

- 1) Consider a radar with a pulsed waveform (sinusoidally varying electric field) with a peak power of 6 kW. What is the absolute maximum power emitted? Does it depend on frequency? Polarization?

The absolute maximum emitted power is 6 kW and it does not depend on the frequency or the polarization.

- 2) Consider a flat plate antenna ($\eta = 0.5$) of area 1 m² operating at mid-C band.

- a) Calculate its gain

Antenna gain follows the following formula for cases such that the wavelength is much less than the aperture diameter:

$$G_0 = G(0, 0) = \frac{4\pi A\eta}{\lambda^2}$$

The question requests the gain in the mid-C band which would be approximately 6 GHz ($\lambda = \frac{c}{f} = 0.03$ m). Since the wavelength is much less than the effective diameter of the square antenna (~ 1 m), then the above equation holds for this condition.

$$G_0 = G(0, 0) = \frac{4\pi A\eta}{\lambda^2} = \frac{4\pi(1)(0.5)}{0.03^2} = 6981$$

- b) Calculate its peak RCS in the midband of L, W, C, X, and Ku. What is the equation relating its gain to RCS, as a function of λ ? (See Tables 3.1 and 3.2.)

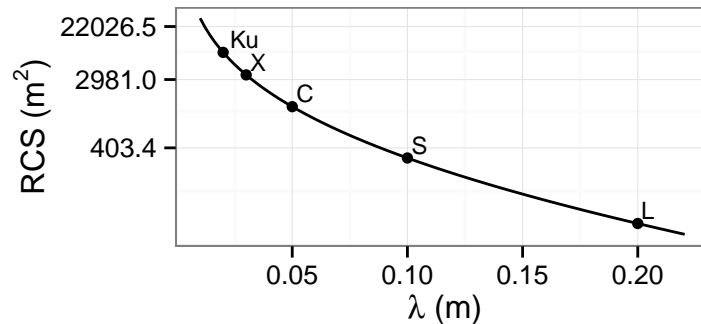
The peak RCS for a square antenna would be normal to the antenna aperture and is conveniently listed in table 3.2 as suggested. The formula is reproduced below:

$$\sigma = \frac{4\pi A^2}{\lambda^2}$$

The above equation does not take into account the geometrical efficiency of the antenna, η . This can be inserted into the cross-section equation by letting $A \rightarrow A\eta$. The resulting equation is the following,

$$\sigma = \frac{4\pi(A\eta)^2}{\lambda^2}$$

The peak RCS value in each of the desired bands is 78.54 m², 314.16 m², 1256.64 m², 3490.66 m², 7853.98 m² for the L, S, C, X, and Ku bands, respectively.



- 3) A mid-C band radar has the following:

$$\begin{aligned}P_{avg} &= 1 \text{ kW} \\G &= 30 \text{ dB} \\\tau_{\text{dwell}} &= 30 \text{ ms} \\T_s &= 580 \text{ K} \\L &= 6 \text{ dB}\end{aligned}$$

Calculate the maximum range at which it can detect a target of $\sigma = -20 \text{ dBm}^2$. Assume the SNR required for detection is 17 dB and $C_B = 1$.

The solution for this problem is accomplished by solving the radar equation for the Range (R) and substituting the above radar parameters after they are converted into matching dB scales.

$$\begin{aligned}\text{SNR} &= \frac{P_{avg} G^2 \lambda^2 \sigma \tau_{\text{dwell}}}{(4\pi)^3 R^4 k T_s C_B L} \\\therefore R &= \left[\frac{P_{avg} G^2 \lambda^2 \sigma \tau_{\text{dwell}}}{(4\pi)^3 (\text{SNR}) k T_s C_B L} \right]^{\frac{1}{4}}\end{aligned}$$

In keeping with the world of sonar and radar, most quantities are calculated in dB space. I now convert the above equation to dB form and substitute all the necessary quantities.

$$\begin{aligned}R &= \left[\frac{P_{avg} G^2 \lambda^2 \sigma \tau_{\text{dwell}}}{(4\pi)^3 (\text{SNR}) k T_s C_B L} \right]^{\frac{1}{4}} \\10 \log_{10}(R) &= 10 \log_{10} \left(\left[\frac{P_{avg} G^2 \lambda^2 \sigma \tau_{\text{dwell}}}{(4\pi)^3 (\text{SNR}) k T_s C_B L} \right]^{\frac{1}{4}} \right) \\&= \frac{1}{4} \cdot 10 [\log_{10}(P_{avg}) + 2 \log_{10}(G) + 2 \log_{10}(\lambda) + \log_{10}(\sigma) + \log_{10}(\tau_{\text{dwell}}) \\&\quad - 3 \log_{10}((4\pi)) - \log_{10}(\text{SNR}) - \log_{10}(k T_s) - \log_{10}(C_B) - \log_{10}(L)] \\&= \frac{1}{4} [40 + 60 + (-13.01) + (-20) + (-15.2) - 33 - 17 - (-201) - 0 - 7] \\&= \frac{1}{4} (195.79) = 48.9 \text{ dBm}\end{aligned}$$

The solution of 48.9 is in the units of dBm which correlates to 77,600 m.

- 4) For the radar used in the Estimate of Signal-to-Noise Ratio, compute, for the received signal, the power, the E field (volts/meter), the B field (tesla; $B = \mu_o H$), and the photons/second.

The parameters of the radar are such that:

$$\begin{aligned}P_{\text{peak}} &= 10^5 \text{ W} & R &= 20,000 \text{ m} \\G &= 40 \text{ dB} & T_s &= 580 \text{ K} \\\lambda &= 0.1 \text{ m} & C_B &= 1 \\\sigma &= 0.01 \text{ m}^2 & L &= 7 \text{ dB} \\\tau &= 100 \text{ ns}\end{aligned}$$

- 5) A lossless radar with gain G and wavelength λ is located at a height H above an infinite, flat planar, perfectly conducting surface, with boresight directed perpendicular to the surface. Compute Γ .

Since the reflector is infinite in size and perfectly conducting, this problem can be reduced to the situation of an emitter that is exactly $2H$ away. The received power is then going to be determined by assuming spherical spreading from the emitter and the cross-section of the receiving antenna. This is taken directly from Equation 1.20 in the text. In this case, $L \rightarrow 0$ since it is a lossless antenna and $R \rightarrow 2H$. The modified equation is:

$$\Gamma = \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 (2H)^4}$$