

# Reconstructing Propositional Proofs in Type Theory

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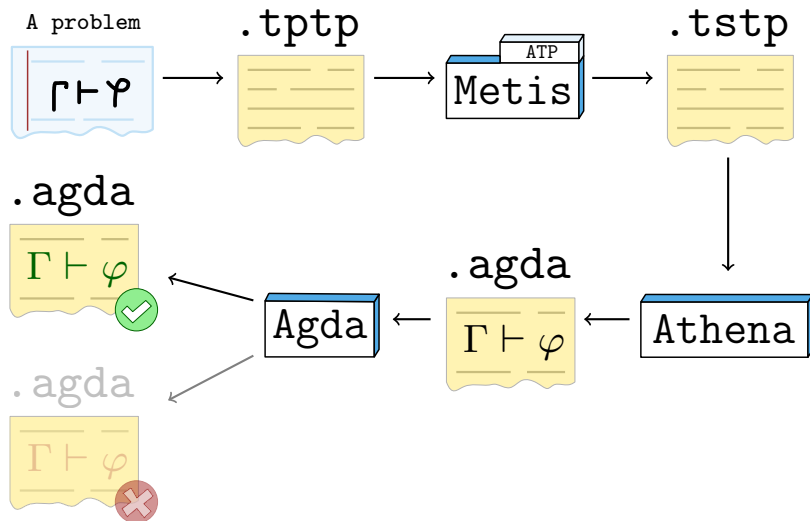
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## Proof Reconstruction: Overview



Metis is an automatic theorem prover for first-order logic with equality developed by Hurd, 2003. For the propositional fragment, Metis has only three inference rules:

$$\frac{}{\Gamma \vdash \varphi} \text{ axiom}$$

$$\frac{}{\Gamma \vdash \varphi \vee \neg \varphi} \text{ assume } \varphi$$

$$\frac{\Gamma \vdash l \vee \varphi \quad \Gamma \vdash \neg l \vee \psi}{\Gamma \vdash \varphi \vee \psi} \text{ resolve } l$$

Where  $\Gamma$  is a set of premises, and  $\_ \vdash \_$  is the syntactical consequence relation between the premises on the right, and the conclusion on the left.

- ▶ Open source implemented in Standard ML
- ▶ Each refutation step is one of *six rules*
- ▶ Reads problem in TPTP format
- ▶ Outputs *detailed* proofs in TSTP format

TSTP derivations by Metis exhibit the following inferences:

Metis rule	Purpose
strip	Strip a goal into subgoals
conjunct	Takes a formula from a conjunction
resolve	A general form of the resolution theorem
canonicalize	Normalization of the formula
clausify	Performs clausification
simplify	Simplify definitions and theorems



Hurd, Joe (2003). “First-order Proof Tactics In Higher-order Logic Theorem Provers”. In: *Design and Application of Strategies/Tactics in Higher Order Logics, number NASA/CP-2003-212448 in NASA Technical Reports*, pp. 56–68. URL: <http://www.gilith.com/research/papers>.