

- 1) Verify that $h(t) = \int_{-\infty}^{\infty} H(f)e^{j2\pi ft}df$.

$$\begin{aligned}
 h(t) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(t')e^{-j2\pi ft'}e^{j2\pi ft}dt'df \\
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(t')e^{j2\pi f(t-t')}dt'df \\
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(t')e^{jp(t-t')}\left(\frac{1}{2\pi}\right)dpdt' && \text{Let } p = 2\pi f \\
 &= \int_{-\infty}^{\infty} h(t')\delta(t-t')dt' && \text{Since } 2\pi\delta(x-a) = \int_{-\infty}^{\infty} e^{ip(x-a)}dp \\
 &= h(t)
 \end{aligned}$$

- 2) Verify that if $h(t) = K, H(f) = K\delta(f)$ and that if $h(t) = K\delta(t), H(f) = K$.

$$\begin{aligned}
 H(f) &= \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}dt \\
 &= \int_{-\infty}^{\infty} Ke^{-j2\pi ft}dt \\
 &= K \int_{-\infty}^{\infty} e^{-j2\pi ft}dt \\
 &= -K\left(\frac{1}{2\pi}\right) \int_{-\infty}^{\infty} e^{jpf}dp \\
 &= -K\delta(f)
 \end{aligned}
 \qquad
 \begin{aligned}
 h(t) &= \int_{-\infty}^{\infty} H(f)e^{j2\pi ft}df \\
 &= \int_{-\infty}^{\infty} Ke^{j2\pi ft}df \\
 &= K \int_{-\infty}^{\infty} e^{j2\pi ft}df \\
 &= K\left(\frac{1}{2\pi}\right) \int_{-\infty}^{\infty} e^{jpt}dp \\
 &= K\delta(t)
 \end{aligned}$$

There seems to be an issue with the minus sign on the first solution. I am not sure how to avoid having a minus sign for at least one of the solutions because of the structure of the transforms.

- 3) Verify that if $h(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT), H(f) = \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T}\right)$.

- 4) Verify that, if $h(t) \leftrightarrow H(f), h(t-t_0) \leftrightarrow H(f)e^{-j2\pi ft_0}$.

$$\begin{aligned}
H(f)e^{-j2\pi ft_0} &= \int_{-\infty}^{\infty} h(t-t_0)e^{-j2\pi ft} dt \\
&= \int_{-\infty}^{\infty} h(t')e^{-j2\pi f(t'+t_0)} dt' && \text{Let } t' = t - t_0 \\
&= \int_{-\infty}^{\infty} h(t')e^{-j2\pi ft'} e^{-j2\pi ft_0} dt' \\
&= \int_{-\infty}^{\infty} h(t')e^{-j2\pi ft'} dt' e^{-j2\pi ft_0} \\
&= H(f)e^{-j2\pi ft_0}
\end{aligned}
\qquad
\begin{aligned}
h(t-t_0) &= \int_{-\infty}^{\infty} H(f)e^{-j2\pi ft_0} e^{j2\pi ft} df \\
&= \int_{-\infty}^{\infty} H(f)e^{-j2\pi f(t-t_0)} df \\
&= h(t-t_0)
\end{aligned}$$

5) Verify that, if $h(t)$ is even, then $H(f)$ is real, and that if $h(t)$ is odd, then $H(f)$ is imaginary.

6) Verify that $x(t) * h(t) = \int_{-\infty}^{\infty} h(t)x(t-\tau)d\tau$.