Ex 6

Chapter 6. Fourier Transform Analysis of Signals and Systems

Date (yy/mm/dd): 2021/. 04./ 30

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Math Problem: (8×5 points)

- Given an ideal low-pass filter frequency response

$$H(\omega) = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & |\omega| > \omega_c \end{cases}$$

1. Find its impulse response $h(t)$

   Hint: 1. use inverse Fourier transform definition to find $h(t)$

   2. *sinc* function is defined in MATLAB as $sinc(t) = \begin{cases} \frac{sin(\pi t)}{\pi t}, & t \neq 0 \\ 1, & t = 0 \end{cases}$

Answer:

By definition, we have

$$h(t) = \mathcal{F}^{-1}(H(\omega)) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\omega}^{\omega_c} e^{j\omega t} d\omega$$

$$= \frac{1}{j2\pi t}\left[e^{j\omega t}\right]_{-\omega_c}^{\omega_c} = \frac{e^{j\omega t} - e^{-j\omega t}}{j2\pi t} = \frac{\sin(\omega_c t)}{\pi t} = \frac{\omega_c}{\pi}\cdot\frac{\sin(\omega_c t)}{\omega_c t} = \frac{\omega_c}{\pi}\cdot sinc\left(\frac{\omega_c t}{\pi}\right)$$

Hence,

$$\frac{\omega_c}{\pi}\cdot\frac{\sin(\omega_c t)}{\omega_c t} \text{ or } \frac{\omega_c}{\pi}\cdot sinc\left(\frac{\omega t}{\pi}\right) \overset{FT}{\Leftrightarrow} \begin{cases} 1, & |\omega| < \omega_c \\ 0, & |\omega| > \omega_c \end{cases}$$

MATLAB Problem: ($3 \times 3 \times 5 + 15$ points)

- Develop a MATLAB program to do the following tasks, and submit your results including 3×3 charts and your MATLAB .m file. ($\omega_c = 2$, ω = -2π:0.1:2π:, n = -40:1:40;)

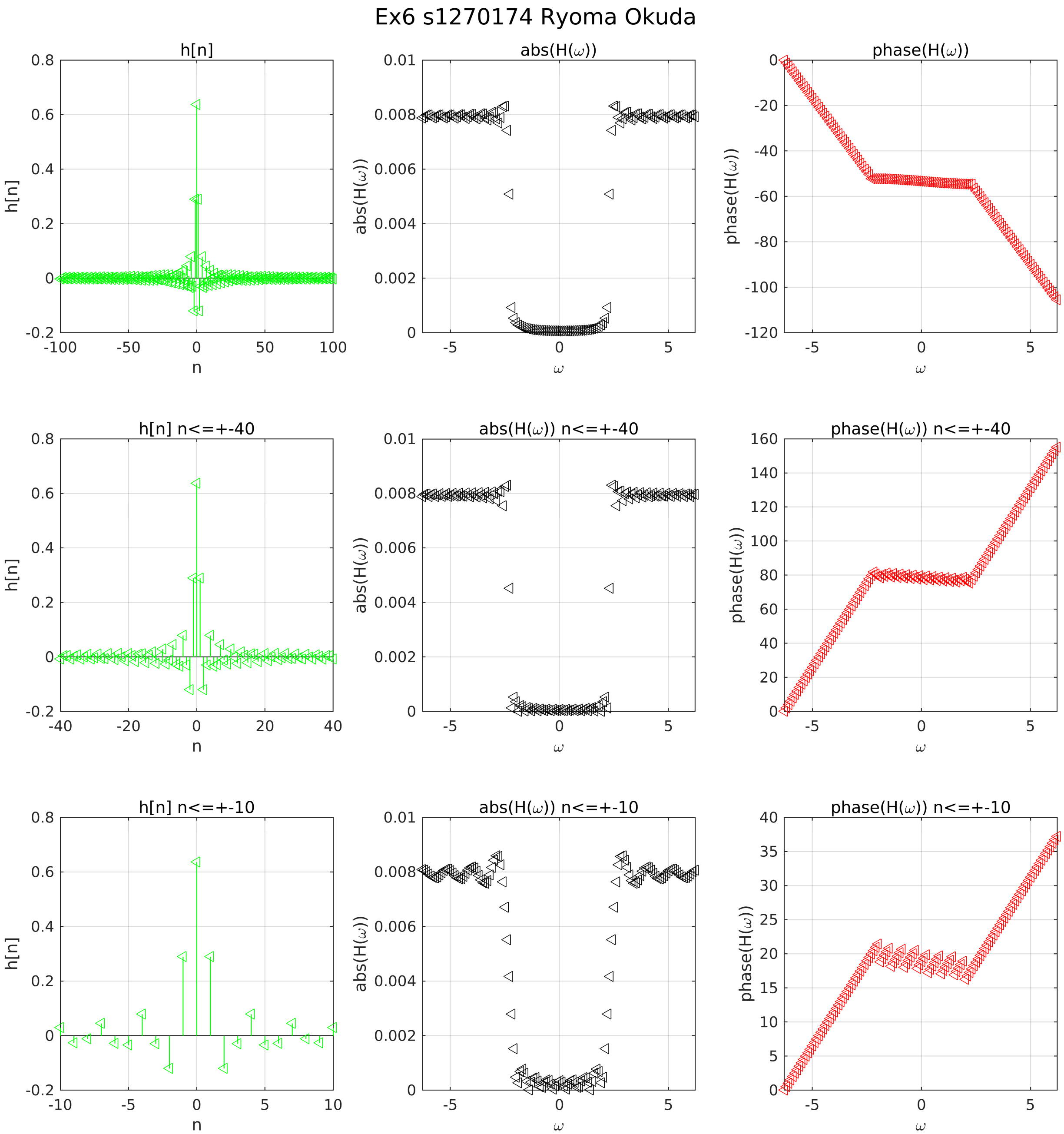
  Hint: use ***sinc*** function for $h(n)$

1. Plot the above $h[n]$, abs($H(\omega)$), phase($H(\omega)$) in subplots 331, 332, 333, respectively;
2. Use limited length $h_{40}[n]$ ($\pm$40) to calculate its $H_{40}(\omega)$, and plot $h_{40}[n]$, abs($H_{40}(\omega)$), phase($H_{40}(\omega)$) in subplots 334, 335, 336, respectively;
3. Use limited length $h_{10}[n]$ ($\pm$10) to calculate its $H_{10}(\omega)$, and plot $h_{10}[n]$, abs($H_{10}(\omega)$), phase($H_{10}(\omega)$) in subplots 337, 338, 339, respectively;
4. Observe the relationship between impulse response and frequency responses (amplitude and phase), and discuss how the length of impulse response gives impact on the frequency responses (amplitude and phase).

```
n=-100:1:100;

mc=2;
m=-2*pi:0.1:2*pi;

h=(mc/pi)*sinc(mc/pi*n);
H=ifft(h,length(m));

% 1

figure;
figure_size = [ 0, 0, 1600, 1600];
set(gcf, 'Position', figure_size);
subplot(3,3,1)
stem(n,h,'g<');
xlabel("n")
ylabel("h[n]")
subtitle("h[n]")
grid on
axis square

% % % % % % % % % % % % % % % %

% 2

subplot(3,3,2)
plot(m,abs(H),'black<');
xlabel("\omega")
ylabel("abs(H(\omega))")
subtitle("abs(H(\omega))")
grid on
axis square

% % % % % % % % % % % % % % % %

% 3

subplot(3,3,3)
plot(m,phase(H),'red<');
xlabel("\omega")
ylabel("phase(H(\omega))")
subtitle("phase(H(\omega))")
grid on
axis square

% % % % % % % % % % % % % % % %

n=-40:1:40;
h=(mc/pi)*sinc(mc/pi*n);
H=ifft(h,length(m));
% 4

subplot(3,3,4)
```

```
stem(n,h,'g<');
xlabel("n")
ylabel("h[n]")
subtitle("h[n] n<=+-40")
grid on
axis square

% % % % % % % % % % % % % % % %

% 5

subplot(3,3,5)
plot(m,abs(H),'black<');
xlabel("\omega")
ylabel("abs(H(\omega))")
subtitle("abs(H(\omega)) n<=+-40")
grid on
axis square

% % % % % % % % % % % % % % % %

% 6

subplot(3,3,6)
plot(m,phase(H),'red<');
xlabel("\omega")
ylabel("phase(H(\omega))")
subtitle("phase(H(\omega)) n<=+-40")
grid on
axis square

% % % % % % % % % % % % % % % %

n=-10:10;
h=(mc/pi)*sinc(mc/pi*n);
H=ifft(h,length(m));

% 7

subplot(3,3,7)
stem(n,h,'g<');
xlabel("n")
ylabel("h[n]")
subtitle("h[n] n<=+-10")
grid on
axis square

% % % % % % % % % % % % % % % %

% 8

subplot(3,3,8)
plot(m,abs(H),'black<');
xlabel("\omega")
```

```
ylabel("abs(H(\omega))")
subtitle("abs(H(\omega)) n<=+-10")
grid on
axis square

% % % % % % % % % % % % % % % %

% 9

subplot(3,3,9)
plot(m,phase(H),'red<');
xlabel("\omega")
ylabel("phase(H(\omega))")
subtitle("phase(H(\omega)) n<=+-10")
grid on
axis square

% % % % % % % % % % % % % % % %

sgtitle("Ex6 s1270174 Ryoma Okuda")
```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h[n]
abs(H(ω))
phase(H(ω))
h[n] n<=+-40
abs(H(ω)) n<=+-40
phase(H(ω)) n<=+-40
h[n] n<=+-10
abs(H(ω)) n<=+-10
phase(H(ω)) n<=+-10
n
ω

> Observe the relationship between impulse response and frequency responses (amplitude and phase), and discuss how the length of impulse response gives impact on the frequency responses (amplitude and phase).

Getting n value smaller, the high frequency seems like high pass filtered since the wave is getting more yoink than original wave.
Also when n<+-40, the phase is inverted.