1.4) A pulsed mid-X band radar has a maximum unambiguous velocity interval $\Delta\nu_\mu = 600$ m/s. (Assume $\tau << \tau_R$.)

a) What is its maximum unambiguous range R_u?

The first operation is to use the $\Delta\mu_\mu$ value stated to determine the f_R. This can then be used to determine the maximum unambiguous range. The value for f_R is found in the following way:

$$\begin{aligned}\Delta\nu_\mu &= \frac{f_R\lambda}{2}\\ \therefore f_R &= \frac{2\Delta\nu_\mu}{\lambda}\\ &= \frac{(2)(600)(10\times 10^9)}{3\times 10^8} \simeq 4000 \text{ Hz}\end{aligned}$$

The maximum ambiguous range can then be found from the above value of f_R.

$$\begin{aligned}R_u &= \frac{c}{2f_R}\\ &= \frac{3\times 10^8}{(2)(4000)} \simeq 37500 \text{ m}\end{aligned}$$

b) Suppose its frequency is changed to mid-Ku band and other parameters remain the same. Calculate $\Delta\nu_\mu$ and R_u.

Altering the frequency to the mid-Ku band does not alter the structure of the pulses and therefore the maximum ambigious range remains the same at 37,500 m. The shift in wavelength however does alter the maximum unambigious velocity interval.

$$\begin{aligned}\Delta\nu_\mu &= \frac{f_R\lambda}{2}\\ &= \frac{4000}{2}\cdot\frac{3\times 10^8}{10\times 10^9} \simeq 60 \text{ m/s}\end{aligned}$$

1.5) Calculate range gate width ΔR in both meters and feet for $\tau = 0.01, 0.1, 1,$ and 10 μsec. What is the appropriate relationship between τ (nsec) and ΔR (feet)?

The range gate is provided with the equation:

$$\Delta R = \frac{c\tau}{2}$$

Results would typically be given in meters but the units can be transformed to feet by using the scaling factor of 3.2808 feet/meter. The range gate width is:

τ (μ s)	ΔR (meters)	ΔR (feet)
0.01	1500	4921
0.1	15000	49212
1	150000	492120
10	1500000	4921200

The equation listed above can have explicit scaling factors inserted to accept nanoseconds and report in feet by applying the appropriate conversion ratio, ρ. The ratio is determined below:

$$\rho = \frac{1\ \text{(m)}}{1\ \text{(sec)}} \cdot \frac{3.2808\ \text{(feet)}}{1\ \text{(m)}} \cdot \frac{1\ \text{(sec)}}{1 \times 10^9\ \text{(nsec)}} \simeq 3.28 \times 10^{-9}$$

The modified equation taking this scaling factor into account is:

$$\Delta R = \frac{c\tau}{2}\rho \rightarrow \frac{\left(3 \times 10^8\right)\tau}{2}\left(3.28 \times 10^{-9}\right) \simeq 0.046\tau\ \text{(feet)}$$

With this relationship, for each nanosecond of time you add to the pulse length, there is a corresponding increase of 0.05 feet in the range gate width.

2.1) Show that $F = (S/N)_{\text{in}}/(S/N)_{\text{out}} = (N_{\text{out}}/N_{\text{in}})(S_{\text{in}}/S_{\text{out}}) = (N_{\text{out}}/N_{\text{in}})(1/G_{\text{LNA}})$ if and only if $T_{\text{ant}} = T_{\text{radar}} = T_0$.

Let's start be going backwards from the equations in the problem to show that that $F = (N_{\text{out}}/N_{\text{in}})(1/G_{\text{LNA}}) = (S/N)_{\text{in}}/(S/N)_{\text{out}}$.

$$\begin{aligned}
F &= \frac{N_{\text{out}}}{N_{\text{in}} G_{\text{LNA}}} \\
&= \frac{N_{\text{out}}}{N_{\text{in}}\left(\frac{S_{\text{out}}}{S_{\text{in}}}\right)} \\
&= \left(\frac{N_{\text{out}}}{N_{\text{in}}}\right)\left(\frac{S_{\text{in}}}{S_{\text{out}}}\right) \\
&= \left(\frac{S_{\text{in}}}{N_{\text{in}}}\right)\left(\frac{N_{\text{out}}}{S_{\text{out}}}\right) \\
&= \left(\frac{S_{\text{in}}}{N_{\text{in}}}\right)\Big/\left(\frac{S_{\text{out}}}{N_{\text{out}}}\right)
\end{aligned}$$

In the strictest sense, the noise factor is the ratio of the noise out of a practical amplifier, in this case the LNA, and the noise out of an ideal amplifier at the standard temperature $T_0 = 290$ K. Up to this point there have been no assumptions on what the form of N_{in} and N_{out} were. Using equation 2.10 from the text we can compose forms for N_{in} and N_{out}. Since the strict definition of F assumes an ideal amplifier, N_1.

In the above equations, there were no attempts to explain what N_{in} and N_{out} were and what they contained. The noise factor is a property of the amplifier, in this case the LNA.

In the first line of the solution, the core definition of the noise factor is used. Namely, this is that the noise factor is equal to the ratio of the measured output noise to the noise that would be generated after the LNA assuming an overall system temperature of $T_0 = 290$ K.

In this work there were no assumptions made on the input noise and it was simply denoted as N_{in}. This noise represents the noise introduced in the system up to the LNA which includes noise from the antenna and the radar itself.

2.2) Show that, if we define $F2 = (S/N)_{\text{in}}/(S/N)_{\text{out}}$ and it $T_{\text{ant}} = T_{\text{radar}}$, then $T_{\text{rcvr}} = (F2 - 1)T_{\text{ant}}$.

2.3) Consider a sensitive radar observing targets against deep space ($T_{\text{ant}} = 3\text{K}$), with an LNA cooled with liquid helium to $T_{\text{rcvr}} = 4.2\text{K}$. If $L_{\text{radar}} = 1$, what is the noise figure in decibels? What is T_{sys}?

2.4) In the front-end circuit shown in Figure 2.11, the designer has put a system gain control, in the form of a variable attenuator, after the first LNA. Sketch the variation of overall system gain and system noise figure as this attenuator is varied over its full range. (Assume that $T_{\text{ant}} = T_{\text{radar}} = T_0$.)

2.5) Show that the standard deviation of the quantization error and an A/D converter is

$$\sigma_\epsilon = \frac{\textit{least significant bit}}{\sqrt{12}}$$