standard Proof $\sqrt{2}$

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The definition of $\sqrt{2}$ is that

$$\sqrt{2}^2 = 2.$$

If $\sqrt{2}$ was rational there must be a fraction $\frac{p}{q}$ where p and q are positive Integers and they have no equal divider, so that

$$\left(\frac{p}{q}\right)^2 = 2.$$

so

$$\frac{p^2}{q^2} = 2$$

Wen we multiply by q^2 we get

$$p^2 = 2q^2$$

So p^2 is an even number and thus p is also even.
let a be $\frac{p}{2}$ so

$$4a^2 = 2q^2$$

and

$$2a^2 = q^2$$

So q^2 is an even number and thus q is also even.
What we see here is, that both p and q are even, but our assumption is, that they have no equal divider. Therefore our assumpton is wrong and

$$\sqrt{2} \notin \mathbb{Q}$$

Q.E.D