INDICES

CHAPTER 3 PART 1

Powers, roots and standard form

Powers, roots and standard form

Square numbers

When we multiply a number by itself we say that we are **squaring** the number.

To square a number we can write a small ^{2} after it.

For example, the number 3 multiplied by itself can be written as

$$3 \times 3 \quad \text{or} \quad 3^2$$

The value of three squared is 9.

The result of any *whole* number multiplied by itself is called a **square number**.

Square roots

Finding the **square root** is the inverse of finding the square:

$$
\begin{tikzcd}
\mathbf{8} \arrow[r, bend left, "\textbf{squared}"] & \mathbf{64} \arrow[l, bend left, "\textbf{square rooted}"]
\end{tikzcd}
$$

We write

$$\sqrt{64} = 8$$

The square root of 64 is 8.

The product of two square numbers

The product of two square numbers is always another square number.

For example,

$$4 \times 25 = 100$$

because

$$2 \times 2 \times 5 \times 5 = 2 \times 5 \times 2 \times 5$$

and

$$(2 \times 5)^2 = 10^2$$

We can use this fact to help us find the square roots of larger square numbers.

Using factors to find square roots

If a number has factors that are square numbers then we can use these factors to find the square root.

For example,

Find $\sqrt{400}$

$\sqrt{400} = \sqrt{(4 \times 100)}$

$= \sqrt{4} \times \sqrt{100}$

$= 2 \times 10$

$= 20$

Find $\sqrt{225}$

$\sqrt{225} = \sqrt{(9 \times 25)}$

$= \sqrt{9} \times \sqrt{25}$

$= 3 \times 5$

$= 15$

Finding square roots of decimals

We can also find the square root of a number can be made be dividing two square numbers.

For example,

Find $\sqrt{0.09}$

$\sqrt{0.09} = \sqrt{(9 \div 100)}$

$= \sqrt{9} \div \sqrt{100}$

$= 3 \div 10$

$= 0.3$

Find $\sqrt{0.0144}$

$\sqrt{0.0144} = \sqrt{(144 \div 10000)}$

$= \sqrt{144} \div \sqrt{10000}$

$= 12 \div 100$

$= 0.12$

Approximate square roots

If a number cannot be written as a product or quotient of two square numbers then its square root cannot be found exactly.

Use the $\sqrt{}$ key on your calculator to find out $\sqrt{2}$.

The calculator shows this as 1.414213562

This is an approximation to 9 decimal places.

The number of digits after the decimal point is infinite and non-repeating.

This is an example of an **irrational** number.

Estimating square roots

What is $\sqrt{50}$?

50 is not a square number but lies between 49 and 64.

Therefore,

$$\sqrt{49} < \sqrt{50} < \sqrt{64}$$

So,

$$7 < \sqrt{50} < 8$$

50 is much closer to 49 than to 64, so $\sqrt{50}$ will be about 7.1

Use the √ key on you calculator to work out the answer.

$\mathbf{\sqrt{50} = 7.07}$ (to 2 decimal places.)

Negative square roots

$5 \times 5 = 25$ **and** $-5 \times -5 = 25$

Therefore, the square root of 25 is 5 or –5.

When we use the √ symbol we usually mean the positive square root.

We can also write ±√ to mean both the positive and the negative square root.

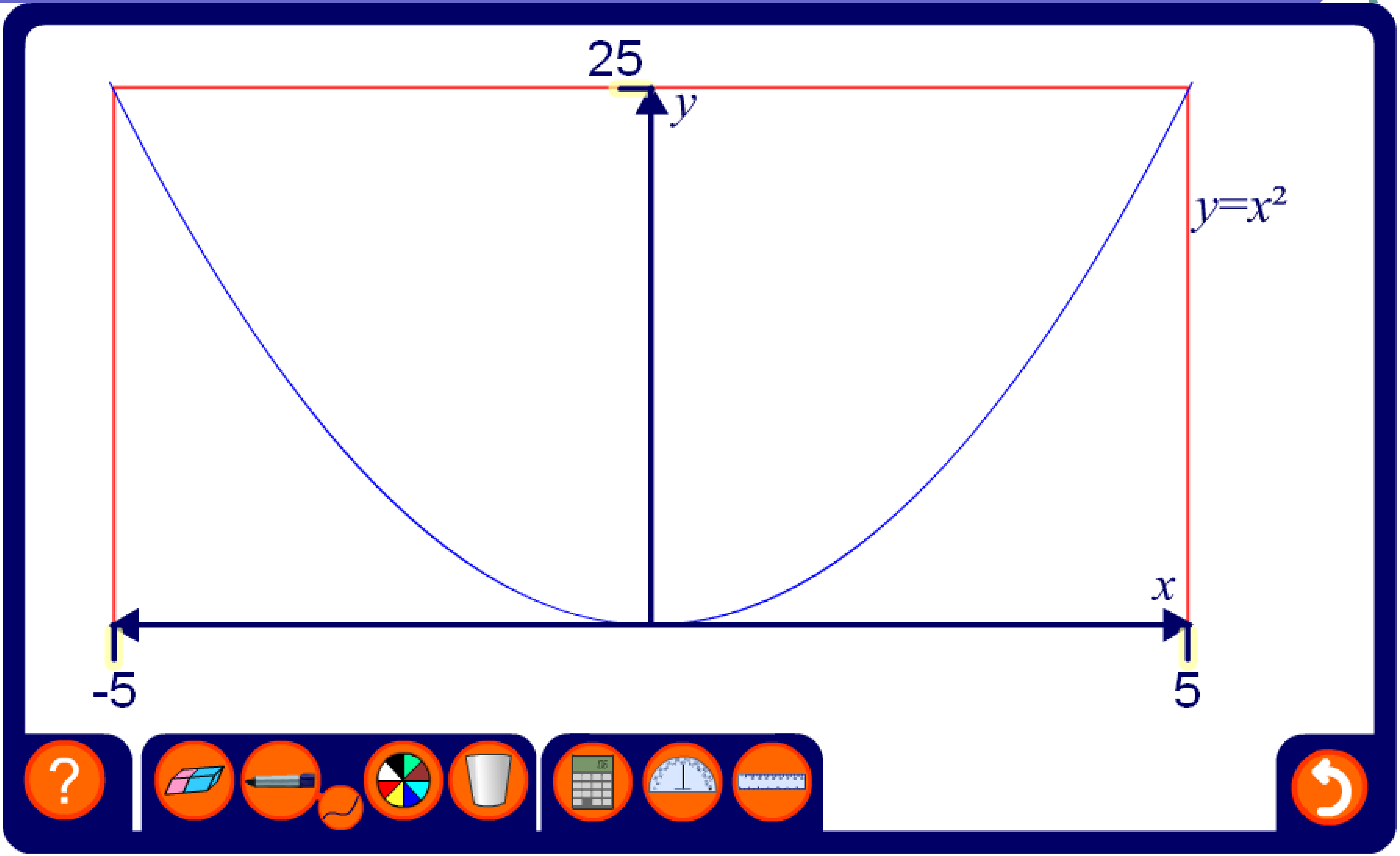
However the equation,

$$x^2 = 25$$

has 2 solutions,

$x = 5$ **or** $x = -5$

Squares and square roots from a graph



Cubes

The numbers 1, 8, 27, 64, and 125 are all:

Cube numbers

$1^3 = 1 \times 1 \times 1 = 1$ ‘1 cubed’ or ‘1 to the power of 3’

$2^3 = 2 \times 2 \times 2 = 8$ ‘2 cubed’ or ‘2 to the power of 3’

$3^3 = 3 \times 3 \times 3 = 27$ ‘3 cubed’ or ‘3 to the power of 3’

$4^3 = 4 \times 4 \times 4 = 64$ ‘4 cubed’ or ‘4 to the power of 3’

$5^3 = 5 \times 5 \times 5 = 125$ ‘5 cubed’ or ‘5 to the power of 3’

Cube roots

Finding the cube root is the inverse of finding the cube:

$$\begin{tikzcd} \mathbf{5} \arrow[r, bend left, "\textbf{cubed}"] & \mathbf{125} \arrow[l, bend left, "\textbf{cube rooted}"] \end{tikzcd}$$

We write

$$\sqrt[3]{125} = 5$$

The cube root of 125 is 5.

Index notation

We use index notation to show repeated multiplication by the same number.

For example

we can use index notation to write **2 × 2 × 2 × 2 × 2** as

$$2^5$$

Index or power

base

This number is read as 'two to the power of five'.

$2^5 = \quad 2 \times 2 \times 2 \times 2 \times 2 = \qquad 32$

Index notation

Evaluate the following:

$6^2 = \quad 6 \times 6 = \quad 36$

$3^4 = \quad 3 \times 3 \times 3 \times 3 = \quad 81$

$(-5)^3 = \quad -5 \times -5 \times -5 = \quad -125$

$2^7 = \quad 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = \quad 128$

$(-1)^5 = \quad -1 \times -1 \times -1 \times -1 \times -1 = \quad -1$

$(-4)^4 = \quad -4 \times -4 \times -4 \times -4 = \quad 256$

When we raise a negative number to an odd power the answer is negative.

When we raise a negative number to an even power the answer is positive.

INDICES

What are Indices?

- Indices provide a way of writing numbers in a more convenient form
- Indices is the plural of Index
- An Index is often referred to as a power

For example

$5 \times 5 \times 5 = 5^3$

$2 \times 2 \times 2 \times 2 = 2^4$

$7 \times 7 \times 7 \times 7 \times 7 = 7^5$

5 is the INDEX

7 is the BASE NUMBER

7^5 & 2^4 are numbers in INDEX FORM

Combining numbers

$5 \times 5 \times 5 \times 2 \times 2 \times 2 \times 2$

$= 5^3 \times 2^4$

We can not write this any more simply

Can ONLY do that if BASE NUMBERS are the same

INDEX LAW

IF ONLY **BASE NUMBERS** ARE **SAME**

Rule 1 : Multiplication

$2^6 \times 2^4 = 2^{10}$

$2^4 \times 2^2 = 2^6$

$3^5 \times 3^7 = 3^{12}$

General Rule

$a^m \times a^n = a^{m+n}$

What Goes In The Box ? 1

Simplify the expressions below :

(1) $6^4 \times 6^3$ $= 6^7$

(2) $9^7 \times 9^2$ $= 9^9$

(3) $11^6 \times 11$ $= 11^7$

(4) $14^9 \times 14^{12}$ $= 14^{21}$

(5) $27^{25} \times 27^{30}$ $= 27^{55}$

(6) $2^2 \times 2^3 \times 2^5$ $= 2^{10}$

(7) $8^7 \times 8^{10} \times 8$ $= 8^{18}$

(8) $5^{20} \times 5^{30} \times 5^{50}$ $= 5^{100}$

Rule 2 : Division

$2^6 \div 2^4 \quad = 3^2$

$2^5 \div 2^2 \quad = 2^3$

$3^5 \div 3^7 \quad = 3^{-2}$

General Rule

$a^m \div a^n = a^{m-n}$

What Goes In The Box ? 2

Simplify the expressions below :

(1) $5^9 \div 5^2$ $=5^7$

(2) $7^{12} \div 7^5$ $= 7^7$

(3) $19^6 \div 19$ $= 19^5$

(4) $36^{15} \div 36^{10}$ $= 36^5$

(5) $18^{40} \div 18^{20}$ $= 18^{20}$

(6) $2^{32} \div 2^{27}$ $= 2^5$

(7) $8^{70} \div 8^{39}$ $= 8^{31}$

(8) $5^{200} \div 5^{180}$ $= 5^{20}$