Reconstructing Propositional Proofs in Type Theory

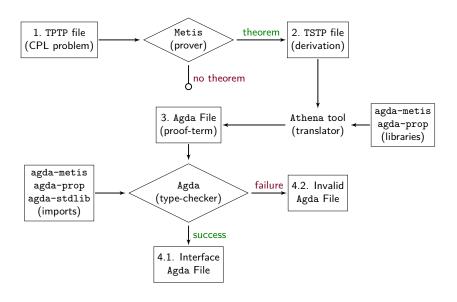
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Proof Reconstruction: Overview



Metis Theorem Prover

Metis is an automatic theorem prover for first-order logic with equality developed by **hurd2003first** [hurd2003first]. For the propositional fragment, Metis has only three inference rules:

$$\cfrac{\cfrac{}{\Gamma \vdash \varphi} \text{ axiom}}{\cfrac{}{\Gamma \vdash \varphi \lor \neg \varphi} \text{ assume } \varphi}$$

$$\cfrac{}{\cfrac{}{\Gamma \vdash l \lor \varphi} \cfrac{}{\Gamma \vdash \neg l \lor \psi} \text{ resolve } l}$$

Where Γ is a set of premises, and $_\vdash_$ is the syntactical consequence relation between the premises on the right, and the conclusion on the left.

Why Metis?

- ▶ Open source implemented in Standard ML
- ▶ Each refutation step is one of *six rules*
- ▶ Reads problem in TPTP format
- ▶ Outputs *detailed* proofs in TSTP format

Inference Rules of Metis

TSTP derivations by Metis exhibit the following inferences:

Metis rule	Purpose
strip	Strip a goal into subgoals
conjunct	Takes a formula from a conjunction
resolve	A general form of the resolution theorem
canonicalize	Normalization of the formula
clausify	Performs clausification
simplify	Simplify definitions and theorems

References I