Water in Agriculture

A Non-Linear Regression Analysis

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March 1, 2018

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Introduction

Background and Motivation

- The discovery and implementation of irrigation for agriculture in the United States has massively accelerated the abilities of farmers to expand and increase their yields
- This has also successfully prevented natural events, like droughts, from interrupting or decreasing the yields.
- Because of the growing populations and higher demands placed on agricultural yields, many of the water sources for irrigation have been exhausted, given reduced ground water, more severe droughts, and competition from industrial sources.

Question of Interest

Question: How do you produce more food using less water?

- The crop water stress index (CWSI) monitors the water needs of crops - values closer to 1 indicate that the crop needs more water, values close to 0 indicate that the crop is well hydrated.
- The soil water content (SWC) informs farmers how much water is needed.
- However, the SWC measure is much more expensive to obtain than the CWSI measure

Goal of Analysis

Goal: Evaluate the relationship between the CWSI and the SWC in order to predict how much water a plant needs (SWC) by only measuring the CWSI.

Exploring the Data

The Data

This dataset has 78 observations of the simultaneous crop water stress index (CWSI) and the soil water constant (SWC) scores of plants.

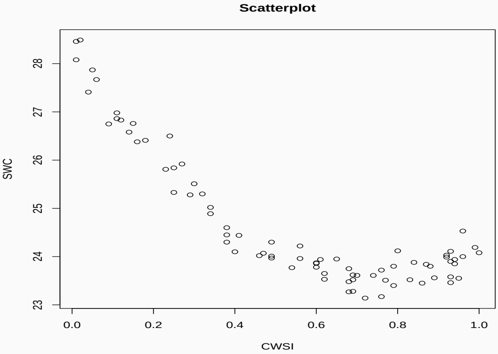
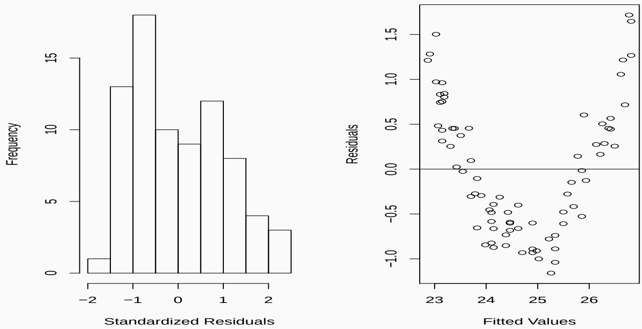


Figure 1: Relationship between CWSI and SWC

Other Linear Model Assumptions

Figure 2: Relationship between CWSI and SWC



The data does not meet the assumptions of a linear model.

Methods

Nonlinear Regression

- We perform nonlinear regression analysis to analyze the data
- Specifically, we use local regression
    - Local regression has good uncertainty quantification properties

Intro to Local Regression

- Local regression fits a weighted least squares model at every point along your x-axis.
- The weights are determined by the distance between the data and the point of interest.
- With weighted least squares:

$$\hat{\beta}_{MLE} = (X'WX)^{-1}(X'Wy)$$

  Where W is a diagonal matrix with how much each point should be weighted.

Model Assumptions

- We are making the assumption that our observed data is independent

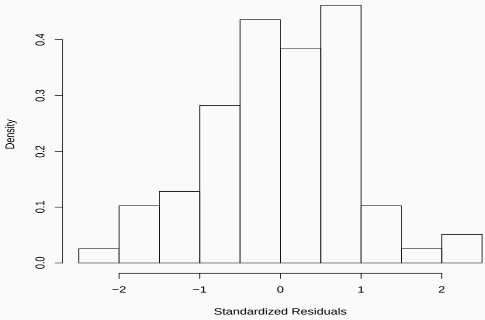


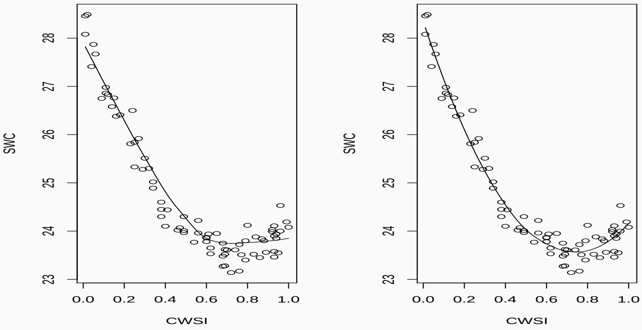
Figure 3: Histogram of Standardized Residuals

Benefits of Local Regression

- Because the regression is re-estimated at each point, it can model non-linear functions.
- The weighting preserves many of the uncertainty properties of linear regression.
- The standard errors aren't fixed, so our confidence bands will be more narrow where we have more data.

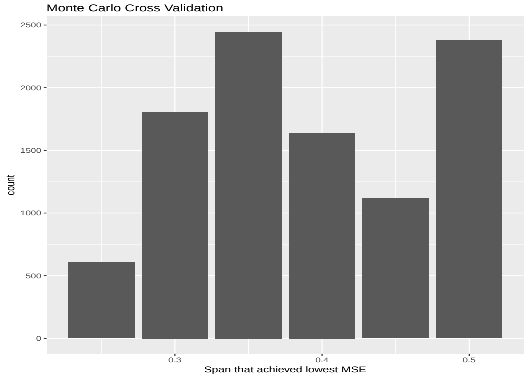
Determining the Degree

Figure 4: Degree of Local Regression: 1 (Linear) vs. 2 (Loess Smoothed)



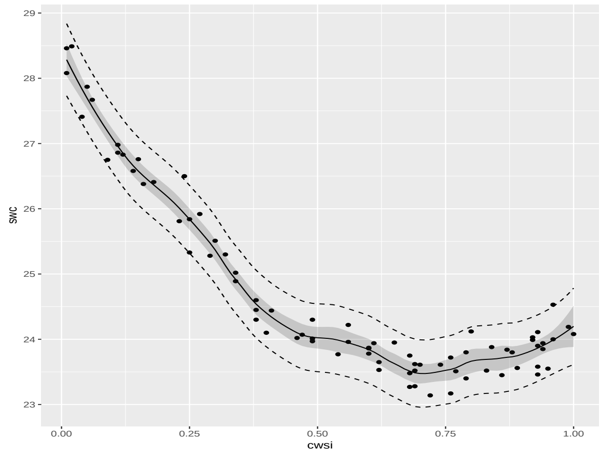
Cross Validation to find Span

Figure 5: Finding the Span through Monte Carlo cross validation



Results

Figure 6: Local Regression Model with Uncertainty Estimates



Prediction Power

- RMSE (using 10-fold cross-validation) = 0.2465
- Psuedo-R^2 = 0.9749

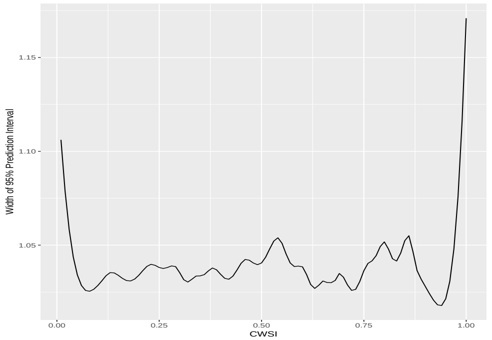


Figure 7: Prediction interval width across domain of CWSI

Application of Model

Table 1: Predictions and 95% Prediction Intervals

CWSI	Lower Bound	SWC Prediction	Upper Bound	Add Water
0.0	27.89045	28.44363	28.99680	0
0.1	26.57014	27.06871	27.56728	0.43129
0.2	25.70501	26.21281	25.70501	1.28719
0.3	24.87299	25.37673	25.88048	2.12327
0.4	23.88196	24.38501	24.88806	3.11499
0.5	23.52527	24.03516	23.52527	3.46484
0.6	23.33893	23.84217	24.34540	3.65783
0.7	22.97121	23.47155	23.97190	4.02845
0.8	23.15453	23.66423	23.15453	3.83577
0.9	23.27358	23.77174	24.26990	3.72826
1.0	23.63005	24.19750	24.76496	3.30250

Significance of Findings and Conclusion

- Farmers can use CWSI scores to predict SWC scores, and be accurate within one unit of soil water content
- **Shortcoming:** Because our model does not provide coefficients, we cannot provide a predictive formula

Future Work

- Explore generalizability of conclusions
    - Different soil types
    - Different plants
    - Different climates