ENGINEERING MATHEMATICS 1

Lecture 1

Topic: Complex Number

Complex Numbers

- **Complex Analysis** - deals with the study of complex numbers – its properties, operations and analytical functions.

- A **complex number** is of the form $z = x + jy$, where x and y are real numbers. The number x is called the real part and y the imaginary part. We write $\mathrm{Re}\, z = x$ and $\mathrm{Im}\, z = y$. The conjugate of z is the complex number . Note that which is a positive real number.

Complex Number Definition

Hence, the variable z denoted by an expression of the form

$$z = x + \mathbf{j}\, y$$

is a complex variable. x is called the real part of z, written as "Re z = x"; y is called the imaginary part of z written as "Im z = y."

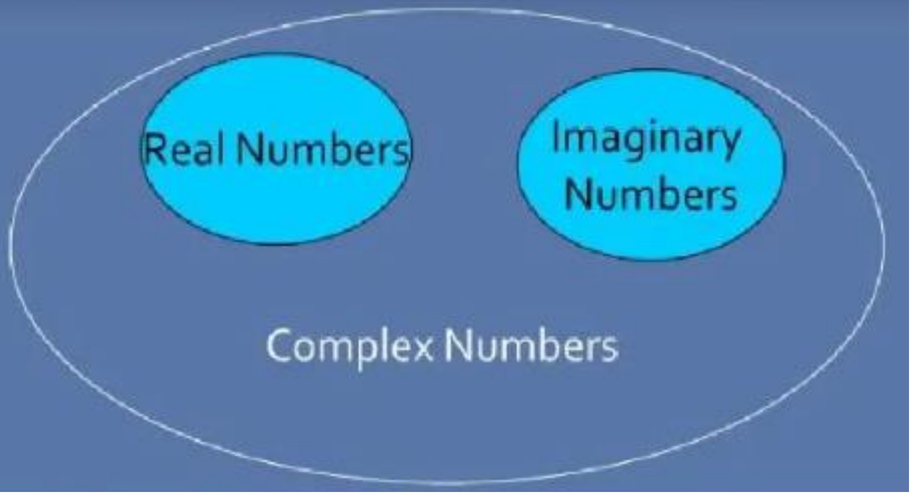
E.g. the following shows a complex number:

$$\underset{\substack{\downarrow \\ \textbf{complex} \\ \textbf{number}}}{z} = \underset{\substack{\downarrow \\ \textbf{real part} \\ \mathbf{Re}\, z}}{\mathbf{0.5}} + \mathbf{j}\, \underset{\substack{\downarrow \\ \textbf{imaginary part} \\ \mathbf{Im}\, z}}{\mathbf{1.323}}$$

With complex numbers every polynomials have a root.

COMPLEX NUMBERS

Real numbers and imaginary numbers are subsets of the set of complex numbers.



Addition and subtraction of complex numbers

To compute the addition or subtraction of complex numbers just collect like terms.

Example 1: Evaluate $z = z_1+z_2$ given;

$$z_1 = -2 + j4\,; \qquad z_2 = 5 + j2$$

Solution: to evaluate $z=z_1+z_2$ collect like terms

$$z = (-2+5) + j(4+2) \quad \Rightarrow \underline{z = 3 + j6}$$

Example 2: Evaluate $z = z_1-z_2$ given;

$$z_1 = 6 + j8\,; \qquad Z_2 = \mathbf{3 - j2}$$

Solution: for $z=z_1-z_2$ also collect like terms

$$z = (6-3) + j(8-(-2)) \quad \Rightarrow \underline{z = 3 + j10}$$

- *Multiplication of complex numbers*

Multiply complex numbers like binomials (you can use FOIL⇒"first, outer, inner, last.")

Example 3: Evaluate $z = z_1 \times z_2$ given;

$$z_1 = -2 + j3; \qquad z_2 = 5 + j$$

Solution: the product $z = (-2 + j3)(5 + j)$

$$\Rightarrow z = -2 \times 5 - j2 + j(3 \times 5) + j^2 3 \qquad (\text{now} \quad j^2 = -1)$$

$$\Rightarrow z = -10 + j(-2 + 15) - 3 = \underline{-13 + j13}$$

Example 4: Compute the product $z = (8 + j6)(3 - j2)$

Solution: $z = (8 \times 3) - j(8 \times 2) + j(6 \times 3) - j^2(6 \times 2)$

$$= 24 + j(-16 + 18) + 12 = \underline{36 + j2}$$

Operations on Complex Numbers

1. Addition:

$$(x_1 + jy_1) + (x_2 + jy_2) = (x_1 + x_2) + j(y_1 + y_2)$$

2. Subtraction:

$$(x_1 + jy_1) - (x_2 + jy_2) = (x_1 - x_2,) + j(y_1 - y_2)$$

3. Multiplication:

$$\begin{aligned}(x_1 + jy_1)(x_2 + jy_2) &= x_1x_2 + jx_1y_2 + jx_2y_1 + j^2y_1y_2 \\ &= (x_1x_2 - y_1y_2) + j(x_1y_2 + x_2y_1)\end{aligned}$$

4. Division:

$$\frac{x_1 + jy_1}{x_2 + jy_2} = \frac{x_1 + jy_1}{x_2 + jy_2} \cdot \frac{x_2 - jy_2}{x_2 - jy_2} = \frac{(x_1x_2 + y_1y_2) + j(x_2y_1 - x_1y_2)}{x_2^2 + y_2^2}$$

Algebraic Properties

Various properties of addition and multiplication of complex numbers are the same as for real numbers.

1. The commutative laws

$$z_1 + z_2 = z_2 + z_1 \qquad z_1 z_2 = z_2 z_1$$

2. The associative laws

$$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3) \qquad (z_1 z_2) z_3 = z_1 (z_2 z_3)$$

3. Distributive law

$$z(z_1 + z_2) = zz_1 + zz_2$$

Algebraic Properties

4. Existence of identity elements for addition and multiplication

   There exist complex numbers 0 and 1 such that $z + 0 = 0 + z = z$ and $z \cdot 1 = 1 \cdot z = z$. The complex number 0 is called the identity element for addition and 1 is the identity element for multiplication.

5. Existence of additive and multiplicative inverses

   For every complex number $z = x + jy$, there exists a complex number $-z = -x - jy$, called the additive inverse of z, such that $z + (-z) = (-z) + z = 0$.

   For every nonzero complex number $z = x + jy$, there exists a complex number z^{-1}, called the multiplicative inverse of z, such that $zz^{-1} = 1$.