

CHAPTER 4

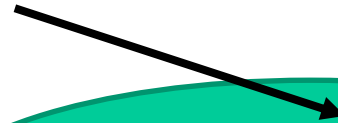
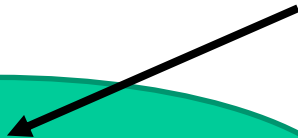
Equations

PART 1

Course Content Outline

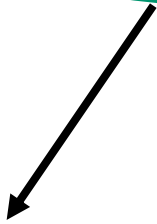
- Linear and Simultaneous Linear Equation.
- Equation and identities
- Linear equation, simplification and solution
- Simultaneous linear equation with two and three unknown

EQUATION



EQUALITY

INEQUALITY



LINEAR

QUADRATIC

Linear Equations vs. Quadratic Equations

- Linear equations are first-degree equations, such as:

$$2x + 1 = 5 \quad \text{or} \quad 4 - 3x = 2$$

- Quadratic equations are second-degree equations, such as:

$$x^2 + 2x - 3 = 0 \quad \text{or} \quad 2x^2 + 3 = 5x$$

LINEAR EQUATIONS

Linear Equations

Linear equation in one variable

can be written in the form $ax + b = c$, $a \neq 0$

Equivalent equations

are equations with the same solutions in the form of
variable = number, or
number = variable

The Addition Property of Equality

Addition Property of Equality

Addition Property of Equality

$a = b$ and $a + c = b + c$ are *equivalent equations*

Example

a.) $8 + z = -8$

$$8 + (-8) + z = -8 + (-8) \quad (\text{Add } -8 \text{ to each side})$$

$$z = -16 \quad (\text{Simplify both sides})$$

$$4p - 11 - p = 2 + 2p - 20$$

Solving Equations

Example

$$4p - 11 - p = 2 + 2p - 20$$

$$3p - 11 = 2p - 18 \quad (\text{Simplify both sides})$$

$$3p + (-2p) - 11 = 2p + (-2p) - 18 \quad (\text{Add } -2p \text{ to both sides})$$

$$p - 11 = -18 \quad (\text{Simplify both sides})$$

$$p - 11 + 11 = -18 + 11 \quad (\text{Add 11 to both sides})$$

$$p = -7 \quad (\text{Simplify both sides})$$

$$5(3 + z) - (8z + 9) = -4z$$

Solving Equations

Example

$$5(3 + z) - (8z + 9) = -4z$$

$$15 + 5z - 8z - 9 = -4z$$

(Use distributive property)

$$6 - 3z = -4z$$

(Simplify left side)

$$6 - 3z + 4z = -4z + 4z$$

(Add $4z$ to both sides)

$$6 + z = 0$$

(Simplify both sides)

$$6 + (-6) + z = 0 + (-6)$$

(Add -6 to both sides)

$$z = -6$$

(Simplify both sides)

The Multiplication Property of Equality

Multiplication Property of Equality

Multiplication property of equality

$a = b$ and $ac = bc$ are equivalent equations

Example

$$-y = 8$$

$$(-1)(-y) = 8(-1) \quad (\text{Multiply both sides by } -1)$$

$$y = -8 \quad (\text{Simplify both sides})$$

$$\frac{1}{7}x = \frac{5}{9}$$

Solving Equations

Example

$$\frac{1}{7}x = \frac{5}{9}$$

$$7\left(\frac{1}{7}x\right) = \left(\frac{5}{9}\right)7$$

(Multiply both sides by 7)

$$x = \frac{35}{9}$$

(Simplify both sides)

$$\frac{8}{3}x = 6$$

Solving Equations

Example

$$\frac{8}{3}x = 6$$

$$\frac{8}{3} \left(\frac{3}{8} \right) x = 6 \left(\frac{8}{3} \right)$$

(Multiply both sides by fraction)

$$x = 16$$

(Simplify both sides)

Solving Equations

Recall that multiplying by a number is equivalent to dividing by its reciprocal

Example

$$3z - 1 = 26$$

$$3z - 1 + 1 = 26 + 1 \quad (\text{Add 1 to both sides})$$

$$3z = 27 \quad (\text{Simplify both sides})$$

$$\frac{3z}{3} = \frac{27}{3} \quad (\text{Divide both sides by 3})$$

$$z = 9 \quad (\text{Simplify both sides})$$

$$12x + 30 + 8x - 6 = 10$$

Solving Equations

Example

$$12x + 30 + 8x - 6 = 10$$

$$20x + 24 = 10$$

(Simplify left side)

$$20x + 24 + (-24) = 10 + (-24)$$

(Add -24 to both sides)

$$20x = -14$$

(Simplify both sides)

$$\frac{20x}{20} = \frac{-14}{20}$$

(Divide both sides by 20)

$$x = \frac{-7}{10}$$

(Simplify both sides)

Further Solving Linear Equations

Solving Linear Equations

Solving linear equations in one variable

- 1) Multiply to clear fractions
- 2) Use distributive property
- 3) Simplify each side of equation
- 4) Get all variable terms on one side and number terms on the other side of equation (addition property of equality)
- 5) Get variable alone (multiplication property of equality)
- 6) Check solution by substituting into original problem

Solving Linear Equations

Example

$$\frac{3(y+3)}{5} = 2y + 6$$

$$\frac{5 \cdot 3(y+3)}{5} = 5(2y+6)$$

(Multiply both sides by 5)

$$3y + 9 = 10y + 30$$

(Simplify)

$$3y + (-3y) + 9 = 10y + (-3y) + 30$$

(Add $-3y$ to both sides)

$$9 + (-30) = 7y + 30 + (-30)$$

(Simplify; add -30 to both sides)

$$\frac{-21}{7} = \frac{7y}{7}$$

(Simplify; divide both sides by 7)

$$-3 = y$$

(Simplify both sides)

An Introduction to Problem Solving

Strategy for Problem Solving

General Strategy for Problem Solving

- 1) Understand the problem
 - Read and reread the problem
 - Choose a variable to represent the unknown
 - Construct a drawing, whenever possible
 - Propose a solution and check
- 2) Translate the problem into an equation
- 3) Solve the equation
- 4) Interpret the result
 - Check proposed solution in problem
 - State your conclusion

Finding an Unknown Number

Example

The product of twice a number and three is the same as the difference of five times the number and $\frac{3}{4}$. Find the number.

1.) *Understand*

Read and reread the problem. If we let

x = the unknown number, then “twice a number” translates to $2x$,

“the product of twice a number and three” translates to $2x \cdot 3$,

“five times the number” translates to $5x$, and

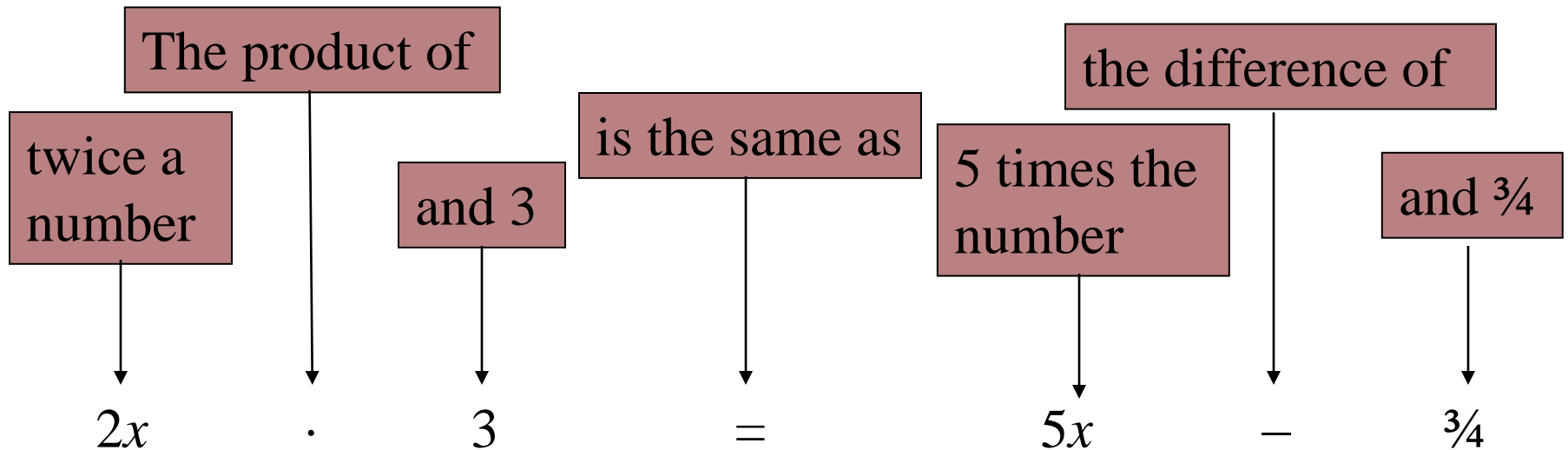
“the difference of five times the number and $\frac{3}{4}$ ” translates to $5x - \frac{3}{4}$.

Continued

Finding an Unknown Number

Example continued

2.) *Translate*



Continued

Finding an Unknown Number

Example continued

3.) *Solve*

$$2x \cdot 3 = 5x - \frac{3}{4}$$

$$6x = 5x - \frac{3}{4}$$

(Simplify left side)

$$6x + (-5x) = 5x + (-5x) - \frac{3}{4}$$

(Add $-5x$ to both sides)

$$x = -\frac{3}{4}$$

(Simplify both sides)

4.) *Interpret*

Check: Replace “number” in the original statement of the problem with $-\frac{3}{4}$. The product of twice $-\frac{3}{4}$ and 3 is $2(-\frac{3}{4})(3) = -4.5$. The difference of five times $-\frac{3}{4}$ and $\frac{3}{4}$ is $5(-\frac{3}{4}) - \frac{3}{4} = -4.5$. We get the same results for both portions.

State: The number is $-\frac{3}{4}$.

Formulas and Problem Solving

Formulas

A *formula* is an equation that states a known relationship among multiple quantities (has more than one variable in it)

$$A = lw \quad (\text{Area of a rectangle} = \text{length} \cdot \text{width})$$

$$I = PRT \quad (\text{Simple Interest} = \text{Principal} \cdot \text{Rate} \cdot \text{Time})$$

$$P = a + b + c \quad (\text{Perimeter of a triangle} = \text{side } a + \text{side } b + \text{side } c)$$

$$d = rt \quad (\text{distance} = \text{rate} \cdot \text{time})$$

$$V = lwh \quad (\text{Volume of a rectangular solid} = \text{length} \cdot \text{width} \cdot \text{height})$$

Solving Formulas

It is often necessary to rewrite a formula so that it is ***solved*** for one of the variables.

This is accomplished by isolating the designated variable on one side of the equal sign.

Solving Equations for a Specific Variable

- 1) Multiply to clear fractions
- 2) Use distributive to remove grouping symbols
- 3) Combine like terms to simplify each side
- 4) Get all terms containing specified variable on the same time, other terms on opposite side
- 5) Isolate the specified variable

Using Formulas

Example

A flower bed is in the shape of a triangle with one side twice the length of the shortest side, and the third side is 30 feet more than the length of the shortest side. Find the dimensions if the perimeter is 102 feet.

1.) *Understand*

Continued