Ex 11

Chapter 11. Structures for Digital Filters II: IIR Filter

Date (yy/mm/dd): 2021. / 05. / 27

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Math Problem: (5×10 points)

- Consider a causal linear shift-invariant filter with transfer function

$$H(z) = \frac{1 + 0.875z^{-1}}{(1 + 0.2z^{-1} + 0.9z^{-2})(1 - 0.7z^{-1})}$$

- Draw a signal flowgraph for this filter using

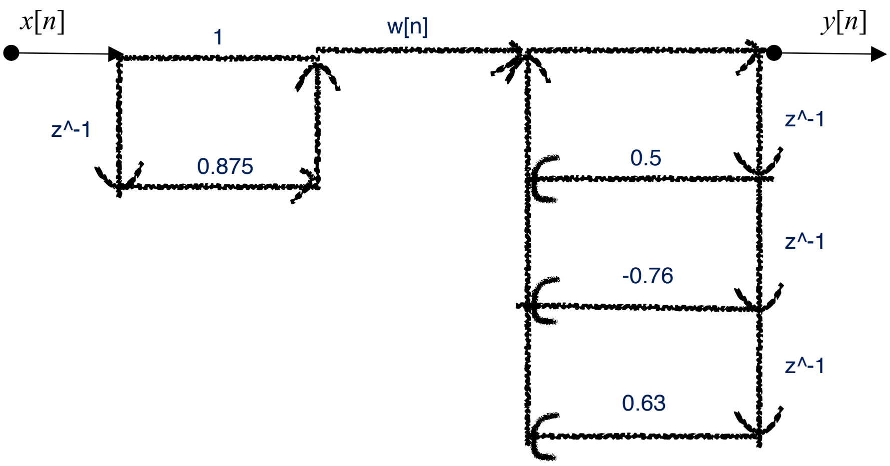
1. Direct form I
2. Direct form II
3. A cascade of first- and second-order filters realized in direct form II
4. A cascade of first- and second-order filters realized in transposed direct form II
5. A parallel connection of first- and second-order filters realized in direct form II

Answers:

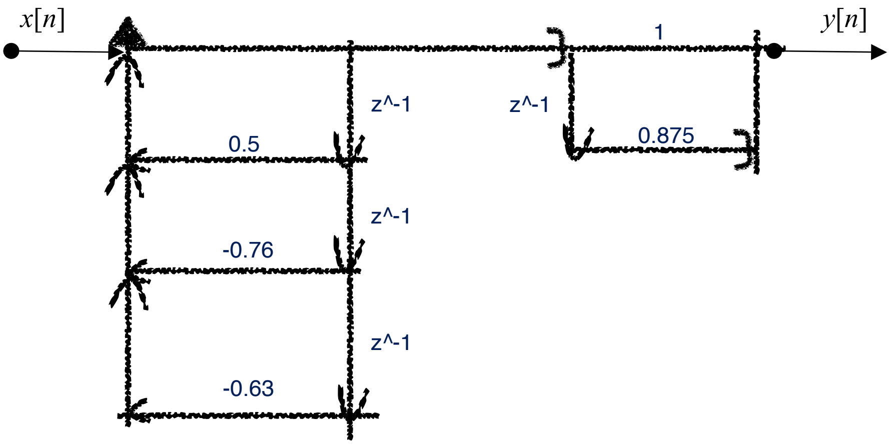
1. Write the transfer function $H(z)$ as a ratio of polynomials in z^{-1},

$$H_1(z) = \frac{1 + 0.875z^{-1}}{1 \text{ -0.5*(z^-1)+0.76*(z^-2)-0.63* } z^{-3}}, \tag{1}$$

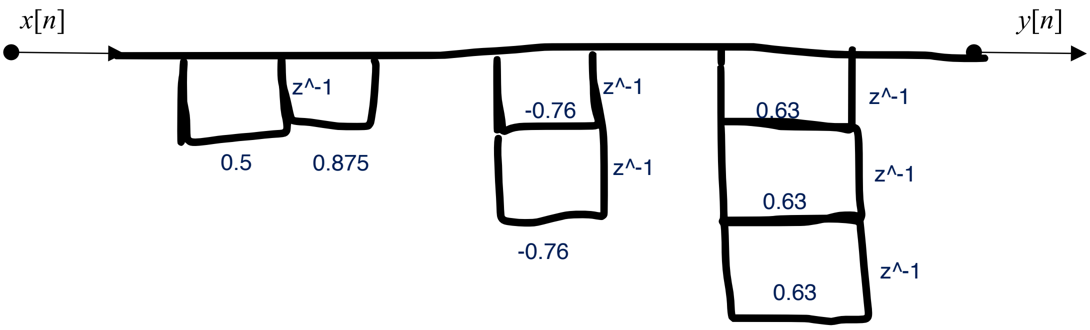
The direct form I realization of $H_1(z)$ is shown below:



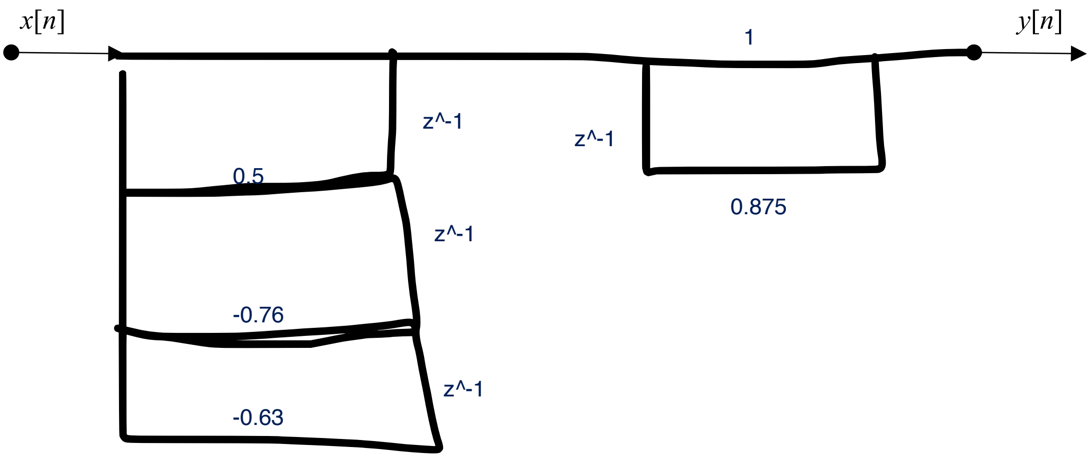
2. The direct form II realization of $H_1(z)$ is shown below:



3. Using a cascade of first- and second-order filters realized in direct form II, we have a choice of either pairing the zero with the first-order factor in the denominator or with the second-order factor. We will pair the zero with the second-order factor. With this pairing, the realization of $H_1(z)$ is as follows:



4. If we change the direct form II filters as showed above to transposed direct form II, we have the realization showed below.



5. For a parallel structure, $H(z)$ must be expanded using a partial fraction expansion:

$$H_2(z) = \frac{1 + 0.875z^{-1}}{(1 + 0.2z^{-1} + 0.9z^{-2})(1 - 0.7z^{-1})} = \frac{A + Bz^{-1}}{1 + 0.2z^{-1} + 0.9z^{-2}} + \frac{C}{1 - 0.7z^{-1}}$$

The constants A, B, and C can be found as follows.

Recombining the two terms in the partial fraction expansion as follows.

$$H_2(z) = \frac{(A + C) + (B + \text{0.2C - 0.7A})z^{-1} + (\text{0.7B + 0.9C})z^{-2}}{(1 + 0.2z^{-1} + 0.9z^{-2})(1 - 0.7z^{-1})}$$

By equating the coefficients in the numerator of this expression with the numerator of $H_2(z)$, we have the following three equations for the three unknowns A, B, and C:

$$\begin{aligned} A + C &= 1 \\ -0.7A + B + 0.2 &= 0.875 \\ -0.7B + 0.9C &= 0 \end{aligned}$$

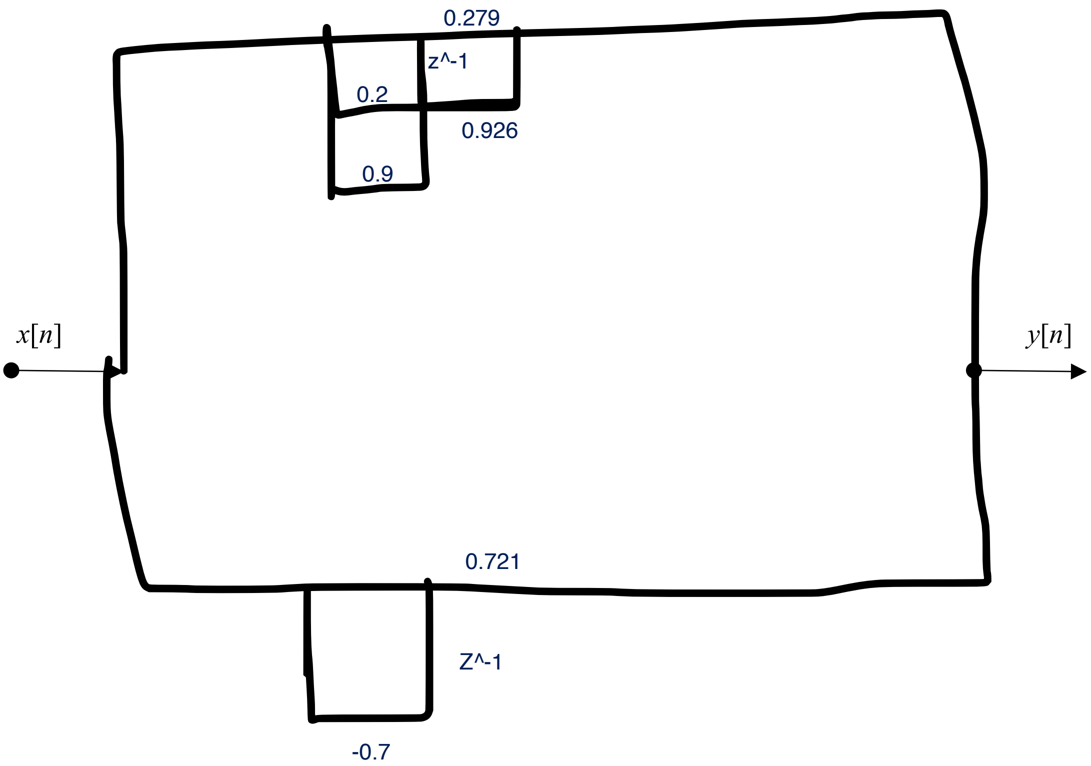
Solving for A, B, and C, we find

A= 0.279 , B= 0.926 , C= 0.721

Therefore, the partial fraction expansion is

$$H_2(z) = \frac{0.279 + 0.926*z\text{^}{-1}}{1 + 0.2z^{-1} + 0.9z^{-2}} + \frac{0.721}{1 - 0.7z^{-1}}, \quad (2)$$

Thus, parallel structure for $H_2(z)$ is showed below:



MATLAB Problem: (5×10 points)

- The frequency response of a digital filter can be interpreted as the transfer function evaluated at $z = e^{j\omega}$.
- MATLAB command “freqz” determines the transfer function of a digital filter from the (real or complex) numerator and denominator polynomials, and returns the complex frequency response $H(e^{j\omega})$.
- Develop a MATLAB program to do the following tasks and submit your results including 4×2 charts and the MATLAB .m file.
- The filter transfer function is $H_1(z)$ as showed in equation (1) in Math Problem.
- The partial fraction expansion is $H_2(z) = H_{21}(z) + H_{22}(z)$ as showed in equation (2) in Math Problem.

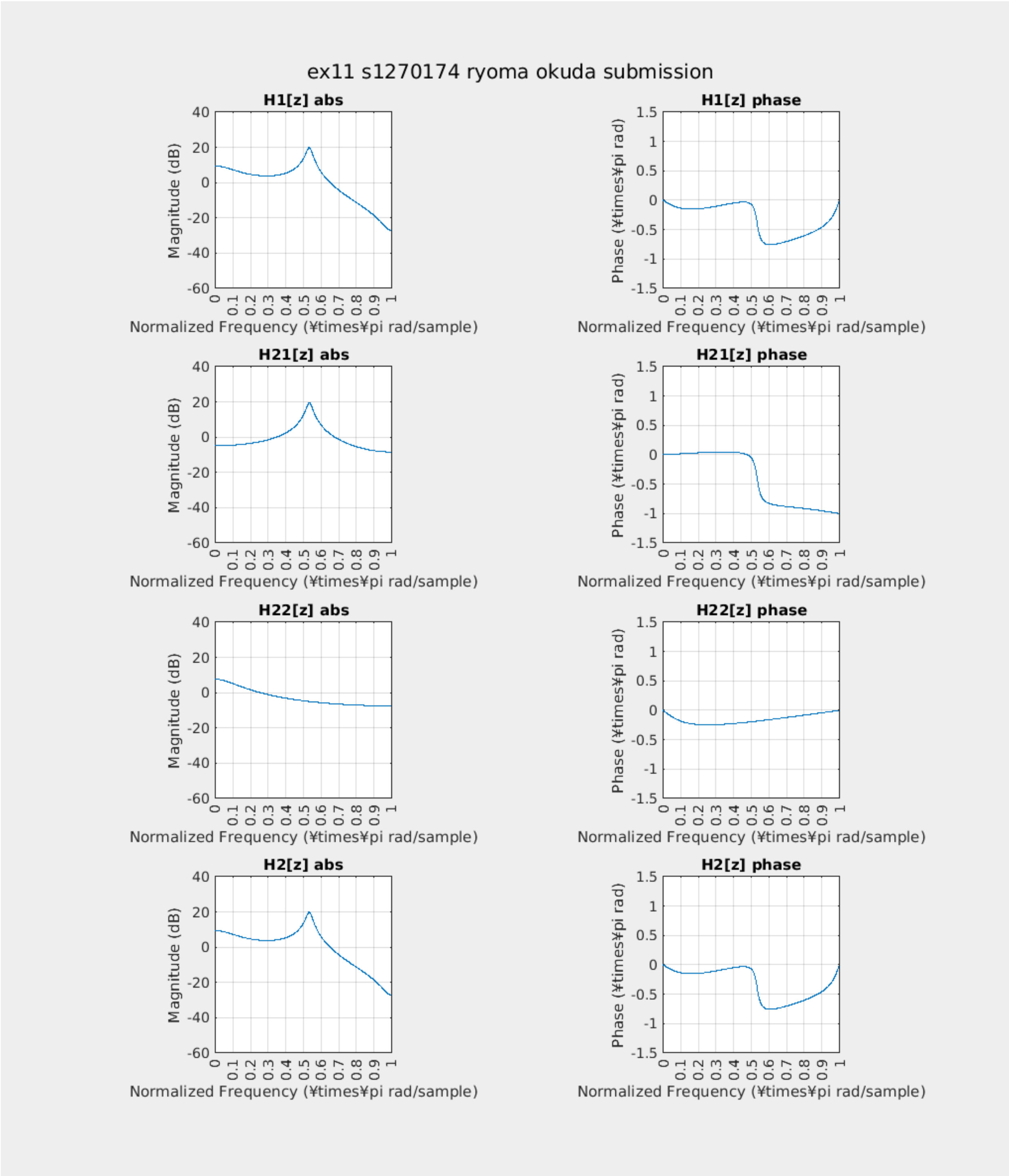
1. Plot the magnitude response and the phase response of $H_1(z)$.
2. Plot the magnitude response and the phase response of $H_{21}(z)$.
3. Plot the magnitude response and the phase response of $H_{22}(z)$.
4. Plot the magnitude response and the phase response of $H_2(z)$.
5. Discuss if the frequency responses of $H_1(z)$ and $H_2(z)$ are same or different, and why?

   Hint: examine the sample code (for cascade form) below before your coding.

```
N = 1024;
b0 = [1]; b1 = [1    0.875]; b2 = [1];
a1 = [1    0.2    0.9]; a2 = [1    -0.7];
b = b0*conv(b1, b2); a = conv(a1, a2);
[h,w] = freqz(b,a,'half', N);
% Plot the magnitude response.
figure; plot(w/pi,20*log10(abs(h))); grid;
ax = gca; ax.YLim = [-60 40]; ax.XTick = 0:.1:1;
xlabel('Normalized Frequency (¥times¥pi rad/sample)'); ylabel('Magnitude (dB)');
% Plot the phase response.
figure; plot(w/pi, phase(h)/pi); grid;
ax = gca; ax.YLim = [-1.5 1.5]; ax.XTick = 0:.1:1;
xlabel('Normalized Frequency (¥times¥pi rad/sample)'); ylabel('Phase (¥times¥pi rad)')
```

```
% variables
figure;
figure_size = [ 0, 0, 3000, 3500];
set(gcf, 'Position', figure_size);
row = 2;
col = 4;
set(gcf,'Color','#eeeeee')

% 1

N = 1024;
b0 = [1]; b1 = [1 0.875]; b2 = [1];
a1 = [1 -0.5 0.76 -0.63];
b = b0*conv(b1, b2); a = a1;
[h,w] = freqz(b,a,'half', N);

subplot(4,2,1)
plot(w/pi,20*log10(abs(h))); grid;
ax = gca; ax.YLim = [-60 40]; ax.XTick = 0:.1:1;
xlabel('Normalized Frequency (¥times¥pi rad/sample)'); ylabel('Magnitude (dB)');
grid on
axis square
title("H1[z] abs")
subplot(4,2,2);
plot(w/pi, phase(h)/pi); grid;
ax = gca; ax.YLim = [-1.5 1.5]; ax.XTick = 0:.1:1;
xlabel('Normalized Frequency (¥times¥pi rad/sample)'); ylabel('Phase (¥times¥pi rad)')
grid on
axis square
title("H1[z] phase")

% 2

N = 1024;
b0 = [1]; b1= [0.279 0.926]; b2 = [1];
a1 = [1 0.2 0.9];
b = b0*conv(b1, b2); a = a1;
[h1,w] = freqz(b,a,'half', N);

subplot(4,2,3)
plot(w/pi,20*log10(abs(h1))); grid;
ax = gca; ax.YLim = [-60 40]; ax.XTick = 0:.1:1;
xlabel('Normalized Frequency (¥times¥pi rad/sample)'); ylabel('Magnitude (dB)');
grid on
axis square
title("H21[z] abs")
subplot(4,2,4);
plot(w/pi, phase(h1)/pi); grid;
ax = gca; ax.YLim = [-1.5 1.5]; ax.XTick = 0:.1:1;
xlabel('Normalized Frequency (¥times¥pi rad/sample)'); ylabel('Phase (¥times¥pi rad)')
grid on
axis square
title("H21[z] phase")
```

```
% 3

N = 1024;
b0 = [1]; b1= [0.721]; b2 = [1];
a1 = [1 -0.7];
bo = b0*conv(b1, b2); ao = a1;
[h2,w] = freqz(bo,ao,'half', N);

subplot(4,2,5)
plot(w/pi,20*log10(abs(h2))); grid;
ax = gca; ax.YLim = [-60 40]; ax.XTick = 0:.1:1;
xlabel('Normalized Frequency (¥times¥pi rad/sample)'); ylabel('Magnitude (dB)');
grid on
axis square
title("H22[z] abs")
subplot(4,2,6);
plot(w/pi, phase(h2)/pi); grid;
ax = gca; ax.YLim = [-1.5 1.5]; ax.XTick = 0:.1:1;
xlabel('Normalized Frequency (¥times¥pi rad/sample)'); ylabel('Phase (¥times¥pi rad)')
grid on
axis square
title("H22[z] phase")


% 4

subplot(4,2,7)
plot(w/pi,20*log10(abs(h1 + h2))); grid;
ax = gca; ax.YLim = [-60 40]; ax.XTick = 0:.1:1;
xlabel('Normalized Frequency (¥times¥pi rad/sample)'); ylabel('Magnitude (dB)');
grid on
axis square
title("H2[z] abs")
subplot(4,2,8);
plot(w/pi, phase(h1 + h2)/pi); grid;
ax = gca; ax.YLim = [-1.5 1.5]; ax.XTick = 0:.1:1;
xlabel('Normalized Frequency (¥times¥pi rad/sample)'); ylabel('Phase (¥times¥pi rad)')
title("H2[z] phase")
grid on
axis square

% % % % % % % % % % % % % % % %

sgtitle("ex11 s1270174 ryoma okuda submission")
```




```
fprintf("multiplication and addition are exchangable each other.")
```

multiplication and addition are exchangable each other. in addition, we can use partial fraction decomposi

```
fprintf("in addition, we can use partial fraction decomposition at the translation,")
fprintf("which means that H1 and H2 is the exactly correct.")
```