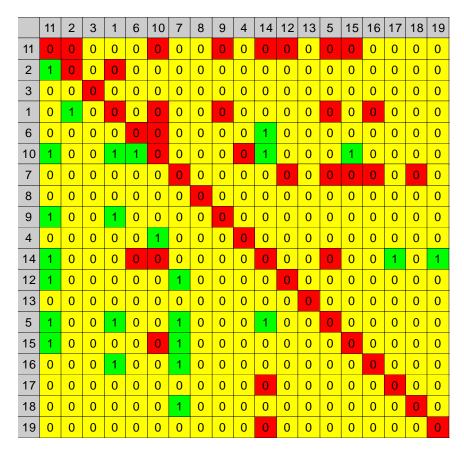


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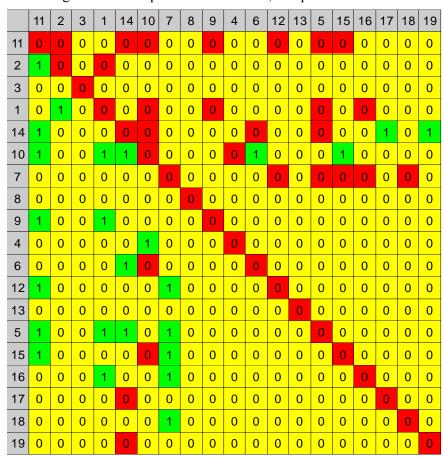


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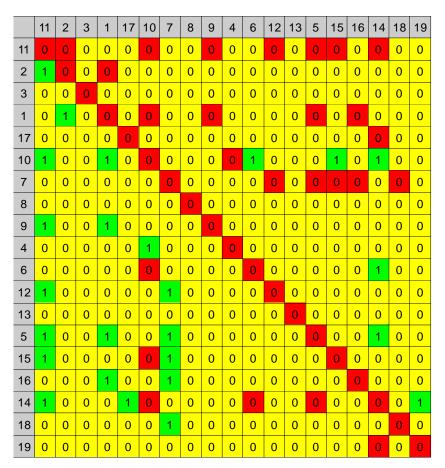


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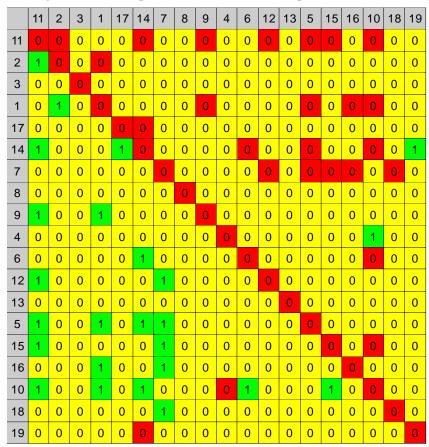


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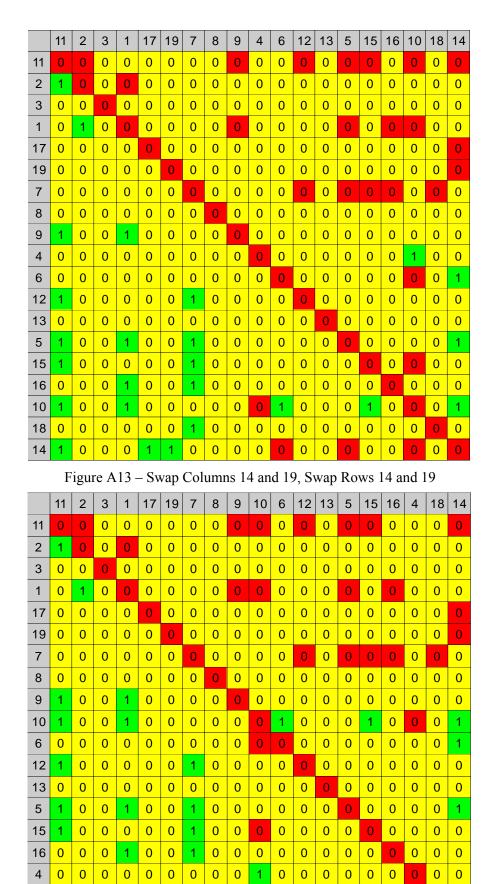


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0 0

0

0 0 0 0 0 0 0

1

0 0 0

0

18 0 0 0 0 0

14

0 0 0

0

0

0

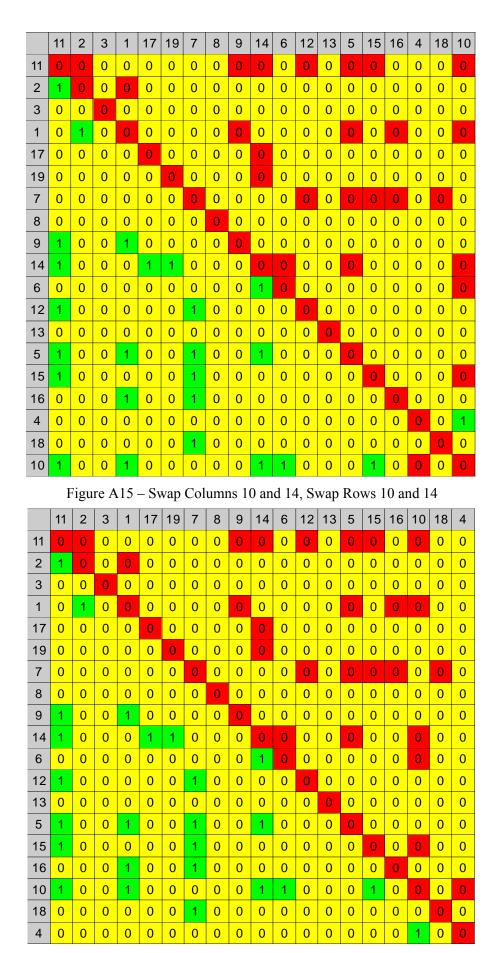


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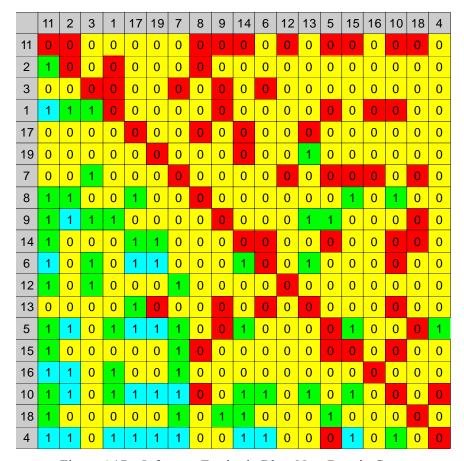


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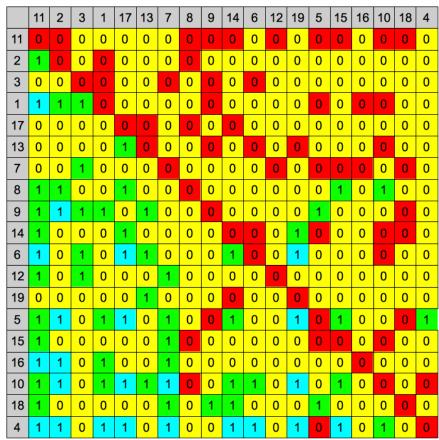


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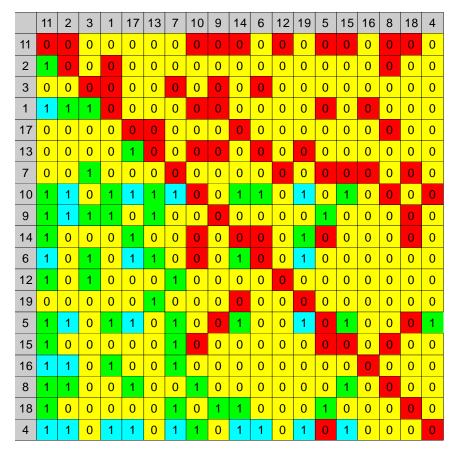


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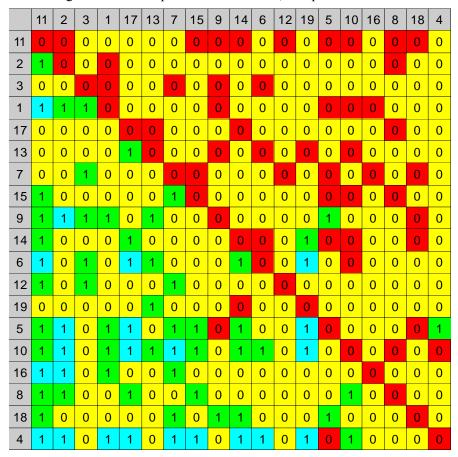


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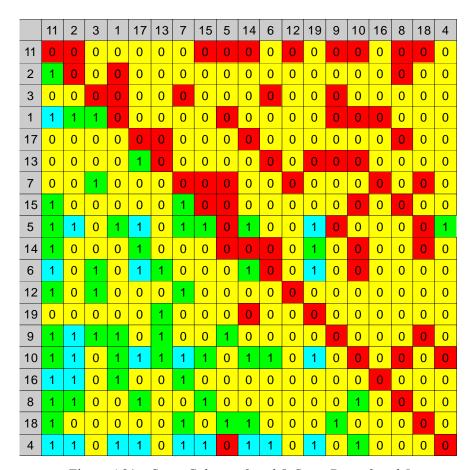


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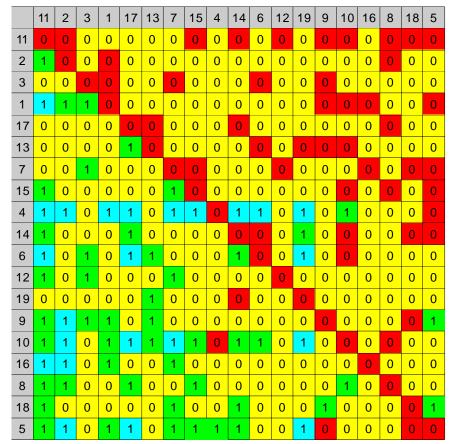


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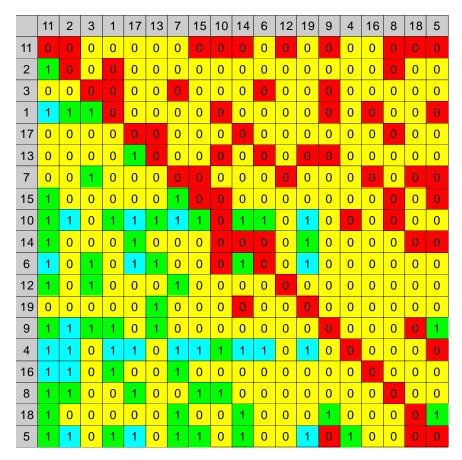


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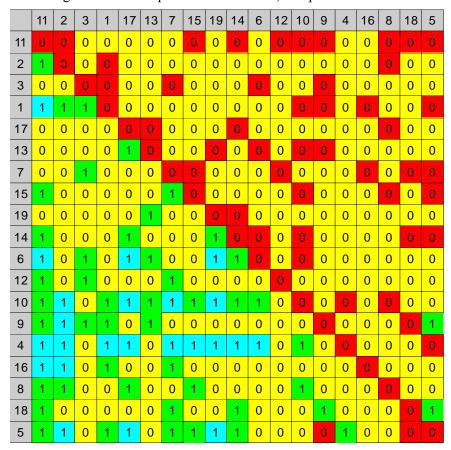


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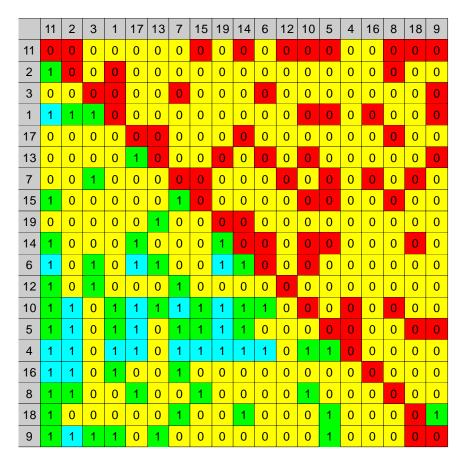


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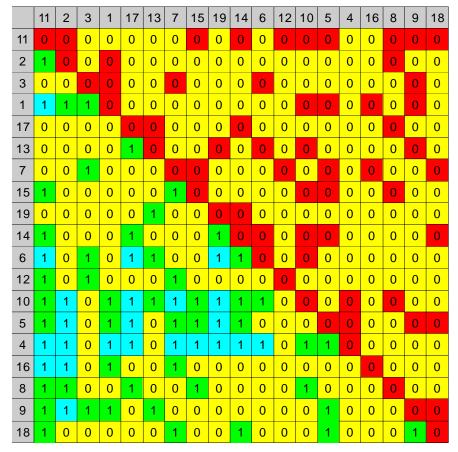


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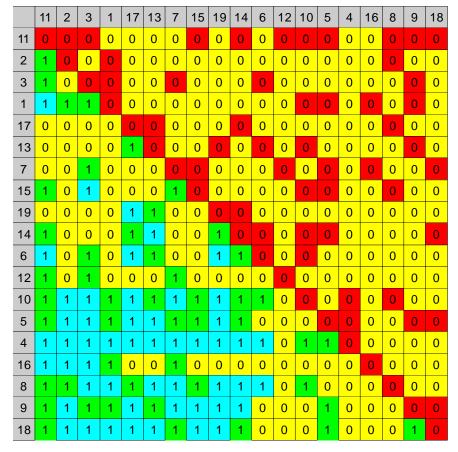


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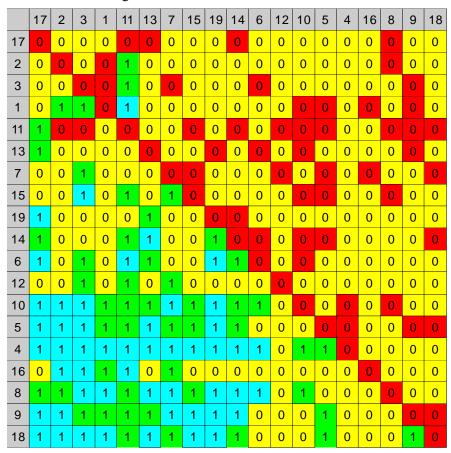


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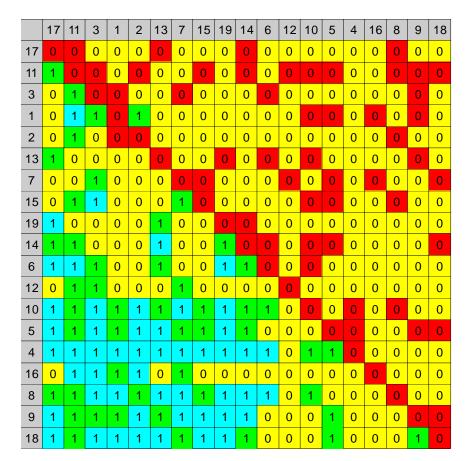


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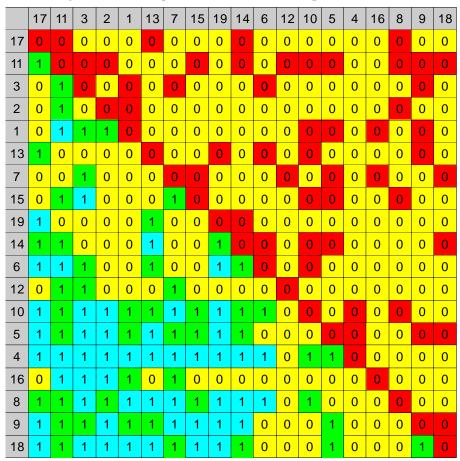


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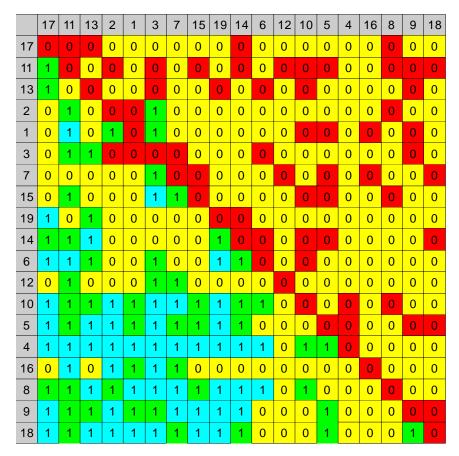


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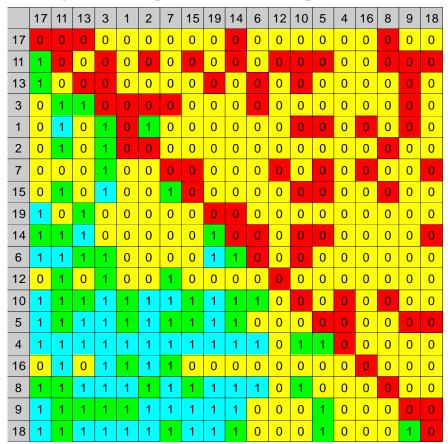


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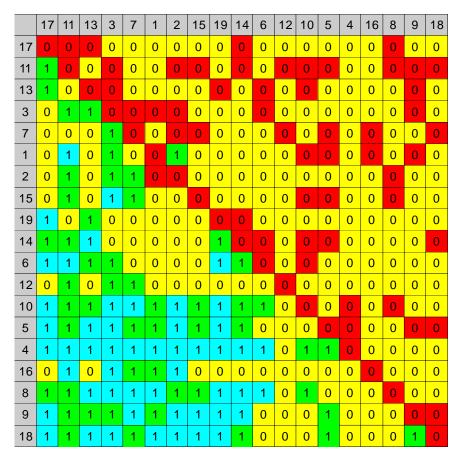


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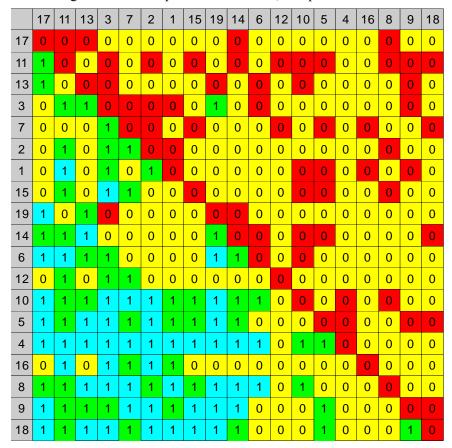


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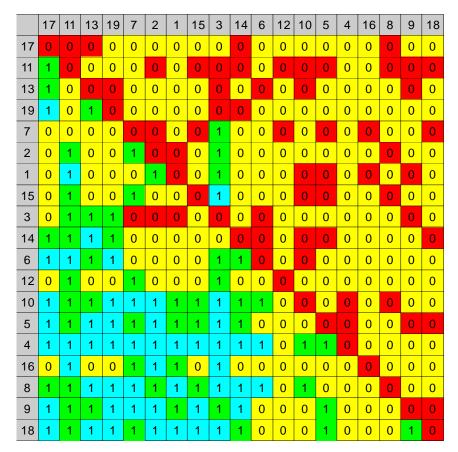


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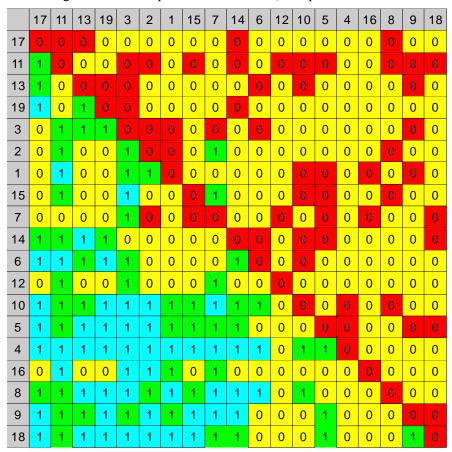


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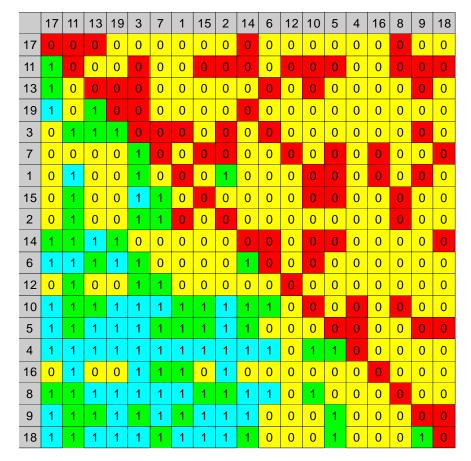


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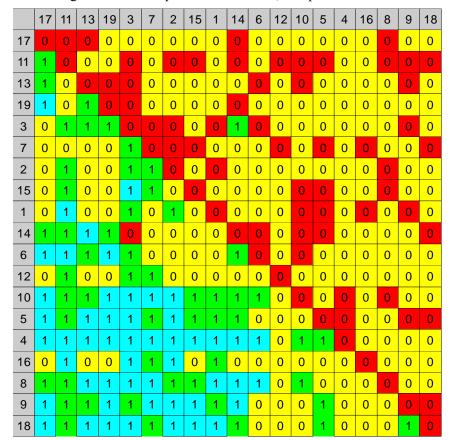


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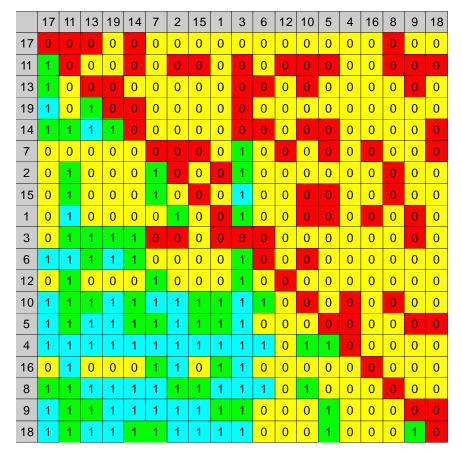


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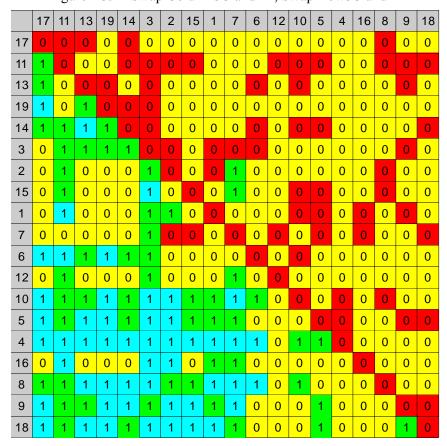


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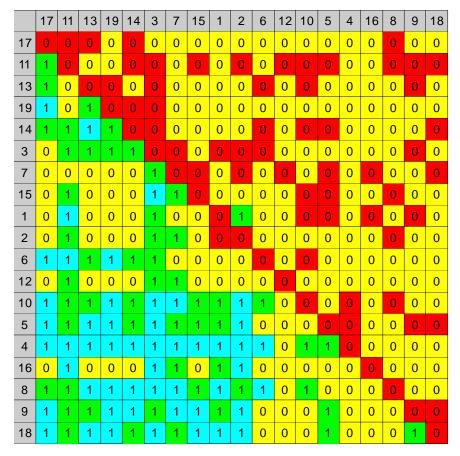


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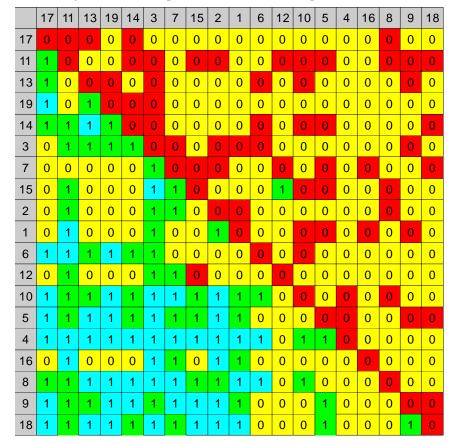


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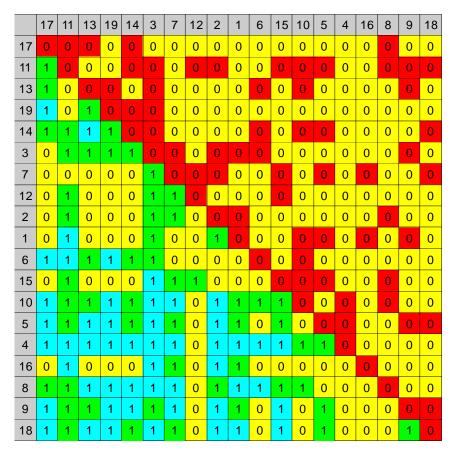


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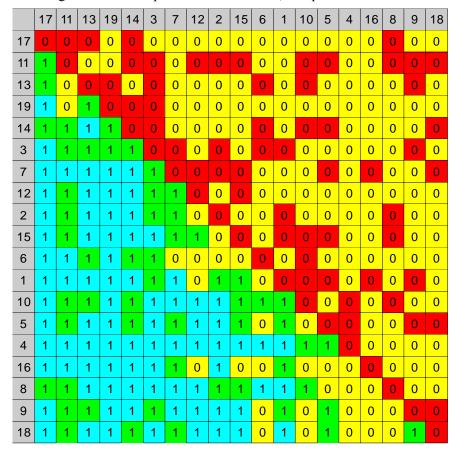


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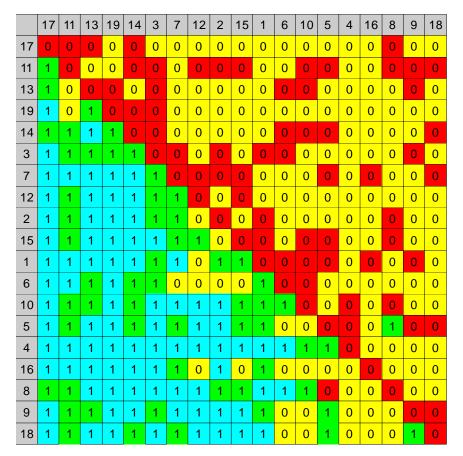


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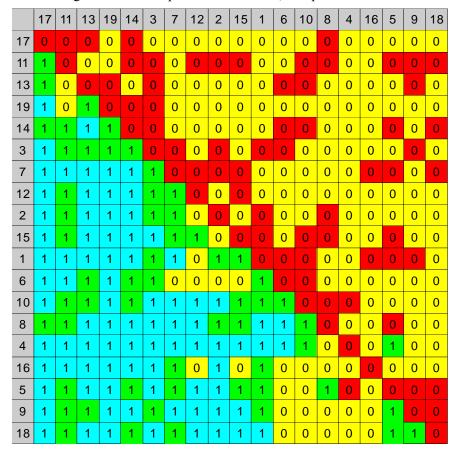


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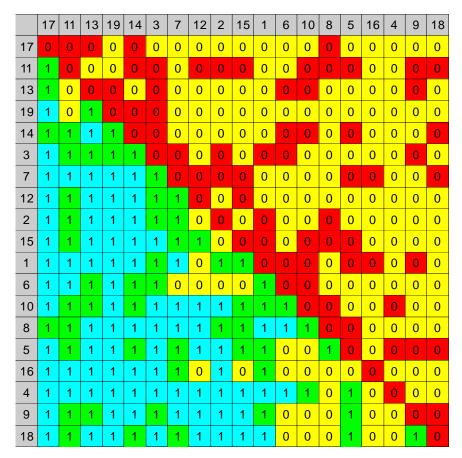


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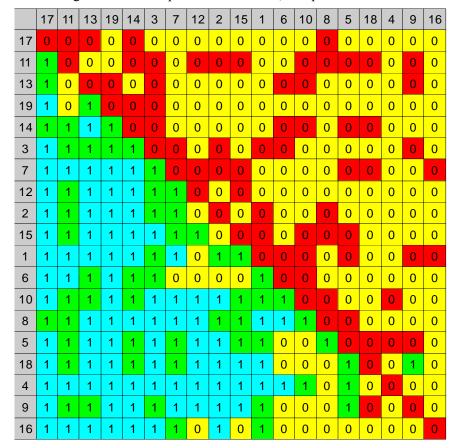


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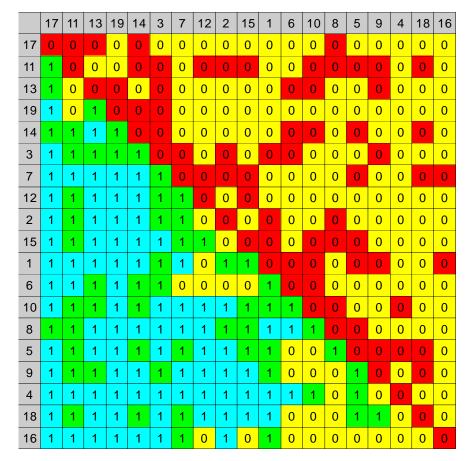


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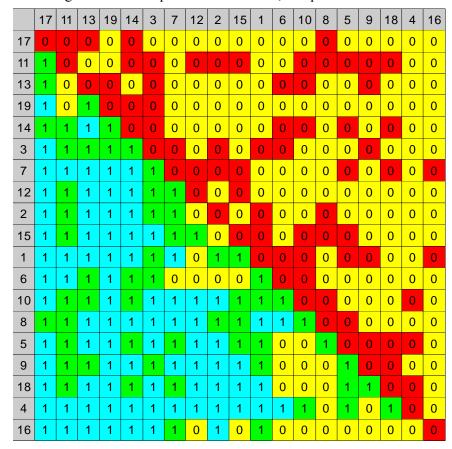


Figure A50 – Swap Columns 4 and 18, Swap Rows 4 and 18; Final System Structure and Information

Boolean Algebra

Boolean algebra operates on two distinct constant values: zero (0) and one (1). There are a number of operators associated with these values in Boolean algebra. These operators are:

- complementation ' (monadic operation)
- addition + (binary operation)
- multiplication · (binary operation)

These operations conform to the following laws:

```
• commutative laws: a + b = b + a, a \cdot b = b \cdot a
```

- distributive laws: $a \cdot (b+c) = (a \cdot b) + (a \cdot c)$, $a + (b \cdot c) = (a+b) \cdot (a+c)$
- identity laws: a + 0 = a, $a \cdot 1 = a$
- complement laws: a + 'a = 1, $a \cdot 'a = 0$

Warfield augmented Boolean algebra by adding the concept of order to the two Boolean constants. The addition of order to Boolean algebra supports the addition of the following new Boolean operators:

• Less than operator: 0 < 1

- Greater than operator: 1 > 0
- Less than or equal to: 0 < 1
- Greater than or equal to: $1 \le 0$
- Boolean subtraction: 1 1

When this new concept of Boolean order is used with matrix operations, the following Boolean matrix operations are added:

- Matrix subtraction: subtract one matrix from another
- Matrix ordering: one matrix less than, greater than, or equal to, another Boolean matrix

Boolean recursion equations and Boolean inequalities are sets of Boolean equations that are used to analyze and evaluate systems. Solution techniques associated with these types of equation sets will be presented later in this document.

Mathematical Sets

A set is a well-defined collection of objects. Each object in a set is called a member or element of the set. Sets may be formed in two ways: **extension** and **intension**. The first way to create a set is to explicitly list all of the elements in the set, which is known as set formation by extension. The second way to create a set is to provide a rule, or set of rules, that describe the set members. This is called set formation by intension.

```
Given X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}
Set X is formed by extension.
```

```
Given Y = \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}
```

Each indexed element of set Y is 10 times the corresponding indexed element of set X.

Set *Y* is formed by **intension**.

A set formed by intension is called well-defined if the rules and criteria for set formation provide the ability to determine if any given element is either included in the set or excluded from the set. If a set is not well-defined, then empirical

procedures may be used to create a variable grade of membership that is assigned to each element in the set. The variable grade of membership is called the element weight. In any specific situation, an element-weight trigger value may be assigned to indicate when the element is considered part of the set. The element weights run from one (1), for full membership, to zero (0) for no membership. Fuzzy sets are sets that contain elements with unequal weights. The sets that will be considered in this report are mostly well-defined.

Well-defined sets may be indexed using an index set if the number of elements in the set is known. Sets may be interrelated in a number of ways, including: **subset, power set, proper subset, complement, union, and intersection**. Sets that are not interrelated are called disjoint, and they have no elements in common.

- Set Z is a **subset** of set W if every element in set Z is also a member of set W. Any set may be a subset of itself.
- The set of all subsets of Z is called the **power set** of Z.
- Given: sets Z and W,

set Z is not equal to set W set Z is a subset of set W

Then Z is a **proper subset** of W

- The **complement** of set X with respect to set Y, is the set of all elements in set Y that are not contained in set X.
- The **union** of set Z and set W consists of all elements that are members of set Z, or set W, or both set Z and set W.
- The **intersection** of set X and set Y consists of all elements that are members of both set X and set Y.

A vector set is ordered using the set indices as a structuring mechanism. When a set is ordered using the set indices, it is called a vector set, or just a vector. A Cartesian product (of sets A and B) consists of the set $A \cdot$ set B of ordered pairs (a, b), where a is an element of A and B is an element of B. The Cartesian product is also a set. Once a vector set has been defined, then the Cartesian product of any given vector set, with itself, may be defined. For example:

- $X = \{1, 2, 3\}$, set X is defined by extension.
- Cartesian product set $Y = X \cdot X = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$, set Y (the Cartesian product of Set X times Set X) is defined by intension.
- Set $W = \{(1, 1), (2, 2), (3, 3)\}$, is a subset of Y.

A partition (Π) on set Z is a collection of disjoint, nonempty subsets of Z whose set union is Z. For example, consider $Z = \{A, B, C, D, E, F\}$. Some possible partitions of set Z are:

- $\Pi_1 = \{A, B, C; D; E, F\}$ (i.e., three partition blocks: A, B, C; D; and E, F)
- $\Pi_2 = [A; B, C; D; E, F]$ (i.e., four partition blocks: A; B, C; D; and E, F)
- $\Pi_3 = [A, B, C, D; E, F]$ (i.e., two partition blocks: A, B, C, D; and E, F)
- $\Pi_4 = [A, B, C, D, E, F]$ (i.e., one partition block: A, B, C, D, E, F)
- $\Pi_5 = [A, B; C, D; E, F]$ (i.e., three partition blocks: A, B; C, D; E, F)
- $\Pi_6 = [A; B; C; D; E; F]$ (i.e., six partition blocks: A; B; C; D; E; F)
- $\Pi_7 = [A, B; C, D; E, F]$ (i.e., three partition blocks A, B; C, D; E, F)

The concepts of **equality**, **product**, **sum**, and **order** are included in partition algebra.

• If every block in one set partition on Z is also a block in another set partition on Z, then the set partitions are equal. As an example:

```
\Pi_5 = [A, B; C, D; E, F]

\Pi_7 = [A, B; C, D; E, F]

\Pi_5 is equal to \Pi_7.
```

• The **product** of two partitions on a set is another partition on the set. The members, element 1 and element 2, of set Z, are in the same block of the product partition only if they are in the same block in both of the argument partitions. As examples:

```
\Pi_1 = [A, B, C; D; E, F]
```

$$\begin{split} \Pi_5 &= [A, B; C, D; E, F] \\ \Pi_1\Pi_5 &= A, B; C; D; E, F \\ \Pi_2 &= [A; B, C; D; E, F] \\ \Pi_6 &= [A; B; C; D; E; F] \\ \Pi_2\Pi_6 &= A; B; C; D; E; F \\ \Pi_4 &= [A, B, C, D, E, F] \\ \Pi_3 &= [A, B, C, D; E, F] \\ \Pi_4\Pi_3 &= A, B, C, D; E, F \end{split}$$

• The **sum** of two partitions on a set is another partition on the set. The members, element 1 and element 2, of set Z are in the same block of the sum partition only if there exists a sequence $x_1, x_2, ..., x_m$, such that element $1 = x_1$, element $2 = x_m$ and all pairs of the form $x_1, x_2, x_3, ..., x_{m-1}$ are in a single block in either argument partition one or argument partition two. As an example, for:

```
    Π<sub>1</sub> = {a, b; c, e, g; d; f, h, j; i}
    Π<sub>2</sub> = {a, b, d; c, g; e, f, h, I; j}
    The first block of the sum is a, b, d. That block is composed of two blocks in Π<sub>1</sub> (a b, and d) and one block in Π<sub>2</sub> (a b d). The second block of the sum is c, e, f, g, h, i, and j. That block is composed of the other blocks from argument partitions Π<sub>1</sub> and Π<sub>2</sub> that are not in a single block.
    The sum of Π<sub>1</sub> and Π<sub>2</sub> is: Π<sub>1</sub> + Π<sub>2</sub> = Π<sub>3</sub> = {a, b, d; c, e, f, g, h, i, j}
```

• Partitions on a set may be **ordered** in value using the following definitions. A partition is less than or equal to another partition on the same set only if every block in the first partition is contained in some block of the second partition. Given two partitions Π_1 and Π_2 , Π_1 is less than or equal to Π_2 only if the product of Π_1 and Π_2 is equal to Π_1 and the sum of Π_1 and Π_2 is equal to Π_2 . The product of Π_1 and Π_2 is always less than or equal to the sum of Π_1 and Π_2 .

If the partition of set A ($\Pi(A)$) consists of a single block, it contains all members of set A, and is called the **identity** partition (Π_I).

• The **product of identity partition** with any other partition Π is equal to Π .

The **zero** partition of a set (Π_Z) consists of a partition that has as many blocks as it has elements.

• The sum of the zero partition, Π_Z , and any other partition Π is equal to Π .

Binary Relations

A binary relation R(A, B) is a subset of the Cartesian product of vector set A and vector set B. A binary relation R(A, A) is a subset of the Cartesian product of vector set A with itself, or $A \times A$. A binary relation R(B, B) is a subset of the Cartesian product of vector set B with itself, or $B \times B$. For example:

```
Set B_1 = \{1, 2, 3\}
Set B_2 = \{4, 5, 6\}
```

Some binary relations on sets $B_1 \times B_2$ are:

$$R_1(B_1, B_2) = \{(1, 4), (1, 6), (2, 4), (3, 6)\}$$

 $R_2(B_1, B_2) = \{(1, 5), (2, 6), (3, 4)\}$
 $R_3(B_1, B_2) = \{(2, 5), (3, 5)\}$
 $R_4(B_1, B_2) = \{(3, 4)\}$

The **complement** (') of the binary relation, R_1 , is composed of all of the elements that are not part of R_1 . The complement of $R_1 = \{(1, 5), (2, 5), (2, 6), (3, 4), (3, 5)\}$

The **transpose** (R^T) of R_1 is produced by exchanging the order in every element pair. The transpose of $R_1 = R^T_1 = \{(4, 1), (6, 1), (4, 2), (6, 3)\}$

A binary relation, R(W, W) on set W may have the following properties (where w_1 and w_2 are elements of W):

reflexivity: w_1Rw_1 irreflexivity: $w_1'Rw_1$ symmetry: if w_1Rw_2 , then w_2Rw_1 asymmetry: if w_1Rw_2 , then $w_2'Rw_1$ antisymmetry: if w_1Rw_2 and w_2Rw_1 , then $w_1 = w_2$ Note that the notation w_iRw_i means that (w_i, w_i)

If a binary relation, *R*, is reflexive and transitive, and *R* and the complement of *R* are antisymmetric, then *R* is called an **order**.

If the complement of *R* is not antisymmetric and all other conditions are met, then *R* is called a **partial order**.

If a binary relation is a partial order, and also identifies a greatest lower bound and a least upper bound, then this partial order is a **lattice**. For example: (page 218)

Let A consist of the binary vector set $\{a_i\} = \{(u_i, v_i, w_i)\}\$, where:

$$a_0 = (0, 0, 0)$$

$$a_1 = (0, 0, 1)$$

$$a_2 = (0, 1, 0)$$

$$a_3 = (0, 1, 1)$$

$$a_4 = (1, 0, 0)$$

$$a_5 = (1, 0, 1)$$

$$a_6 = (1, 1, 0)$$

$$a_7 = (1, 1, 1)$$

Define binary relation, *R*, on set *A* X *A* using the following two conditions:

- (1) $a_i R a_i$ if and only if $u_i \le u_i$, $v_i \le v_j$, $w_i \le w_j$, where, by definition 0 < 1, 0 = 0 and 1 = 1
- (2) a_i can participate in R if and only if a_i is a solution of the Boolean recursion equation set:

$$u = u$$

$$v + u = v$$

$$w + v + u = w$$

The only elements that can participate in R are the elements of subset $A_1 = \{a_0, a_1, a_3 \text{ and } a_7\}$. Applying condition (1), it is seen that the binary relation defined by the two conditions is

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R = \{(a_0, a_0), (a_0, a_1), (a_0, a_3), (a_0, a_7), (a_1, a_1), (a_1, a_3), (a_1, a_7), (a_3, a_3), (a_3, a_7), (a_7, a_7)\}
This relation is not a partial order for the set A, since it is not reflexive. However, if it is reinterpreted as a binary relation on A_1 \times A_1, it is both a partial order and a lattice.
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In this way, a set of Boolean recursive equations can constrain a binary relation to a subspace that is both a partial order, and a lattice.