

- 1) Consider a radar with a pulsed waveform (sinusoidally varying electric field) with a peak power of 6 kW. What is the absolute maximum power emitted? Does it depend on frequency? Polarization?

The absolute maximum emitted power is 6 kW and it does not depend on the frequency or the polarization.

- 2) Consider a flat plate antenna ( $\eta = 0.5$ ) of area 1 m<sup>2</sup> operating at mid-C band.

- a) Calculate its gain

Antenna gain follows the following formula for cases such that the wavelength is much less than the aperture diameter:

$$G_0 = G(0, 0) = \frac{4\pi A\eta}{\lambda^2}$$

The question requests the gain in the mid-C band which would be approximately 6 GHz ( $\lambda = \frac{c}{f} = 0.03$  m). Since the wavelength is much less than the effective diameter of the square antenna ( $\sim 1$  m), then the above equation holds for this condition.

$$G_0 = G(0, 0) = \frac{4\pi A\eta}{\lambda^2} = \frac{4\pi(1)(0.5)}{0.03^2} = 6981$$

- b) Calculate its peak RCS in the midband of L, W, C, X, and Ku. What is the equation relating its gain to RCS, as a function of  $\lambda$ ? (See Tables 3.1 and 3.2.)

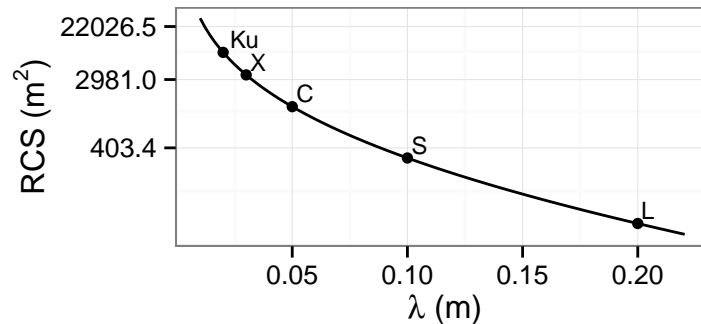
The peak RCS for a square antenna would be normal to the antenna aperture and is conveniently listed in table 3.2 as suggested. The formula is reproduced below:

$$\sigma = \frac{4\pi A^2}{\lambda^2}$$

The above equation does not take into account the geometrical efficiency of the antenna,  $\eta$ . This can be inserted into the cross-section equation by letting  $A \rightarrow A\eta$ . The resulting equation is the following,

$$\sigma = \frac{4\pi(A\eta)^2}{\lambda^2}$$

The peak RCS value in each of the desired bands is 78.54 m<sup>2</sup>, 314.16 m<sup>2</sup>, 1256.64 m<sup>2</sup>, 3490.66 m<sup>2</sup>, 7853.98 m<sup>2</sup> for the L, S, C, X, and Ku bands, respectively.



- 3) A mid-C band radar has the following:

$$\begin{aligned}P_{avg} &= 1 \text{ kW} \\G &= 30 \text{ dB} \\ \tau_{\text{dwell}} &= 30 \text{ ms} \\T_s &= 580 \text{ K} \\L &= 6 \text{ dB}\end{aligned}$$

Calculate the maximum range at which it can detect a target of  $\sigma = -20 \text{ dBm}^2$ . Assume the SNR required for detection is 17 dB and  $C_B = 1$ .

The solution for this problem is accomplished by solving the radar equation for the Range ( $R$ ) and substituting the above radar parameters after they are converted into matching dB scales.

$$\begin{aligned}\text{SNR} &= \frac{P_{avg} G^2 \lambda^2 \sigma \tau_{\text{dwell}}}{(4\pi)^3 R^4 k T_s C_B L} \\ \therefore R &= \left[ \frac{P_{avg} G^2 \lambda^2 \sigma \tau_{\text{dwell}}}{(4\pi)^3 (\text{SNR}) k T_s C_B L} \right]^{\frac{1}{4}}\end{aligned}$$

In keeping with the world of sonar and radar, most quantites are calculated in dB space. I now convert the above equation to dB form and substitute all the nessessary quantities.

$$\begin{aligned}R &= \left[ \frac{P_{avg} G^2 \lambda^2 \sigma \tau_{\text{dwell}}}{(4\pi)^3 (\text{SNR}) k T_s C_B L} \right]^{\frac{1}{4}} \\ 10 \log_{10}(R) &= 10 \log_{10} \left( \left[ \frac{P_{avg} G^2 \lambda^2 \sigma \tau_{\text{dwell}}}{(4\pi)^3 (\text{SNR}) k T_s C_B L} \right]^{\frac{1}{4}} \right) \\ &= \frac{1}{4} \cdot 10 [\log_{10}(P_{avg}) + 2 \log_{10}(G) + 2 \log_{10}(\lambda) + \log_{10}(\sigma) + \log_{10}(\tau_{\text{dwell}}) \\ &\quad - 3 \log_{10}((4\pi)) - \log_{10}(\text{SNR}) - \log_{10}(k T_s) - \log_{10}(C_B) - \log_{10}(L)] \\ &= \frac{1}{4} [40 + 60 + (-13.01) + (-20) + (-15.2) - 33 - 17 - (-201) - 0 - 7] \\ &= \frac{1}{4} (195.79) = 48.9 \text{ dBm}\end{aligned}$$

The solution of 48.9 is in the units of dBm which correlates to 77,600 m which is approximatly 42 NM.

- 4) For the radar used in the Estimate of Signal-to-Noise Ratio, compute, for the received signal, the power, the  $E$  field (volts/meter), the  $B$  field (tesla;  $B = \mu_o H$ ), and the photons/second.

The parameters of the radar are such that:

$$\begin{aligned}P_{\text{peak}} &= 10^5 \text{ W} & R &= 20,000 \text{ m} \\ G &= 40 \text{ dB} & T_s &= 580 \text{ K} \\ \lambda &= 0.1 \text{ m} & C_B &= 1 \\ \sigma &= 0.01 \text{ m}^2 & L &= 7 \text{ dB} \\ \tau &= 100 \text{ ns}\end{aligned}$$

**Power:**

The power of the received signal is derived from the following equation:

$$P_R = P_T \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L}$$

Since most of the inputs are in mks, I will continue to use that unit system and convert the gain and the losses to ratios of  $10^4$  and  $10^{0.7}$ , respectively. With these conversions the result is as follows:

$$\begin{aligned} P_R &= 10^5 \frac{(10^4)^2 (0.1)^2 (0.01)}{(4 * \pi)^3 (20000)^4 (10^{0.7})} \\ &= 6.28 \times 10^{-13} \text{ W} \end{aligned}$$

**Electric and Magnetic Fields:**

The electric and magnetic fields can be expressed as:

$$E = \sqrt{\frac{S}{c\epsilon_0}} \quad B = \sqrt{\frac{S}{c\mu_0^3}} \quad \{\text{given that } B = \mu_0 H\}$$

Since  $S$  is the magnitude of the Poynting vector which is simply the power which was derived in the previous step, the result is easily obtained as  $1.54 \times 10^{-5}$  V/m and 0.03 T for the electric field and the magnetic field, respectively.

**Number of Photons:**

The flux can be determined by taking the power, which is in J/s, and dividing it by the energy of the photon. The energy of the photon is given by  $E = \frac{hc}{\lambda}$  where  $h$  is the plank constant of  $6.626 \times 10^{-34}$  Js.

$$N_{\text{Photons}} = \frac{S\lambda}{hc} = \frac{(6.28 \times 10^{-13}) (0.1)}{(6.626 \times 10^{-34}) (3 \times 10^8)} = 3.16 \times 10^{11} \text{ photons/s/m}^2$$

Though this seems like a lot we need to factor in the effective area of the antenna. This can be determined by using the following equation:

$$A_e = \frac{G\lambda^2}{4\pi} = \frac{(10^4) (0.1^2)}{4\pi} = \frac{25}{\pi} = 7.96 \text{ m}^2$$

Scaling the above result for the number of photons per area per second results in a total of  $2.5 \times 10^{13}$  photons per second reaching the antenna. This seems a bit large to me which indicates that either I have a lack of understanding in scale or I have a very strong understanding of the probability of a mistake in the previous work.

- 5) A lossless radar with gain  $G$  and wavelength  $\lambda$  is located at a height  $H$  above an infinite, flat planar, perfectly conducting surface, with boresight directed perpendicular to the surface. Compute  $\Gamma$ .

Since the reflector is infinite in size and perfectly conducting, this problem can be reduced to the situation of an emitter that is exactly  $2H$  away. The received power is then going to be determined by assuming spherical spreading from the emitter and the cross-section of the receiving antenna. This is taken directly from Equation 1.20 in the text. In this case,  $L \rightarrow 0$  since it is a lossless antenna and  $R \rightarrow 2H$ . The modified equation is:

$$\Gamma = \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 (2H)^4}$$