Belief Propagation Neural Networks

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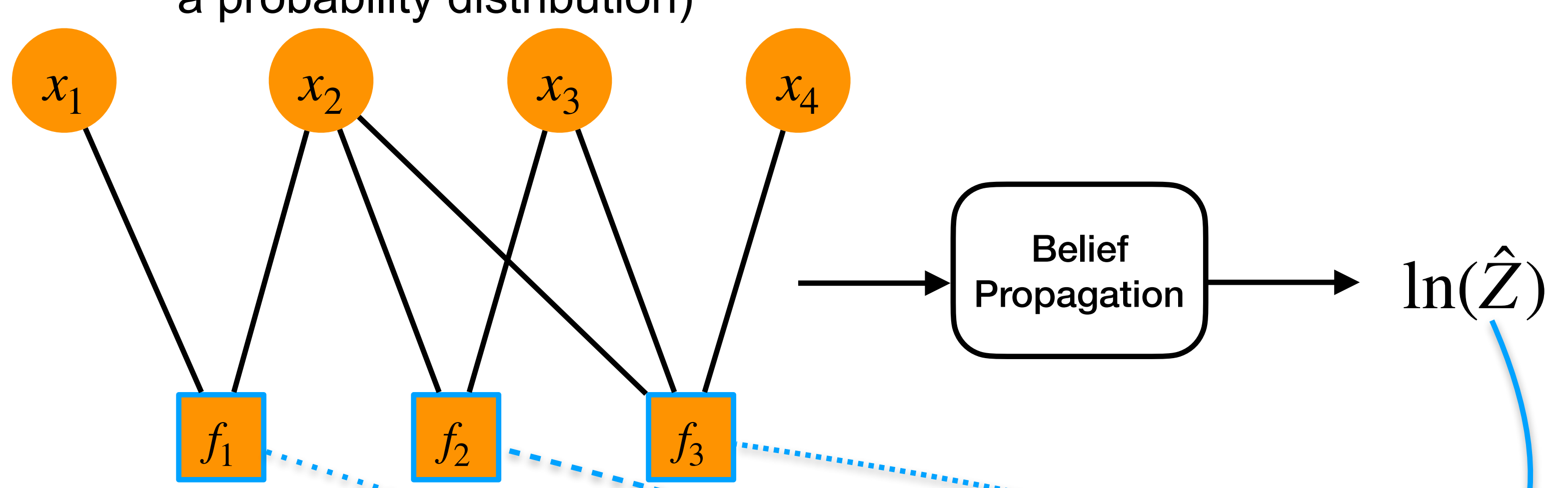
TLDR: Belief propagation (BP) is an efficient variational inference algorithm with theoretical guarantees. However, its estimates can have poor accuracy on certain domains. We introduce a neural architecture (BPNN) that generalizes BP, while ***preserving guarantees****. Empirically, BPNN learns from a small training set to perform* ***more accurate inference*** *than BP and* ***converge faster****.*

Problem: Improve the accuracy of belief propagation (BP) while maintaining guarantees
Approach: Design neural networks that generalize BP
- Iterative neural architecture that converges to fixed points of BP (BPNN-D)
- Generalization of Bethe approximation that respects factor graph isomorphism (BPNN-B)

Background: Belief Propagation

Input: factor graph (compact representation of a probability distribution)

Output: estimate of normalization constant Z

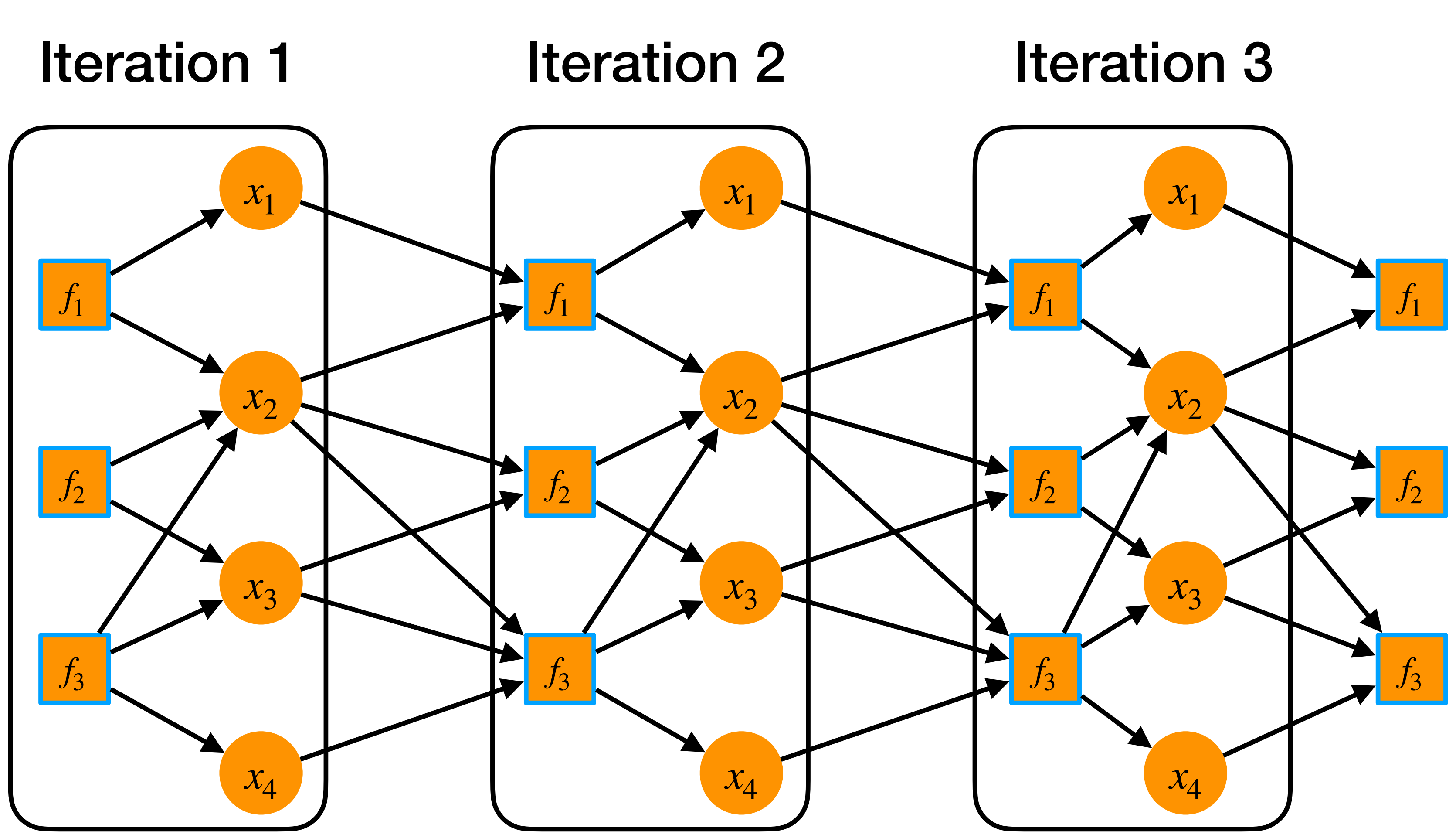


$$p(x_1, x_2, x_3, x_4) = \frac{1}{Z} f_1(x_1, x_2) f_2(x_2, x_3) f_3(x_2, x_3, x_4)$$

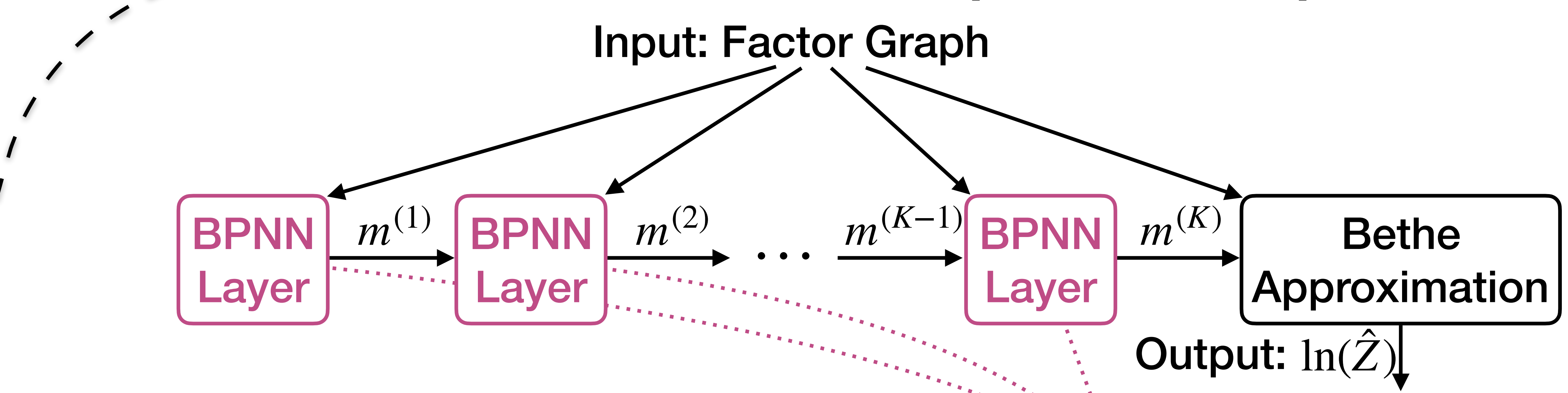
BP is efficient and provides accurate estimates in certain domains

BP: Unrolled Computation Graph

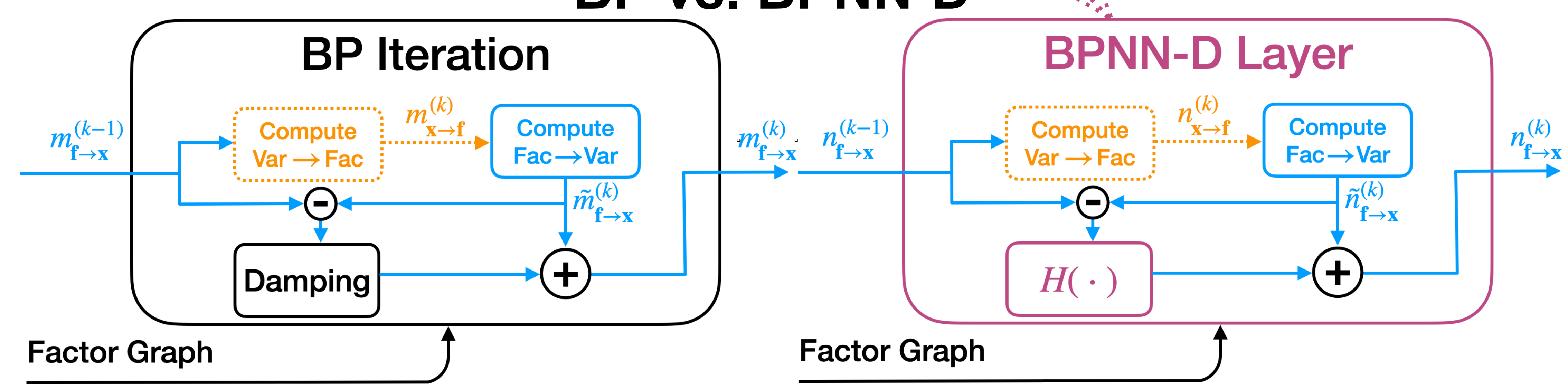
- BP performs iterative message passing between variable and factor nodes.
- This computation can be unrolled (as shown below).
- ***Limitation: computation is fixed and BP estimates can be bad***



BPNN: Differentiable Computation Graph



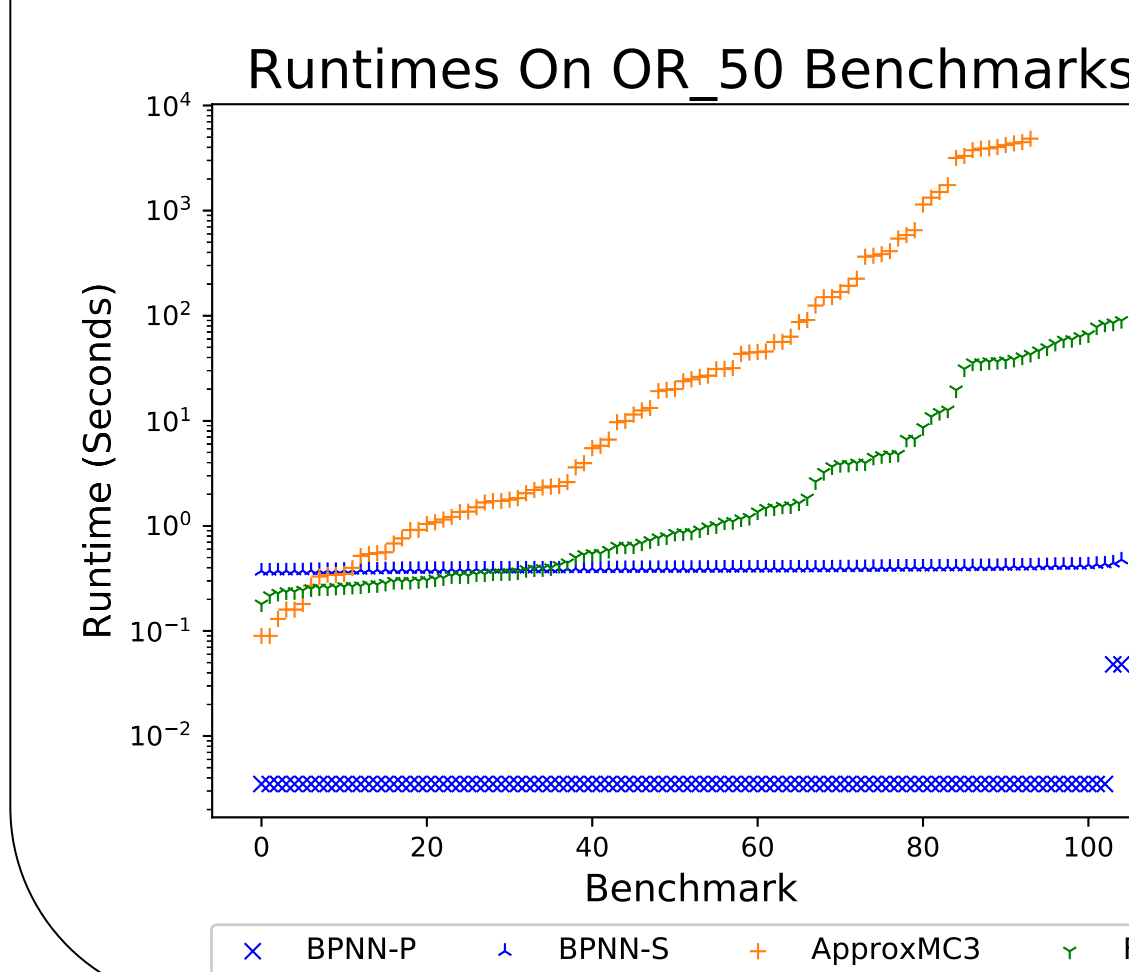
BP vs. BPNN-D

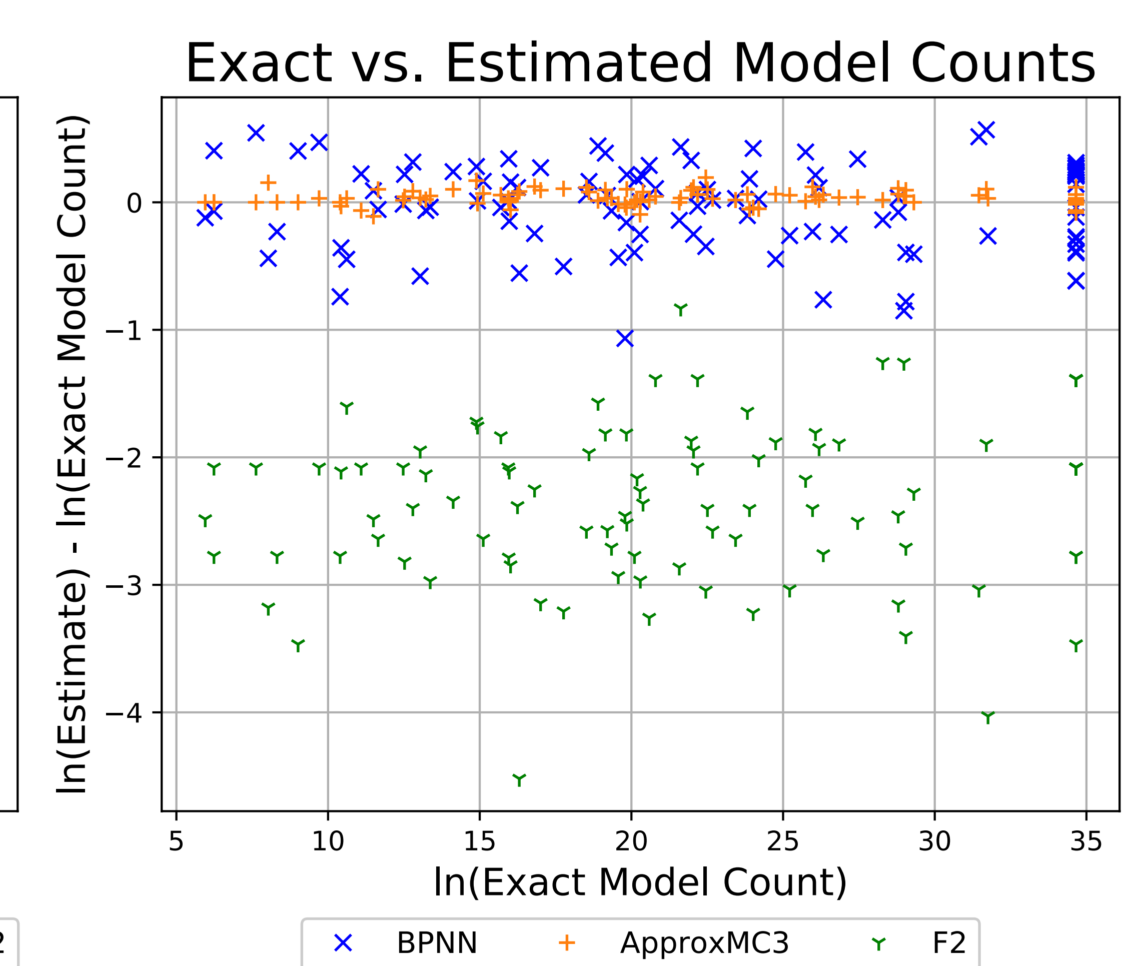


- We replace standard BP damping with learnable operator $H(\cdot)$, ***generalizing BP***
- Set of fixed points is unchanged for certain $H(\cdot)$, ***preserving guarantees***

Experiments: Propositional Model Counting

- Comparison of BPNN with state of the art randomized hashing algorithms F2[1] and ApproxMC3[2]
- Left: BPNN provides median ***speedups of 250x and over 3,000x*** (with parallel GPU batching)
- Right: Estimation error on each benchmark. BPNN provides good estimates for its runtime.





1. Achlioptas, Dimitris, Zayd Hammoudeh, and Panos Theodoropoulos. "Fast and flexible probabilistic model counting." *International Conference on Theory and Applications of Satisfiability Testing*, 2018.
2. Soos, Mate, and Kuldeep S. Meel. "Bird: Engineering an efficient cnf-xor sat solver and its applications to approximate model counting." *AAAI*, 2019.