

Proof $10=9.99999\dots$

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1 Introduction

In the following Proof I will show, that

$$9.\bar{9} = 10.$$

As we know from the geometric series: for $|x| < 1$

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$$

We will need this later.

2 Proof

let $M = 9.\bar{9}$

$$M = 9 + \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} \dots$$

so when we divide by 9

$$\frac{1}{9}M = 1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} \dots$$

and now we can write the infinite addition as a geometric series

$$\frac{1}{9}M = \sum_{i=0}^{\infty} \left(\frac{1}{10}\right)^i$$

so as we can see from the introduction

$$\frac{1}{9}M = \frac{1}{1 - \frac{1}{10}} = \frac{10}{9}$$

so when we multiply by 9

$$M = 10 = 9.\bar{9}$$

Q.E.D.