higher-grade ODE

Objetive: The student recognizes the solutions of a higher grade ordinary differential equation, through appropriate substitutions reduces order and finds solutions from a known solution ,solves ODE using the Method of Undetermined Coefficients and Variation of Parameter

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I. in the following differential equations, make the appropriate substitution to reduce them to first order differential equations and then solve them

1 $x^2y'' + (y')^2 - xy' = 0$

2 $y'' = y'e^y$

II. Use the method of undetermined coefficients to find a solution to the given higher-order equation.

1.- $y'' - 2y' + y = te^t$

2.- $y''' - 3y'' + 2y' = t + e^t \quad \text{con} \quad y(0) = 1 \;,\; y'(0) = -\frac{1}{4} \;,\; y''(0) = -\frac{3}{2}$

III. Use the method of parameter variation to find a solution to the given higher-order equation.

1.- $y'' - 2y' + y = e^x \ln x$

2.- $y''' + y' = \sec x \quad \text{con} \quad y(0) = 2 \;,\; y'(0) = 1 \;,\; y''(0) = -2$

IV. Solve the following Cauchy- Euler equations

1.- $x^2y'' + xy' + y = 4\sin x\,(\ln x)$

2.- $x^2y'' + 4xy' + 2y = 4\ln x$

3.- $16x^2y'' - 8xy' + 9y = 4x - 8 \quad \text{con} \quad y(1) = 4 \;,\; y'(1) = -1$

V. A series circuit has a capacitor of $0.25 \times 10^{-6} F$ and an inductor of $1H$. If the initial charge on the capacitor is $10^{-6} F$ and there is no initial current, find the charge Q on the capacitor at any time t. Graph the solution in Geogebra

VI. Find the steady-state current in an LRC-series circuit when $L = \frac{1}{2} H$, $R = 20\Omega$, $C = 0.001 f$, and $E(t) = 100 \sin 60t + 200 \cos 40t V$. Graph the solution in Geogebra

VII. A large tank initially contains 100 gallons of brine in which 10 pounds of salt have been dissolved. Starting at $t = 0$, pure water enters the tank at a rate of 5 gallons per minute. The mixture is kept uniform by stirring and with the mixture well stirred, it flows out simultaneously at a rate of 2 gallons per minute.

a) Determine the concentration of salt in the tank at any instant t.

b) How much salt will be in the tank after 15 minutes and what will be the concentration at that time?

c) If the capacity of the tank is 250 gallons, what will be the concentration in the tank at the instant it is filled?

VIII. The differential equation:

$$m\frac{dP}{dt} = P\,(a - b \ln P).$$

It is a modification of the logistic equation and is known as the differential equation of Gompertz. This ODE is sometimes used as a model in the study of population growth or decay, the growth of solid tumours and certain kinds of actuarial predictions.

a. Solve the DE subject to $P(0) = P_0$.

b. Suppose that $a = 1$, $b = 1$, draw the solution curves corresponding to the cases:$P_0 > e$.