

1) Prove the four properties of the ambiguity function given in section 4.2.4

A) The matched filter output when the target is just as expected ($\tau = 0, v = 0$) is

$$|\chi(0, 0)| = 1$$

B) When the target is not just as expected, the filter output cannot be greater than $|\chi(0, 0)|$ and generally is less:

$$0 \leq |\chi(\tau, v)| \leq 1$$

C) The integral of $|\chi(\tau, v)|^2 = 1$:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\chi(\tau, v)|^2 d\tau dv = 1$$

D) $|\chi(-\tau, -v)| = |\chi(\tau, v)|$.

2) For a single monochromatic pulse, show that

$$\chi(\tau, v) = \left(1 - \frac{|\tau|}{t_p}\right) \text{sinc} \left[\pi v t_p \left(1 - \frac{|\tau|}{t_p}\right) \right], \quad |\tau| \leq t_p; 0 \text{ elsewhere.}$$