Spatial and competing dynamics in opinion dynamics modelling

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Risk Physics

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Introduction

Temporal evolution of the system is the usual approach to modelling opinion dynamics. This approach gives rise to a few innate problems – temporal empirical data is limited, e. g. presidential elections take place only each 4-5 years, while spatial distributions over districts is usually abundant. In addition, evolution by state change process does not account for spatial movement (commuting, migration) of agents present in real systems. Novel ways have been proposed to overcome such issues in [1, 2] by introducing spatial mobility into the dynamics.

This research studies approaches to account for competing dynamics in opinion modelling. It provides a methodology on how spatial heterogeneity can be measured by considering system scaling properties. Furthermore, following examples of competing dynamics in Ising model we introduce the competition of spatial and temporal dynamics to Compartmental voter model (CVM) [2] and study its properties. The original CVM is further studied through the lens of rank-dynamics [3] and approximation to Fokker-Planck equation.

Measuring spatial heterogeneity

An alternative to temporal dynamics of the system can be considered spatial dynamics. Spatial dynamics are determined by the spatial heterogeneity of the vote turnout (or i.e. census statistic). Such quantitative evaluation is attained via scaling procedure which is easily applicable to empirical data as it is available in different scales – streets, cities, regions, etc. This method is advantageous in cases when data is hierarchical (i.e. census data) and works without any knowledge of underlying geography.

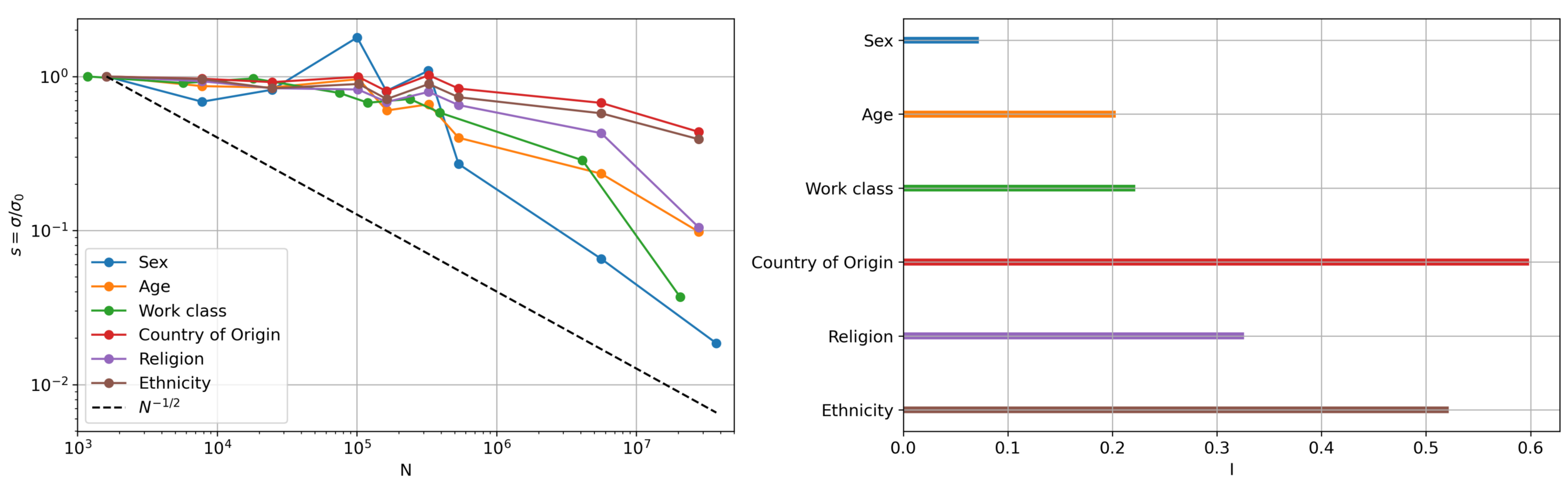

Figure 1. Spatial heterogeneity scores for UK 2011 census data. Left graph shows scaling curves for selected characteristics, y axis shows relative change in standard deviation, x axis displays population per compartment for selected scales. Dashed line shows $N^{-1/2}$ power law for randomly distributed values. Right graph shows scores of heterogeneity index I for said characteristics. Higher positive values signal bigger spatial autocorrelation.

For each spatial unit starting from the finest resolution one can calculate a desired statistic (i.e. standard deviation) and get its mean value. The procedure resembles renormalization group technique. Such mean value is available for each scale and thus a scaling curve is available as seen in Fig. 1 (left). It can be shown that **for independent and randomly distributed values this scaling curve would follow a power law of $N^{-1/2}$**. Similar scaling curves can be obtained for highly (un)correlated states like configuration snapshots of (anti)ferromagnetic states in Ising lattice. The deviation from power law to either of cases gives a quantitative result of spatial heterogeneity, evaluated as in Fig. 1 (right). A heterogeneity index *I* quantifies the deviation from $N^{-1/2}$ power law.

Competing temporal and spatial dynamics

A case of competing dynamics for spin models has been studied in the context of nonequilibrium Ising model. In competing dynamics Ising model we employ two processes acting with probabilities *p* and *1-p* interchangeably. Temporal dynamics are simulated with state change Glauber (Metropolis) dynamics and spatial dynamics correspond to Kawasaki dynamics. Results of spatial heterogeneity for lattice configurations are shown in Fig. 3. **The spatial heterogeneity index *I* score high in scale invariance temperature regions and can be applied to detect such changes.**

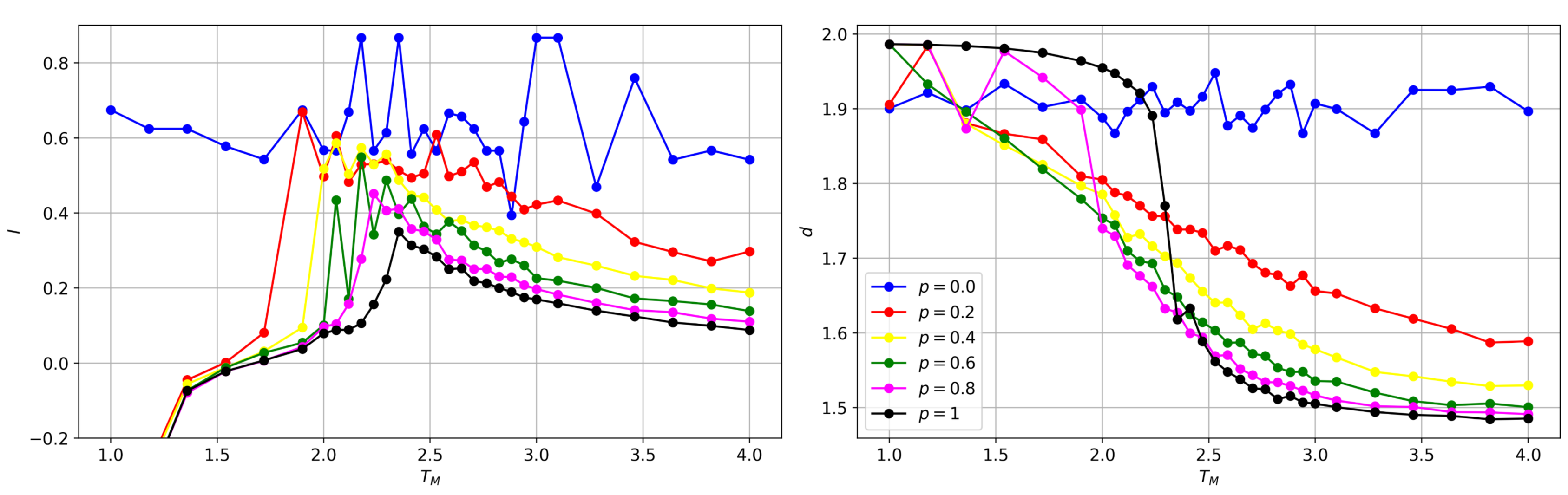

Figure 2. Heterogeneity index I and box-counting dimension *d* scores for competing Metropolis and Kawasaki dynamics in Ising model. T_M is the temperature of Metropolis dynamics, Kawasaki dynamics set to zero temperature, *p* is a probability to select Metropolis dynamics as next Monte Carlo step.

In another implementation of competing dynamics, we extend CVM dynamics by introducing state change process. The introduction of this term results in state migration rate *ν* and state change rate *λ* (*i, j* marks compartments and *k, m* marks agent types, total compartments *M* and types *T*, with population of *N* and comp. capacity *C*):

$$\nu_{i\to j}^{k\to m} = \begin{cases} X_i^k\left(\epsilon^{(m)} + X_j^{(m)}\right) & i \neq j, k = m, N_j < C \\ 0 & \text{other cases} \end{cases}$$

$$\lambda_{i\to j}^{k\to m} = \begin{cases} X_i^k\left(\mu^{(m)} + X_j^{(m)}\right) & i = j, k \neq m \\ 0 & \text{other cases} \end{cases}$$

While the original CVM belongs to the class of voter models, only specific cases of the extension to competing dynamics do. Its entry and exit rates take more complicated form that does not necessarily lead to agent of type k in compartment i population to be Beta distributed. Beta binomial distribution is possible when rates *ε* and *μ* are all equal. However **analyzing agent population X_i^k normalized to agent type (N^k) or total compartment (N_i) population reveals the effects of state change or spatial dynamics to the agent population distribution.**

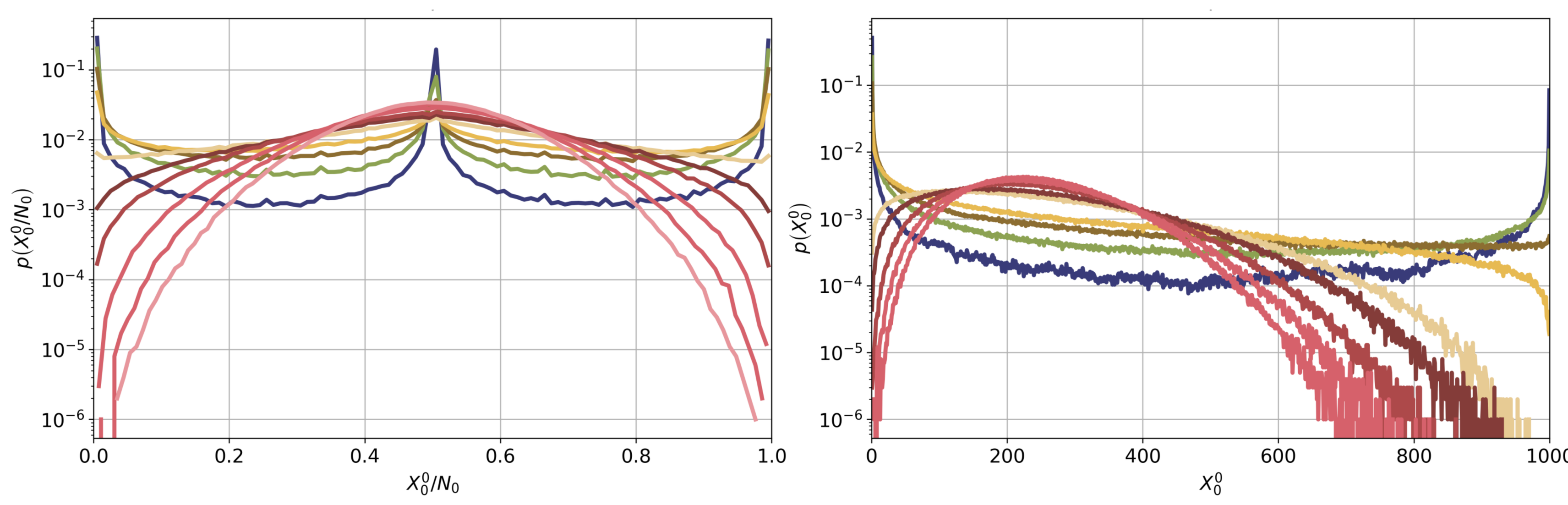

Figure 3. Probability distributions for state change and competing CVM. Left graph shows pdf of agent population in **state-change-only** dynamics. The x axis is normalized to total compartment population which display the effects of state change dynamics on population pdf. Right graph shows **competing dynamics** where *ε=μ* for each agent type and compartment. The distributions in this particular case still follow Beta binomial distribution as in original CVM.

Rank-dynamics and approximation to Fokker-Plank equation

Rank-size description is an alternative to cumulative distribution point of view for data which can be ordered, like territorial units. Recently a general framework has been proposed to model dynamics of ranking [3]. The study of ranking dynamics of CVM compartments reveals that dynamics of a single compartment are not sufficient to describe dynamics of system – **first mean passage times for rank change do not follow power-law of $t^{-3/2}$ thus making the model not a one-dimensional Markov process.**

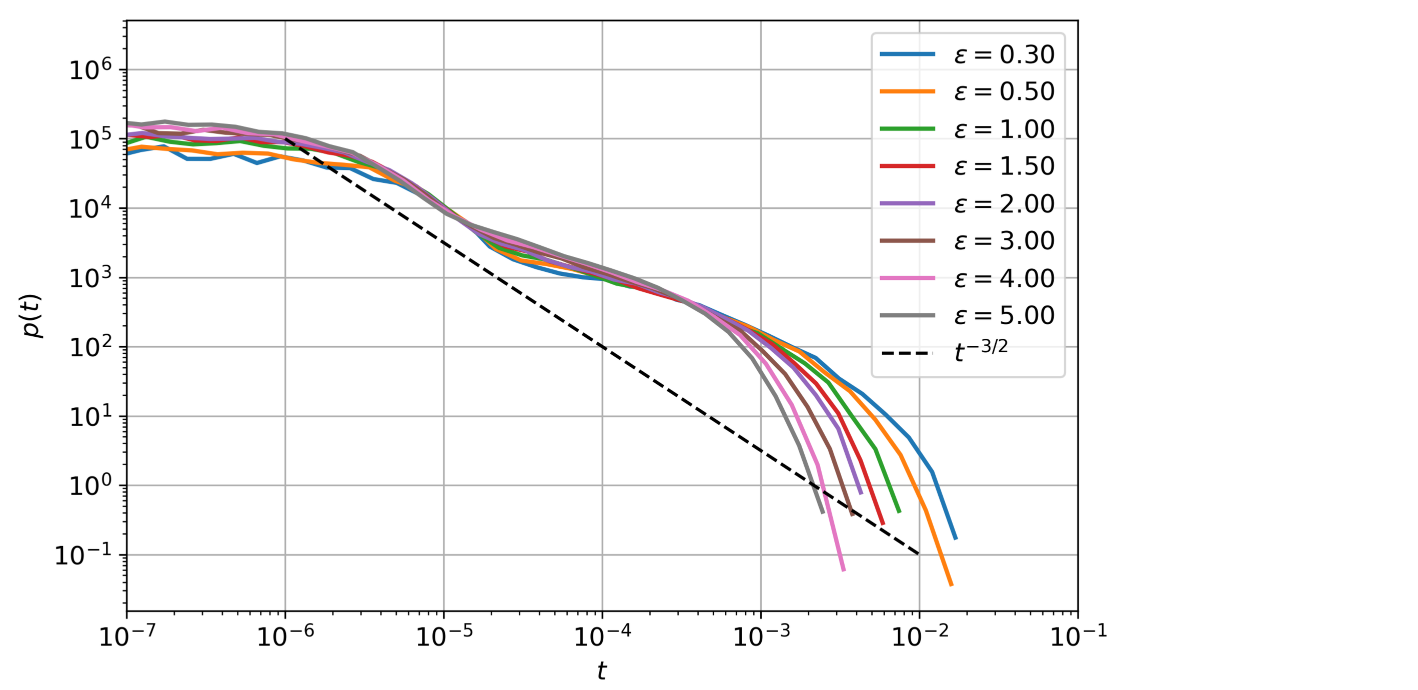

Figure 4. Pdf of first mean passage times (x axis) for ranks of compartments in original CVM. The passage times are for jumps from last rank (10) to first rank (1) in a *M*=10 compartment model with varying proportional migration rates *ε*. The dashed line represents a 1D Markov process case which is a result for 1D Brownian motion.

Further approximation of both rank and agent population dynamics are made by numerically calculating Kramers-Moyal coefficients. The coefficients are of the form:

$$D^{(n)}(X) = \frac{1}{n!\Delta t}\langle[x(t+\Delta t) - x(t)]^n\rangle|_{x(t)=X}$$

We find that these coefficients diminish linearly for first 10 powers indicating that the approximation to second-order is inaccurate.

Acknowledgements

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References

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