ENGINEERING MATHEMATICS 1

LECTURE 3 (PART 1)

POLYNOMIALS EVALUATION
&
FACTORIZATION

POLYNOMIAL FUNCTIONS

A <u>POLYNOMIAL</u> is a monomial or a sum of monomials.

A <u>POLYNOMIAL IN ONE VARIABLE</u> is a polynomial that contains only one variable.

Example: $5x^2 + 3x - 7$

POLYNOMIAL FUNCTIONS

The <u>DEGREE</u> of a polynomial in one variable is the greatest exponent of its variable.

A <u>LEADING COEFFICIENT</u> is the coefficient of the term with the highest degree.

What is the degree and leading coefficient of $3x^5 - 3x + 2$?

A polynomial equation used to represent a function is called a POLYNOMIAL FUNCTION.

Polynomial functions with a degree of 1 are called LINEAR POLYNOMIAL FUNCTIONS

Polynomial functions with a degree of 2 are called QUADRATIC POLYNOMIAL FUNCTIONS

Polynomial functions with a degree of 3 are called CUBIC POLYNOMIAL FUNCTIONS

POLYNOMIAL FUNCTIONS

EVALUATING A POLYNOMIAL FUNCTION

Find $f(-2)$ if $f(x) = 3x^2 - 2x - 6$

$$f(-2) = 3(-2)^2 - 2(-2) - 6$$

$$f(-2) = 12 + 4 - 6$$

$$f(-2) = 10$$

POLYNOMIAL FUNCTIONS

EVALUATING A POLYNOMIAL FUNCTION

Find $f(2a)$ if $f(x) = 3x^2 - 2x - 6$

$$f(2a) = 3(2a)^2 - 2(2a) - 6$$

$$f(2a) = 12a^2 - 4a - 6$$

POLYNOMIAL FUNCTIONS

EVALUATING A POLYNOMIAL FUNCTION

Find $f(m + 2)$ if $f(x) = 3x^2 – 2x – 6$

$$f(m + 2) = 3(m + 2)^2 – 2(m + 2) – 6$$

$$f(m + 2) = 3(m^2 + 4m + 4) – 2(m + 2) – 6$$

$$f(m + 2) = 3m^2 + 12m + 12 – 2m – 4 – 6$$

$$f(m + 2) = 3m^2 + 10m + 2$$

POLYNOMIAL FUNCTIONS

EVALUATING A POLYNOMIAL FUNCTION

Find $2g(-2a)$ if $g(x) = 3x^2 - 2x - 6$

$$2g(-2a) = 2[3(-2a)^2 - 2(-2a) - 6]$$

$$2g(-2a) = 2[12a^2 + 4a - 6]$$

$$2g(-2a) = 24a^2 + 8a - 12$$

POLYNOMIAL FUNCTIONS

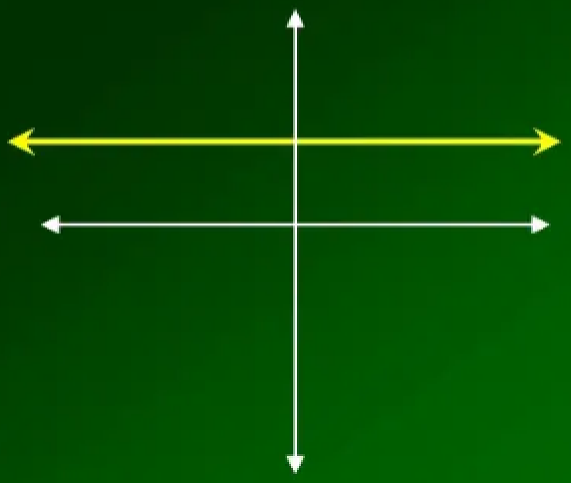
GENERAL SHAPES OF POLYNOMIAL FUNCTIONS

$f(x) = 3$

Constant Function

Degree = 0

Max. Zeros: 0



POLYNOMIAL FUNCTIONS

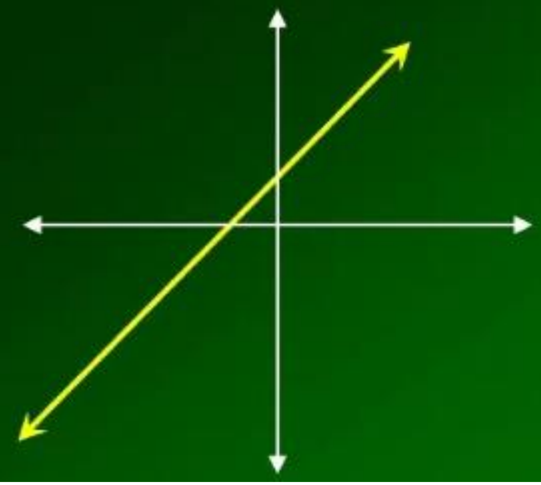
GENERAL SHAPES OF POLYNOMIAL FUNCTIONS

$f(x) = x + 2$

Linear Function

Degree = 1

Max. Zeros: 1



POLYNOMIAL FUNCTIONS

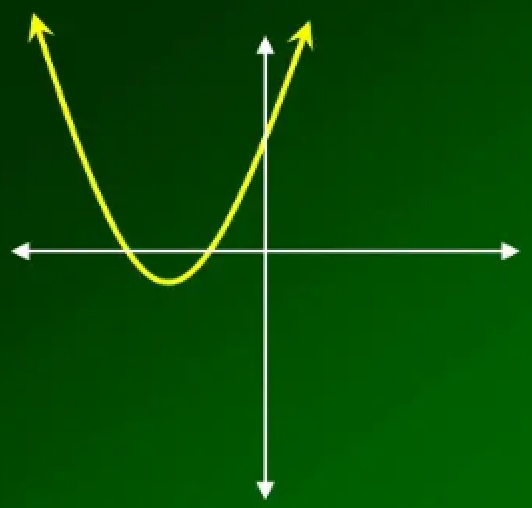
GENERAL SHAPES OF POLYNOMIAL FUNCTIONS

$f(x) = x^2 + 3x + 2$

Quadratic Function

Degree = 2

Max. Zeros: 2



POLYNOMIAL FUNCTIONS

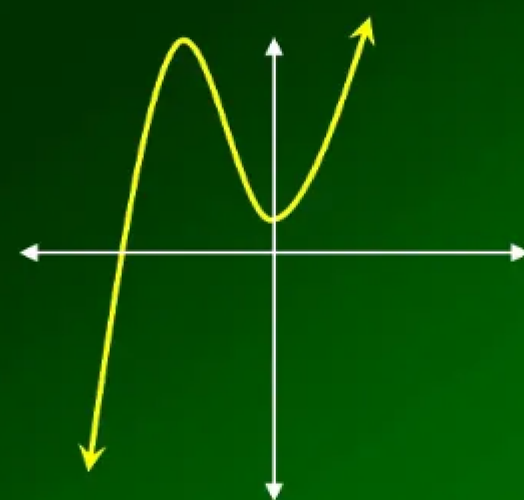
GENERAL SHAPES OF POLYNOMIAL FUNCTIONS

$f(x) = x^3 + 4x^2 + 2$

Cubic Function

Degree = 3

Max. Zeros: 3



POLYNOMIAL FUNCTIONS

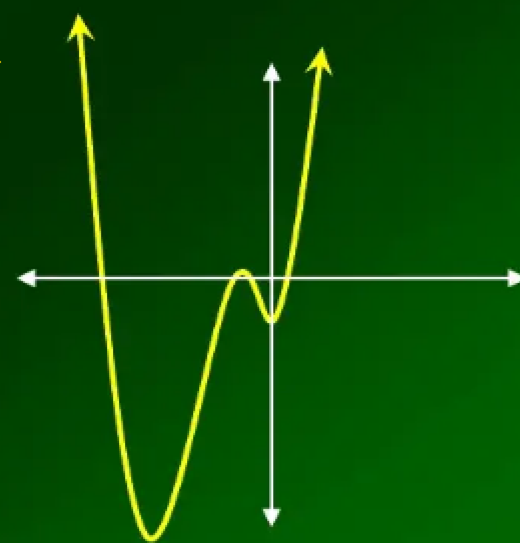
GENERAL SHAPES OF POLYNOMIAL FUNCTIONS

$f(x) = x^4 + 4x^3 - 2x - 1$

Quartic Function

Degree = 4

Max. Zeros: 4



POLYNOMIAL FUNCTIONS

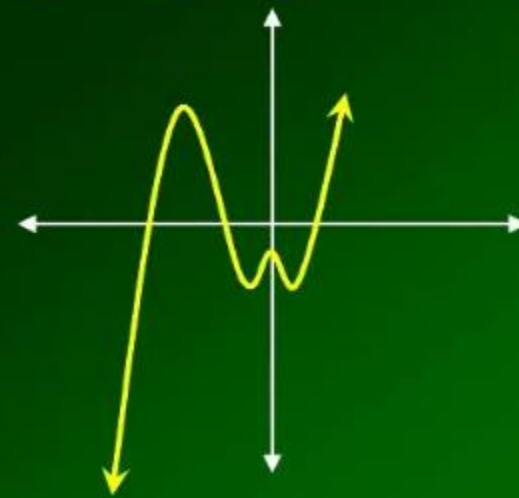
GENERAL SHAPES OF POLYNOMIAL FUNCTIONS

$f(x) = x^5 + 4x^4 - 2x^3 - 4x^2 + x - 1$

Quintic Function

Degree = 5

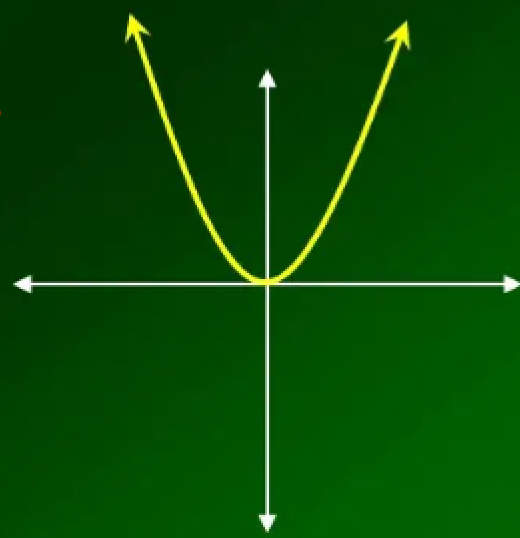
Max. Zeros: 5



POLYNOMIAL FUNCTIONS

END BEHAVIOR

$f(x) = x^2$



Degree: Even

Leading Coefficient: +

End Behavior:

As $x \rightarrow -\infty$; $f(x) \rightarrow +\infty$

As $x \rightarrow +\infty$; $f(x) \rightarrow +\infty$

POLYNOMIAL FUNCTIONS

END BEHAVIOR

$f(x) = -x^2$

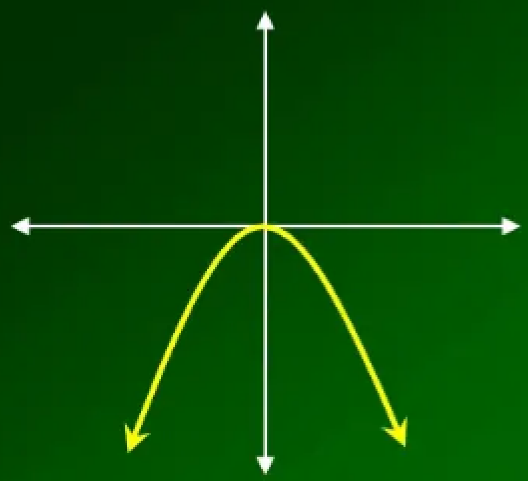
Degree: Even

Leading Coefficient: –

End Behavior:

As $x \rightarrow -\infty$; $f(x) \rightarrow -\infty$

As $x \rightarrow +\infty$; $f(x) \rightarrow -\infty$



POLYNOMIAL FUNCTIONS

END BEHAVIOR

$f(x) = x^3$

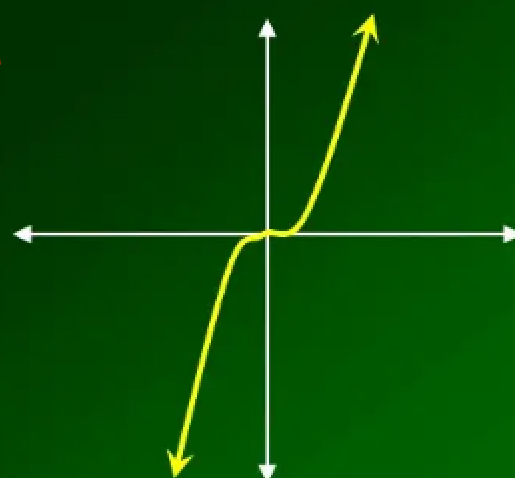
Degree: Odd

Leading Coefficient: +

End Behavior:

As $x \to -\infty$; $f(x) \to -\infty$

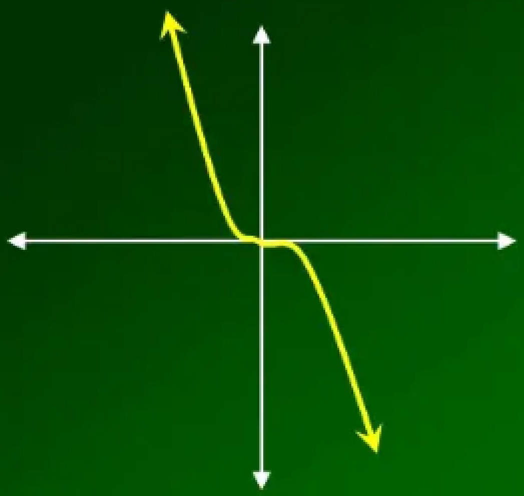
As $x \to +\infty$; $f(x) \to +\infty$



POLYNOMIAL FUNCTIONS

END BEHAVIOR

$f(x) = -x^3$



Degree: Odd

Leading Coefficient: –

End Behavior:

As $x \to -\infty$; $f(x) \to +\infty$

As $x \to +\infty$; $f(x) \to -\infty$

REMAINDER AND FACTOR THEOREMS

$f(x) = 2x^2 - 3x + 4$

Divide the polynomial by $x - 2$

$$\begin{array}{r|rrr} 2 & 2 & -3 & 4 \\ & & 4 & 2 \\ \hline & 2 & 1 & 6 \end{array}$$

Find f(2)

$f(2) = 2(2)^2 - 3(2) + 4$

$f(2) = 8 - 6 + 4$

$f(2) = 6$

REMAINDER AND FACTOR THEOREMS

When synthetic division is used to evaluate a function, it is called **SYNTHETIC SUBSTITUTION**.

Try this one:

Remember – Some terms are missing

$$f(x) = 3x^5 - 4x^3 + 5x - 3$$

Find f(-3)

REMAINDER AND FACTOR THEOREMS

FACTOR THEOREM

The binomial x – a is a factor of the polynomial f(x) if and only if f(a) = 0.

REMAINDER AND FACTOR THEOREMS

Is $x - 2$ a factor of $x^3 - 3x^2 - 4x + 12$

$$\begin{array}{r|rrr|r} 2 & 1 & -3 & -4 & 12 \\ & & 2 & -2 & -12 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

Yes, it is a factor, since $f(2) = 0$.

Can you find the two remaining factors?

REMAINDER AND FACTOR THEOREMS

$$(x + 3)(\ ?\ )(\ ?\ ) = x^3 - x^2 - 17x - 15$$

Find the two unknown (?) quantities.

Factoring and Finding Roots of Polynomials

Factoring Polynomials

Terms are Factors of a Polynomial if, when they are multiplied, they equal that polynomial:

$$x^2 + 2x - 15 = (x-3)(x+5)$$

- $(x - 3)$ and $(x + 5)$ are Factors of the polynomial

$$x^2 + 2x - 15$$

Solving a Polynomial Equation

Rearrange the terms to have zero on one side:

$$x^2 + 2x = 15 \quad \Rightarrow \quad x^2 + 2x - 15 = 0$$

Factor:

$$(x+5)(x-3) = 0$$

Set each factor equal to zero and solve:

$$(x+5) = 0 \quad \text{and} \quad (x-3) = 0$$

$$x = -5 \qquad\qquad x = 3$$

The only way that $x^2 + 2x - 15$ can $= 0$ is if $x = -5$ or $x = 3$

Solutions/Roots a Polynomial

Setting the Factors of a *Polynomial Expression* equal to zero gives the Solutions to the *Equation* when the polynomial expression equals zero. Another name for the Solutions of a Polynomial is the Roots of a *Polynomial* !

Zeros of a Polynomial Function

A Polynomial Function is usually written in function notation or in terms of x and y.

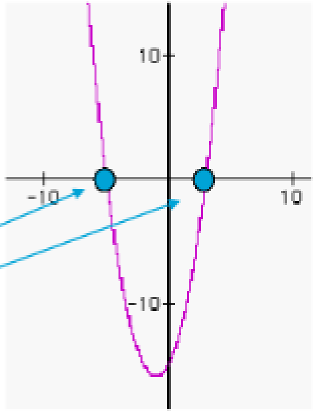
$$f(x) = x^2 + 2x - 15 \quad \text{or} \quad y = x^2 + 2x - 15$$

The Zeros of a *Polynomial Function* are the *solutions* to the equation you get when you set the polynomial equal to zero.

Graph of a Polynomial Function

Here is the graph of our polynomial function:

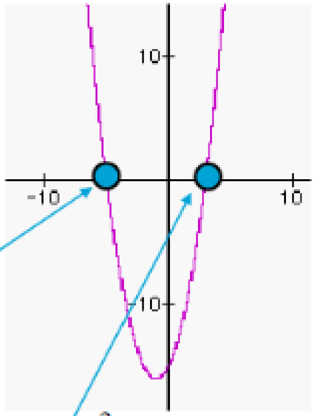
$$y = x^2 + 2x - 15$$



The *Zeros* of the Polynomial are the values of x when the polynomial equals zero. In other words, the *Zeros* are the x-values where *y equals zero*.

x-Intercepts of a Polynomial

The points where y = 0 are called the *x-intercepts* of the graph.



$y = x^2 + 2x - 15$

The *x-intercepts* for our graph are the points...

(-5, 0) and (3, 0)

Factors, Roots, Zeros

For our *Polynomial Function*:

$$y = x^2 + 2x - 15$$

The ***Factors*** are: $(x + 5)$ & $(x - 3)$

The ***Roots/Solutions*** are: $x = -5$ and 3

The ***Zeros*** are at: $(-5, 0)$ and $(3, 0)$

Factoring a polynomial means expressing it as a *product* of other polynomials.

Factoring Method #1

Factoring polynomials with a common *monomial* factor (using GCF).

**Always look for a GCF before using any other factoring method.

Steps

1. Find the greatest common factor (GCF).

2. Divide the polynomial by the GCF. The quotient is the other factor.

3. Express the polynomial as the product of the quotient and the GCF.

$$\text{Example}: \ 6c^3d - 12c^2d^2 + 3cd$$

Step 1: $GCF = 3cd$

Step 2: *Divide by GCF*

$$(6c^3d - 12c^2d^2 + 3cd) \div 3cd =$$

$$2c^2 - 4cd + 1$$

The answer should look like this:

$$Ex{:}\quad 6c^3d - 12c^2d^2 + 3cd$$

$$= \boxed{3cd(2c^2 - 4cd + 1)}$$

Factor these on yc
own looking for a

1. $6x^3 + 3x^2 - 12x$

2. $5x^2 - 10x + 35$ =

Factoring Method #2

Factoring polynomials that are a *difference of squares*.

A "Difference of Squares" is a *binomial* (*2 terms only*) and it factors like this:

$$a^2 - b^2 = (a+b)(a-b)$$

To factor, express each term as a square of a monomial then apply the rule... $a^2 - b^2 = (a+b)(a-b)$

$$Ex:\quad x^2 - 16 =$$

$$x^2 - 4^2 =$$

$$\boxed{(x+4)(x-4)}$$

Here is another example:

$$\frac{1}{49}x^2 - 81 =$$

$$\left(\frac{1}{7}x\right)^2 - 9^2 = \boxed{\left(\frac{1}{7}x+9\right)\left(\frac{1}{7}x-9\right)}$$

Try these o

1. $x^2 - 121 = ($

2. $9y^2 - 169x^2$

3. $x^4 - 16 = (x$

Factoring Method #3

Factoring a *trinomial* in the form:

$$ax^2 + bx + c$$

Factoring a trinomial: $ax^2 + bx + c$

1. Write two sets of parenthesis, (   )(   ). These will be the *factors* of the trinomial.

2. Product of first terms of both binomials must equal first term of the trinomial. (ax^2)

Factoring a trinomial: $ax^2 + bx + c$

3. The product of last terms of both binomials must equal last term of the trinomial (c).

4. Think of the FOIL method of multiplying binomials, the sum of the outer and the *inner* products must equal the middle term (bx).

$$Example: \ x^2 - 6x + 8$$

$(\boxed{x} \quad)(\boxed{x} \quad) \longrightarrow x \cdot x = x^2$

$(x \ \boxed{-2})(x \ \boxed{-4}) \longrightarrow$ O + I = bx ?

Factors of +8: 1 & 8 $\longrightarrow$ $1x + 8x = 9x$

2 & 4 $\longrightarrow$ $2x + 4x = 6x$

-1 & -8 $\longrightarrow$ $-1x - 8x = -9x$

-2 & -4 $\longrightarrow$ $-2x - 4x = -6x$

$$x^2 - 6x + 8 = \boxed{(x-2)(x-4)}$$

Check your answer by using FOIL

$$\begin{array}{ccccccc} & & \text{F} & & \text{O} & & \text{I} & & \text{L} \end{array}$$

$$(x-2)(x-4) = x^2 - 4x - 2x + 8$$

$$= x^2 - 6x + 8$$

Lets do another example:

$$6x^2 - 12x - 18$$

Don't Forget Method #1.

Always check for GCF before you do anything else.

$$6(x^2 - 2x - 3)$$ *Find a GCF*

$$6(x-3)(x+1)$$ *Factor trinomial*

When $a>1$ and $c<1$, there may be more combinations to try!

Example: $6x^2 + 13x - 5$

Step 1:

Find the factors of $6x^2$:

$3x \cdot 2x$

$6x \cdot x$

Example: $6x^2 + 13x - 5$

Step 2: Find the factors of -5:

5 □ -1

-5 □ 1

$$\left(\begin{array}{c} -1 \;\square\; 5 \\ 1 \;\square\; -5 \end{array}\right)$$

Order can make a difference!

$Example: \quad 6x^2 + 13x - 5$

Step 3: Place the factors inside the parenthesis until O + I = bx.

Try: $(6x - 1)(x + 5)$

F O I L

$$6x^2 + 30x - x - 5$$

O + I = $30x - x = 29x$

This doesn't work!!

$Example: \quad 6x^2 + 13x - 5$

Switch the order of the second terms and try again.

$$(6x+5)(x-1)$$

F O I L

$$6x^2 - 6x + 5x - 5$$

O + I = -6x + 5x = -
x

This doesn't work!!

Try another combination:

Switch to $3x$ and $2x$

$$(3x - 1)(2x + 5)$$

F O I L

$$6x^2 + 15x - 2x - 5$$

O+I = $15x - 2x = 13x$ IT WORKS!!

$$6x^2 + 13x - 5 = \boxed{(3x - 1)(2x + 5)}$$