# Proof Reconstruction in Classical Propositional Logic

Jonathan Prieto-Cubides

Advisor: Andrés Sicard-Ramírez

Universidad EAFIT
Medellín, Colombia

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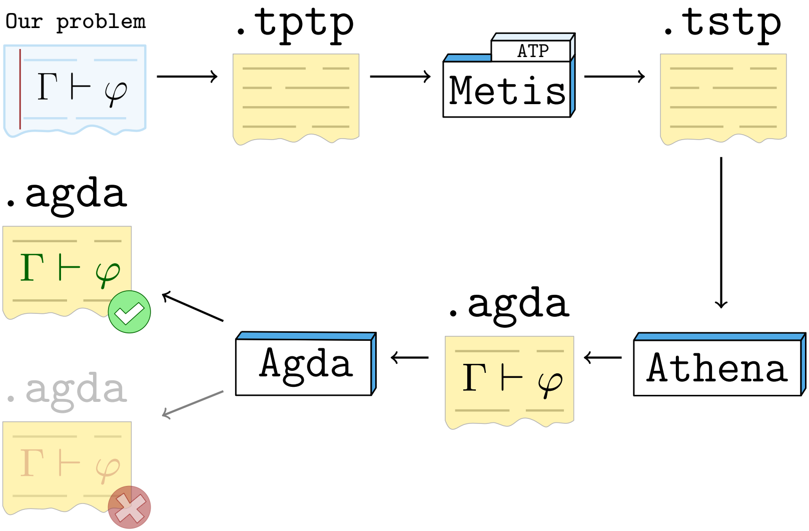
## Outline

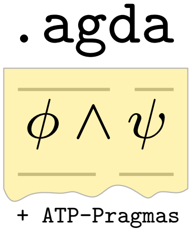
## Proof Reconstruction



## Previous Work: Apia

*Proving first-order theorems written in Agda using automatic theorem provers for first-order logic*

At the moment, the communication between Agda and the ATPs is unidirectional because the ATPs are being used as oracles (Sicard-Ramírez, 2015).



```
$ cat Or.agda
module Or where

data _or_ (A B : Set) : Set where
  inj1 : A -> A or B
  inj2 : B -> A or B

postulate
  A B     : Set
  or-comm : A or B -> B or A
{-# ATP prove or-comm #-}
```

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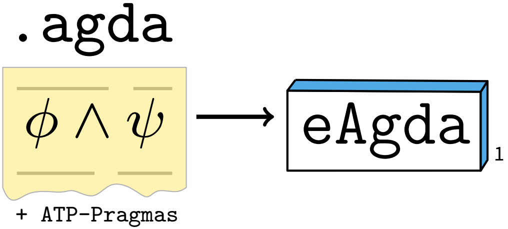

[1] Development version of Agda in order to handle a new built-in ATP-pragma. `https://github.com/asr/eagda`

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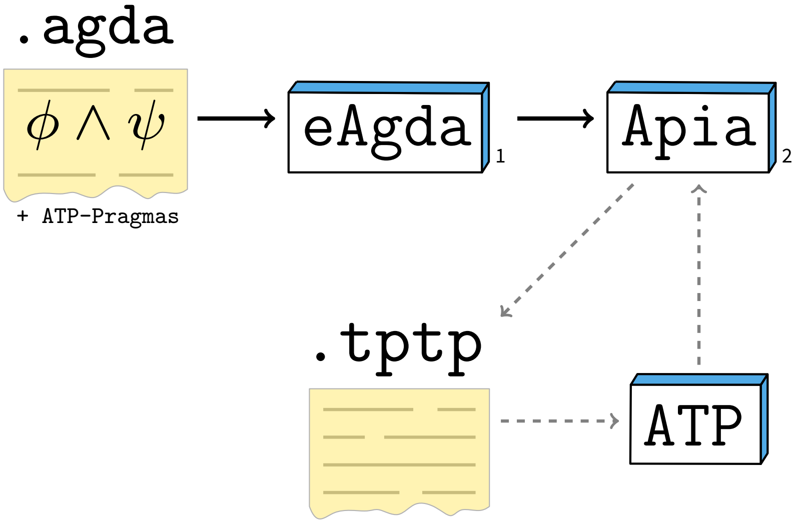


[1] Development version of Agda in order to handle a new built-in ATP-pragma. `https://github.com/asr/eagda`

[2] Haskell program for proving first-order theorems written in Agda using ATPs. `https://github.com/asr/apia`

## TPTP Syntax

*Thousands of Problems for Theorem Provers*

.tptp

- Is a language[3] to encode problems in text files
- Is the input of the ATPs
- his problems contains formulas with the form

  **`language(name, role, formula).`**

| | |
|---|---|
| `language` | FOF, or CNF |
| `name` | to identify the formula within the problem |
| `role` | axiom, definition, hypothesis, conjecture, among others |
| `formula` | the logic formula in the language |

Go to Reconstruction

[3] Is available at `http://www.cs.miami.edu/~tptp/TPTP/SyntaxBNF.html`

## TPTP Examples

*From Prop-Pack repository*[4]

Problems in Propositional Logic:

- $p \vdash p$

```
$ cat basic-4.tptp
fof(a, axiom, p).
fof(goal, conjecture, p).
```

- $p \wedge q \vdash q \wedge p$

```
$ cat conj-3.tptp
fof(a, axiom, p & q).
fof(goal, conjecture, q & p).
```

- $\vdash \neg(p \wedge \neg p) \vee (q \wedge \neg q)$

```
$ cat neg-7.tptp
fof(goal, conjecture, ~ ((p & ~ p) | (q & ~ q))).
```

[4] Is available at `http://github.com/jonaprieto/pro-pack`

## TSTP Syntax

.tstp

A TSTP derivation [5]

- Is a Directed Acyclic Graph where

| | |
|---|---|
| `leaf` | is a formulae from the TPTP input |
| `node` | is a formulae inferred from parent formulae |
| `root` | the final derived formulae |

- Is a list of annotated formulae:

```
language(name, role, formula, source [,useful info]).
```

where **source** typically is a inference record:

```
inference(rule, useful information, parents)
```

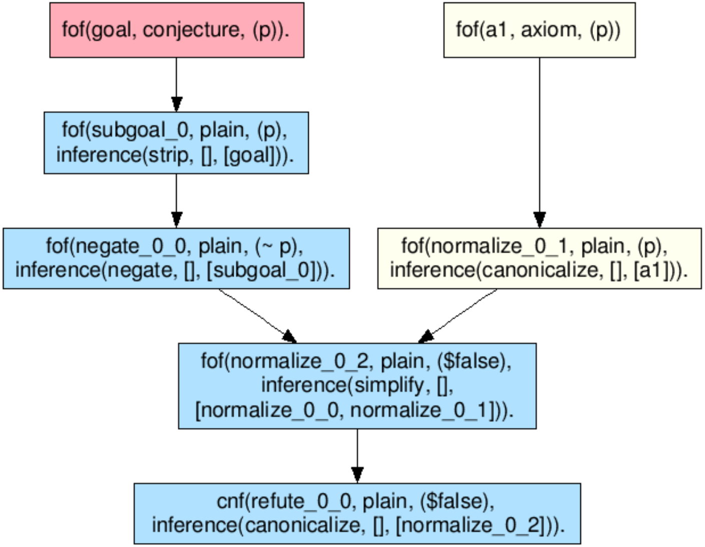
[5] `http://www.cs.miami.edu/~tptp/TPTP/QuickGuide/Derivations.html`

## TSTP Example

- Proof found by **Metis** ATP for the problem $p \vdash p$

```
$ metis --show proof basic-4.tptp
fof(a, axiom, (p)).
fof(goal, conjecture, (p)).
fof(subgoal_0, plain, (p),
  inference(strip, [], [goal])).
fof(negate_0_0, plain, (~ p),
  inference(negate, [], [subgoal_0])).
fof(normalize_0_0, plain, (~ p),
  inference(canonicalize, [], [negate_0_0])).
fof(normalize_0_1, plain, (p),
  inference(canonicalize, [], [a])).
fof(normalize_0_2, plain, ($false),
  inference(simplify, [],
    [normalize_0_0, normalize_0_1])).
cnf(refute_0_0, plain, ($false),
    inference(canonicalize, [], [normalize_0_2])).
```

## DAG for the previous TSTP derivation found by `Metis` ATP


fof(goal, conjecture, (p)).
fof(a1, axiom, (p))
fof(subgoal_0, plain, (p), inference(strip, [], [goal])).
fof(negate_0_0, plain, (~ p), inference(negate, [], [subgoal_0])).
fof(normalize_0_1, plain, (p), inference(canonicalize, [], [a1])).
fof(normalize_0_2, plain, ($false), inference(simplify, [], [normalize_0_0, normalize_0_1])).
cnf(refute_0_0, plain, ($false), inference(canonicalize, [], [normalize_0_2])).

## Design Decisions for the Reconstruction Tool

Athena

**`Haskell`**

- Parsing
- AST construction
- Creation and analysis of DAG derivations
- Analysis of inference rules used
- Generation of Agda code of the proof

## Design Decisions for the Reconstruction Tool

*Programming Languages*

**Agda** is a dependently typed functional programming language and it also a proof assistant.
Our usages:

- Logic framework for Classical Propositional Logic
- Type-Checker for the Agda code of the proofs generated

## References

Sicard-Ramírez, Andrés (2015). *Reasoning about functional programs by combining interactive and automatic proofs.* PEDECIBA Informática, Universidad de la República.