Ex 10

Chapter 10. Digital Filter Design I: FIR Filter

Date (yy/mm/dd): 2021 / 05. / 25

Student ID: s1270174 Name: Ryoma Okuda Score: ______/100

Background

- A desired LPF $H_{\mathrm{LPF}}(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \omega_c \\ 0, & \omega_c < |\omega| \le \pi \end{cases}$
- IDTFT of $H_{\mathrm{LPF}}(e^{j\omega})$ is computed to get the corresponding impulse response $h_{\mathrm{LPF}}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{\mathrm{LPF}}(e^{j\omega})e^{j\omega n}d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega n}d\omega = \frac{1}{2\pi}\left(\frac{e^{j\omega_c n}}{jn} - \frac{e^{-j\omega_c n}}{jn}\right) = \frac{\sin\omega_c n}{\pi n}, \quad -\infty < n < \infty, n \neq 0$
- For n=0, $h_{\mathrm{LPF}}[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{\mathrm{LPF}}(e^{j\omega})d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} d\omega = \frac{\omega_c}{\pi}$
- In summary, we have the LPF impulse response $h_{\mathrm{LPF}}[n] = \begin{cases} \frac{\omega_c}{\pi} & n = 0 \\ \frac{\sin\omega_c n}{\pi n} & n \neq 0, -\infty < n < \infty \end{cases}$
- $h_{\mathrm{LPF}}[n]$ is doubly infinite, not absolutely summable, and therefore unrealizable.
- Setting $h_{\mathrm{LPF}}[n]$ outside the range $-M \le n \le M$ to zero gets a finite-length non-causal approximation of length $N = 2M + 1$.
- Shifting $h_{\mathrm{LPF}}[n]$ to the right yields a finite-length causal LPF:

$$\hat{h}_{\mathrm{LPF}}[n] = \begin{cases} \frac{\omega_c}{\pi} & n = M \\ \frac{\sin\omega_c(n-M)}{\pi(n-M)} & 0 \le n \le N-1 \end{cases}$$

- A variety of approaches have been proposed to design FIR digital filters. A direct and straightforward method is based on truncating the Fourier series representation of the prescribed frequency response.
- Given the desired frequency response $H_{\mathrm{d}}(e^{j\omega})$, we can compute the corresponding impulse response sequence

$$h_{\mathrm{d}}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{\mathrm{d}}(e^{j\omega})e^{j\omega n}d\omega, \quad -\infty < n < \infty$$

- In this case, $h_{\mathrm{d}}[n]$ is of infinite length and noncausal, and impossible to realize in practical application.
- Our objective is to find a finite-duration impulse response sequence $h_{\mathrm{t}}[n]$ of length $2M+1$ whose DTFT $H_{\mathrm{t}}(e^{j\omega}) = \sum_{n=-M}^{M} h_{\mathrm{t}}[n]e^{-j\omega n}$ approximates $H_{\mathrm{d}}(e^{j\omega})$ in some sense.
- One commonly used approximation criterion is to minimize the integral-squared error

$$\Phi = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|H_{\mathrm{t}}(e^{j\omega}) - H_{\mathrm{d}}(e^{j\omega})\right|^2 d\omega$$

- Using Parseval's relation, we have

$$\Phi = \sum_{n=-\infty}^{\infty} |h_{\mathrm{t}}[n] - h_{\mathrm{d}}[n]|^2 = \sum_{n=-M}^{M} |h_{\mathrm{t}}[n] - h_{\mathrm{d}}[n]|^2 + \sum_{n=-\infty}^{-M-1} |h_{\mathrm{d}}[n]|^2 + \sum_{n=M+1}^{\infty} |h_{\mathrm{d}}[n]|^2$$

- It is evident that Φ is minimum when $h_{\mathrm{t}}[n] = h_{\mathrm{d}}[n]$ for $-M \le n \le M$.
- It means that the best finite-length approximation $h_{\mathrm{t}}[n]$ to the desired infinite-length impulse response $h_{\mathrm{d}}[n]$ in the mean-square error sense is simply obtained by truncation, or multiplying a rectangular window $w[n]$ in time domain $h_{\mathrm{t}}[n] = h_{\mathrm{d}}[n] \cdot w[n]$, or convoluting the Fourier transform $H_{\mathrm{w}}(e^{j\omega})$ of the rectangular window in frequency domain $H_{\mathrm{t}}(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{\mathrm{d}}(e^{j\varphi}) H_{\mathrm{w}}(e^{j(\omega-\varphi)}) d\varphi$
- Simply truncating $h_{\mathrm{d}}[n]$ to get a causal FIR filter $h_{\mathrm{t}}[n]$ leads an oscillatory behavior in its magnitude response (Gibbs phenomenon).
- The Gibbs phenomenon can be reduced by either using tapered windows such as Bartlett, Hann, Hamming, Blackman (tapers smoothly to zero at each end) or providing a smooth transition band.
- Using a tapered window reduces the height of the side-lobes, but increases the main-lobe width and results in a wider transition at the discontinuity.
- A causal FIR filter with an impulse response $h[n]$ can be derived from $h_{\mathrm{t}}[n]$ by delaying the latter sequence by *M* samples, $h[n] = h_{\mathrm{t}}[n-M]$
- Note that the causal filter $h[n]$ has the same magnitude response as that of the noncausal filter $h_{\mathrm{t}}[n]$ and its phase response has a linear phase shift of ωM radians with respect to that of the noncausal filter $h_{\mathrm{t}}[n]$.

Math Problem: (5×10 points)

- Given the magnitude response of a desired LPF below, where $\omega_c = 0.5\pi$ and *M*=10

$$|H_{\mathrm{LPF}}(e^{j\omega})| = \begin{cases} 1, & |\omega| \le \omega_c \\ 0, & \omega_c < |\omega| \le \pi \end{cases}$$

1. Determine its desired impulse response $h_{\mathrm{LPF}}[n]$.
2. Determine its truncated impulse response $h_{\mathrm{LPFRect}}[n]$ by using a rectangular window.
3. Determine its truncated impulse response $h_{\mathrm{LPFHann}}[n]$ by using a Hann window.
4. Determine its truncated impulse response $h_{\mathrm{LPFHamming}}[n]$ by using a Hamming window.
5. Determine its truncated impulse response $h_{\mathrm{LPFBlackman}}[n]$ by using a Blackman window.

Answers:

1. IDFT of $H_{\mathrm{LPF}}(e^{j\omega})$ is computed to get the corresponding impulse response

   For *n*=0, $h_{\mathrm{LPF}}[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{\mathrm{LPF}}(e^{j\omega}) d\omega$

   For *n*≠0, $h_{\mathrm{LPF}}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{\mathrm{LPF}}(e^{j\omega}) e^{j\omega n} d\omega$

   Therefore, we have the desired LPF impulse response $h_{\mathrm{LPF}}[n] = \begin{cases} \frac{\text{sin(wc * n)}}{\pi \text{ * n}}, & n = 0 \\ & n \ne 0, -\infty < n < \infty \end{cases}$

2. Multiplying a rectangular window $w_{\mathrm{Rect}}[n] = \begin{cases} 1 & 0 \le n \le 20 \\ 0 & \text{else} \end{cases}$

   We have $h_{\mathrm{LPFRect}}[n] = h_{\mathrm{LPF}}[n] \cdot w_{\mathrm{Rect}}[n]$

$$h_{\text{LPFRect}}[n] = \begin{cases} \dfrac{\sin(wc * n)}{\pi * n}, & n = 0 \\ & 0 < n <= 20 \end{cases}$$

3. Multiplying a Hann window $w_{\text{Hann}}[n] = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{20}\right) & 0 \leq n \leq 20 \\ 0 & \text{else} \end{cases}$

   We have $h_{\text{LPFHann}}[n] = h_{\text{LPF}}[n] \cdot w_{\text{Hann}}[n]$

$$h_{\text{LPFHann}}[n] = \begin{cases} \dfrac{\sin(wc * n)}{\pi * n} \left[0.5 - 0.5 * \cos\left(\dfrac{2\pi * n}{20}\right)\right] & n = 0 \\ & 0 < n <= 20 \end{cases}$$

4. Multiplying a Hamming window $w_{\text{Hamming}}[n] = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{20}\right) & 0 \leq n \leq 20 \\ 0 & \text{else} \end{cases}$

   We have $h_{\text{LPFHamming}}[n] = h_{\text{LPF}}[n] \cdot w_{\text{Hamming}}[n]$

$$h_{\text{LPFHamming}}[n] = \begin{cases} \dfrac{\sin(wc * n)}{\pi * n} \left[0.54 - 0.46 * \cos\left(\dfrac{2\pi * n}{20}\right)\right] & n = 0 \\ & 0 < n <= 20 \end{cases}$$

5. Multiplying a Blackman window $w_{\text{Blackman}}[n] = \begin{cases} 0.42 - 0.5\cos\left(\frac{2\pi n}{20}\right) + 0.08\cos\left(\frac{4\pi n}{20}\right) & 0 \leq n \leq 20 \\ 0 & \text{else} \end{cases}$

   We have $h_{\text{LPFBlackman}}[n] = h_{\text{LPF}}[n] \cdot w_{\text{Blackman}}[n]$

$$h_{\text{LPFBlackman}}[n] = \begin{cases} \dfrac{\sin(wc * n)}{\pi * n} \left[0.42 - 0.5 * \cos\left(\dfrac{2\pi * n}{20}\right) + 0.08 * \cos\left(\dfrac{2\pi * n}{20}\right)\right] & n = 0 \\ & 0 < n <= 20 \end{cases}$$

MATLAB Problem: (5 × 10 points)

- Develop a MATLAB program to do the following tasks and submit your results including 5 × 3 charts and the MATLAB .m file.

  Hint: run and check the sample code before your MATLAB coding.

1. Develop a MATLAB program "tripletplot.m" to implement a triplet of subplots in one row by referring the sample code.
2. Use "tripletplot.m" to plot the results when using Hann window.
3. Use "tripletplot.m" to plot the results when using Hamming window.
4. Use "tripletplot.m" to plot the results when using Blackman window.
5. Discuss features of passband, stopband and transition band when using different windows in LPF implementation.

$\dfrac{\sin(wc * n)}{\pi * n}$

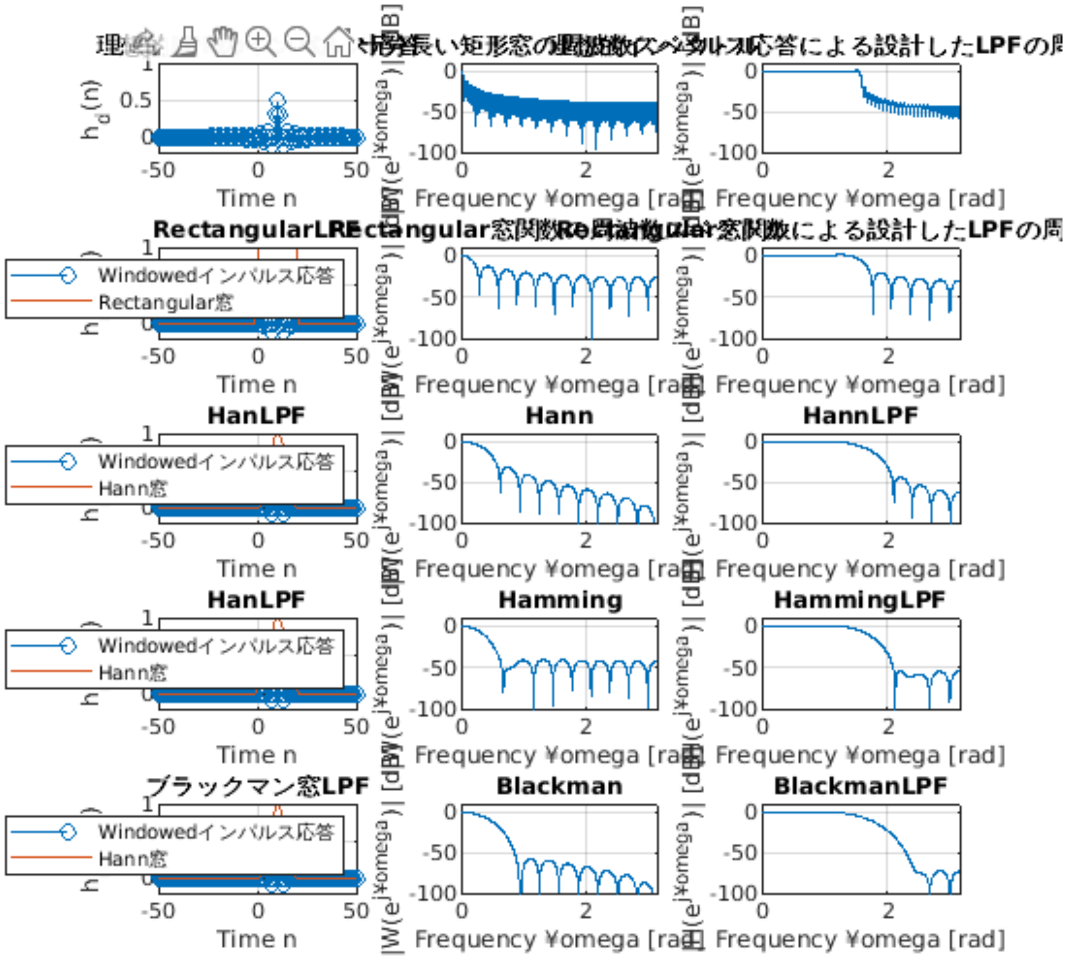
$\dfrac{\sin(wc * n)}{\pi * n}$

```
clf; clear;
L = 50;
n = -L:L;
M = 10;
N = 2*M+1;
w = linspace(0,pi,1024);
wc = 0.5*pi;

figure
figure_size = [ 0, 0, 1600, 1600];
set(gcf, 'Position', figure_size);

hd = (wc/pi)*sinc((n-M)*wc/pi);
subplot(5,3,1);
stem(n, hd);
axis([-L L -0.2 1]);
grid;
xlabel('Time n');
ylabel('h_d(n)');
title('###LPF########');

win= rectwin(2*L+1);
Win= freqz(win,1,w);
maxWin = max(abs(Win));
subplot(5,3,2);
plot(w, 20*log10(abs(Win)/maxWin));
axis([0 pi -100 10]);
grid;
xlabel('Frequency ¥omega [rad]');
ylabel('|W(e^{j¥omega})| [dB]');
title('################');

H = freqz(hd,1,w);
subplot(5,3,3);
plot(w, 20*log10(abs(H)));
axis([0 pi -100 10]);
grid;
xlabel('Frequency ¥omega [rad]');
ylabel('|H(e^{j¥omega})| [dB]');
title('################LPF######');


win= rectwin(N);
winz= [zeros(1, L-M) win' zeros(1, L-M)];
winz= wshift('1D', winz, -M);
h = hd.*winz;
subplot(5,3,4);
stem(n, h);
hold on;
plot(n, winz);
axis([-L L -0.2 1]);
grid;
```

```
xlabel('Time n');
ylabel('h_r_e_c_t(n)');
title('RectangularLPF');
legend('Windowed#######','Rectangular#');

Win=  freqz(win,1,w);
maxWin = max(abs(Win));
subplot(5,3,5);
plot(w, 20*log10(abs(Win)/maxWin));
axis([0 pi -100 10]);
grid;
xlabel('Frequency ¥omega [rad]');
ylabel('|W(e^{j¥omega})| [dB]');
title('Rectangular###########');
H = freqz(h,1,w);
subplot(5,3,6);
plot(w, 20*log10(abs(H)));
axis([0 pi -100 10]);
grid;
xlabel('Frequency ¥omega [rad]');
ylabel('|H(e^{j¥omega})| [dB]');
title('Rectangular#########LPF######');

win= hann(N);
winz= [zeros(1, L-M) win' zeros(1, L-M)];
winz= wshift('1D', winz, -M);
h = hd.*winz;
subplot(5,3,7);
stem(n, h);
hold on;
plot(n, winz);
axis([-L L -0.2 1]);
grid;
xlabel('Time n');
ylabel('h_r_e_c_t(n)');
title('HanLPF');
legend('Windowed#######','Hann#');

Win= freqz(win,1,w);
maxWin = max(abs(Win));
subplot(5,3,8);
plot(w, 20*log10(abs(Win)/maxWin));
axis([0 pi -100 10]);
grid;
xlabel('Frequency ¥omega [rad]');
ylabel('|W(e^{j¥omega})| [dB]');
title('Hann');H = freqz(h,1,w);
subplot(5,3,9);
plot(w, 20*log10(abs(H)));
axis([0 pi -100 10]);
grid;
xlabel('Frequency ¥omega [rad]');
ylabel('|H(e^{j¥omega})| [dB]');
title('HannLPF');
```

```
win= hamming(N);
winz= [zeros(1, L-M) win' zeros(1, L-M)];
winz= wshift('1D', winz, -M);
h = hd.*winz;
subplot(5,3,10);
stem(n, h);
hold on;
plot(n, winz);
axis([-L L -0.2 1]);
grid;
xlabel('Time n');
ylabel('h_r_e_c_t(n)');
title('HanLPF');
legend('Windowed#######','Hann#');

Win= freqz(win,1,w);
maxWin = max(abs(Win));
subplot(5,3,11);
plot(w, 20*log10(abs(Win)/maxWin));
axis([0 pi -100 10]);
grid;
xlabel('Frequency ¥omega [rad]');
ylabel('|W(e^{j¥omega})| [dB]');
title('Hamming');


H = freqz(h,1,w);
subplot(5,3,12);
plot(w, 20*log10(abs(H)));
axis([0 pi -100 10]);
grid;
xlabel('Frequency ¥omega [rad]');
ylabel('|H(e^{j¥omega})| [dB]');
title('HammingLPF');

win= blackman(N);
winz= [zeros(1, L-M) win' zeros(1, L-M)];
winz= wshift('1D', winz, -M);
h = hd.*winz;
subplot(5,3,13);
stem(n, h);
hold on;
plot(n, winz);
axis([-L L -0.2 1]);
grid;
xlabel('Time n');
ylabel('h_r_e_c_t(n)');
title('#######LPF');
legend('Windowed#######','Hann#');

Win= freqz(win,1,w);
maxWin = max(abs(Win));
subplot(5,3,14);
```

```
plot(w, 20*log10(abs(Win)/maxWin));
axis([0 pi -100 10]);
grid;
xlabel('Frequency ¥omega [rad]');
ylabel('|W(e^{j¥omega})| [dB]');
title('Blackman');


H = freqz(h,1,w);
subplot(5,3,15);
plot(w, 20*log10(abs(H)));
axis([0 pi -100 10]);
grid;
xlabel('Frequency ¥omega [rad]');
ylabel('|H(e^{j¥omega})| [dB]');
title('BlackmanLPF');
```



Published with MATLAB® R2021a