# Ex 3

Chapter 3. Linear Time-Invariant Systems (Discrete-Time)

Date (yy/mm/dd): 2021/. 04. 20

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Math Problem: (6 × 10 points)

- The unit step sequence input $x[n]$ and the impulse response $h[n]$ of a discrete-time LTI system are given by

$$x[n] = u[n], \qquad h[n] = \alpha^n u[n], \qquad 0 < \alpha < 1$$

Hint: $\sum_{n=0}^{N-1} \alpha^n = \begin{cases} \frac{1-\alpha^N}{1-\alpha}, & \alpha \neq 1 \\ N, & \alpha = 1 \end{cases}$

1. Compute the convolution sum, or the system output $y[n] = x[n] * h[n]$.

Answer:

By the definition, we have

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

For $n < 0$, $x[k]$ and $h[n-k]$ do not overlap, $y[n] = 0$

For $n \geq 0$, they overlap from $k = 0$ to $k = n$, we have

$$y[n] = x[n] * h[n] = \sum_{k=0}^{n} x[k]h[n-k] = \sum_{k=0}^{n} h[n-k]$$

Changing the variable of summation $k$ to $m = n - k$ and using the given hint, we have

$$y[n] = \sum_{m=n}^{0} h[m] = \sum_{m=0}^{n} \alpha^{n-k} = \frac{1-\alpha^{n+1}}{1-\alpha}$$

Thus, we can write the output $y[n]$ as

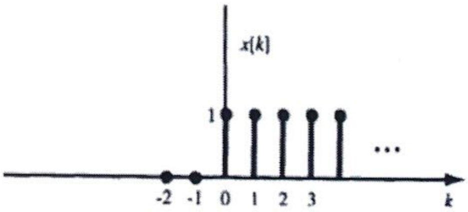
$$y[n] = \frac{1-\alpha^{n+1}}{1-\alpha} \cdot u(n)$$

MATLAB Problem: (4 × 10 points)

- Develop a MATLAB program to do the following tasks and submit your results including 4 charts and the MATLAB program ($\alpha$=0.5; $k$=-5:1:5).

Hint: use stem($k$, $x$) to draw the discrete time signal (sequence) $x[k]$

1. Plot a chart indicating $x[k]$ sequence as right chart
2. Plot a chart indicating $h[-2-k]$ sequence
3. Plot a chart indicating $h[3-k]$ sequence
4. Calculate $y[k]$ and plot a chart indicating $y[k]$ sequence



```
k=-5:1:5;
a=0.5;
mid=int8(length(k)/2);
x=heaviside(k);
x(mid)=1;
% % % % % % % % % % % % % % % %

% 1

figure;
subplot(2,4,1)
stem(k,x,'g<');
ylabel("Amplitude")
xlim([-10 10])
ylim([-1.5 1.5])
subtitle("x[k]")
grid on
axis square

% % % % % % % % % % % % % % % %

% 2

h1=a.^(-2-k).*heaviside(-2-k);
h1(-2+mid)=1;

subplot(2,4,2)
stem(k,h1,'black<');
ylabel("Amplitude")
xlim([-10 10])
ylim([-1.5 1.5])
subtitle("h[-2-k]")
grid on
axis square

% % % % % % % % % % % % % % % %

% 3

h2=a.^(3-k).*heaviside(3-k);
h2(3+mid)=1;

subplot(2,4,3)
stem(k,h2,'red<');
ylabel("Amplitude")
xlim([-10 10])
ylim([-1.5 1.5])
subtitle("h[3-k]")
grid on
axis square

% % % % % % % % % % % % % % % %
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% 4

y=(1-a.^(k+1))/(1-a).*heaviside(k);
y(mid)=1;

subplot(2,4,4)
stem(k,y,'blue<');
ylabel("Amplitude")
xlim([-10 10])
ylim([-3 3])
subtitle("y[k]")
grid on
axis square


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sgtitle("Ex2 s1270174 Ryoma Okuda")
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