ENGINEERING MATHEMATICS 1

Lecture 1

Topic: Number System

By: Ts. Dr. Intan Irieyana Zulkepli

Course content outline

- Type of numbers
- Number system; binary, decimal (denary), octal, hexadecimal
- Converting an octal number to its equivalent binary number, binary number to its equivalent hexadecimal number, hexadecimal number to its equivalent binary number
- Approximation- “rounding off” significant figures
- Decimal point number systems

Natural Numbers

- a.k.a "Counting Numbers"
- 1, 2, 3, 4, 5, . . .
- At some point, the idea of "zero" came to be considered as a number.
- Eg: If the farmer does not have any sheep, then the number of sheep that the farmer owns is zero.
- We call the set of natural numbers plus the number zero the *whole numbers*.

Numeral and place value

- The whole numbers or natural numbers are written using the ten numerals 0, 1, .. ., 9 where the position of a numeral dictates the value that it represents.
- 246 stands for 2 hundreds and 4 tens and 6 units.
- That is 200 + 40 + 6. Here the numerals 2, 4 and 6 are called the hundreds, tens and unit coefficients respectively.

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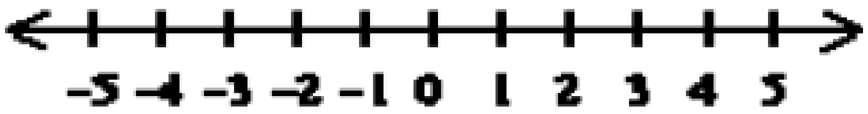
The Number Line

- The ordered nature of the real numbers lets us arrange them along a line (imagine that the line is made up of an infinite number of points all packed so closely together that they form a solid line). The points are **ordered** so that points to the **right are greater** than points to the left:



- Every real number corresponds to a distance on the number line, starting at the center (zero).
- Negative numbers represent distances to the left of zero, and positive numbers are distances to the right.
- The arrows on the end indicate that it keeps going forever in both directions.

Estimating

- For many engineering problems, we cannot obtain analytical solutions.
- Numerical methods yield approximate results, results that are close to the exact analytical solution. We cannot exactly compute the errors associated with numerical methods.
    - Only rarely given data are exact, since they originate from measurements. Therefore there is probably error in the input information.
    - Algorithm itself usually introduces errors as well, e.g., unavoidable round-offs, etc …
    - The output information will then contain error from both of these sources.

Rounding numbers

A round number is mathematically defined as an integer which is the product of a considerable number of comparatively small factors as compared to its neighbouring numbers.

Ten

- 72 - 70
- 59 - ?
- 126 -?
- 349 - ?
- 1257 -?

Hundred

- 369 - 400
- 2349 - ?
- 4258 - ?
- 13526 -?

Tenth

5.367 - 5.4

12.342 - ?

8.1476 - ?

132.3591 - ?

Whole number – Ones

34.235 - 34

146.581 -?

Hundredth

5.7246 – 5.72

14.6899 -?

Thousandth

3.25614 - 3.256

17.87463 - ?

Decimal point

3 units + 1 tenth + 2 hundredths + 5 thousandths

$$3+\frac{1}{10}+\frac{2}{100}+\frac{5}{1000}$$

0 . 7 6

$$1s+\frac{1}{10s}+\frac{1}{100s}$$

0 ones + 7 tenths + 6 hundredth

$$0+\frac{7}{10}+\frac{6}{100}=\frac{76}{100}$$

Significant Figures

- Significant figures are counted from the first non-zero numeral encountered starting from the left of the number. When the required number of significant figures has been counted off, the remaining numerals are deleted.

53,8<u>00</u> How many significant figures?

5.38×10^4	3
5.380×10^4	4
5.3800×10^4	5

Trailing zeros: Zeros are sometimes used to locate the decimal point not significant figures.

0.00001753	4
0.0001753	4
0.001753	4

Non-positional Number Systems

Characteristics

-Use symbols such as I for 1, II for 2, III for 3, IIII for 4, IIIII for 5, etc.

-Each symbol represents the same value regardless of its position in the number

-The symbols are simply added to find out the value of a particular number

Difficulty

-It is difficult to perform arithmetic with such a number system

Positional Number Systems

Characteristics

-Use only a few symbols called digits

-These symbols represent different values depending on the position they occupy in the number

Positional Number Systems (Cont..)

- The value of each digit is determined by
  1. The digit itself
  2. The position of the digit in the number
  3. The base of the number system

- **Base** = total number of digits in the number system
- The maximum value of a single digit is always equal to **one less than the value of the base**

Number system

- Defines a set of values used to represent quantity

Name	Base (N)	
• Binary	2	0 to (N-1); 0,1
• Octal	8	0 to (N-1); 0,1,2,3,4,5,6,7
• Decimal	10	0 to (N-1);0,1,2,3,4…..9
• Duodecimal	12	0 to (N-1); 0,1,2,3,4…..9,A,B
• Hexadecimal	16	0 to (N-1); 0,1,2,3,4…….F

Binary Number System

- A positional number system
- Base 2
- Two Digits: 0, 1
- Example: 1010110_2
- Positional Number System

$$
\begin{array}{c}
2^{n-1} \cdots 2^4 2^3 2^2 2^1 2^0 \\
b_{n-1} \cdots b_4\, b_3\, b_2\, b_1\, b_0
\end{array}
$$

- **B**inary Dig**its** are called Bits
- Bit b_o is the least significant bit (LSB).
- Bit b_{n-1} is the most significant bit (MSB).

Example

$$10101_2 = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) \times (1 \times 2^0)$$

$$= 16 + 0 + 4 + 0 + 1$$

$$= 21_{10}$$

Decimal Number System

- A positional number sytem
- Base 10
- Ten Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- Example: 1045_{10}
- Positional Number System

$$10^{n-1} \cdots 10^4 10^3 10^2 10^1 10^0$$

$$d_{n-1} \cdots d_4\, d_3\, d_2\, d_1\, d_0$$

- Digit d_0 is the least significant digit (LSD).
- Digit d_{n-1} is the most significant digit (MSD).

Example

$$2586_{10} = (2 \times 10^3) + (5 \times 10^2) + (8 \times 10^1) + (6 \times 10^0)$$

$$= 2000 + 500 + 80 + 6$$

Octal Number System

Characteristics

- A positional number system
- Has total 8 symbols or digits (0, 1, 2, 3 ,4, 5, 6, 7)
- Hence, its base = 8
- The maximum value of a single digit is 7 (one less than the value of the base
- Each position of a digit represents a specific power of the base (8)

$$8^{n-1} \quad \ldots \quad 8^4 \quad 8^3 \quad 8^2 \quad 8^1 \quad 8^0$$

$$O_{n-1} \quad \ldots \quad O_4 \quad O_3 \quad O_2 \quad O_1 \quad O_0$$

$$2057_8 = (2 \times 8^3) + (0 \times 8^2) + (5 \times 8^1) + (7 \times 8^0)$$

$$= 1024 + 0 + 40 + 7$$

$$= 1071_{10}$$

Hexadecimal Number System

- A positional number sytem
- Base 16
- Sixteen Digits: 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F
- Example: $EF56_{16}$
- Positional Number System
- $$16^{n-1} \cdots 16^4 16^3 16^2 16^1 16^0$$

0000	0
0001	1
0010	2
0011	3

0100	4
0101	5
0110	6
0111	7

1000	8
1001	9
1010	A
1011	B

1100	C
1101	D
1110	E
1111	F

Example

$$
\begin{aligned}
1AF_{16} &= (1 \times 16^2) + (A \times 16^1) + (F \times 16^0) \\
&= 1 \times 256 + 10 \times 16 + 15 \times 1 \\
&= 256 + 160 + 15 \\
&= 431_{10}
\end{aligned}
$$

Representing Numbers in Different Number Systems

- In order to be specific about which number system we are referring to, it is a common practice to indicate the base as a subscript. Thus, we write:

- $10101_2 = 21_{10}$

Converting a number of another base to a decimal number

METHOD

- STEP 1 : Determine the column (positional) value of each digit
- STEP 2 : Multiply the obtained column values by the digits in the corresponding columns
- STEP 3 : Calculate the sum of these products

Example

$$4706_8 = ?_{10}$$

$$4706_8 = 4 \times 8^3 + 7 \times 8^2 + 0 \times 8^1 + 6 \times 8^0$$

Common values multiplied by the corresponding digits

$$= 4 \times 512 + 7 \times 64 + 0 + 6 \times 1$$

$$= 2048 + 448 + 0 + 6$$

Sum of these products

$$= 2502_{10}$$

Converting a decimal number to a number of another base

Division- Remainder Method

- STEP 1 : Divide the decimal number to be converted by the value of the new base
- STEP 2 : Record the remainder from step 1 as the rightmost digit (least significant digit) of the new base number
- STEP 3 : Divide the quotient of the previous divide by the new base
- STEP 4: Record the remainder from step 3 as the next digit (to the left) of the new base number

- Repeat steps 3 and 4, recording remainders from right to left, until the quotient becomes zero in step 3
- Note that the last remainder thus obtained will be the most significant digit (MSD) of the new base number

2	245_{10}	
2	122	— 1
2	61	— 0
2	30	— 1
2	15	— 0
2	7	— 1
2	3	— 1
2	1	— 1
	0	— 1

Now write all the remainders in the reverse order, i.e. from bottom to top.

Then $245_{10} = 11110101_2$

$$
\begin{array}{r|c l}
8 & 524_{10} & \\ \cline{2-2}
8 & 65 & -4 \\ \cline{2-2}
8 & 8 & -1 \\ \cline{2-2}
8 & 1 & -0 \\ \cline{2-2}
 & 0 & -1
\end{array}
$$

As before, write the remainders in order, i.e. from bottom to top.

$\therefore\ 524_{10} = 1014_8$

Example

$952_{10} = ?_8$

Solution:

		Remainders
8	952	
	119	0
	14	7
	1	6
	0	1

Hence, $952_{10} = 1670_8$

Decimal to binary (Successive division)

75 - 1001011

$75 \div 2 = 37$	R	1	LSB
$37 \div 2 = 18$	R	1	
$18 \div 2 = 9$	R	0	
$9 \div 2 = 4$	R	1	
$4 \div 2 = 2$	R	0	
$2 \div 2 = 1$	R	0	
$1 \div 2 = 0$	R	1	MSB

142 - 10001110

$142 \div 2 = 71$	R	0	LSB
$71 \div 2 = 35$	R	1	
$35 \div 2 = 17$	R	1	
$17 \div 2 = 8$	R	1	
$8 \div 2 = 4$	R	0	
$4 \div 2 = 2$	R	0	
$2 \div 2 = 1$	R	0	
$1 \div 2 = 0$	R	1	MSB

339 - 101010011

$339 \div 2 = 169$	R	1	LSB
$169 \div 2 = 84$	R	1	
$84 \div 2 = 42$	R	0	
$42 \div 2 = 21$	R	0	
$21 \div 2 = 10$	R	1	
$10 \div 2 = 5$	R	0	
$5 \div 2 = 2$	R	1	
$2 \div 2 = 1$	R	0	
$1 \div 2 = 0$	R	1	MSB

Decimal to Hexadecimal

479 - 1 D F_{16}

479	÷ 16	= 29.9375	= 29 R 15	⇨	F
29	÷ 16	= 1.8125	= 1 R 13	⇨	D
1	÷ 16	= 0.0625	= 0 R 1	⇨	1

LSD ⇧ MSD

8 9 4 - 3 7 E_{16}

894	÷ 16	= 55.875	= 55 R 14	⇨	E
55	÷ 16	= 3.4375	= 3 R 7	⇨	7
3	÷ 16	= 0.1875	= 0 R 3	⇨	3

LSD ⇧ MSD

Converting a number of some **base** to a number of another **base**

METHOD

- STEP 1 : Convert the original number to a decimal number (base 10)
- STEP 2 : Convert the decimal number so obtained to the new base number

Converting a Number of Some Base to a Number of Another Base

(Continued from previous slide..)

Example

$545_6 = ?_4$

Solution:

Step 1: Convert from base 6 to base 10

$$\begin{aligned} 545_6 &= 5 \times 6^2 + 4 \times 6^1 + 5 \times 6^0 \\ &= 5 \times 36 + 4 \times 6 + 5 \times 1 \\ &= 180 + 24 + 5 \\ &= 209_{10} \end{aligned}$$

Converting a Number of Some Base to a Number of Another Base

(Continued from previous slide..)

Step 2: Convert 209_{10} to base 4

4	209	Remainders
	52	1
	13	0
	3	1
	0	3

Hence, $209_{10} = 3101_4$

So, $545_6 = 209_{10} = 3101_4$

Thus, $545_6 = 3101_4$

Shortcut Method for Converting a Binary Number to its Equivalent Octal Number

(Continued from previous slide..)

Example

$1101010_2 = ?_8$

Step 1: Divide the binary digits into groups of 3 starting from right

$\underline{001}$ $\underline{101}$ $\underline{010}$

Step 2: Convert each group into one octal digit

$001_2 = 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1$
$101_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 5$
$010_2 = 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 2$

Hence, $1101010_2 = 152_8$

Shortcut method for converting an **octal** number to its **equivalent binary** number

METHOD

- STEP 1 : Convert each octal digit to a 3 digit binary number (the octal digits may be treated as decimal for this conversion)
- STEP 2 : Combine all the resulting binary groups (of digits each) into a single binary number

Shortcut Method for Converting an Octal Number to Its Equivalent Binary Number

(Continued from previous slide..)

Example

$562_8 = ?_2$

Step 1: Convert each octal digit to 3 binary digits

$5_8 = 101_2, \quad 6_8 = 110_2, \quad 2_8 = 010_2$

Step 2: Combine the binary groups

$562_8 = \underbrace{101}_{5} \quad \underbrace{110}_{6} \quad \underbrace{010}_{2}$

Hence, $562_8 = 101110010_2$

Shortcut method for converting a **binary** number to its **equivalent hexadecimal** number

METHOD

- STEP 1 : Divide the binary digits into groups of four starting from the right
- STEP 2 : Combine each group of four binary digits to one hexadecimal digit

Shortcut Method for Converting a Binary Number to its Equivalent Hexadecimal Number

(Continued from previous slide..)

Example

$111101_2 = ?_{16}$

Step 1: Divide the binary digits into groups of four starting from the right

$\underline{0011}$ $\underline{1101}$

Step 2: Convert each group into a hexadecimal digit

$0011_2 = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3_{10} = 3_{16}$

$1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 13_{10} = D_{16}$

Hence, $111101_2 = 3D_{16}$

Shortcut method for converting a **hexadecimal** number to its **equivalent binary** number

METHOD

- STEP 1 : Convert the decimal equivalent of each hexadecimal digit to a 4 digit binary number
- STEP 2 : Combine all the resulting binary groups (of 4 digits each) in a single binary number

Shortcut Method for Converting a Hexadecimal Number to its Equivalent Binary Number

(Continued from previous slide..)

Example

$2AB_{16} = ?_2$

Step 1: Convert each hexadecimal digit to a 4 digit binary number

$2_{16} = 2_{10} = 0010_2$
$A_{16} = 10_{10} = 1010_2$
$B_{16} = 11_{10} = 1011_2$

Shortcut Method for Converting a Hexadecimal Number to its Equivalent Binary Number

(Continued from previous slide..)

Step 2: Combine the binary groups

$$2AB_{16} = \underset{2}{\underline{0010}} \quad \underset{A}{\underline{1010}} \quad \underset{B}{\underline{1011}}$$

Hence, $2AB_{16} = 001010101011_2$

THANK YOU