

- 1) Verify that $h(t) = \int_{-\infty}^{\infty} H(f) e^{j2\pi ft} df$.

$$\begin{aligned}
 h(t) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(t') e^{-j2\pi ft'} e^{j2\pi ft} dt' df \\
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(t') e^{j2\pi f(t-t')} dt' df \\
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(t') e^{jp(t-t')} \left(\frac{1}{2\pi} \right) dp dt' && \text{Let } p = 2\pi f \\
 &= \int_{-\infty}^{\infty} h(t') \delta(t-t') dt' && \text{Since } 2\pi\delta(x-a) = \int_{-\infty}^{\infty} e^{ip(x-a)} dp \\
 &= h(t)
 \end{aligned}$$

- 2) Verify that if $h(t) = K$, $H(f) = K\delta(f)$ and that if $h(t) = K\delta(t)$, $H(f) = K$.

$$\begin{aligned}
 H(f) &= \int_{-\infty}^{\infty} h(t) e^{-j2\pi ft} dt \\
 &= \int_{-\infty}^{\infty} K e^{-j2\pi ft} dt \\
 &= K \int_{-\infty}^{\infty} e^{-j2\pi ft} dt \\
 &= -K \left(\frac{1}{2\pi} \right) \int_{-\infty}^{\infty} e^{jpf} dp \\
 &= -K\delta(f)
 \end{aligned}
 \qquad
 \begin{aligned}
 h(t) &= \int_{-\infty}^{\infty} H(f) e^{j2\pi ft} df \\
 &= \int_{-\infty}^{\infty} K e^{j2\pi ft} df \\
 &= K \int_{-\infty}^{\infty} e^{j2\pi ft} df \\
 &= K \left(\frac{1}{2\pi} \right) \int_{-\infty}^{\infty} e^{jpt} dp \\
 &= K\delta(t)
 \end{aligned}$$

There seems to be an issue with the minus sign on the first solution. I am not sure how to avoid having a minus sign for at least one of the solutions because of the structure of the transforms.

- 3) Verify that if $h(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$, $H(f) = \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T}\right)$.

- 4) Verify that, if $h(t) \leftrightarrow H(f)$, $h(t-t_0) \leftrightarrow H(f)e^{-j2\pi ft_0}$.

- 5) Verify that, if $h(t)$ is even, then $H(f)$ is real, and that if $h(t)$ is odd, then $H(f)$ is imaginary.

6) Verify that $x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau$.