1) Consider a radar with a pulsed waveform (sinusoidally varying electric field) with a peak power of 6 kW. What is the absolute maximum power emitted? Does it depend on frequency? Polarization?

The absolute maximum emitted power is 6 kW and it does not deppend on the frequency or the polorization.

2) Consider a flat plate antenna ($\eta = 0.5$) of area 1 m^2 operating at mid-C band.

a) Calculate its gain

Antenna gain follows the following formula for cases such that the wavelength is much less than the aperature diameter:

$$G_0 = G(0,0) = \frac{4\pi A\eta}{\lambda^2}$$

The question requests the gain in the mid-C band which would be approximately 6 GHz ($\lambda = \frac{c}{f} = 0.03$ m). Since the wavelength is much less than the effective diameter of the square antenna ($\sim$ 1m), then the above equation holds for this condition.

$$G_0 = G(0,0) = \frac{4\pi A\eta}{\lambda^2} = \frac{4\pi(1)(0.5)}{0.03^2} = 6981$$

b) Calculate its peak RCS in the midband of L, W, C, X, and Ku. What is the equation relating its gain to RCS, as a function of λ? (See Tables 3.1 and 3.2.)

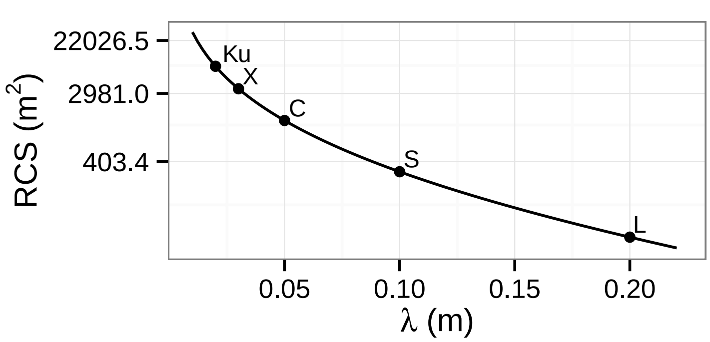
The peak RCS for a square antenna would be normal to the antenna aperture and is conveintly listed in table 3.2 as suggested. The formula is reproduced below:

$$\sigma = \frac{4\pi A^2}{\lambda^2}$$

The above equation does not take into account the geometrical efficiney of the antenna, η. This can be inserted into the cross-section equation by letting $A \to A\eta$. The resulting equation is the following,

$$\sigma = \frac{4\pi (A\eta)^2}{\lambda^2}$$

The peak RCS value in each of the desired bands is 78.54 m^2, 314.16 m^2, 1256.64 m^2, 3490.66 m^2, 7853.98 m^2 for the L, S, C, X, and Ku bands, respectively.



3) A mid-C band radar has the following:

$$\begin{aligned} P_{avg} &= 1 \text{ kW} \\ G &= 30 \text{ dB} \\ \tau_{\text{dwell}} &= 30 \text{ ms} \\ T_s &= 580\text{K} \\ L &= 6 \text{ dB} \end{aligned}$$

Calculate the maximum range at which it can detect a target of $\sigma = -20\text{dBm}^2$. Assume the SNR required for detection is 17 dB and $C_B = 1$.

The solution for this problem is accomplished by solving the radar equation for the Range (R) and substituting the above radar parameters after they are converted into matching dB scales.

$$\begin{aligned} \text{SNR} &= \frac{P_{avg} G^2 \lambda^2 \sigma \tau_{\text{dwell}}}{(4\pi)^3 R^4 k T_s C_B L} \\ \therefore R &= \left[\frac{P_{avg} G^2 \lambda^2 \sigma \tau_{\text{dwell}}}{(4\pi)^3 (\text{SNR}) k T_s C_B L} \right]^{\frac{1}{4}} \end{aligned}$$

In keeping with the world of sonar and radar, most quantites are calculated in dB space. I now convert the above equation to dB form and substitute all the nessessary quantities.

$$\begin{aligned} R &= \left[\frac{P_{avg} G^2 \lambda^2 \sigma \tau_{\text{dwell}}}{(4\pi)^3 (\text{SNR}) k T_s C_B L} \right]^{\frac{1}{4}} \\ 10 \log_{10}(R) &= 10 \log_{10} \left(\left[\frac{P_{avg} G^2 \lambda^2 \sigma \tau_{\text{dwell}}}{(4\pi)^3 (\text{SNR}) k T_s C_B L} \right]^{\frac{1}{4}} \right) \\ &= \frac{1}{4} \cdot 10 \left[\log_{10}(P_{avg}) + 2\log_{10}(G) + 2\log_{10}(\lambda) + \log_{10}(\sigma) + \log_{10}(\tau_{\text{dwell}}) \right. \\ &\quad \left. -3\log_{10}((4\pi)) - \log_{10}(\text{SNR}) - \log_{10}(kT_s) - \log_{10}(C_B) - \log_{10}(L) \right] \\ &= \frac{1}{4} \left[40 + 60 + (-13.01) + (-20) + (-15.2) - 33 - 17 - (-201) - 0 - 7 \right] \\ &= \frac{1}{4}(195.79) = 48.9 \text{ dBm} \end{aligned}$$

The solution of 48.9 is in the units of dBm which correlates to 77,600 m which is approximatly 42 NM.

4) For the radar used in the Estimate of Signal-to-Noise Ratio, compute, for the received signal, the power, the E field (volts/meter), the B field (tesla; $B = \mu_o H$), and the photons/second.

The parameters of the radar are such that:

$$\begin{aligned} P_{\text{peak}} &= 10^5 \text{ W} \\ G &= 40 \text{ dB} \\ \lambda &= 0.1 \text{ m} \\ \sigma &= 0.01 \text{ m}^2 \\ \tau &= 100 \text{ ns} \end{aligned} \qquad \begin{aligned} R &= 20,000 \text{ m} \\ T_S &= 580 \text{ K} \\ C_B &= 1 \\ L &= 7 \text{ dB} \end{aligned}$$

Power:

The power of the received signal is derived from the following equation:

$$P_R = P_T \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L}$$

Since most of the inputs are in mks, I will continue to use that unit system and convert the gain and the losses to ratios of 10^4 and $10^{0.7}$, respectivley. With these conversions the result is as follows:

$$\begin{aligned} P_R &= 10^5 \frac{\left(10^4\right)^2 (0.1)^2 (0.01)}{(4 * pi)^3 (20000)^4 (10^{0.7})} \\ &= 6.28 \times 10^{-13} \text{ W} \end{aligned}$$

Electric and Magnetic Fields:

The electric and magnetic fields can be expressed as:

$$E = \sqrt{\frac{S}{c\varepsilon_0}} \qquad B = \sqrt{\frac{S}{c\mu_0^3}} \quad \{\text{given that } B = \mu_0 H\}$$

Since S is the magnitude of the Poynting vector which is simply the power which was derived in the previous step, the result is easily obtained as 1.54×10^{-5} V/m and 0.03 T for the electric field and the magnetic field, respectively.

Number of Photons:

The flux can be determined by taking the power, which is in J/s, and dividing it by the energy of the photon. The energy of th photon is given by $E = \frac{hc}{\lambda}$ where h is the plank constant of 6.626×10^{-34} Js.

$$N_{\text{Photons}} = \frac{S\lambda}{hc} = \frac{\left(6.28 \times 10^{-13}\right)(0.1)}{\left(6.626 \times 10^{-34}\right)\left(3 \times 10^8\right)} = 3.16 \times 10^{11} \text{ photons/s/m}^2$$

Though this seems like a lot we need to factor in the effective area of the antenna. This can be determined by using the following equation:

$$A_e = \frac{G\lambda^2}{4\pi} = \frac{\left(10^4\right)\left(0.1^2\right)}{4\pi} = \frac{25}{\pi} = 78.5 \text{ m}^2$$

Scaling the above result for the number of photons per area per second results in a total of 2.5×10^{13} photons per second reaching the antenna. This seems a bit large to me which indicates that either I have a lack of understanding in scale or I have a very strong understanding of the probability of a mistake in the previous work.

5) A lossless radar with gain G and wavelength λ is located at a height H above an infinite, flat planar, perfectly conducting surface, with boresight directed perpendicular to the surface. Compute Γ.

Since the reflector is infinite in size and perfectly conducting, this problem can be reduced to the situation of an emitter that is exactly $2H$ away. The received power is then going to be determined by assuming spherical spreading from the emitter and the cross-section of the receiving antenna. This is taken directly from Equation 1.20 in the text. In this case, $L \to 0$ since it is a lossless antenna and $R \to 2H$. The modified equation is:

$$\Gamma = \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 (2H)^4}$$