Performance Evaluation, Homework 2

Joël M. Fonseca 227334, Francis Damachi 217575

March 18, 2019

1 Statistics Warmup

1.1 Problem 1

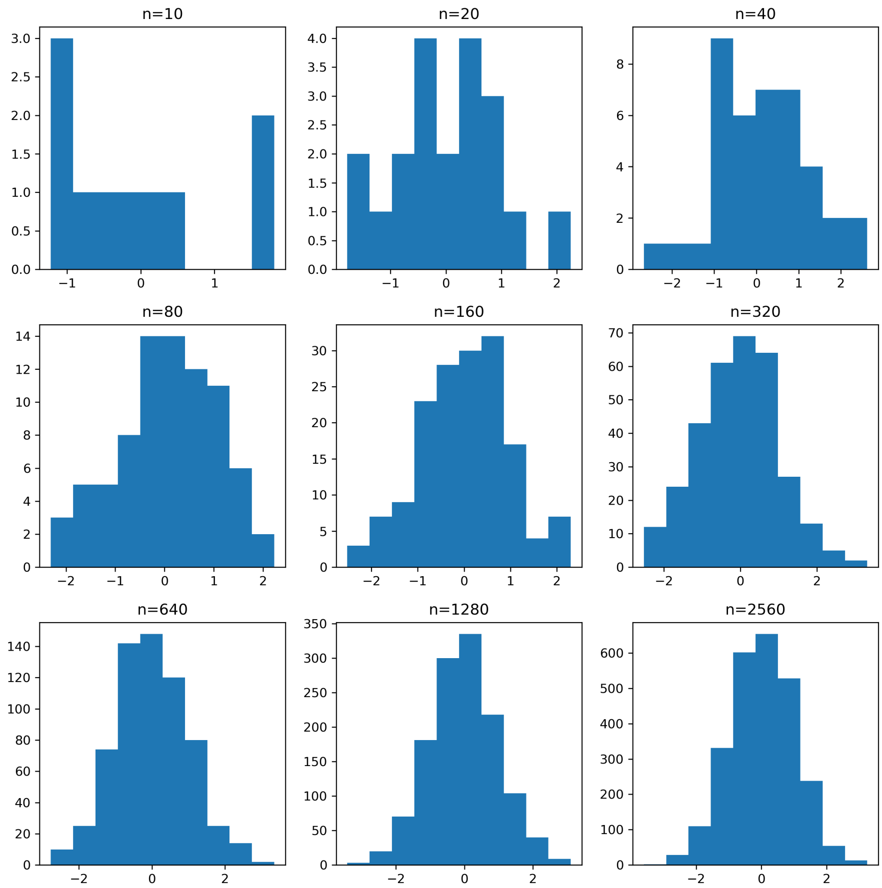

Figure 1: Histograms for a sample of different n iid standard normal variables.

We can see from Figure 1 that as n gets bigger, the shape of the empirical distribution converges towards the true (theoretical) distribution of the standard normal distribution.

1.2 Problem 2

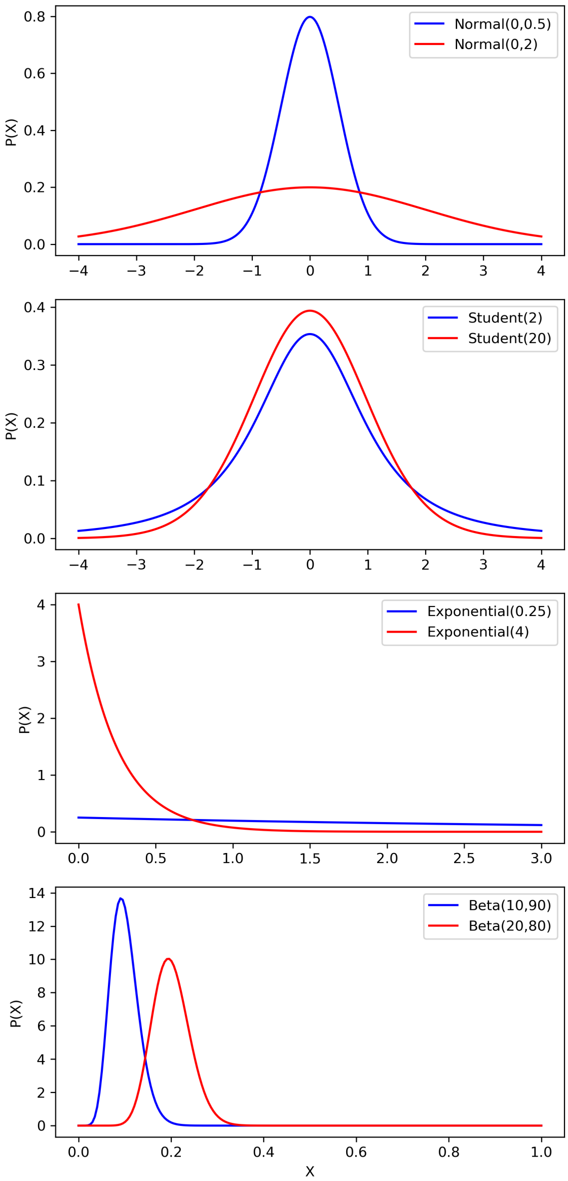

Figure 2: Densities of different distributions.

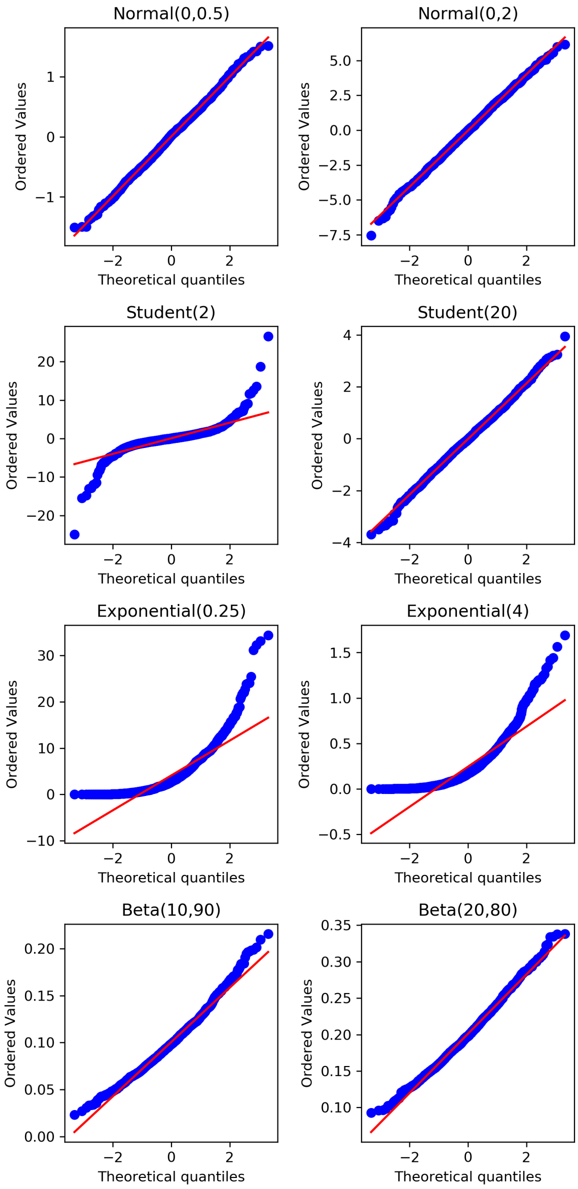

Figure 3: Corresponding standard normal QQ-plots for densities of different distributions.

1.2.1 S-shape

An *S-shape* in a standard normal QQ-plot, like for the Student(2) distribution from Figure 3, is interpreted like a Bell curve where both tails are fatter with smaller concentration of points around the origin. If we look at a very small theoretical quantile, then, as the left tail is more heavy, the corresponding Student(2) data quantile will be smaller as more data occurs until this theoretical quantile. A similar reasoning can be established for a large theoretical quantile: as the right tail is also heavier for the Student(2) distribution, then the corresponding Student(2) data quantile will be larger compared to the theoretical quantile as there is more data after it.
The opposite *S-shape* (like for the Normal(0,0.5) distribution - very subtle) means that both tails are thinner compared to a standard normal distribution.

1.2.2 U-shape

A *U-shape* in standard normal QQ-plot, like for the Exponential(0.25) distribution from Figure 3, is interpreted like a distribution that is right-skewed or that has positive skewness. First, if we look at the distribution of the Exponential(0.25) distribution from Figure 2, we observe that the distribution has positive support only. We can also see this fact from Figure 3 as we have no quantile

in the negative support of the Exponential(0.25) data quantile. Hence, the Exponential(0.25) data quantiles ares greater for some negative theoretical quantiles. Secondly, from Figure 2 we see that the Exponential(0.25) distribution has an heavier tail extending out to the right compared to a Gaussian distribution. This explains why the last Exponential(0.25) data quantiles from Figure 3 are greater than 20 while the theoretical quantiles are around 3.
Similarly, the opposite *U-Shape* (upside down) means that the distribution is left-skewed or that it has negative skewness.

2 Simulate Random Waypoint

2.1 Measurements/Plots

We closely follow the example 6.5 from the book. We simply simulate the random waypoint using the same stochastic recurrence. The various results can be shown in Table 1 and Figure 4. The course and the waypoints of one 1 user (resp. 8 users) are shown in Figure 5 (resp. Figure 6).

Metric	*Value*
Mean	341.64
Minimum	304
Maximum	386

Table 1: Mean, minimum and maximum number of waypoints reached by the different mobiles for one simulation run (86400s).

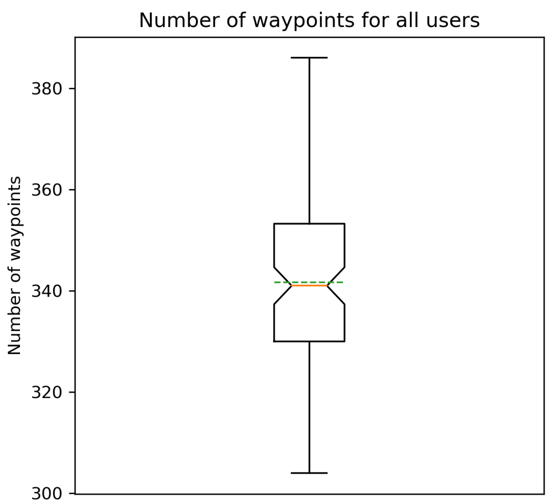

Figure 4: Box plot for the number of waypoints for 100 users. The mean is in green while the median in yellow.

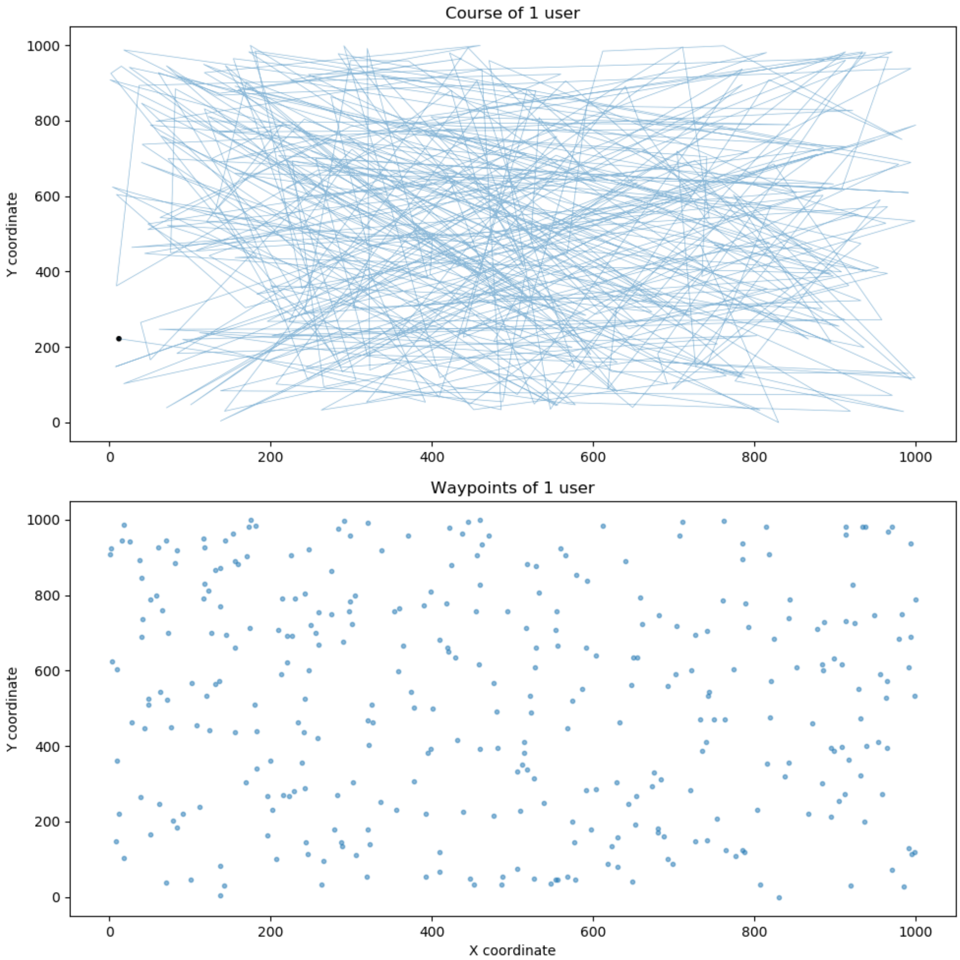

Figure 5: Course and waypoints of 1 user with the black dot as the starting waypoint.

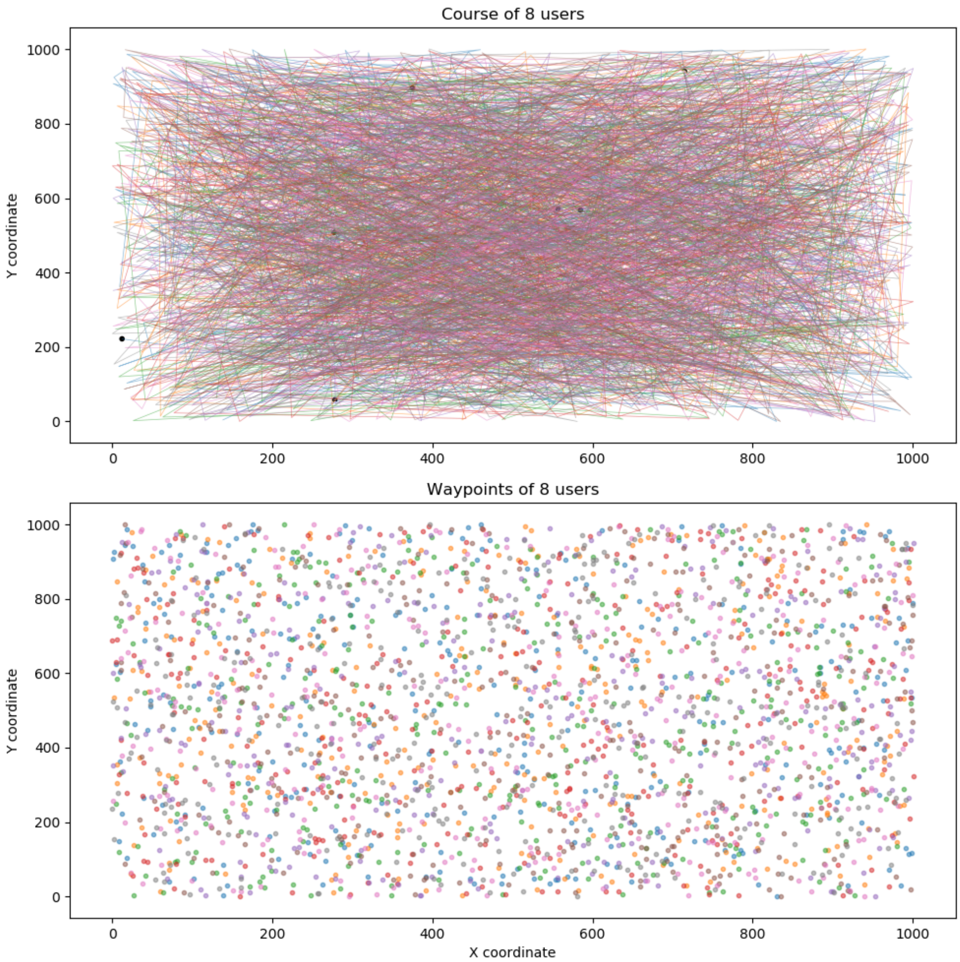

Figure 6: Course and waypoints of 8 users. This time we cannot see the black dots correctly because of the large number of points.

The real time our program takes to generate 1 day of simulated time highly depends on the configuration of the computer that is used. We managed to run it in 0.43 seconds on a i7 4th generation with 32Gb of RAM.

3 Different Viewpoints

3.1 Event Average Viewpoint

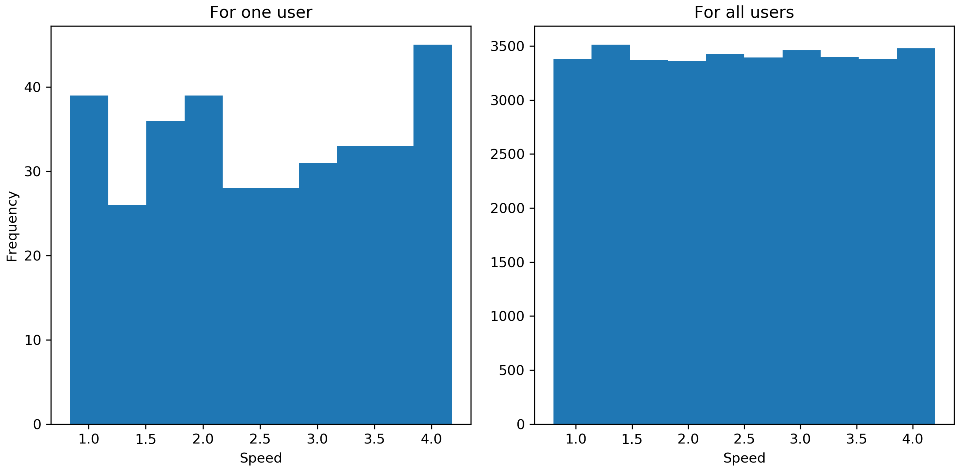

Figure 7: Histogram of speeds sampled at transition epochs T_n.

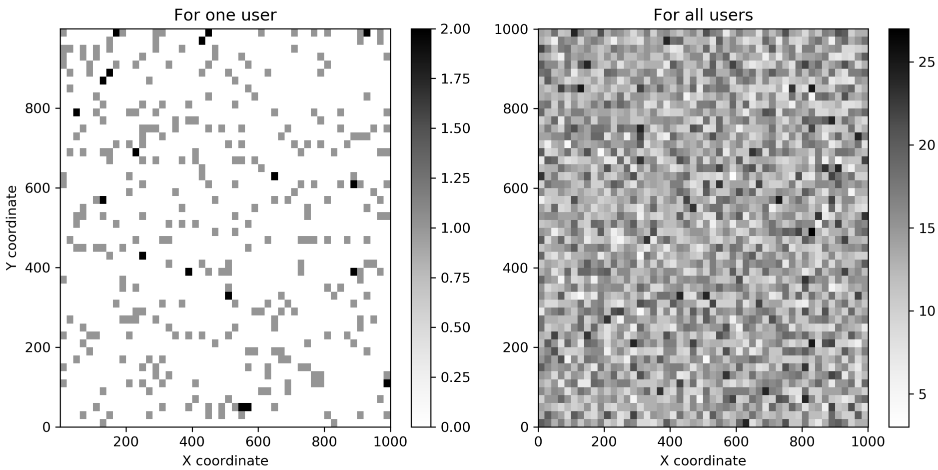

Figure 8: Histogram of positions sampled at transition epochs T_n.

3.2 Time Average Viewpoint

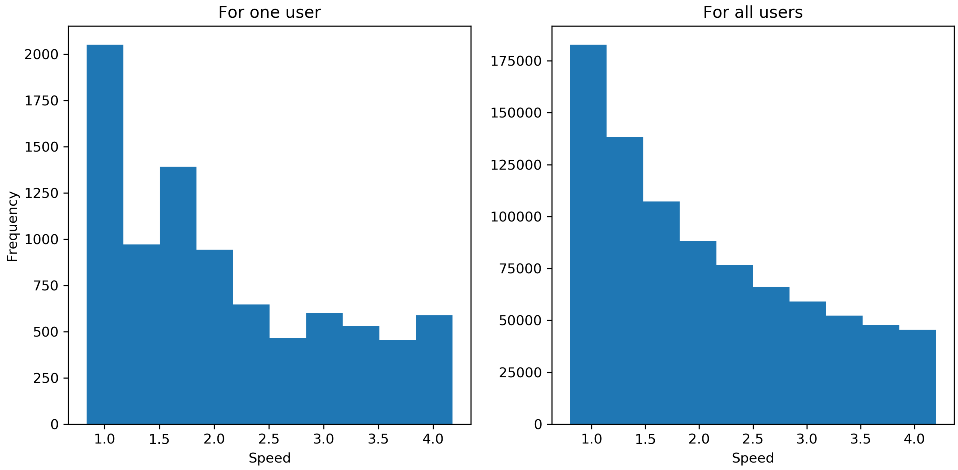

Figure 9: Histogram of speeds sampled every 10 seconds.

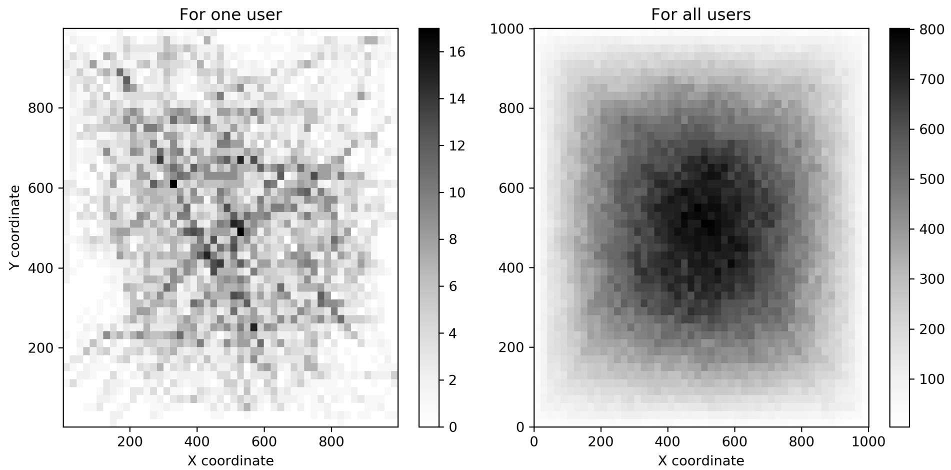

Figure 10: Histogram of positions sampled every 10 seconds.

We see that with the time viewpoint, the speeds and positions are no longer uniformly distributed. These results can be formally proved by the Palm's inversion formula, but we will rather go through an intuitive justification, as we have not yet discussed this subject in the lectures.
Figure 7 shows that the distribution of speed is uniformly distributed between v_{min} and v_{max} when sampled every transition epochs T_n (by construction). The shape of Figure 9 can be explained by the fact that we spend more time at low speeds compared to higher speeds.
Similarly, Figure 8 shows that the distribution of positions is uniformly distributed between the square with $l = 1000$ and $L = 1000$ when sampled every transition epochs T_n (by construction). The shape of Figure 10 can be explained by the fact that we are more likely to pass by the center of the square between two waypoints.

4 Confidence Intervals

4.1 Confidence intervals for the medians and means

4.1.1 Assumptions

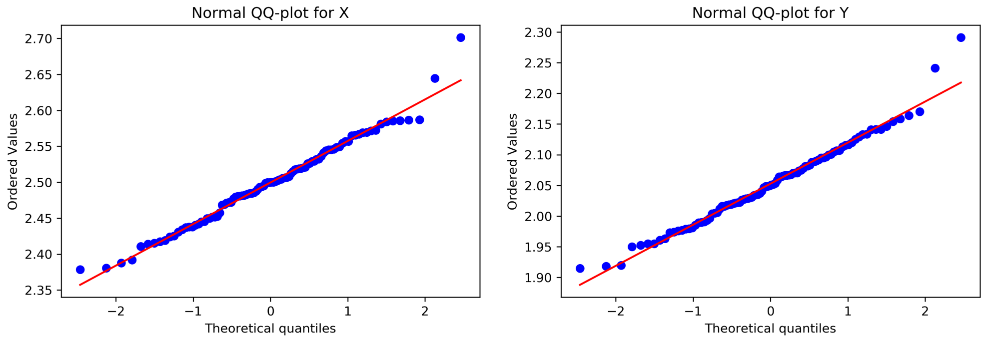

Figure 11: QQ-plots for X and Y.

In order to apply theorem 2.1 of the book to calculate the confidence intervals for metrics of X and Y, key asssumptions have to be made. First of all, the random variables $X_1, ..., X_n$ (resp. $Y_1, ..., Y_n$) must be independent and identically distributed. They are independent because the values of X_i (resp. Y_i) does not influence the value of X_{i+1} (resp. Y_{i+1}). This is due to the fact that each i corresponds to a different user. They are identically distributed because for each user a new simulation is being run with the same configuration. We can conclude that this assumption is verified.
Now that we have stated and verified the iid assumption, we can use theorem 2.3. We assume that the data comes from a normal distribution. A common way to verify this assumption is to make a QQ-plot of both X and Y (Figure 11). As one can see, since most of the points of both plots lie on the red line, we are safe to say that both X and Y have comparable quantiles compared to the normal distribution. Hence, our assumptions are verifed.

4.1.2 Results

The exact values of the different metrics for the confidence intervals can be found in Table 2 and 3. We also plot some metrics in Figure 12 for a better visualization and to make the comparison easier between the different configurations.

Random variable	*N*	*Mean*	*Lower bound*	*Upper bound*	*Interval width*
X	100	2.49954	2.48999	2.50909	0.01910
	30	2.50047	2.48001	2.52094	0.04093
Y	100	2.05271	2.04163	2.06379	0.02215
	30	2.04854	2.02523	2.07184	0.04661

Table 2: Confidence interval metrics of the mean for X and Y for N={100,30}.

Random variable	*N*	*Median*	*Lower bound*	*Upper bound*	*Interval width*
X	100	2.49986	2.48462	2.51223	0.02761
	30	2.49053	2.48134	2.54411	0.06277
Y	100	2.05003	2.02989	2.06759	0.03770
	30	2.03361	2.01740	2.07017	0.05277

Table 3: Confidence interval metrics of the median for X and Y for N={100,30}.

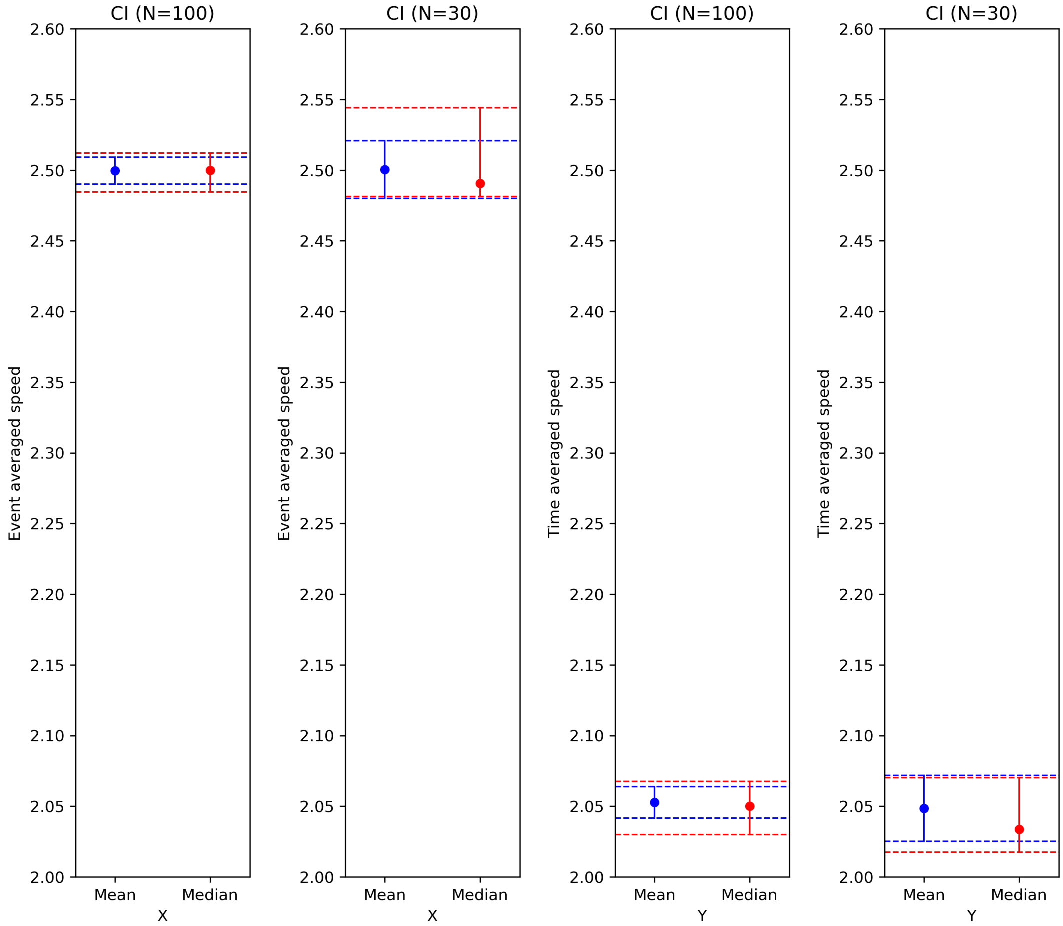

Figure 12: Confidence intervals of the mean and median of X and Y for N=$\{100,30\}$.

4.1.3 Discussion

We can see a significant difference on the confidence intervals between the event averaged speed and the time averaged speed for both values of N. This is what we can expect and follows from the analysis already done in the previous section: in the time average viewpoint, we spend more time in low speeds, consequently the mean will be lower compared to the event average viewpoint.

We can see that with a smaller sample size, both the mean and the median confidence intervals are wider. Hence, a larger sample size reduces the variability of the metrics.

4.2 Prediction intervals for samples

4.2.1 Assumptions

In order to use theorem 2.6 which is about prediction interval, we must make sure that the sequence $Y_1, ..., Y_n$ is an iid sequence. Moreover, it must follow a normal distribution. These assumptions have already been verified in the previous subsection. Additionally for the prediction interval with the order statistic, there are no assumptions one must make other than the sequences must be iid.

4.2.2 Results

The exact values of the prediction intervals can be found in Table 4 and 5 for respectively the normal assumption and the order statistic technique. We also plot these intervals in Figure 12 for a better visualization and to make the comparison easier between the different configurations.

N	*Lower bound*	*Upper bound*	*Interval width*
100	1.92196	2.18346	0.26150
60	1.91535	2.18399	0.26864
30	1.90130	2.19577	0.29447

Table 4: Prediction intervals assuming normal distribution for Y for N={100,60,30}.

N	*Lower bound*	*Upper bound*	*Interval width*
100	1.91808	2.24126	0.32318
60	1.91466	2.29093	0.37627
30	-	-	-

Table 5: Prediction intervals using order statistic for Y for N={100,60,30}.

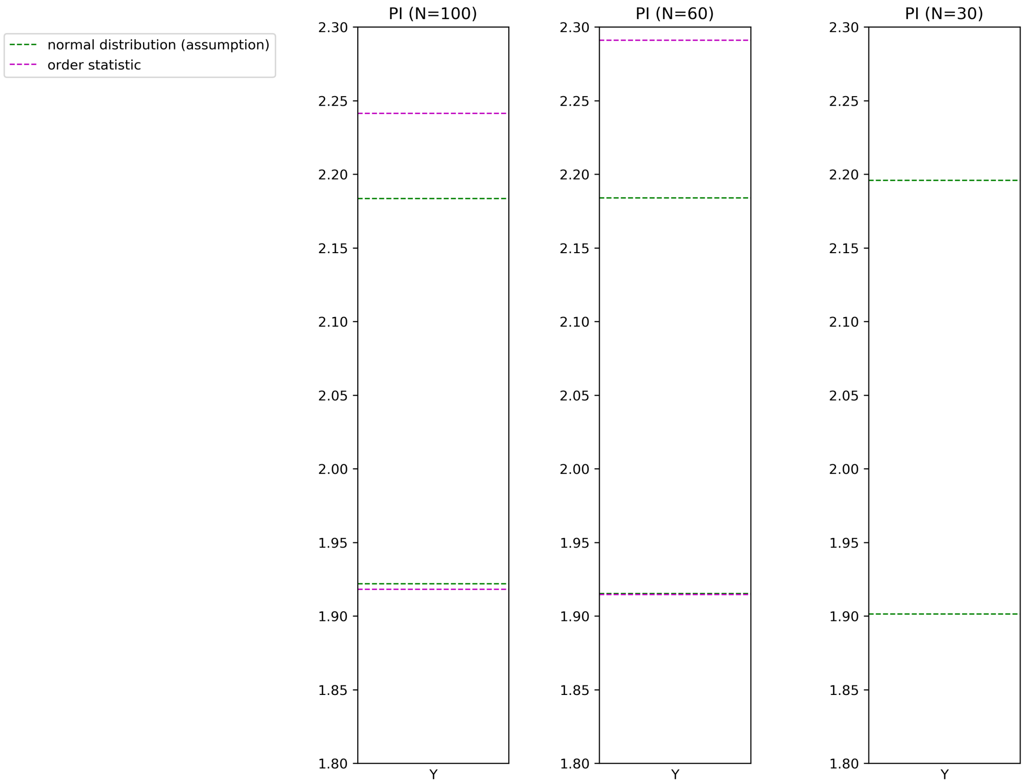

Figure 13: Prediction intervals of Y for N={100, 60, 30}.

4.2.3 Discussion

We can see that there is no significant difference between the two techniques that are used. Recall that the first technique assumes that the samples are normally distributed. The order statistic technique makes no particular assumption about the distribution of the samples. For this reason, if the samples are not normally distributed, then we should notice a significant difference. But because this is not the case, we can say that the samples are indeed normally distributed (which was verified in our assumption). Note that for N=30 we cannot use the order statistic method. Indeed, this technique is not valid for $N < 39$ at level 0.95.

Finally, we see that changing the value of N does not change significantly the prediction interval if we use the technique that assumes normal distribution. However, we need a minimal number of samples in order to use correctly the order statistic technique for a prediction interval at level 0.95.