Performance Evaluation, Homework 3

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1 Simulate

From Figure 1 we see that the requests are served at a similar rate as their arrival. If we zoom in, we can observe a small delay, but it is not large enough to generate a significant difference.

From Figure 2 we can see how both types of jobs coincide. This is due to the fact that each type 1 job is followed by a type 2 job.

From Table 1 we see that $\overline{R}$ is different for type 1 and type 2 jobs. This is due to each job type following a different distribution for their service time. Indeed, the type 1 job distribution has a mean greater than the type 2 job service time distribution. This is why $\overline{R}$ is bigger for the type 1 job. However, the average number of jobs served per second is the same for both types of job due to a type 2 job needing to wait for its type 1 job to be served before being able to be served itself.

Type job	$\overline{R}$ **[ms]**	**Average number of jobs served [s]**
1	8.686	73.616
2	4.898	73.616

Table 1: Average response time $\overline{R}$ (in milliseconds) and average number of jobs served (in seconds) for both type 1 and type 2 jobs.

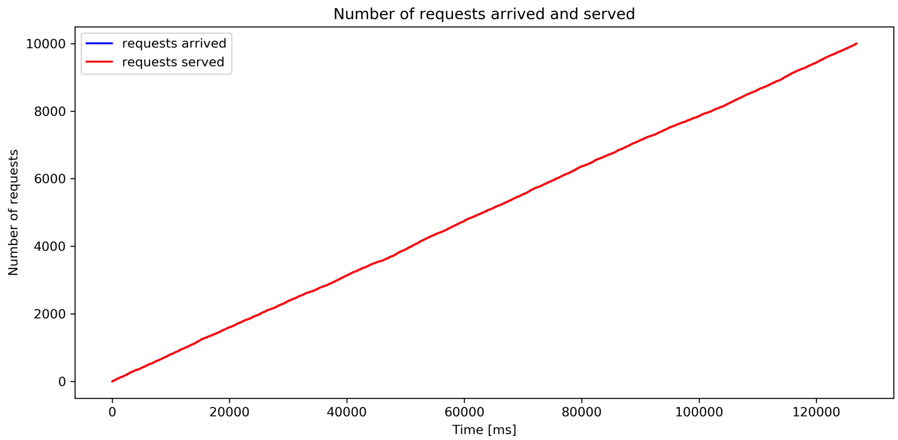

Figure 1: Number of requests arrived and served.

Figure 2: Number of type 1 and type 2 jobs in the processor queue.

2 Stationarity

For this question, we plot the total number of jobs for various values of $\lambda \in [25, 250]$. Results are shown on Figure 4. We can see that for values of $\lambda \in [30, 150]$, the system still has a stationary regime. However, for values of $\lambda \geq 180$, the total number of jobs seem to increase as time goes on. In other words, the system doesn't seem to achieve a stationary regime. In the following subsection, we try to define the exact boundary of λ beyond which the system is no longer stationary.

2.1 Analytic result

Recall that in order to have a stationary system, the following condition should be satisfied

$$\rho = \lambda \bar{S} < 1, \tag{1}$$

where λ is the arrival rate, and $\bar{S}$ is the average service time. In our case, we can define λ as

$$\lambda = \lambda_1 + \lambda_2, \tag{2}$$

and

$$\bar{S} = \frac{\bar{S}_1 + \bar{S}_2}{2}, \tag{3}$$

as in our system we are dealing with both type 1 and type 2 jobs. From Question 1, we know that both types of jobs have the same rate for the average number of jobs served per second. Hence, we can define λ_2 as

$$\lambda_2 = \frac{1}{\bar{S}_1 + \bar{S}_2}, \tag{4}$$

which transforms Equation 2 into

$$\lambda = \lambda_1 + \lambda_2 = \lambda_1 + \frac{1}{\bar{S}_1 + \bar{S}_2}. \tag{5}$$

Equation 3 can easily be solved as we know the distribution of the service time for each type of job. The type 1 job follows a Log-Normal distribution with parameters $\mu = 1.5$ and $\sigma = 0.6$, therefore

$$\bar{S}_1 = \exp\left(1.5 + \frac{0.6^2}{1}\right) = 5.3656. \tag{6}$$

Similarly, the type 2 job follows a Uniform distribution with parameters $a = 0.6$ and $b = 1$, hence

$$\bar{S}_2 = \frac{1 - 0.6}{2} = 0.8. \tag{7}$$

Finally, rewriting Equation 1 with the above results gives

$$\begin{aligned}
&\lambda \bar{S} < 1 \\
&\iff (\lambda_1 + \lambda_2)\bar{S} < 1 \\
&\iff (\lambda_1 + \frac{1}{\bar{S}_1 + \bar{S}_2}) < \frac{1}{\bar{S}} \\
&\iff \lambda_1 < \frac{1}{\bar{S}_1 + \bar{S}_2} \\
&\iff \lambda_1 < \frac{1}{5.3656 + 0.8} = 0.16219.
\end{aligned} \tag{8}$$

Hence, for $\lambda < 162.19$ request/s the system will be stationary. Figure 3 supports our result.

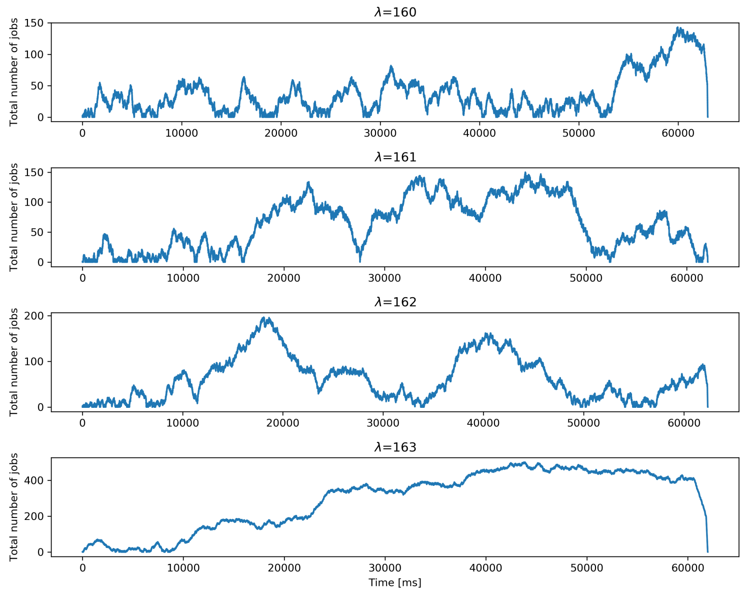

Figure 3: Verifying with values around the boundary supports our result.

3 Remove Transients

As explained in the book, in order to calculate the convergence of the stationary regime, we have to retrieve the value of the second eigenvalue modulus the transition matrix of the markov chain in our simulation. However, in practice a method that works would be simply by visual inspection of the produced graph. Meaning that we wish to omit the points in graph where a trend begins to occur. So we choose the limit value λ for which we still achieve a stationary regime, however an initial transient period appears to start occurring. This limit value λ is simply the one we had calculated in the previous question. When we run the simulation with $\lambda = 162$, the output of the graph changes quite drastically from one simulation to the other. For that reason we simulate 10 runs with $\lambda = 161$ and try to visually detect the time we are still at a transient state.

From Figure 4 we see that the simulations are very different from one to the other and that it's difficult to infer the transient period from these simulations. To be on the safe side, we get rid of the first quarter of data and assume the remaining data to be on the stationary regime. Also this makes sure that we still have enough data points.

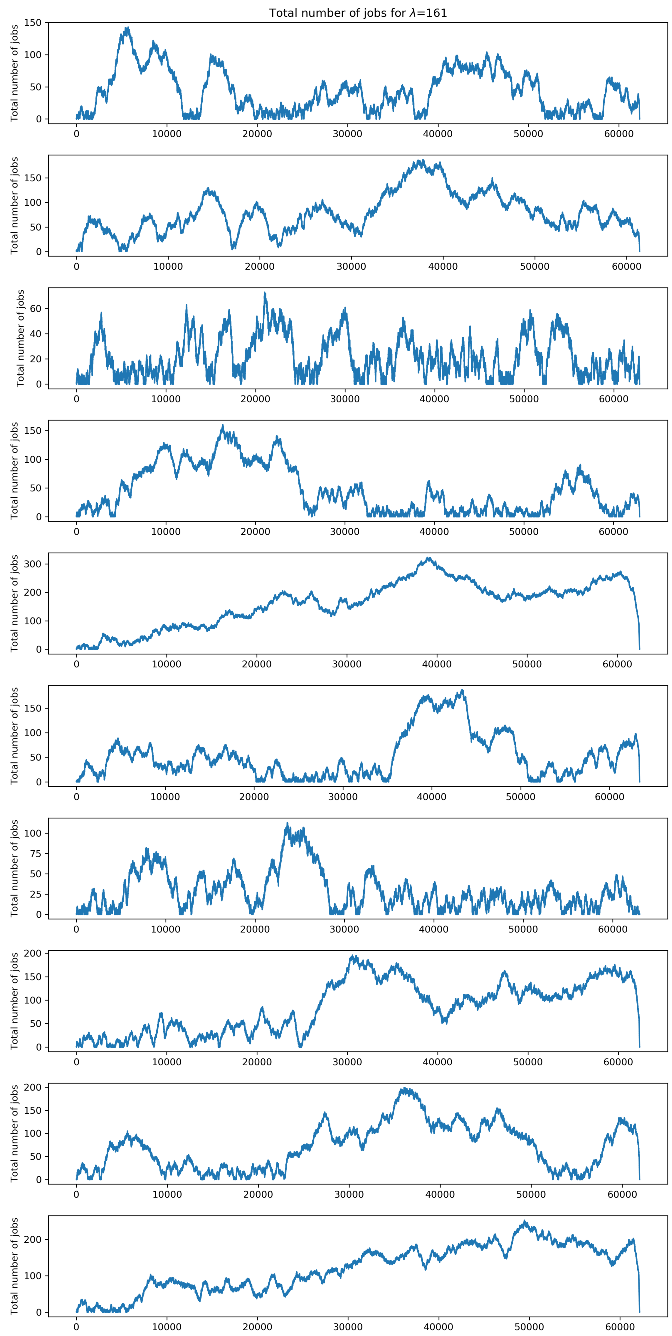

Figure 4: 10 simulations for λ=161.

Figures 5 and 6 shows the confidence interval at level $\gamma = 0.95$ for the mean and the median of the type 1 and type 2 jobs for $N = 30$ simulations for $\lambda = 60$ and $\lambda = 160$ respectively, with and without the transient period.

To compute the confidence intervals we use theorem 2.1 and 2.2 from the exam booklet for the median and the mean respectively. Both assume iid random variables which is the case as we ran $N = 30$ independent simulations with the same configuration.

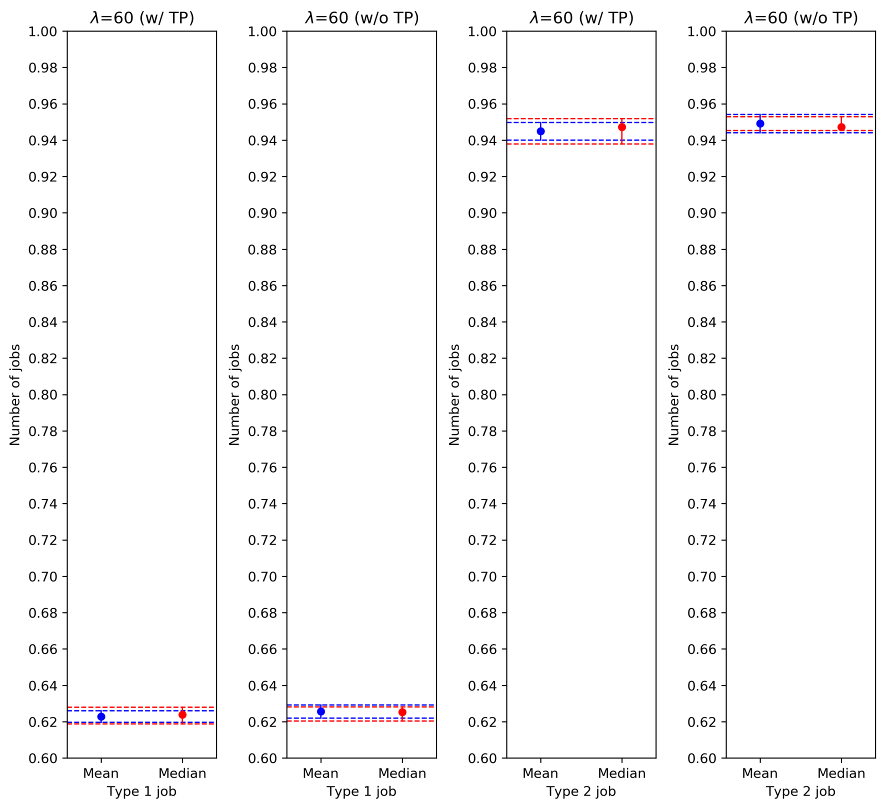

Figure 5: Confidence intervals of the mean and median of type 1 and type 2 jobs for $\lambda = 60$ with and without transient period (TP) for $N = 30$ simulations.

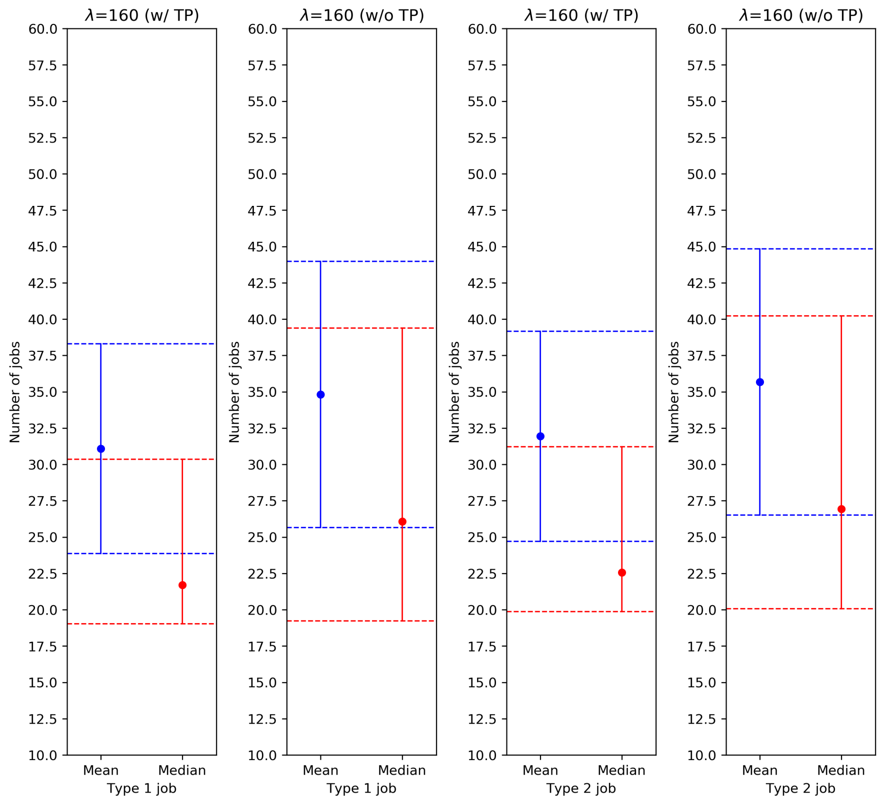

Figure 6: Confidence intervals of the mean and median of type 1 and type 2 jobs for $\lambda = 160$ with and without transient period (TP) for $N = 30$ simulations.

From Figures 5 and 6 we can observe a small increase for both metrics in the configuration where we removed the transient period. This corresponds to the expected behaviour: the system needs some time before it reaches the stationary regime.

4 Little's Law

We want to see if Little's Law holds by computing the following equation.

$$\lambda \bar{R} = \bar{N} \tag{9}$$

$$\bar{R} = \bar{R}_1 + \bar{R}_2 \tag{10}$$

where λ is the arrival rate and $\bar{R}_i$ is the average response time of job i. This value is used to calculate the average response time $\bar{R}$ of a job in Equation 9. Finally, $\bar{N}$ corresponds to the average number of jobs in the system. With this formula, we proceed to verifying the values we had calculated. We also want to remind the reader that these measure averages are retrieved for $N = 30$ simulations.

With Transient Period						
λ	$\bar{R}_1$	$\bar{R}_2$	$\bar{R}$	Measured $\bar{N}$	Calculated $\bar{N}$	Delta
60e-3	7.446	3.564	11.011	1.091	0.661	0.431
160e-3	230.965	228.377	459.293	73.269	73.487	0.190

Without Transient Period						
λ	$\bar{R}_1$	$\bar{R}_2$	$\bar{R}$	Measured $\bar{N}$	Calculated $\bar{N}$	Delta
60e-3	7.456	3.566	11.022	1.093	0.661	0.431
160e-3	191.966	188.583	380.559	59.060	60.889	1.830

Table 2: Showing the application of the Little's Law with the measured values. The first table is with transient period and the other is without it.

5 Parameter estimation and confidence interval

5.1 First experiment

To compute the confidence interval of the parameter ϵ at level $\gamma = 0.95$ for $n = 10$ we use theorem 2.4 from the exam booklet. This gives

$$p_0(10) = 1 - \left(\frac{1-0.95}{2}\right)^{\frac{1}{10}} = 0.3085. \tag{11}$$

Hence, the corresponding confidence interval is $[0, 0.3085]$.

To find the corresponding confidence interval for the stability region we simply add the probabilities $1-\epsilon$ and ϵ in Equation 8. The total arrival rate λ will be changed as follows

$$\begin{aligned} \lambda &= (\lambda_1 + \lambda_2)P(\text{"req is of type 1"}) + \lambda_1 P(\text{"req is of type 2"}) \\ &= \lambda_1 + \lambda_2(1-\epsilon). \end{aligned} \tag{12}$$

Then, we get

$$\begin{aligned} &\lambda\bar{S} < 1 \\ &\iff (\lambda_1 + \lambda_2(1-\epsilon))\bar{S} < 1 \\ &\iff \lambda_1 < \frac{1}{\bar{S}} - \lambda_2(1-\epsilon) \\ &\iff \lambda_1 < 0.21222 \end{aligned} \tag{13}$$

where in the last inequality we set $\epsilon = 0.3085$. Hence, the confidence interval at level $\gamma = 0.95$ for the stability region is [162.19, 212.22] [request/s]. Observe that for $\epsilon = 0$ we get the same boundary as for Question 2. Obviously with $n = 10$, the estimate of our confidence interval is not reliable enough. In order to infer strong conclusions we should take n much larger.

5.2 Second experiment

For this experiment we want to find the value of n such that

$$p_0(n) < 0.01. \tag{14}$$

We found that

$$\begin{aligned}
&1 - \left(\frac{1-0.95}{2}\right)^{\frac{1}{n}} < 0.01 \\
&\iff 0.025^{\frac{1}{n}} > 0.99 \\
&\iff \frac{1}{n}\ln(0.025) > ln(0.99) \\
&\iff n > \frac{ln(0.025)}{ln(0.99)} \\
&\iff n > 367.04.
\end{aligned} \tag{15}$$

Hence, we need to pick $n = 368$ samples of type 1 job so that we can assure at level $\gamma = 0.95$ that $\epsilon \in [0, 0.01]$.