

Proof Reconstruction in Classical Propositional Logic

(Work in Progress)

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Joint work with Andrés Sicard-Ramírez

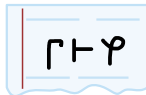
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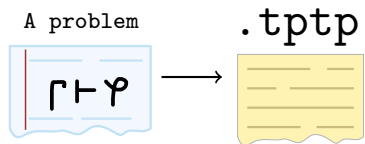
Agda Implementors' Meeting XXV
May 9-15th



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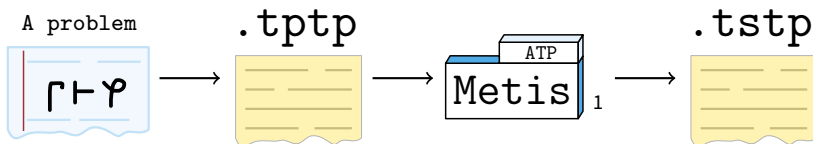
A problem



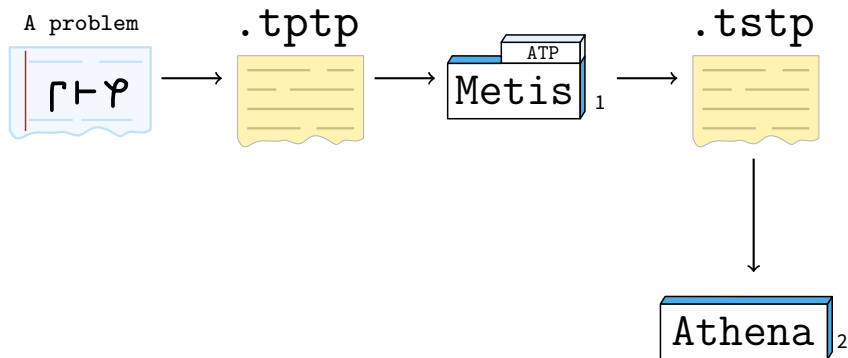




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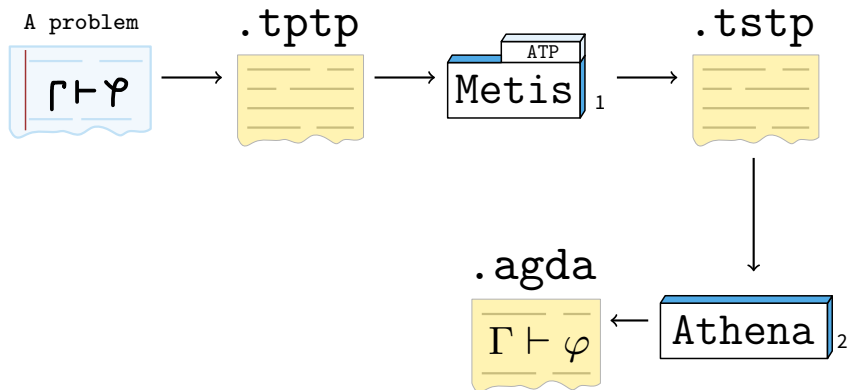


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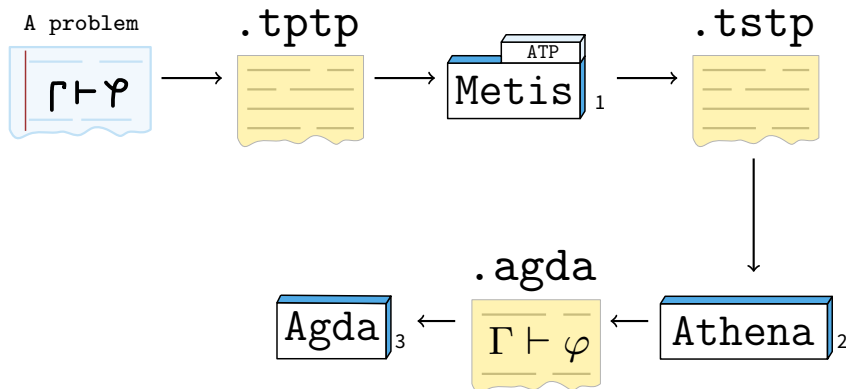
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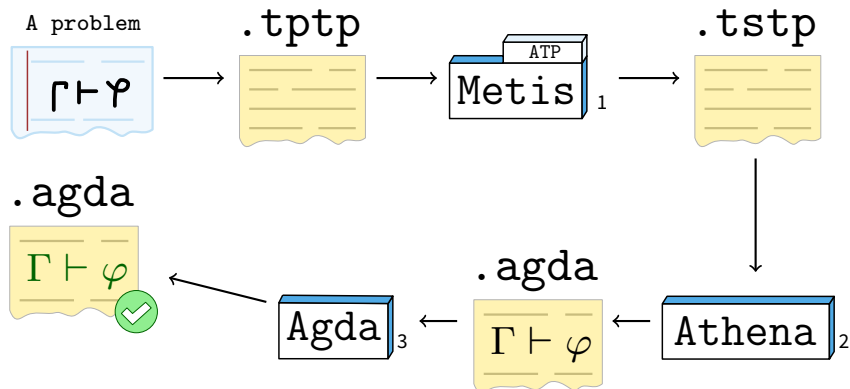
Proof Reconstruction: Overview



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³ It is available at <http://github.com/agda/agda>

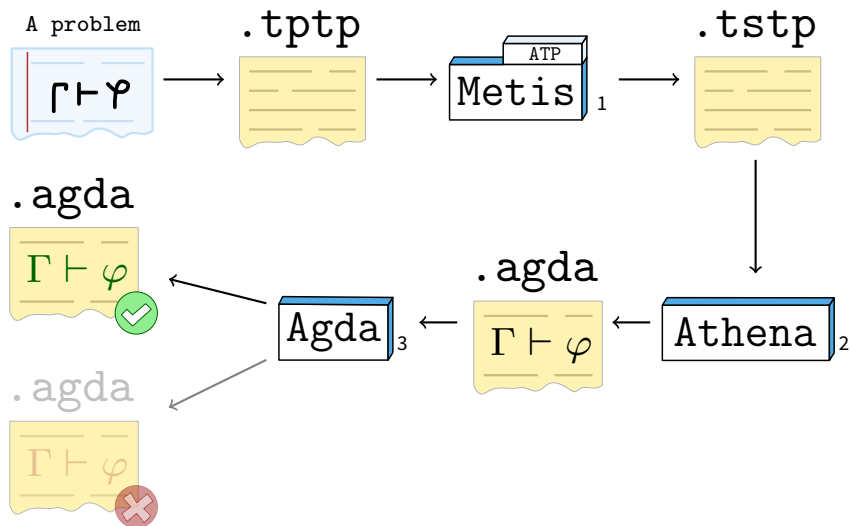


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.tptp



- ▶ Is a language⁴ to encode problems (Sutcliffe, 2009)
- ▶ Is the input of the ATPs
- ▶ Annotated formulas with the form

```
language(name, role, formula).
```

language FOF or CNF

name to identify the formula within the problem

role axiom, definition, hypothesis, conjecture, among others

formula version in TPTTP format

⁴Is available at <http://www.cs.miami.edu/~tptp/TPTP/SyntaxBNF.html>

► $p \vdash p$

```
fof(a, axiom, p).  
fof(goal, conjecture, p).
```

► $\vdash \neg(p \wedge \neg p) \vee (q \wedge \neg q)$

```
fof(goal, conjecture, ~ ((p & ~ p) | (q & ~ q))).
```

⁵Is available at <http://github.com/jonaprieto/prop-pack>

.tstp



A TSTP derivation⁶

- ▶ Is a **Directed Acyclic Graph** where
 - leaf** is a formula from the TPTP input
 - node** is a formula inferred from parent formula
 - root** the final derived formula
- ▶ Is a list of annotated formulas with the form:

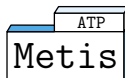
```
language(name, role, formula, source [,useful info]).
```

where **source** typically is an inference record:

```
inference(rule, useful info, parents)
```

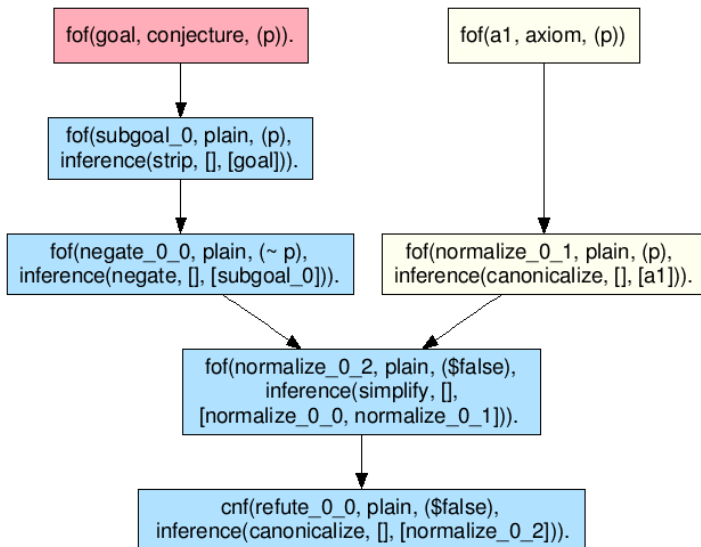
⁶<http://www.cs.miami.edu/~tptp/TPTP/QuickGuide/Derivations.html>

- Proof found by **Metis** ATP for the problem $p \vdash p$



```
$ metis --show proof basic-4.tptp
fof(a, axiom, (p)).
fof(goal, conjecture, (p)).
fof(subgoal_0, plain, (p),
    inference(strip, [], [goal])).
fof(negate_0_0, plain, (~ p),
    inference(negate, [], [subgoal_0])).
fof(normalize_0_0, plain, (~ p),
    inference(canonicalize, [], [negate_0_0])).
fof(normalize_0_1, plain, (p),
    inference(canonicalize, [], [a])).
fof(normalize_0_2, plain, ($false),
    inference(simplify, [],
        [normalize_0_0, normalize_0_1])).
cnf(refute_0_0, plain, ($false),
    inference(canonicalize, [], [normalize_0_2])).
```

DAG for the previous TSTP derivation found by Meti's ATP

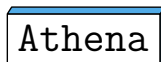


Athena

Is a Haskell program that translates proofs given by Metis Prover in TSTP format to Agda code.

It depends on:

- ▶ **agda-prop** Classical Logic within Agda: Axioms + Theorems
- ▶ **agda-metis** Theorems of the inference rules of Metis Prover



Haskell

- ▶ Parsing
- ▶ AST construction
- ▶ Creation and analysis of DAG derivations
- ▶ Analysis of inference rules used
- ▶ Generation of Agda code of the proof

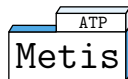


Agda

Is a dependently typed functional programming language and it also a proof assistant.

We used it to type-check the proofs found by Metis Prover

- ▶ Agda-Prop Library: Logic framework for Classical Propositional Logic
- ▶ Agda-Metis Library: theorems based on the inference rules of Metis Prover



Metis is an automatic theorem prover for First-Order Logic with equality

Why Metis?

- ▶ Open source implemented in Standard ML
- ▶ Reads problem in **TPTP** format
- ▶ Outputs detailed proofs in **TSTP** format
- ▶ Each refutation step is one of **6 simple rules**

TSTP derivations exhibit these inferences:

Rule	Task
canonicalize	transforms formulas to CNF, DNF or NNF
clausify	performs clausification
conjoinct	extracts a formula from a conjunction
negate	applies negation to the formula
resolve	applies theorems of resolution
simplify	applies over a list of formula to simplify them
strip	splits a formula into subgoals

- ▶ In TSTP
- ▶ Function:

```
conjunct : Prop → Prop → Prop
conjunct (φ ∧ ψ) ω with [ eq φ ω ] | [ eq ψ ω ]
... | true | _      = φ
... | false | true  = ψ
... | false | false = conjunct φ ω
conjunct φ ω      = φ
```

- ▶ Theorem

```
atp-conjunct
: ∀ {Γ} {φ}
→ (ω : Prop)
→ Γ ⊢ φ
→ Γ ⊢ conjunct φ ω
```

Example goes here

Verified Example goes here

Failure Example goes here

SledgeHammer

- ▶ Isabelle/HOL tool
- ▶ Metis ported within Isabelle
- ▶ Reconstruct proofs of well-known ATPs: EProver, Vampire, among others

Integrating Waldmeister and Agda

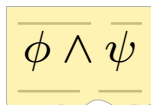
- ▶ Source code not available
- ▶ Equational Logic
- ▶ Reflection Layers

Related Work: Apia

Proving first-order theorems written in Agda using automatic theorem provers for first-order logic

At the moment, the communication between Agda and the ATPs is unidirectional because the ATPs are being used as oracles (Sicard-Ramírez, 2015).

. agda



+ ATP-Pragma

```
module Or where

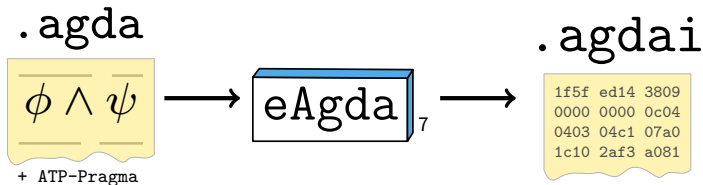
data _∨_ (A B : Set) : Set where
  inj1 : A → A ∨ B
  inj2 : B → A ∨ B

postulate
  A B      : Set
  ∨-comm   : A ∨ B → B ∨ A
  {-# ATP prove ∨-comm #-}
```

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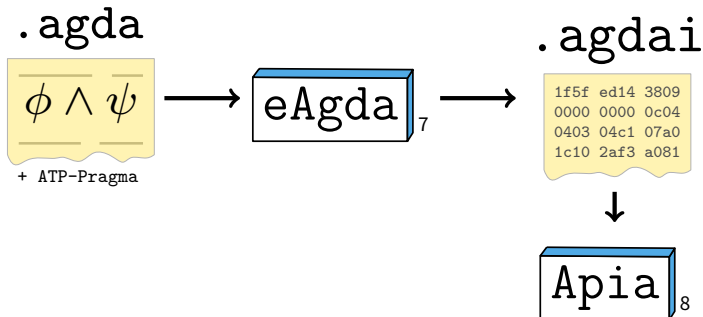
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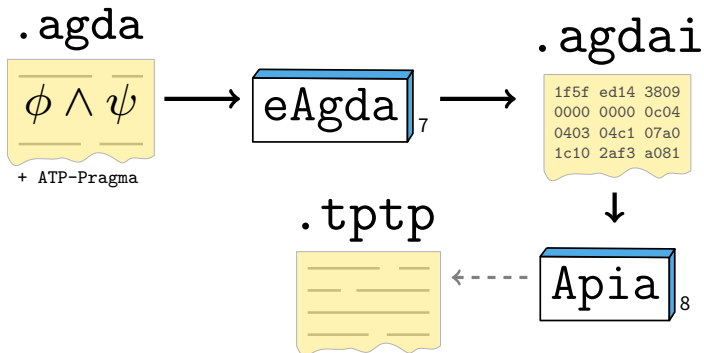


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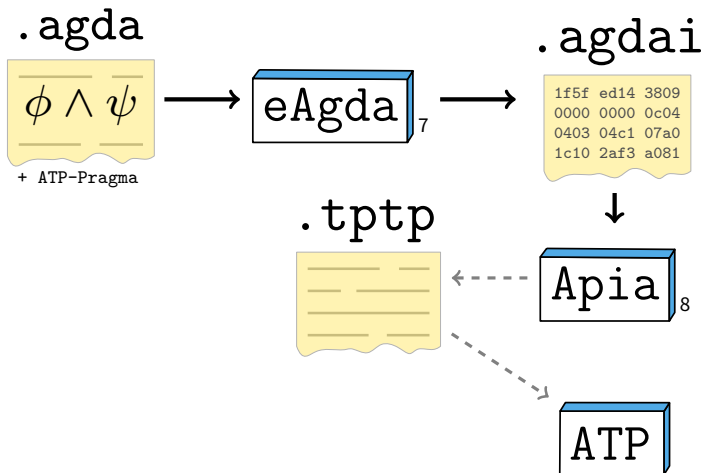


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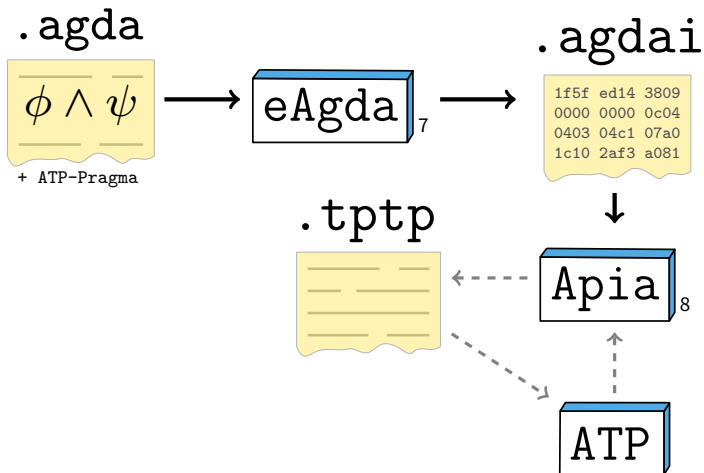


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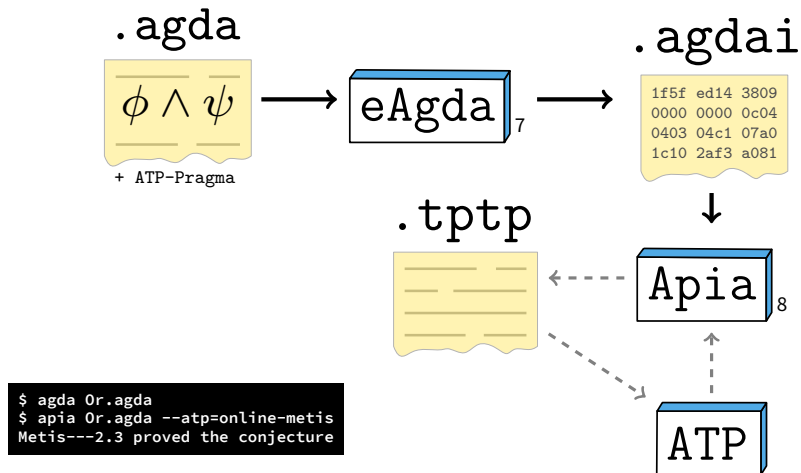


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- ▶ Add shallow embedding in order to work with **Apia**
- ▶ Support First-Order Logic with Equality
- ▶ Support another prover like **EProver**



Sicard-Ramírez, Andrés (2015). *Reasoning about functional programs by combining interactive and automatic proofs*. PEDECIBA Informática, Universidad de la República.



Sutcliffe, G. (2009). “The TPTP Problem Library and Associated Infrastructure: The FOF and CNF Parts, v3.5.0”. In: *Journal of Automated Reasoning* 43.4, pp. 337–362.