Quantum Communication Complexity

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Contents

Qubit/Qudit Model

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- Logic gates $\rightarrow$ unitary operators

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- Weyl-Heisenberg Operators

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- Gottesman-Knill Theorem

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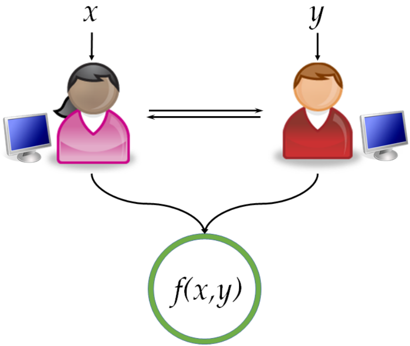
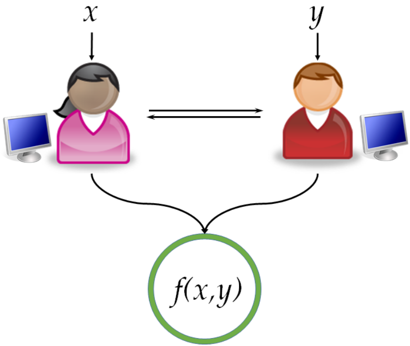
- Clifford $\implies W_U(\beta|\alpha) \geq 0 \forall \alpha, \beta \in \mathbb{Z}_d^2$
- $\mathcal{M}_\rho = \sum_{\alpha \in \mathbb{Z}_d^2} |W_\rho(\alpha)|$
- $\mathcal{M}_U = \max_{\alpha \in \mathbb{Z}_d^2} \sum_{\beta \in \mathbb{Z}_d^2} |W_U(\beta|\alpha)|$

Raz's Problems



Raz's Problems



- Measure information exchanged

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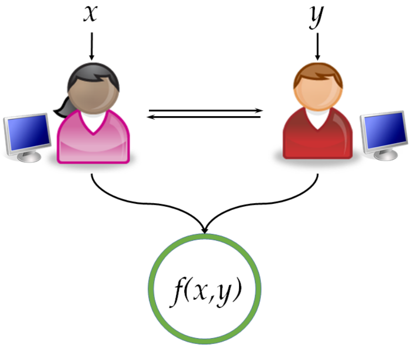

- Measure information exchanged
- Early proof of exponential separation

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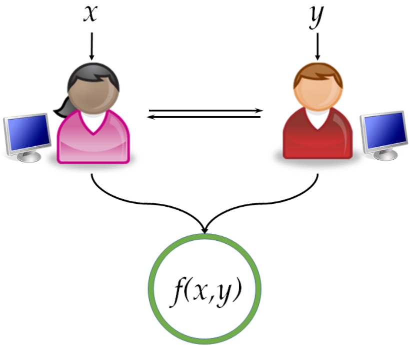

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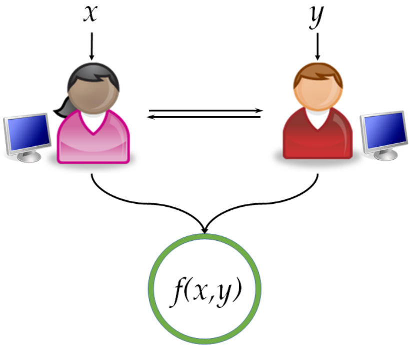
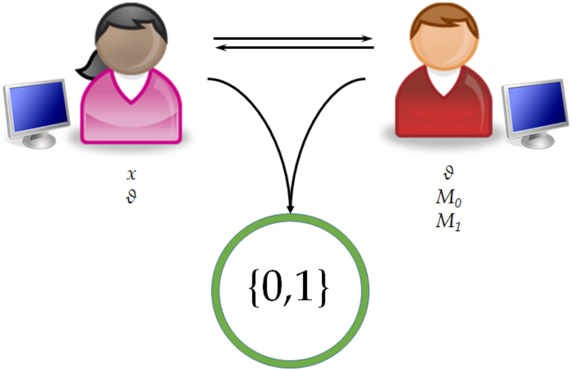


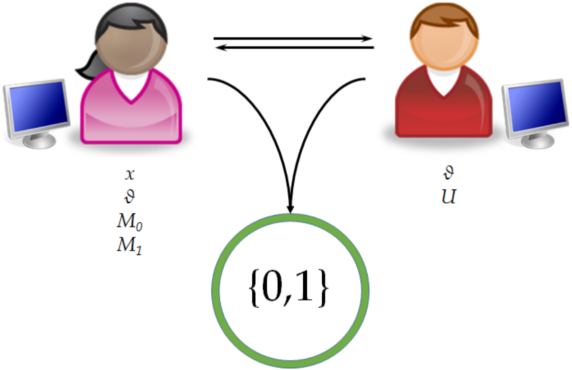
- Measure information exchanged
- Early proof of exponential separation
- Two main variants
- $O(\log d)$ communication complexity for quantum algorithm

Raz's Problem: Variant 1



- Return 0 if $d(x, M_0) < \vartheta$, 1 otherwise

Raz's Problem: Variant 2



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Convergence and Complexity

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- $\Pr(|\hat{R} - \mathbb{E}(R)|) \geq \epsilon) \leq 2e^{\frac{-T\epsilon^2}{2\mathcal{M}^2}}$

Acknowlegments

- Joseph Emerson
- Joel Wallman
- Mark Howard

Refrences

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