MA677 HW2

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1. "The shortest distance between two points is a taxi."

```
ey1 <- (3*3/42+8*8/42+8*8/42+7*7/42+3*3/42+6*6/42+2*2/42+1*1/42+4*4/42)
print(ey1)
```

```
## [1] 6
```

2. $$f(x,y) = 12y^2$$

   for 0 <= y <= x <= 1

$$g(x,y) = xy$$

$$g(x,y)f(x,y) = xy * 12y^2 = 12xy^3$$

```
funxy2 <- function(x,y) 12*x*y^3
exy2 <- integral2(funxy2, xmin = 0, xmax = 1, ymin = 0, ymax = 1)
print(exy2)
```

```
## $Q
## [1] 1.5
##
## $error
## [1] 3.191891e-16
```

3. $$E[(X_1 - 2X_2 + X_3)^2] = E[(X_1 - 2X_2 + X_3)]^2$$
$$= [E(X_1) - 2E(X_2) + E(X_3)]^2$$

```
ex4 <- (.5 - 2*.5 + .5)^2
print(ex4)
```

```
## [1] 0
```

4. $$f(x) = e^{-x}$$

$$Y = e^{.75X} = e^{.75e^{-x}}$$

$$g(x)f(x) = e^x * e^{.75e^{-x}} = e^{.75e^{-x}+x}$$

```
funy4 <- function(x) exp(.75*exp(-x) + x)
ey4 <- integrate(funy4, upper = 1, lower = 0)
print(ey4)
```

```
## 2.681179 with absolute error < 3e-14
```

5. $$Y = g(X) = 2X^2 + 1$$

```
## Possible values of X: 1,2,3,4,5,6
ey5 <- ((2*1+1) + (2*4+1) + (2*9+1) + (2*16+1) + (2*25+1) + (2*36+1))/6
print(ey5)
```

```
## [1] 31.33333
```

6.

$$X = 2(1 - X)$$

$$Y = 2X + 1 = 2 * 2(1 - X) + 1 = 4 - 4X + 1 = 5 - 4X$$

$$g(x)f(x) = (5 - 4x)(2x + 1) = 10x + 5 - 8x^2 - 4x = -8x^2 + 6x + 5$$

```
funy6 <- function(x) -8*x*x + 6*x + 5
ey6 <- integrate(funy6, lower = 0, upper = 1)
print(ey6) ##E(Y)
```

```
## 5.333333 with absolute error < 5.9e-14
```

```
ey6_2 <- 5.333333^2 ##E(Y^2) = (E(Y))^2
print(ey6_2)
```

```
## [1] 28.44444
```

7. Prove:

$$E[(ax + b)^n] = sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i E(X^{n-i})$$

Proof:

$$E[(ax + b)^n] = sum_{i=0}^{n} (ax + b)^i$$

##Expected value of an arbritary equation

$$= sum_{i=0}^{n} \binom{n}{i} (ax)^{n-i} b^i$$

##Binomial theorem

$$= sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i x^{n-i}$$

$$= sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i E[X^{n-i}]$$

8.

$$E(X) = np$$

$$E(Y) = n(1 - p)$$

$$E(X - Y) = E(X) - E(Y)$$

```
ex8 <- 20*.05
ey8 <- 20*.95
print(ex8 - ey8)
```

```
## [1] -18
```

```
cat("In a sample of n parts, you could expect to find 90% fewer bad parts than good parts.")
```

```
## In a sample of n parts, you could expect to find 90% fewer bad parts than good parts.
```