Competing dynamics in compartmental voter model

Justas Kvedaravicius[1], Aleksejus Kononovicius[2]

[1]Faculty of Physics, Vilnius University, Lithuania
[2]Institute of Theoretical Physics and Astronomy, Faculty of Physics, Vilnius University, Lithuania
justas.kvedaravicius@ff.stud.vu.lt

Modelling opinion dynamics requires making assumptions that would be simple enough mathematically yet sufficient to recreate real populations statistical characteristics. Coming up with new sorts of agent interactions and states proved fruitful but difficult to compare against empirical data [1]. A way to overcome this was proposed in [2] through the inclusion of agent migration in spatial domain. The idea was generalized in [3] by proposing a compartmental voter model (CVM) - its dynamics described by agent migration to and from compartments. In this research we investigate an extension of CVM by including state changes leading to competing dynamics. The generalization of dynamics resembles competition of Kawasaki and Glauber dynamics in Ising model.

The CVM proposes entry and exit transition rates which follow the form of noisy voter model. Thus indicating that the model belongs to the group of voter models and that number of k type agents in compartment i ($X_i^{(k)}$) are Beta distributed. The CVM entry and exit rates read:

$$^{+}\lambda_i^{(k)} = \left[N^{(k)} - X_i^{(k)}\right]\left(\varepsilon^{(k)} + X_i^{(k)}\right), \tag{1}$$

$$^{-}\lambda_i^{(k)} = X_i^{(k)}\left((M-1)\varepsilon^{(k)} + \left[N^{(k)} - X_i^{(k)}\right]\right), \tag{2}$$

here $^{\pm}\lambda_i^{(k)}$ are entry and exit rates for agents of type k to compartment i, M is number of compartments, $\varepsilon^{(k)}$ is migration rate for agents of type k, $N^{(k)}$ marks total number of agents of type k.

Competing dynamics are introduced by assuming additional state change rate of the same form as migration rate in CVM:

$$\eta_{ij}^{km} = \begin{cases} X_i^{(k)}\left(\mu^{(m)} + X_j^{(m)}\right) & i = j, k \neq m, \\ 0 & \text{other cases.} \end{cases} \tag{3}$$

Here $\mu^{(m)}$ marks transition rates to agent type m. Inclusion of these rates changes total entry/exit rates. We mark them ν and note that they define transitions to and from compartment as well as transition within the compartment via type change:

$$^{+}\nu_i^{(k)} = \left[N^{(k)} - X_i^{(k)}\right]\left(\varepsilon^{(m)} + X_i^{(k)}\right) + \left[N_i - X_i^{(k)}\right]\left(\mu^{(k)} + X_i^{(k)}\right), \tag{4}$$

$$^{-}\nu_i^{(k)} = X_i^{(k)}\left((M-1)\varepsilon^{(k)} + \left[N^{(k)} - X_i^{(k)}\right]\right) + X_i^{(k)}\left(\sum_{m\neq k}^{T}\mu^{(m)} + \left[N_i - X_i^{(k)}\right]\right). \tag{5}$$

Here T is a total number of agent types, $^{\pm}\nu_i^{(k)}$ marks entry and exit rates for agent type k in compartment i. The rates of competing dynamics take more complex form - they cannot be simplified to noisy voter model rates due to two different sorts of factors $N^{(k)} - X_i^{(k)}$ and $N_i - X_i^{(k)}$. $N^{(k)}$ corresponds to the total number of k type agents and N_i equals total number of any type agents in compartment i. The model can be viewed as two coupled voter models with varying total number of agents N.

In the research we further explore $X_i^{(k)}/N_i$ and $X_i^{(k)}/N^{(k)}$ distributions dependency on model parameters M, T, $\mu^{(k)}$, $\varepsilon^{(k)}$. Further, calculations of Kramers-Moyal coefficients allow to approximate the model to Fokker-Planck equation. A finite form of the equation can suggest coefficient values for a simplified form of competing CVM rates.

[1] A. Peralta, J. Kertész, G. Iñiguez, *Opinion dynamics in social networks: From models to data*. arXiv preprint arXiv:2201.01322 (2022).
[2] J. Fernandez-Gracia, K. Suchecki, J. J. Ramasco, M. Miguel, V. Eguiluz, *Is the voter model a model for voters?*, Physical Review Letters 112, 158701 (2014).
[3] A. Kononovicius, *Compartmental voter model*, Journal of Statistical Mechanics: Theory and Experiment 2019, 103402 (2019).