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Computer Science
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THE THEORY OF MATRICES

SECOND EDITION

WITH APPLICATIONS

Peter Lancaster and Miron Tismenetsky

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