Proof Reconstruction in Classical Propositional Logic

(Work in Progress)

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Agda Implementors' Meeting XXV
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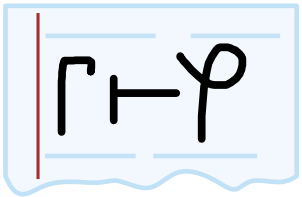
UNIVERSIDAD EAFIT

Proof Reconstruction: Overview

A problem



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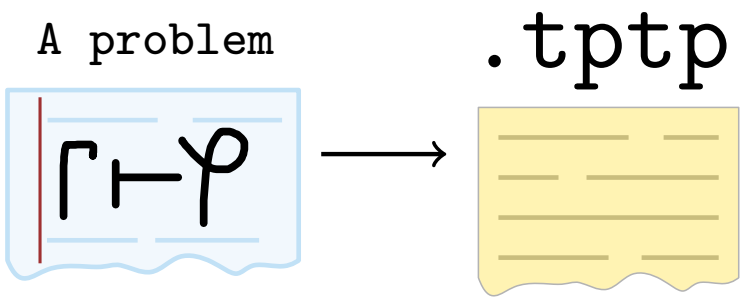

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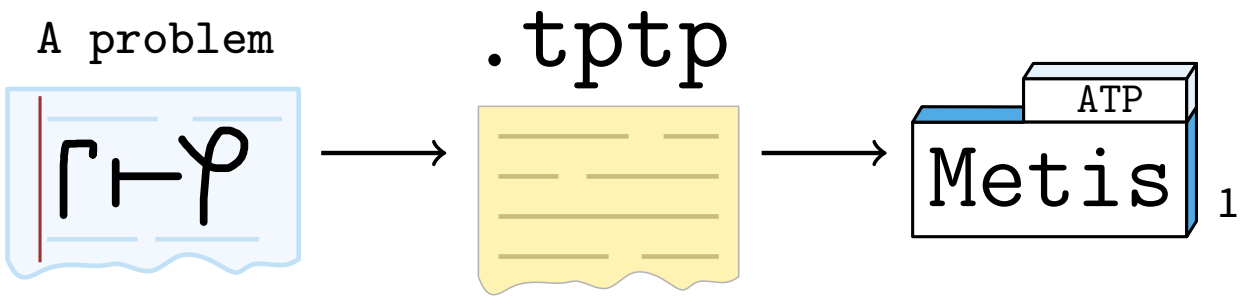

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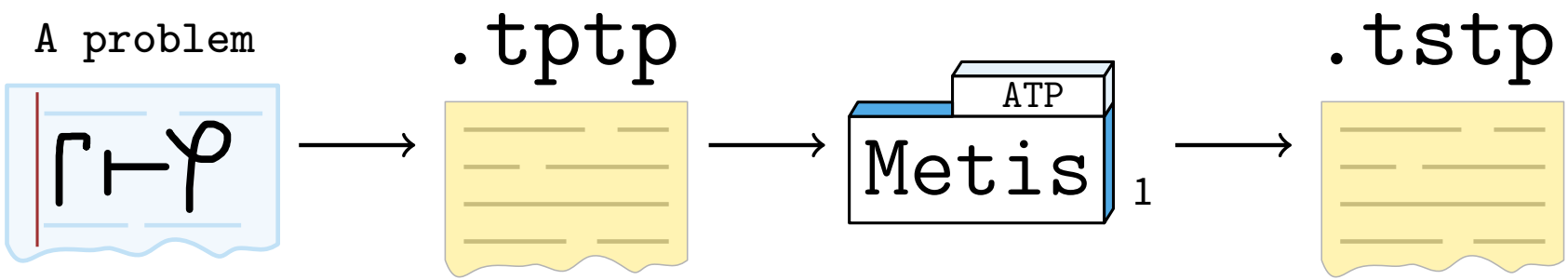

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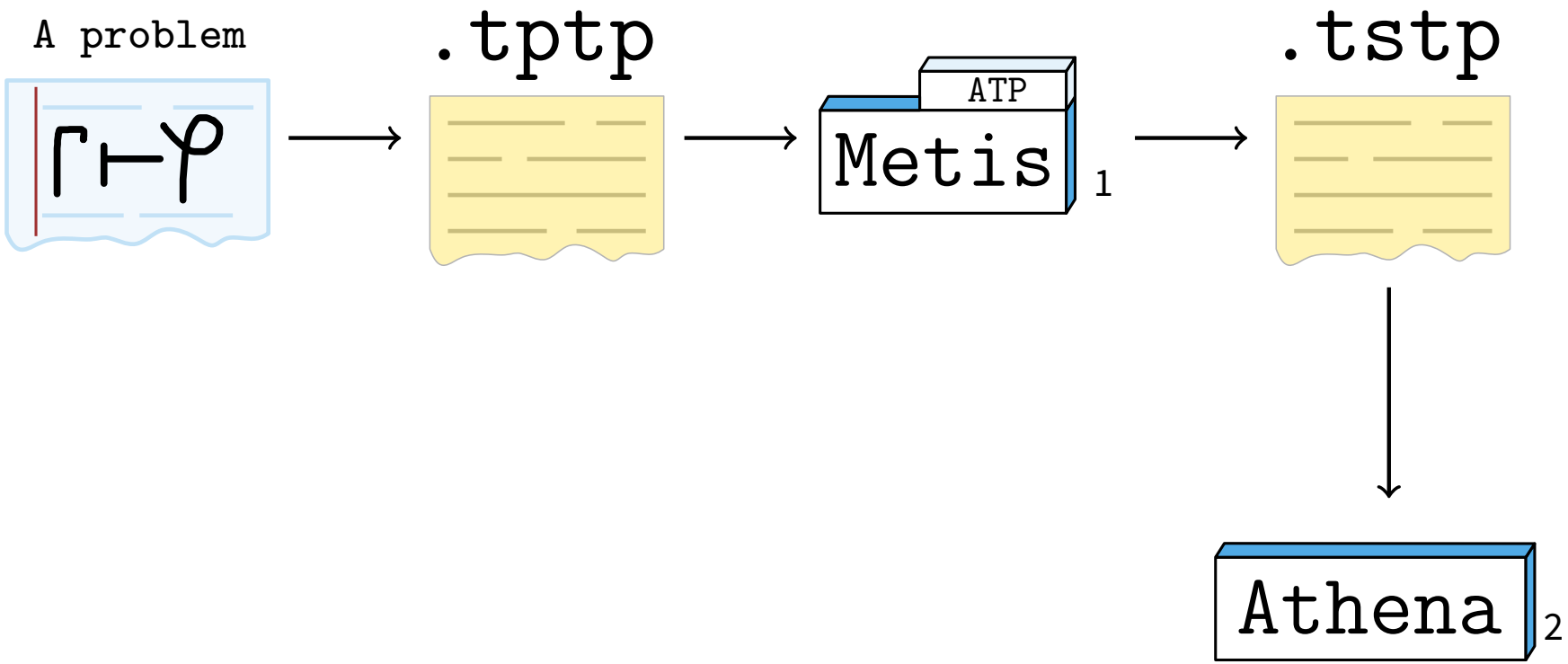

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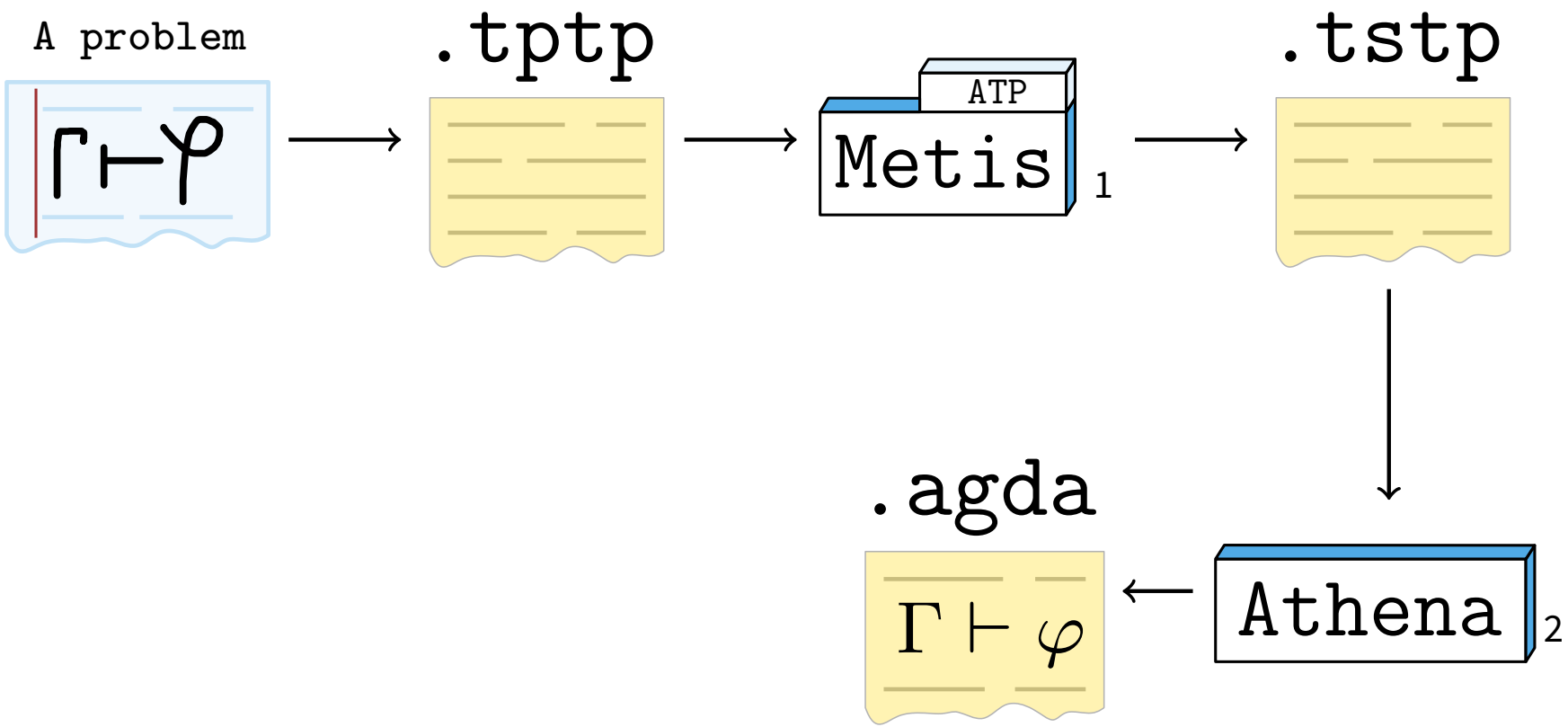

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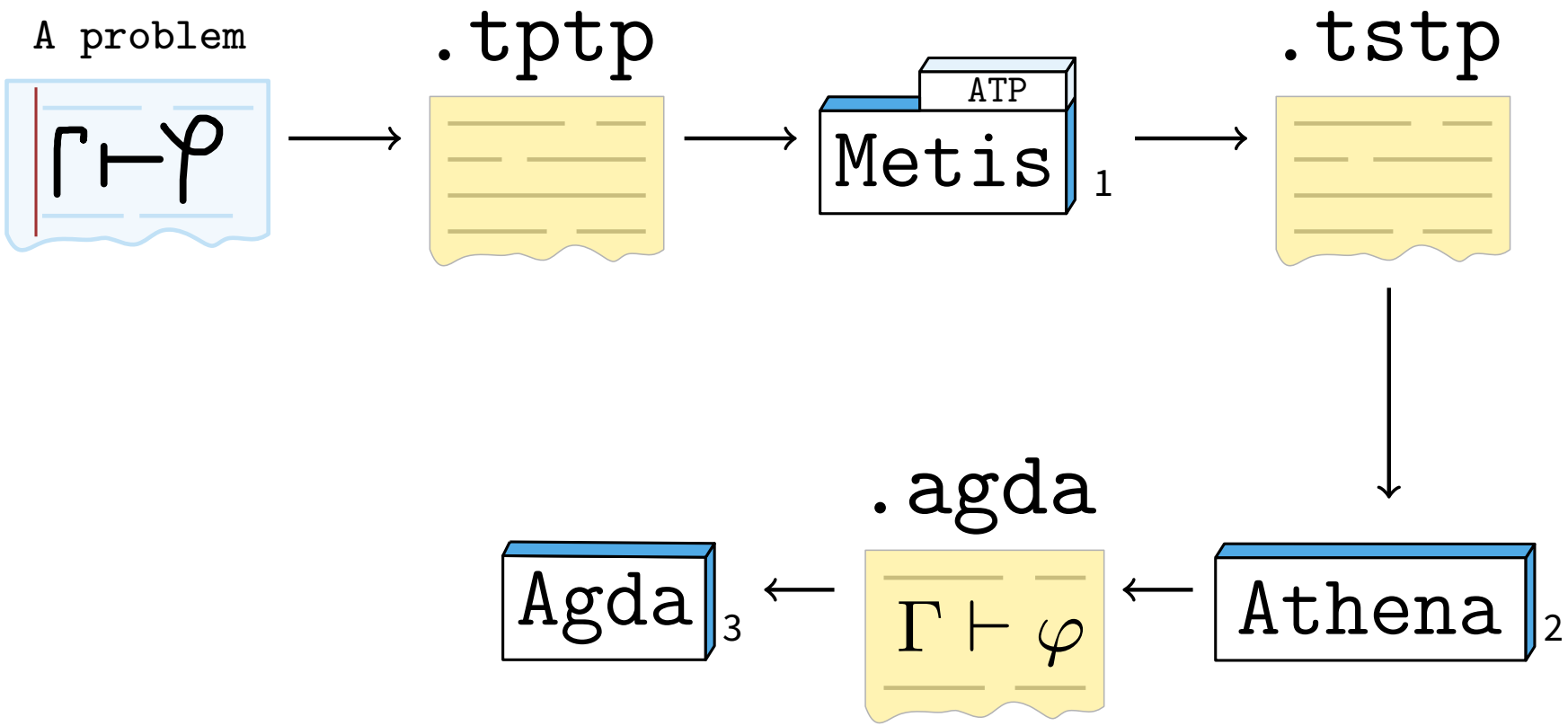

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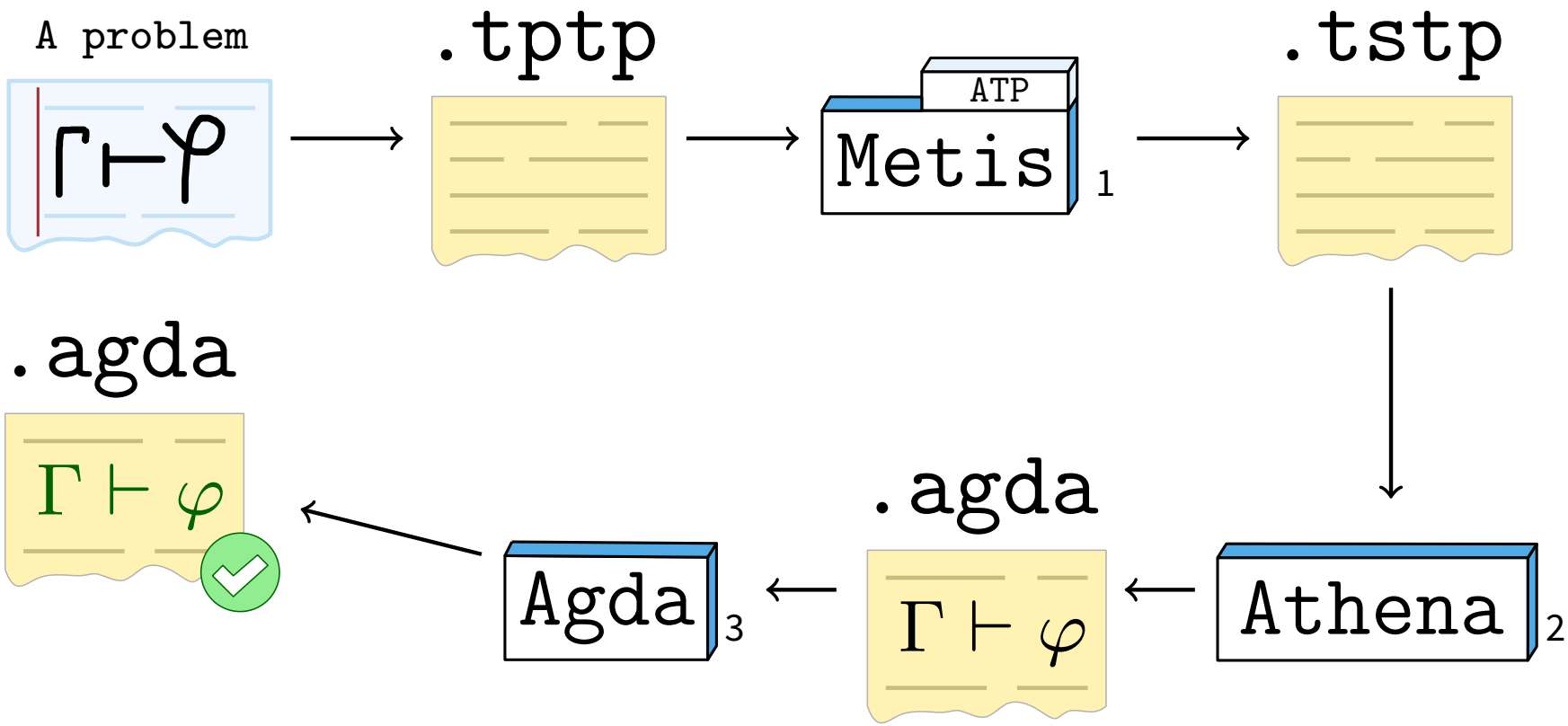

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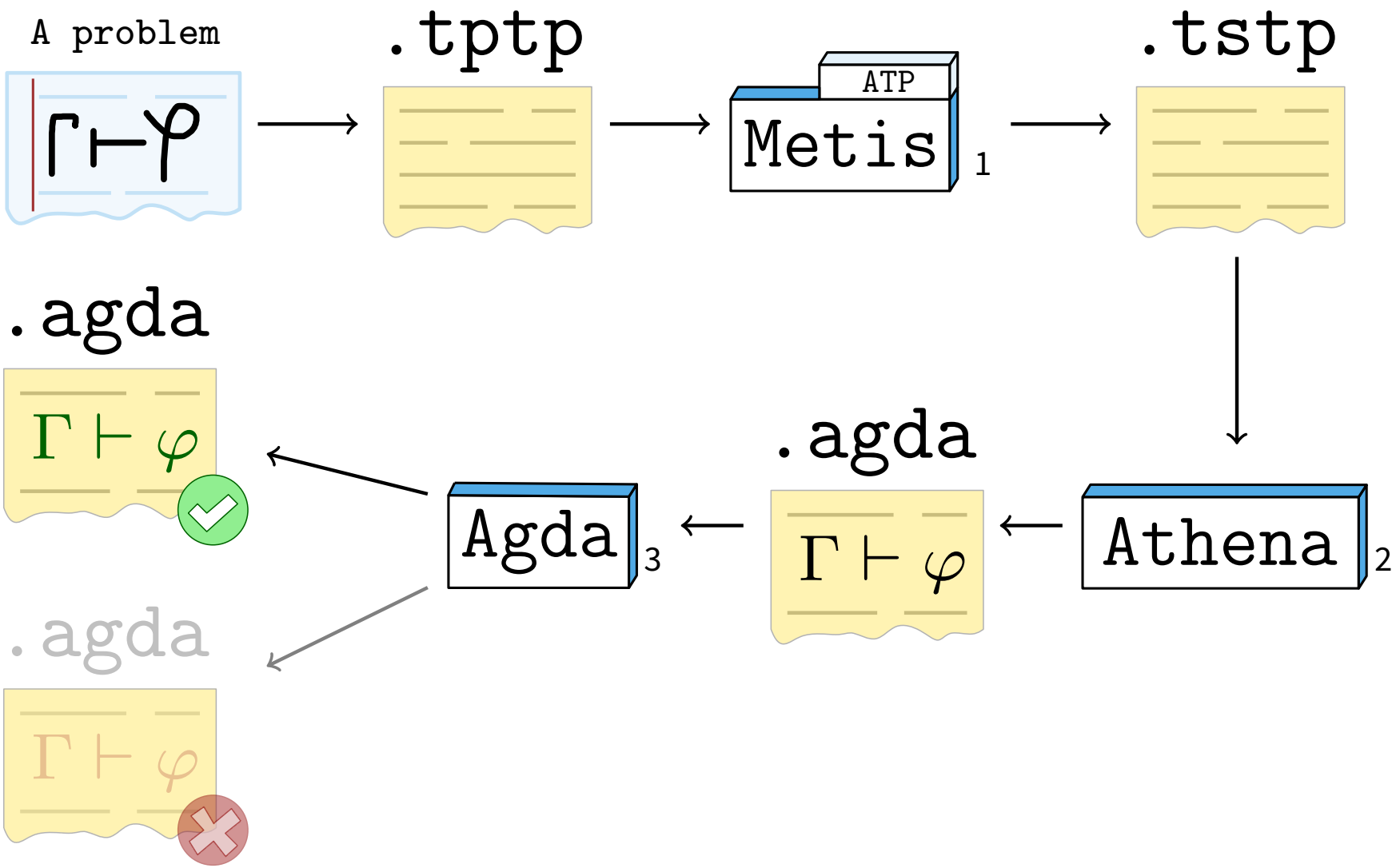

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Bonus Slides

.tptp

- Is a language[4] to encode problems (Sutcliffe, 2009)
- Is the input of the ATPs
- Annotated formulas with the form

```
language(name, role, formula).
```

`language`	FOF or CNF
`name`	to identify the formula within the problem
`role`	axiom, definition, hypothesis, conjecture, among others
`formula`	version in TPTTP format

[4] Is available at `http://www.cs.miami.edu/~tptp/TPTP/SyntaxBNF.html`

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TPTP Examples

From Prop-Pack repository [5]

- $p \vdash p$

```
fof(a, axiom, p).
fof(goal, conjecture, p).
```

- $\vdash \neg(p \wedge \neg p) \vee (q \wedge \neg q)$

```
fof(goal, conjecture, ~ ((p & ~ p) | (q & ~ q))).
```

[5] Is available at `http://github.com/jonaprieto/prop-pack`

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.tstp

A TSTP derivation [6]

- Is a **D**irected **A**cyclic **G**raph where
  - `leaf` is a formula from the TPTP input
  - `node` is a formula inferred from parent formula
  - `root` the final derived formula
- Is a list of annotated formulas with the form:

```
language(name, role, formula, source [,useful info]).
```

where **source** typically is an inference record:

```
inference(rule, useful info, parents)
```

[6] http://www.cs.miami.edu/~tptp/TPTP/QuickGuide/Derivations.html

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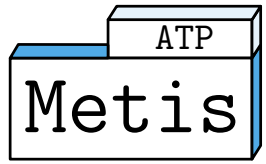
- Proof found by **Metis** ATP for the problem $p \vdash p$




```
$ metis --show proof basic-4.tptp
fof(a, axiom, (p)).
fof(goal, conjecture, (p)).
fof(subgoal_0, plain, (p),
  inference(strip, [], [goal])).
fof(negate_0_0, plain, (~ p),
  inference(negate, [], [subgoal_0])).
fof(normalize_0_0, plain, (~ p),
  inference(canonicalize, [], [negate_0_0])).
fof(normalize_0_1, plain, (p),
  inference(canonicalize, [], [a])).
fof(normalize_0_2, plain, ($false),
  inference(simplify, [],
    [normalize_0_0, normalize_0_1])).
cnf(refute_0_0, plain, ($false),
    inference(canonicalize, [], [normalize_0_2])).
```

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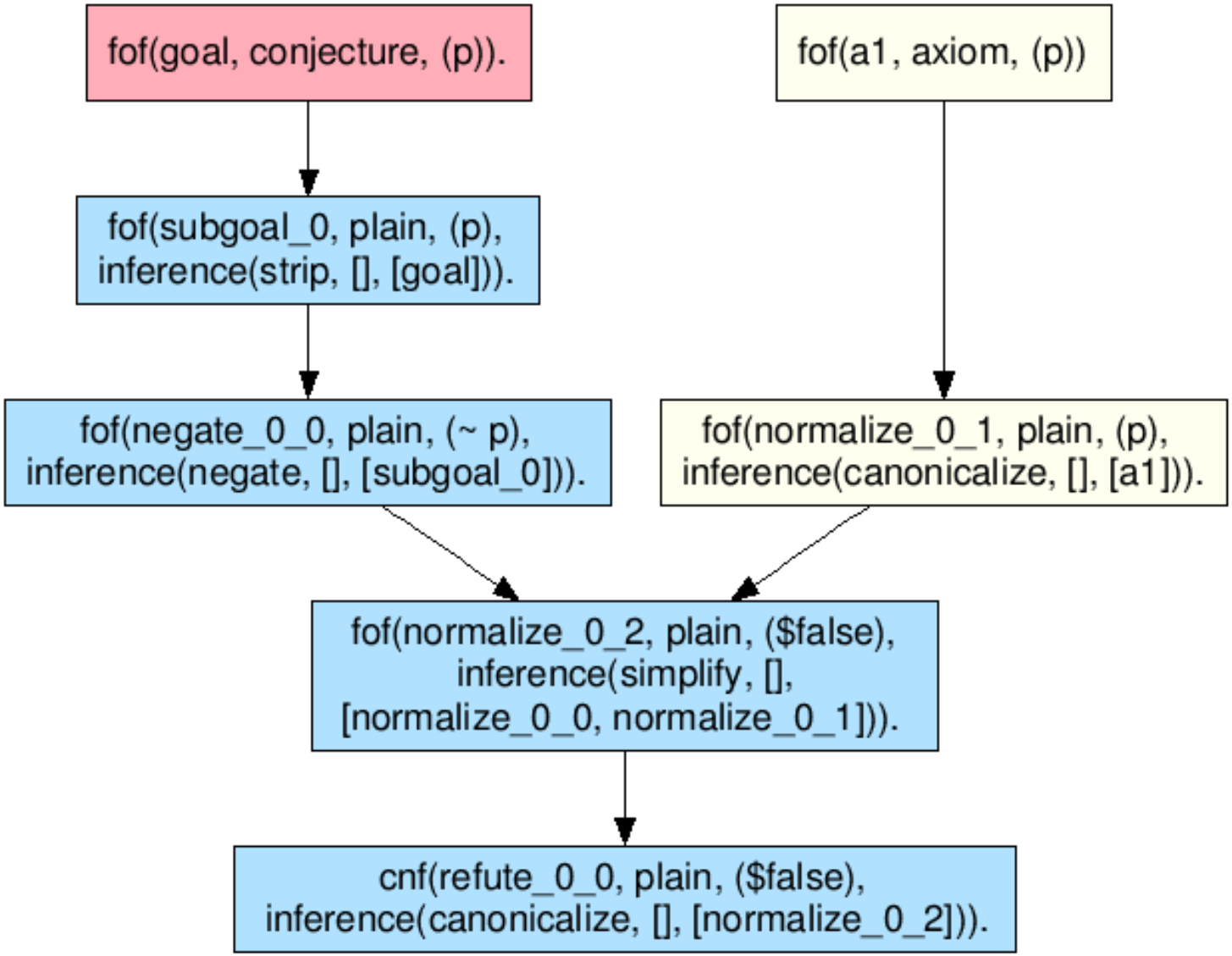
DAG for the previous TSTP derivation found by `Metis` ATP



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Athena

Is a Haskell program that translates proofs given by Metis Prover in TSTP format to Agda code.

It depends on:

- **`agda-prop`** Classical Logic within Agda: Axioms + Theorems
- **`agda-metis`** Theorems of the inference rules of Metis Prover

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Design Decisions for the Reconstruction Tool

Haskell

- Parsing
- AST construction
- Creation and analysis of DAG derivations
- Analysis of inference rules used
- Generation of Agda code of the proof

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Agda

Is a dependently typed functional programming language and it also a proof assistant.
We used it to type-check the proofs found by Metis Prover

- Agda-Prop Libary: Logic framework for Classical Propositional Logic
- Agda-Metis Library: theorems based on the inference rules of Metis Prover

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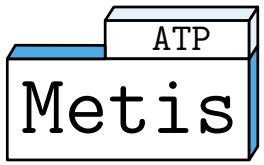

Metis is an automatic theorem prover for First-Order Logic with equality

Why Metis?

- Open source implemented in Standard ML
- Reads problem in **TPTP** format
- Outputs detailed proofs in **TSTP** format
- Each refutation step is one of **6 simple rules**

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TSTP derivations exhibit these inferences:

Rule	Task
`canonicalize`	transforms formulas to CNF, DNF or NNF
`clausify`	performs clausification
`conjunct`	extracts a formula from a conjunction
`negate`	applies negation to the formula
`resolve`	applies theorems of resolution
`simplify`	applies over a list of formula to simplify them
`strip`	splits a formula into subgoals

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conjunct

```
conjunct : Prop → Prop → Prop
conjunct ϕ( ⊠ ψ) ω with ⊠ eq ϕ ω ⊠ | ⊠ eq ψ ω ⊠
... | true  | _     = ϕ
... | false | true  = ψ
... | false | false = conjunct ϕ ω
conjunct ϕ ω        = ϕ

atp-conjunct
  : ∀ Γ{} ϕ{}
  → ω( : Prop)
  → Γ ⊠ ϕ
  → Γ ⊠ conjunct ϕ ω
```

Agda Code

Generated by Athena Tool

Example goes here

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Type-checked Proof

Verified Example goes here

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Type-checked Proof

Failure Example goes here

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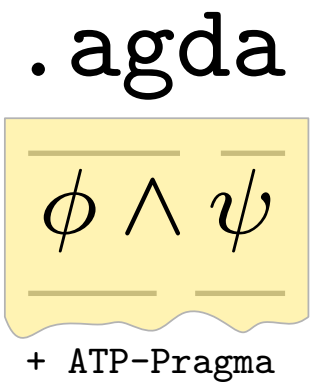
SledgeHammer

- Isabelle/HOL tool
- Metis ported within Isabelle
- Reconstruct proofs of well-known ATPs: EProver, Vampire, among others

Integrating Waldmeister and Agda

- Source code not available
- Equational Logic
- Reflection Layers

At the moment, the communication between Agda and the ATPs is unidirectional because the ATPs are being used as oracles (Sicard-Ramírez, 2015).




```
module Or where

data _∨_ (A B : Set) : Set where
  inj1 : A → A ∨ B
  inj2 : B → A ∨ B

postulate
  A B    : Set
  ∨-comm : A ∨ B → B ∨ A
{-# ATP prove ∨-comm #-}
```

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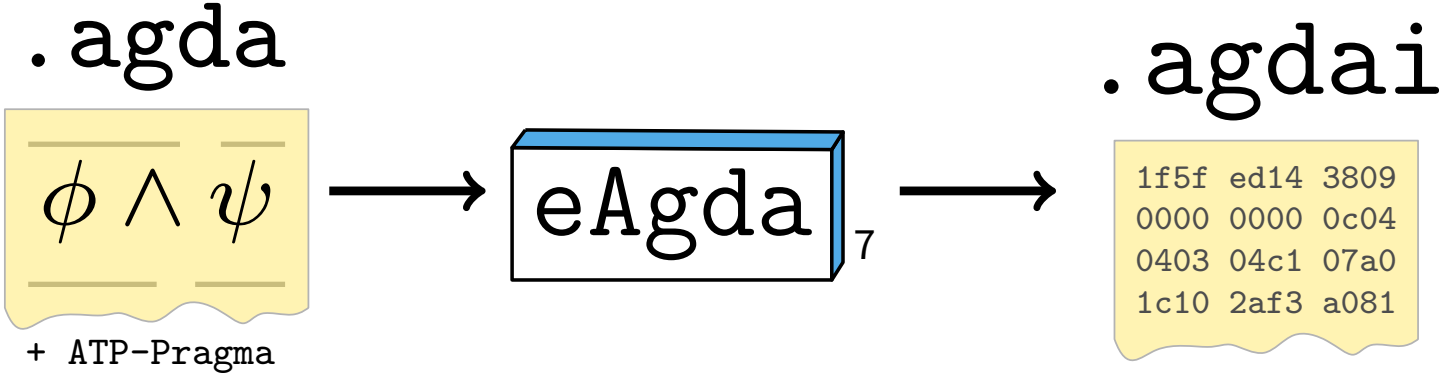
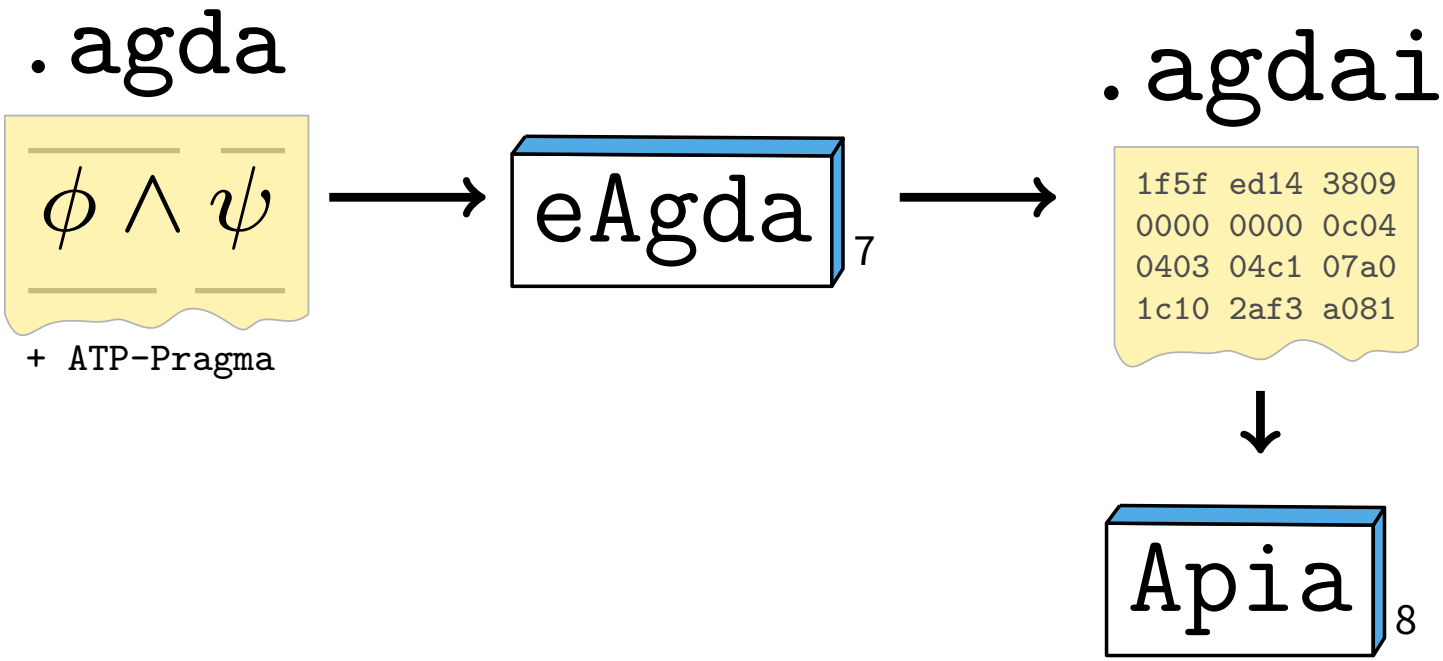


[7] Development version of Agda in order to handle a new built-in ATP-pragma. `https://github.com/asr/eagda`

Related Work: Apia

Proving first-order theorems written in Agda using automatic theorem provers for first-order logic



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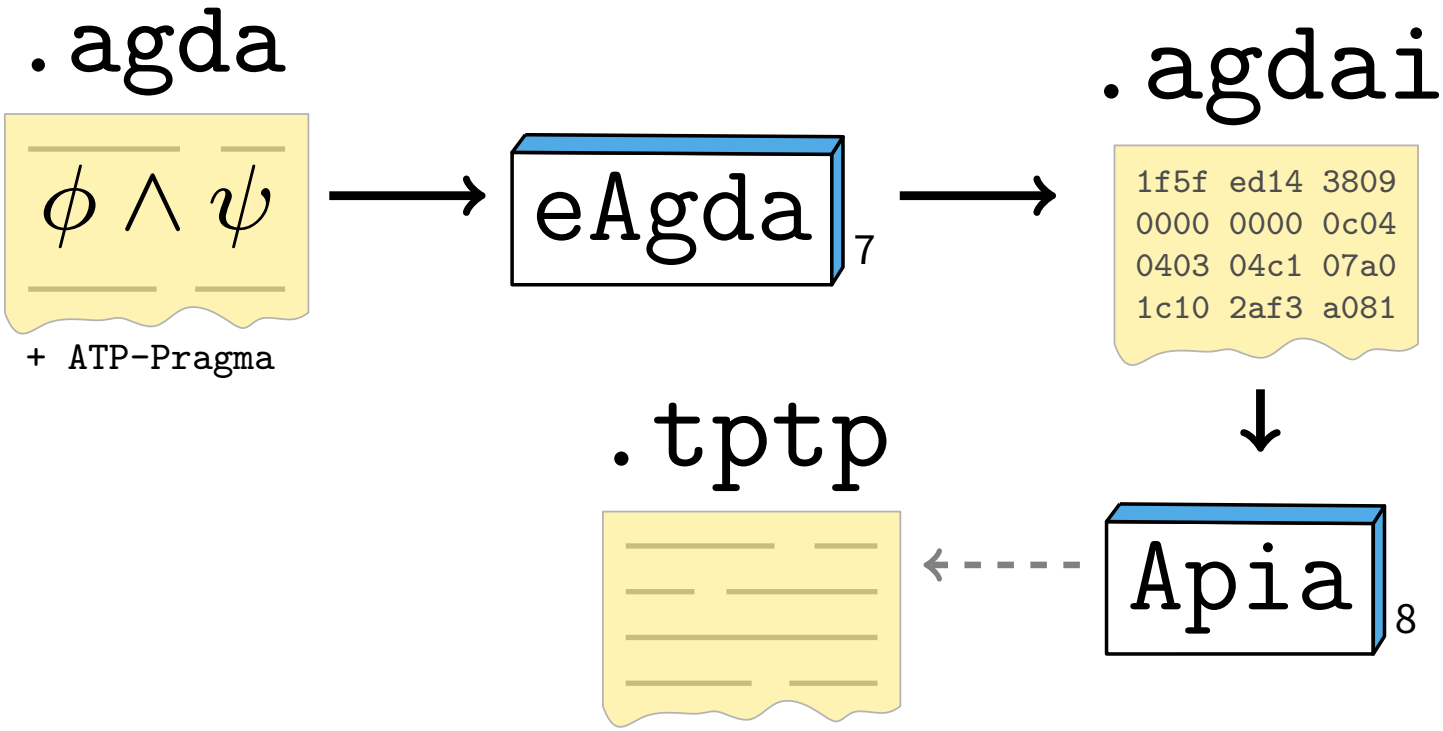

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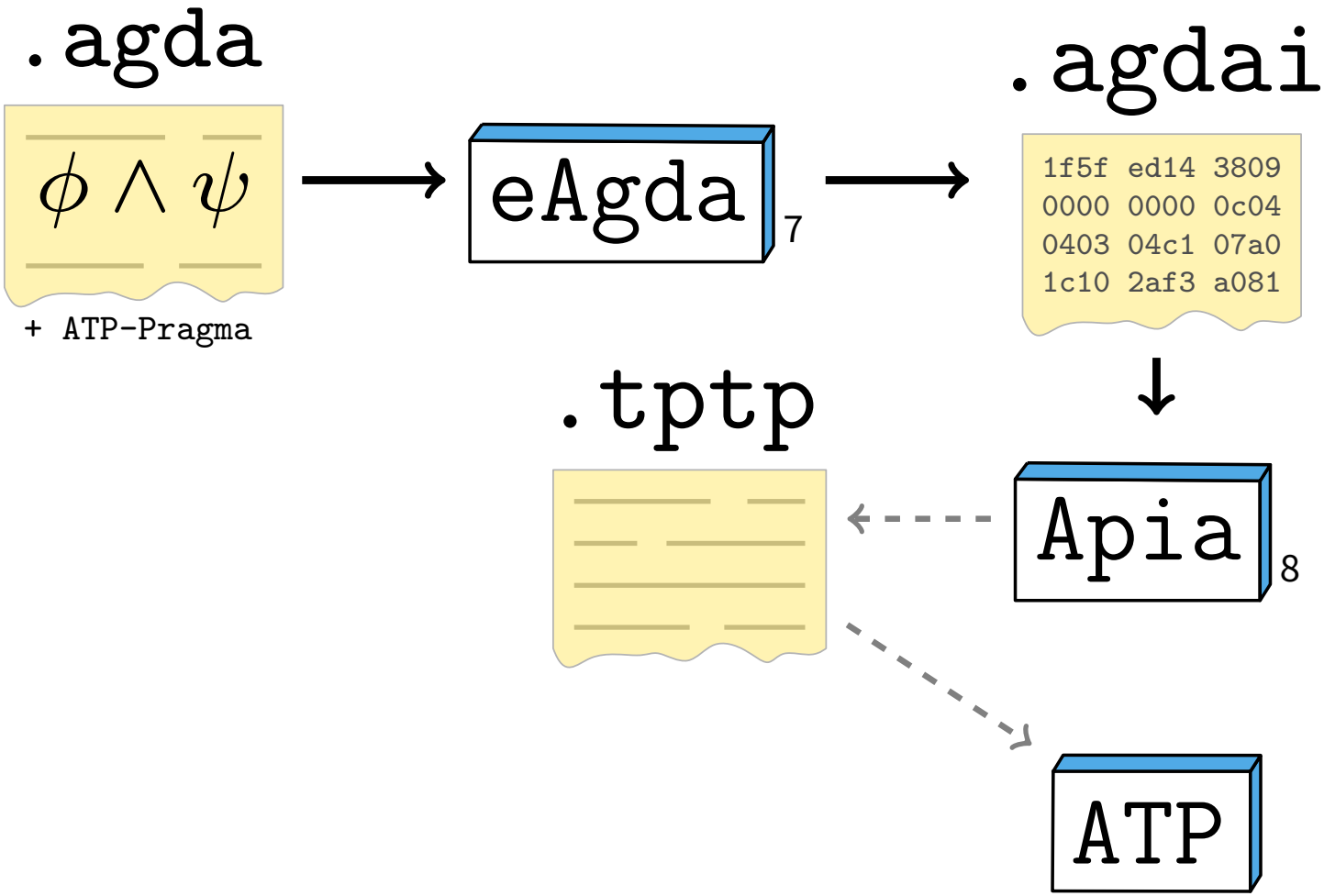

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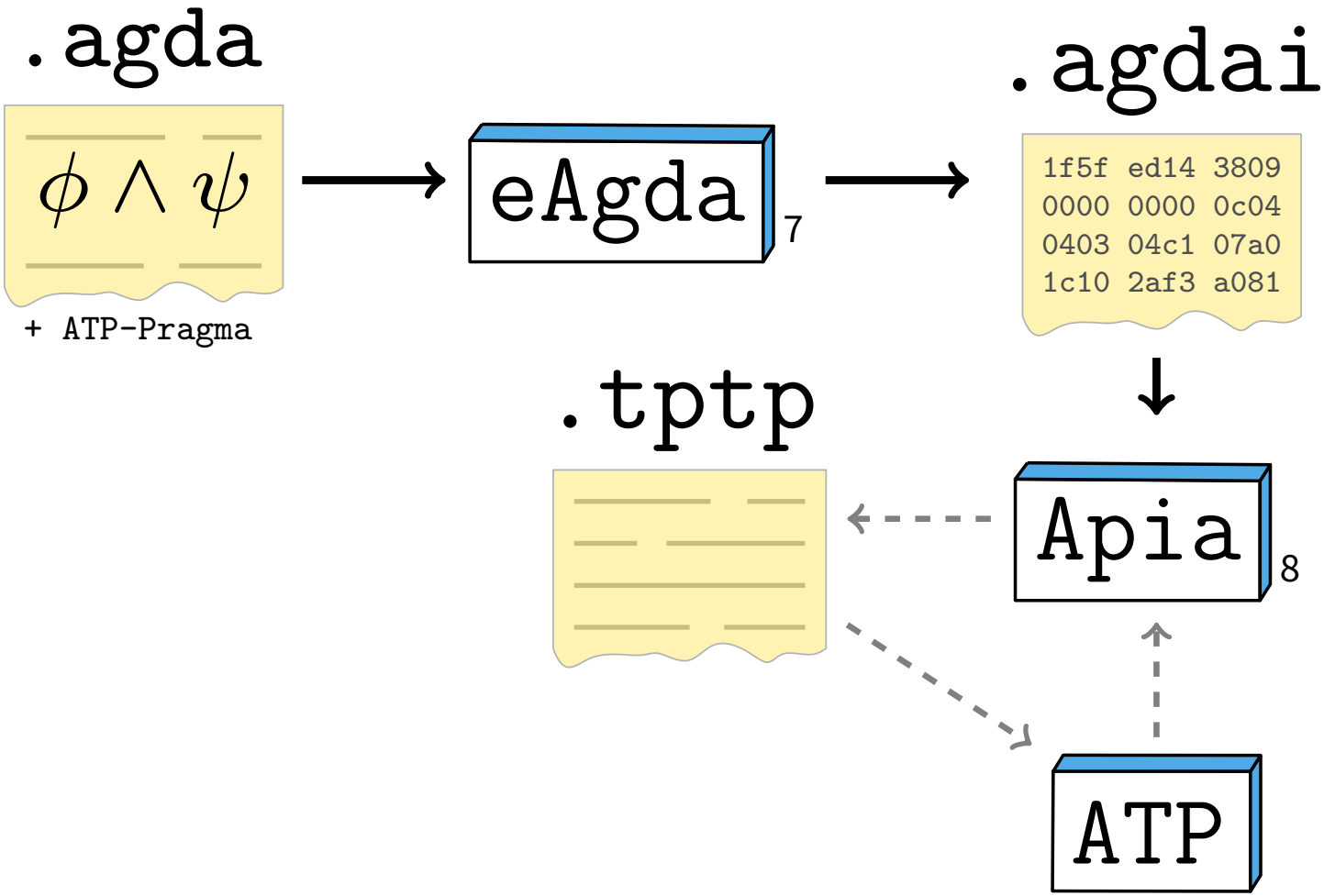

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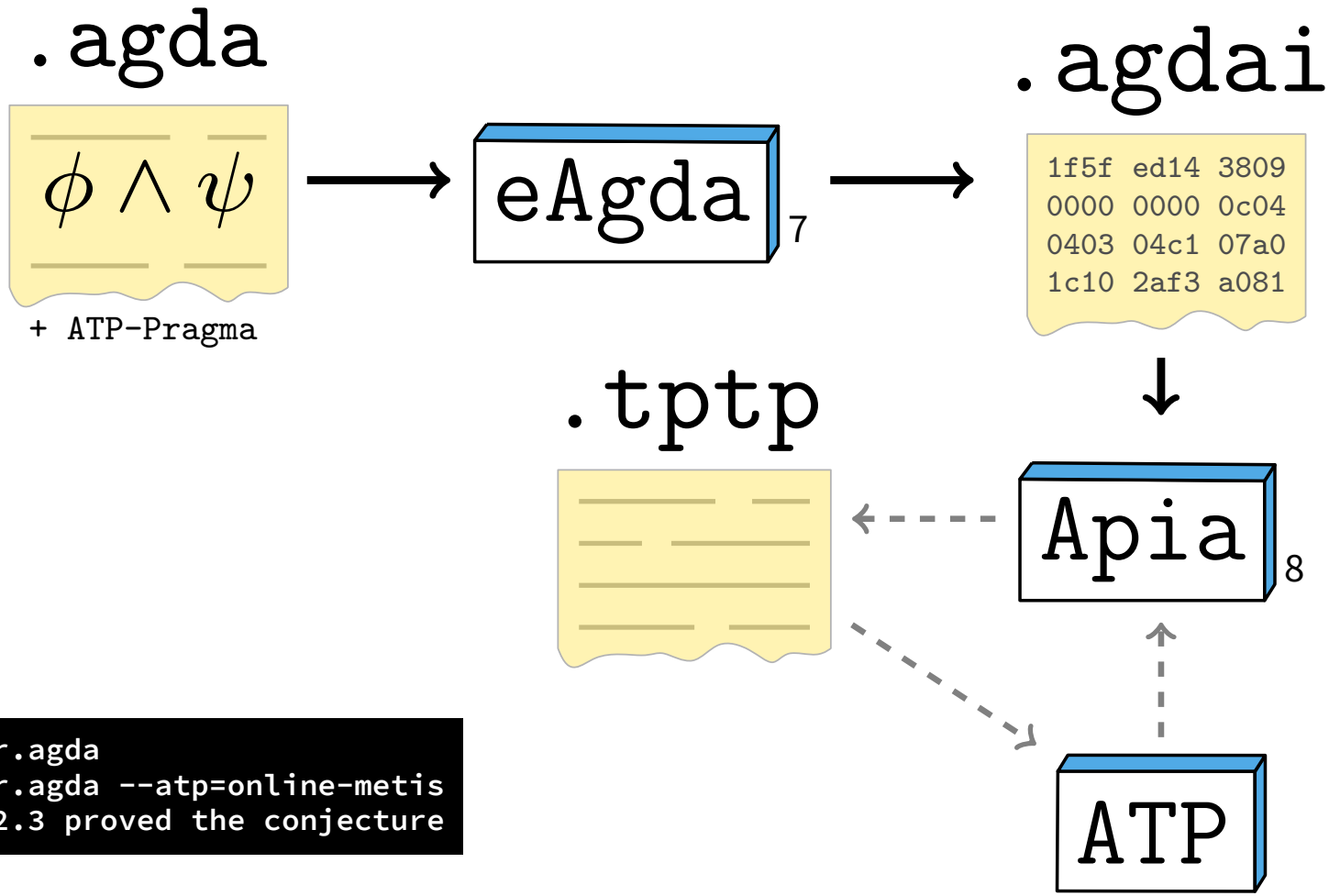


```
$ agda Or.agda
$ apia Or.agda --atp=online-metis
Metis---2.3 proved the conjecture
```

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Future Work

- Add shallow embedding in order to work with **`Apia`**
- Support First-Order Logic with Equality
- Support another prover like **`EProver`**