Ex 7

Chapter 7. Laplace Transform

Date (yy/mm/dd): 2021, 5, 6

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Math Problem: (10 × 5 points)

- Given a signal $x(t)$:

$$x(t) = e^{-3t}u(t) + e^{2t}u(-t)$$

1. Find the Laplace transform $X(s)$
2. Sketch the ~~pole-zero~~ plot and the ROC

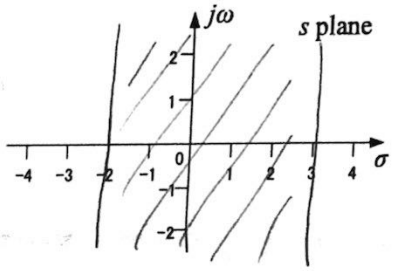
   Hint: use the tables and properties in slides 18-23 to derive the Laplace transform of the given signal $x(t)$

Answer:

From the tables and properties, we have

$$e^{-3t}u(t) \leftrightarrow \frac{1}{s-3}, \quad \mathrm{Re}(s) < 3$$

$$e^{2t}u(-t) \leftrightarrow \frac{1}{s+2}, \quad \mathrm{Re}(s) > -2$$



Hence, we see that the ROCs in two parts overlap, and thus

$$X(s) = \frac{1}{s-3} + \frac{1}{s+2} = \frac{2s-1}{(s-3)(s+2)}, \quad -2 < \mathrm{Re}(s) < 3$$

From the above $X(s)$, we see that it has poles $s = -3, 3$ and zeros $s = \frac{1}{2}$, and the ROC is sketched as above.

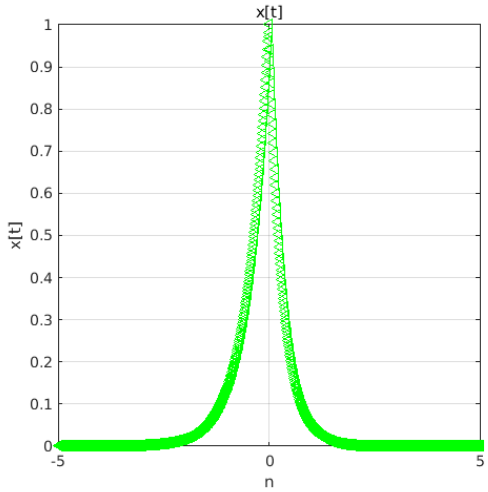
MATLAB Problem: (5 × 10 points)

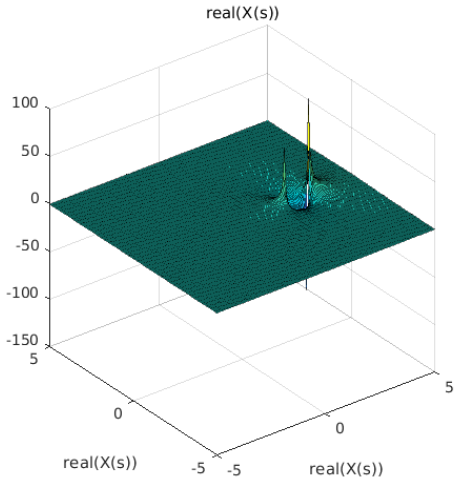
- Develop a MATLAB program to do the following tasks and submit your results including the MATLAB .m file and a 2 × 2 plot. (t = -5:0.01:5) (σ = -5:0.05:5, ω = -5:0.05:5)

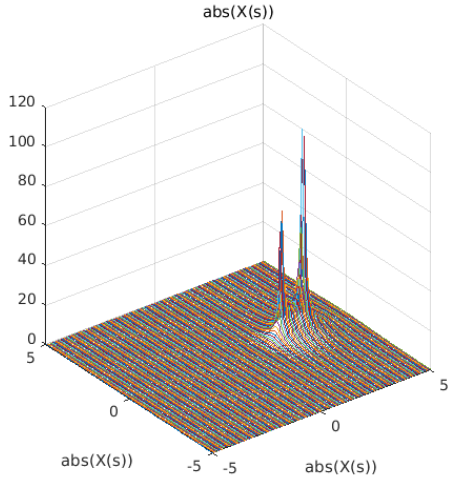
  Hint: use "help" to check commands such as flip, meshgrid, ones, plot3, and surf.

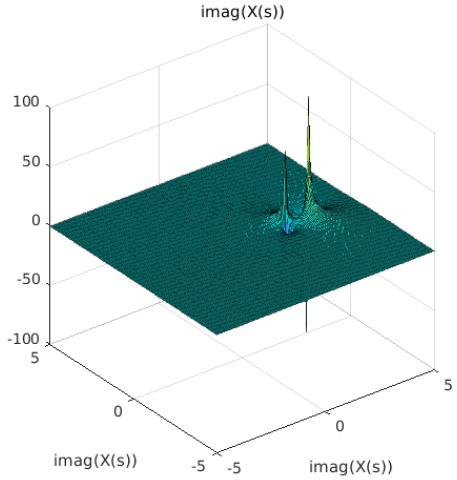
1. plot($x(t)$) in subplot 221;
2. plot3(σ, ω, abs($X(s)$)) on s-plane in subplot 223;
3. surf(σ, ω, real($X(s)$)) on s-plane in subplot 222;
4. surf(σ, ω, imag($X(s)$)) on s-plane in subplot 224;
5. find the coordinates of poles and zeros on s-plane.

```
t=-5:0.01:5;
[gm,mg]=meshgrid(-5:0.05:5, -5:0.05:5);
x=exp(2*t).*heaviside(-t)+exp(-3*t).*heaviside(t);
X=(2*(gm+1i*mg)-1)./(((gm+1i*mg)-3).*((gm+1i*mg)-2));

% 1

figure;
figure_size = [ 0, 0, 1600, 1600];
set(gcf, 'Position', figure_size);
subplot(2,2,1)
plot(t,x,'g<');
xlabel("n")
ylabel("x[t]")
subtitle("x[t]")
grid on
axis square

% % % % % % % % % % % % % % % %

% 2

subplot(2,2,2)
surf(gm,mg,real(X), 'EdgeAlpha',0.5);
xlabel("real(X(s))")
ylabel("real(X(s))")
subtitle("real(X(s))")
grid on
axis square

% % % % % % % % % % % % % % % %

% 3

subplot(2,2,3)
plot3(gm, mg, abs(X))
xlabel("abs(X(s))")
ylabel("abs(X(s))")
subtitle("abs(X(s))")
grid on
axis square

% % % % % % % % % % % % % % % %

% 4

subplot(2,2,4)
surf(gm,mg,imag(X), 'EdgeAlpha',0.5);
xlabel("imag(X(s))")
ylabel("imag(X(s))")
subtitle("imag(X(s))")
grid on
axis square
```

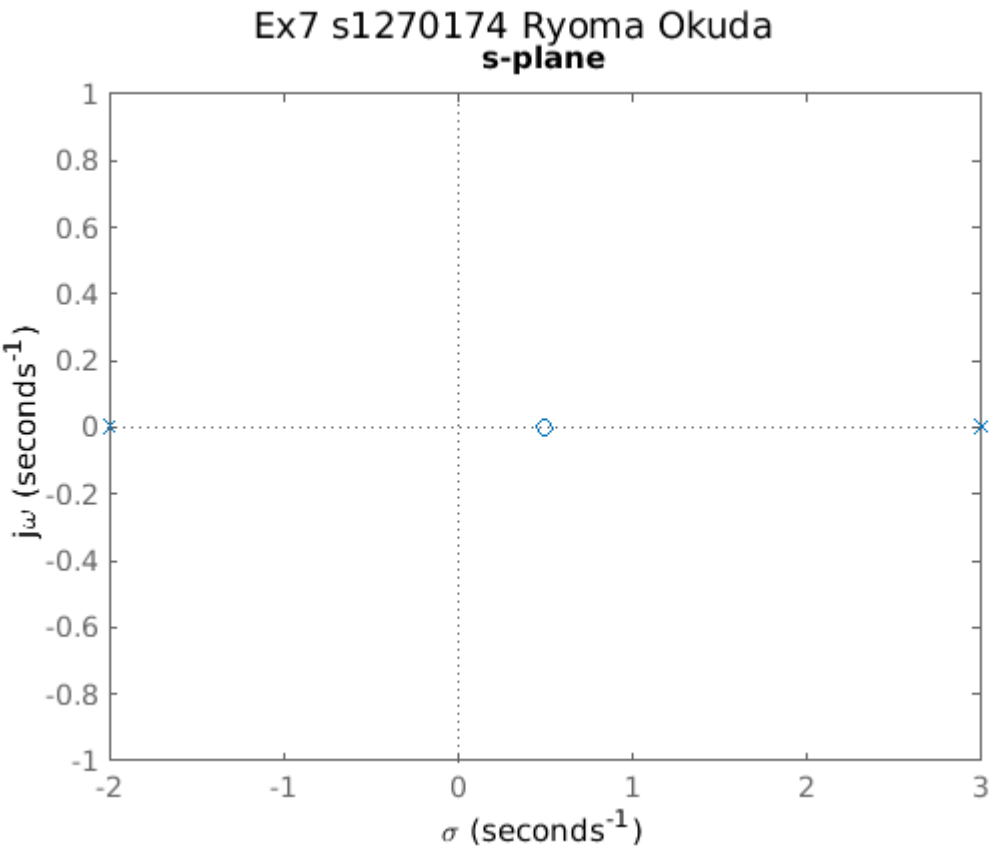









```
% % % % % % % % % % % % % % % %

% s plane

H = tf([0 2 -1],[1 -1 -6]);
figure;
pzplot(H);
xlabel("\sigma");
ylabel("j\omega");
title("s-plane");

sgtitle("Ex7 s1270174 Ryoma Okuda")
```


Ex7 s1270174 Ryoma Okuda
s-plane
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-2
-1
0
1
2
3
jω (seconds-1)
σ (seconds-1)