1.4) A pulsed mid-X band radar has a maximum unambiguous velocity interval $\Delta\nu_\mu = 600$ m/s. (Assume $\tau << \tau_R$.)

a) What is its maximum unambiguous range R_u?

The first operation is to use the $\Delta\mu_\mu$ value stated to determine the f_R. This can then be used to determine the maximum unambiguous range. The value for f_R is found in the following way:

$$\begin{aligned}
\Delta\nu_\mu &= \frac{f_R\lambda}{2} \\
\therefore f_R &= \frac{2\Delta\nu_\mu}{\lambda} \\
&= \frac{(2)(600)(10\times 10^9)}{3\times 10^8} \simeq 4000 \text{ Hz}
\end{aligned}$$

The maximum ambiguous range can then be found from the above value of f_R.

$$\begin{aligned}
R_u &= \frac{c}{2f_R} \\
&= \frac{3\times 10^8}{(2)(4000)} \simeq 37500 \text{ m}
\end{aligned}$$

b) Suppose its frequency is changed to mid-Ku band and other parameters remain the same. Calculate $\Delta\nu_\mu$ and R_u.

Altering the frequency to the mid-Ku band does not alter the structure of the pulses and therefore the maximum ambigious range remains the same at 37,500 m. The shift in wavelength however does alter the maximum unambigious velocity interval.

$$\begin{aligned}
\Delta\nu_\mu &= \frac{f_R\lambda}{2} \\
&= \frac{4000}{2}\cdot\frac{3\times 10^8}{10\times 10^9} \simeq 60 \text{ m/s}
\end{aligned}$$

1.5) Calculate range gate width ΔR in both meters and feet for $\tau = 0.01, 0.1, 1,$ and 10 μsec. What is the appropriate relationship between τ (nsec) and ΔR (feet)?

The range gate is provided with the equation:

$$\Delta R = \frac{c\tau}{2}$$

Results would typically be given in meters but the units can be transformed to feet by using the scaling factor of 3.2808 feet/meter. The range gate width is:

τ (μ s)	ΔR (meters)	ΔR (feet)
0.01	1500	4921
0.1	15000	49212
1	150000	492120
10	1500000	4921200

The equation listed above can have explicit scaling factors inserted to accept nanoseconds and report in feet by applying the appropriate conversion ratio, ρ. The ratio is determined below:

$$\rho = \frac{1 \text{ (m)}}{1 \text{ (sec)}} \cdot \frac{3.2808 \text{ (feet)}}{1 \text{ (m)}} \cdot \frac{1 \text{ (sec)}}{1 \times 10^9 \text{ (nsec)}} \simeq 3.28 \times 10^{-9}$$

The modified equation taking this scaling factor into account is:

$$\Delta R = \frac{c\tau}{2}\rho \rightarrow \frac{\left(3 \times 10^8\right)\tau}{2}\left(3.28 \times 10^{-9}\right) \simeq 0.046\tau \text{ (feet)}$$

With this relationship, for each nanosecond of time you add to the pulse length, there is a corresponding increase of 0.05 feet in the range gate width.

2.1) Show that $F = (S/N)_{\text{in}}/(S/N)_{\text{out}} = (N_{\text{out}}/N_{\text{in}})(S_{\text{in}}/S_{\text{out}}) = (N_{\text{out}}/N_{\text{in}})(1/G_{\text{LNA}})$ if and only if $T_{\text{ant}} = T_{\text{radar}} = T_0$.

Using the chain of equalities above, the ratio of the input and the output S/N from the point of view of the signal amplifier can be written as a ratio of the output noise from the amplier to the noise from its input.

$$(S/N)_{\text{in}}/(S/N)_{\text{out}} = \frac{N_{\text{out}}}{N_{\text{in}}G_{\text{LNA}}}$$

In the above context I will let $N_{\text{in}} = k\left[\frac{T_{\text{ant}}}{L_{\text{Radar}}} + T_{\text{Radar}}\left(1 - \frac{1}{L_{\text{Radar}}}\right)\right]B = k\alpha B$ and $N_{\text{out}} = N_{\text{in}}G + kT_{\text{rcvr}}BG$

$$\begin{aligned}
\frac{N_{\text{out}}}{N_{\text{in}}G_{\text{LNA}}} &= \frac{k\alpha BG + kT_{\text{rcvr}}BG}{k\alpha BG} \\
&= \frac{k\alpha BG}{k\alpha BG} + \frac{kT_{\text{rcvr}}BG}{k\alpha BG} \\
&= 1 + \frac{T_{\text{rcvr}}}{\alpha} \\
&= 1 + \frac{(F-1)T_0}{\alpha} \\
\therefore \quad \frac{(S/N)_{\text{in}}}{(S/N)_{\text{out}}} &= 1 + \frac{(F-1)T_0}{\alpha}
\end{aligned}$$

This can now be solved for F in terms of the other quantites and the α term can be resubstituted for a final form.

$$\begin{aligned}
F &= \frac{\alpha}{T_0}\left[\frac{(S/N)_{\text{in}}}{(S/N)_{\text{out}}} - 1\right] + 1 \\
&= \frac{\frac{T_{\text{ant}}}{L_{\text{Radar}}} + T_{\text{Radar}}\left(1 - \frac{1}{L_{\text{Radar}}}\right)}{T_0}\left[\frac{(S/N)_{\text{in}}}{(S/N)_{\text{out}}} - 1\right] + 1
\end{aligned}$$

As can be seen in the final form of the equation, $F = \frac{(S/N)_{\text{in}}}{(S/N)_{\text{out}}}$ only when the numerator of the first fraction is equal to T_0. This can only occur if $T_{\text{ant}} = T_{\text{Radar}} = T_0$.

2.2) Show that, if we define $F2 = (S/N)_{\text{in}}/(S/N)_{\text{out}}$ and if $T_{\text{ant}} = T_{\text{radar}}$, then $T_{\text{rcvr}} = (F2-1)T_{\text{ant}}$.

Repurposing steps from Problem 2.1, the following is a good starting point.

$$\frac{(S/N)_{\text{in}}}{(S/N)_{\text{out}}} = 1 + \frac{T_{\text{rcvr}}}{\alpha} \ ,$$

where $\alpha = \frac{T_{\text{ant}}}{L_{\text{Radar}}} + T_{\text{Radar}}\left(1 - \frac{1}{L_{\text{Radar}}}\right)$. If $T_{\text{ant}} = T_{\text{Radar}}$, then I can replace α with T_{ant}. This results in the following:

$$\begin{aligned}
\frac{(S/N)_{\text{in}}}{(S/N)_{\text{out}}} &= 1 + \frac{T_{\text{rcvr}}}{T_{\text{ant}}} \\
\therefore \quad T_{\text{rcvr}} &= \left[\frac{(S/N)_{\text{in}}}{(S/N)_{\text{out}}} - 1\right] T_{\text{ant}} \\
&= (F2-1)\, T_{\text{ant}}
\end{aligned}$$

2.3) Consider a sensitive radar observing targets against deep space ($T_{\text{ant}} = 3\text{K}$), with an LNA cooled with liquid helium to $T_{\text{rcvr}} = 4.2\text{K}$. If $L_{\text{radar}} = 1$, what is the noise figure in decibels? What is T_{sys}?

The noise figure can be determined soley from the T_{rcvr} value of 4.2K. By inverting the general formula for T_{rcvr} in terms of F and substituting the values the result can be found.

$$\begin{aligned}
T_{\text{rcvr}} &= (F-1)T_0 \\
\therefore \quad F = \frac{T_{\text{rcvr}}}{T_0} + 1 & \\
&= \frac{4.2}{290} + 1 \\
&\simeq 1.014
\end{aligned}$$

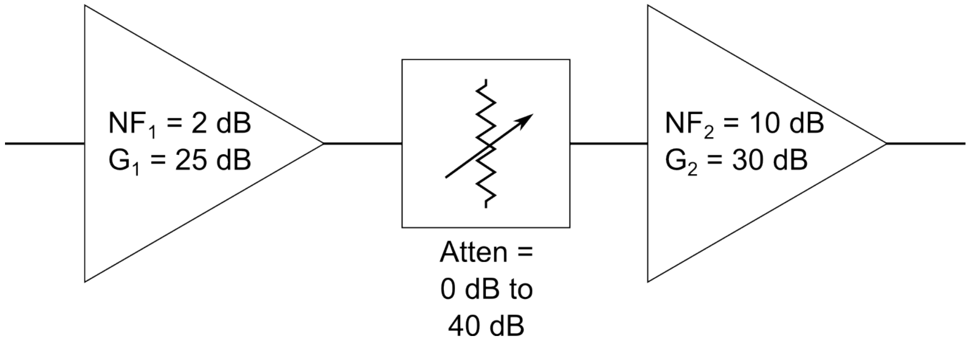
The value of 1.014 in decibels is found by applying the function $g(x) = 10log_{10}(x)$. The final noise figure in decibels is 0.06 dB.

The value of T_{sys} is found by substitution of the given values.

$$\begin{aligned}
T_{\text{sys}} &= \frac{T_{\text{ant}}}{L_{\text{Radar}}} + T_{\text{Radar}}\left(1 - \frac{1}{L_{\text{Radar}}}\right) + T_{\text{rcvr}} \\
&= 3 + 4.2 = 7.2\ \text{K}
\end{aligned}$$

Please note that in the above, T_{Radar} is not needed since the losses from the radar are equal to 1.

2.4) In the front-end circuit shown in Figure 2.11, the designer has put a system gain control, in the form of a variable attenuator, after the first LNA. Sketch the variation of overall system gain and system noise figure as this attenuator is varied over its full range. (Assume that $T_{\text{ant}} = T_{\text{radar}} = T_0$.)



The overall system gain is simply found by adding the decible versions of G_1, G_2, and the attenuation:

$$G_{\text{sys}} = G_1 + G_2 + A \qquad \text{(all in decibles)},$$

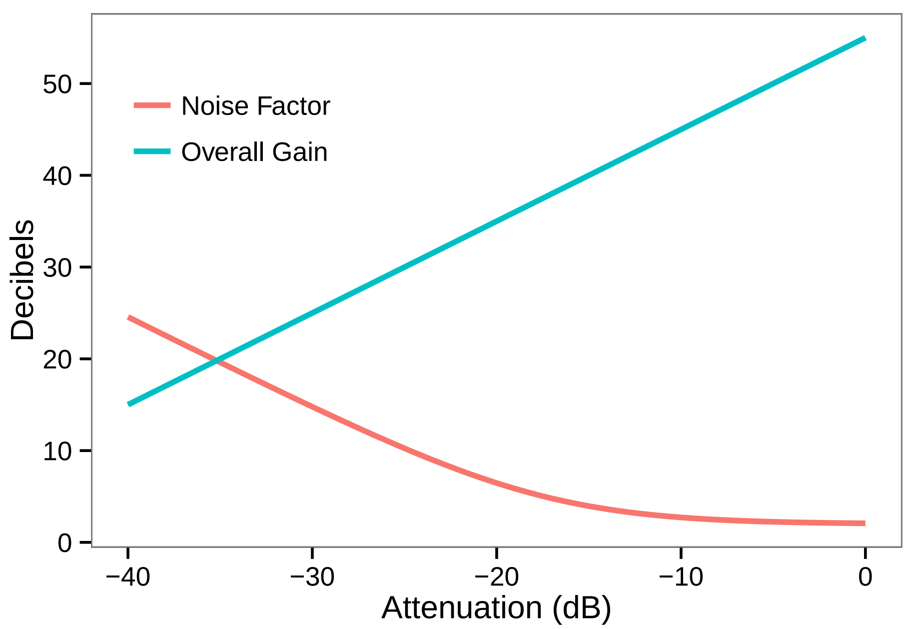
where the attenuation is identified as A.The overall system noise factor can be found by mimicing the proceedure found in section 2.2.11 of the text where it discusses the cascading of multiple amplifiers. The primary difference is that the attenuation needs to be added into the proceedure in the correct place. The output noise of the first amplifier is $G_1 k T_0 B + G_1 N_1$. This noise then needs to be scaled by the attenuation and then passed to the second amplifier as the input. The final noise out of the second amplifier is the following:

$$\begin{aligned} N_o &= G_2 A \left(G_1 k T_0 B + G_1 N_1\right) + G_2 N_2 \\ &= G_1 G_2 A k T_0 B + G_1 G_2 A N_1 + G_2 N_2 \end{aligned}$$

By dividing this by the noise all of the way out of the dual amplifier setup, $G_1 G_2 A k T_0 B$, the overall noise factor is found:

$$\begin{aligned} F &= 1 + \frac{N_1}{k T_0 B} + \frac{N_2}{G_1 A k T_0 B} \\ &= F_1 + \frac{F_2 - 1}{G_1 A} \end{aligned}$$

The overall system gain and the overall system noise factor is presented in the Figure below.



2.5) Show that the standard deviation of the quantization error and an A/D converter is

$$\sigma_\epsilon = \frac{\textit{least significant bit}}{\sqrt{12}}$$

The can be determined by calculating the expectation value of the squared voltage error as the input voltage to the A/D transits through its dynamic range. Letting the variable x denote time, one cycle of the sawtooth shaped voltage error is bound by $-\frac{E_0}{2} \leq x \leq \frac{E_0}{2}$. While the signal transits this space, the probabilty distribution of the error is uniform and encoded as $p(x) = \frac{1}{E_0}$. The function of the error voltage in this space is $f(x) = x$. Putting all of this together, to find the mean squared error we simply need to determine the expectation value of $f(x)^2$.

$$\begin{aligned}
\text{MSE} &= \int_{-\frac{E_0}{2}}^{\frac{E_0}{2}} f(x)^2 p(x) dx \\
&= \int_{-\frac{E_0}{2}}^{\frac{E_0}{2}} x^2 \frac{1}{E_0} dx \\
&= \frac{1}{E_0} \left[\frac{x^3}{3} \right]_{-\frac{E_0}{2}}^{\frac{E_0}{2}} \\
&= \frac{1}{3E_0} \left[\frac{E_0^3}{8} + \frac{E_0^3}{8} \right] \\
&= \frac{1}{3E_0} \frac{2E_0^3}{8} \\
&= \frac{E_0^2}{12}
\end{aligned}$$

The standard deviation is the square root of the mean squared error and therefore, based on the above result for the MSE, the standard deviation is then:

$$\sigma = \frac{E_0}{\sqrt{12}}$$