



Computational examples in transport phenomena

with Python

This document was prepared as part of the course *The Basics of Transport Phenomena* from Delft University of Technology, available on edX.org as DelftX: TP101x.

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camillejr.github.io/science-docs/
To contact me personally drop me a line at:
kamilazdybal@gmail.com

Cover picture: *Forêt de Soignes, Belgium*
Photo taken on: 16 Jan 2017
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1 General remarks on transport of physical properties

The general equation for the transport of a property ψ within a control volume is:

$$\frac{d\psi}{dt} = \Psi_{in} - \Psi_{out} + \Psi_{production} - \Psi_{consumption} \quad (1)$$

where $\frac{d\psi}{dt} = \dot{\psi}$ is the flux of the property ψ .

2 Steady-state conduction in a lengthwise-insulated rod with internal heat production

We derive a one-dimensional temperature distribution function $T(x)$ for a steady-state heat conduction in a straight rod of length L . We assume that the internal heat production Q_p in units of W/m^3 is present at every point inside the rod volume. This may for instance simulate heating up of a wire due to electrical current. The rod is perfectly insulated along its length and it loses heat only through its endpoints which in a steady-state case are kept at a fixed temperature T_0 .

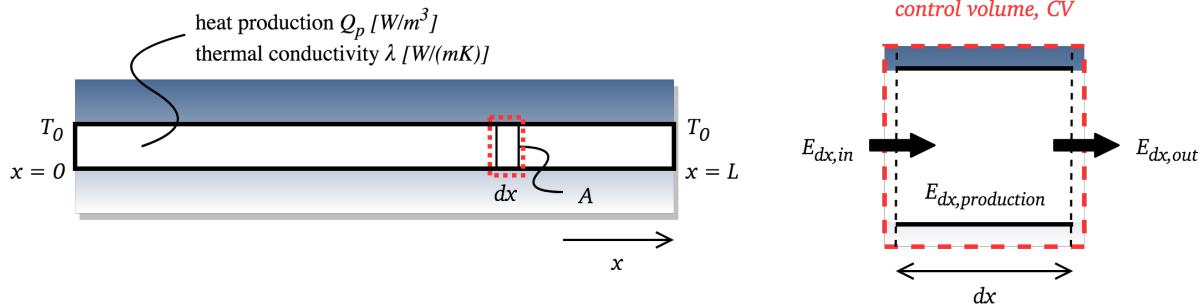


Figure 1: Conduction in a rod with internal heat production.

We will take for the control volume a slice dx from the rod. The energy balance for the rod element dx :

$$\frac{dE_{dx}}{dt} = E_{dx,in} - E_{dx,out} + E_{dx,production} \quad (2)$$

Note here that $E_{dx,in}$, $E_{dx,out}$ and $E_{dx,production}$ are energies per unit time and so have the units of W .

The heat flux ϕ which has units of W can be modeled using the Fourier's law for one-dimensional heat conduction:

$$\phi = \lambda A \left(-\frac{dT}{dx} \right) \quad (3)$$

where λ is the thermal conductivity and is a property of the material, A is the rod's cross-sectional area and $\frac{dT}{dx}$ is a temperature gradient which plays a role of the driving force for thermal energy transport.

Hence:

$$E_{dx,in} = \lambda A \left(-\frac{dT}{dx} \right)_x \quad (4)$$

$$E_{dx,out} = \lambda A \left(-\frac{dT}{dx} \right)_{x+dx} \quad (5)$$

The energy per unit time coming from the heat production can be written as Q_p multiplied by the volume of the slice dx :

$$E_{dx,production} = Q_p A dx \quad (6)$$

In the steady-state $\frac{dE}{dt} = 0$ and the energy balance becomes:

$$\lambda A \left(-\frac{dT}{dx} \right)_x - \lambda A \left(-\frac{dT}{dx} \right)_{x+dx} + Q_p A dx = 0 \quad (7)$$

Simplifying the above energy balance we get:

$$\frac{\left(\frac{dT}{dx}\right)_{x+dx} - \left(\frac{dT}{dx}\right)_x}{dx} = -\frac{Q_p}{\lambda}$$

It is interesting to note here that we have lost the dependence on the cross-sectional surface area of the rod.

If we now substitute some function $f(x) = \frac{dT}{dx}$ we notice that we have:

$$\frac{f(x+dx) - f(x)}{dx} = -\frac{Q_p}{\lambda}$$

in other words:

$$\frac{df(x)}{dx} = -\frac{Q_p}{\lambda} \quad (8)$$

With the above substitution, the differential equation that we are about to solve becomes:

$$\frac{d^2T}{dx^2} = -\frac{Q_p}{\lambda} \quad (9)$$

Applying the boundary conditions from both ends of the rod, the solution to the above differential equation is:

$$T(x) = -\frac{Q_p}{2\lambda}(x^2 - Lx) + T_0 \quad (10)$$

2.1 Final remarks

Note that even though the heat flow was assumed to be one-dimensional in this exercise, it does not mean that the geometry of the problem needs to be one-dimensional. In fact, we assumed the rod to be a three-dimensional object having length and a cross-sectional area. Rather, the one-dimensionality of the problem means that it is practical to assume only one of the three directions as an important direction for heat transport. Since the rod was perfectly insulated along its length, the temperature gradient along directions perpendicular to the x -axis is zero.

2.2 Computational example

As a computational example we will draw the graph of the temperature distribution in a copper rod 200m long. We assume that the thermal conductivity for this rod is $400 \frac{W}{m \cdot K}$. The internal heat production in the entire rod is 20W.

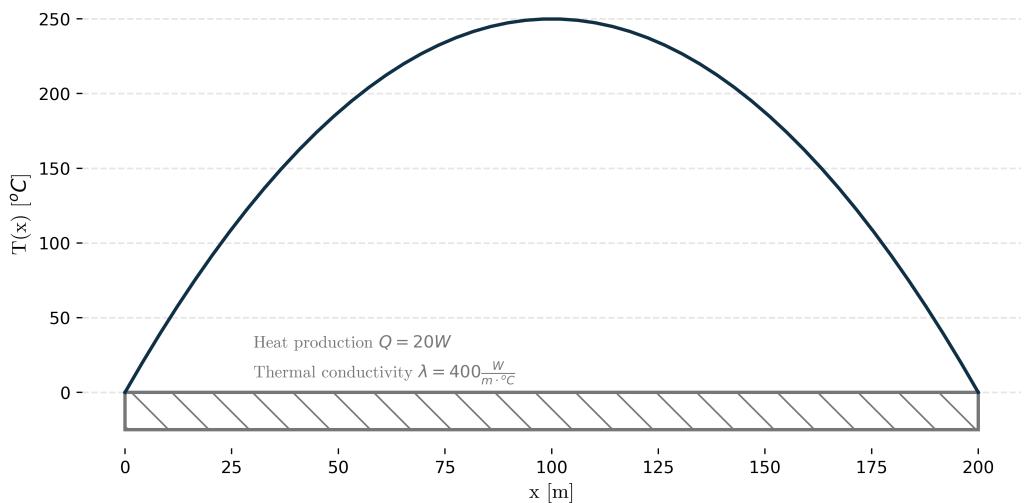


Figure 2: Temperature distribution in a rod with internal heat production of 20W.

3 Laminar flows of Newtonian fluids

4 Lumped system analysis of a steel plate

5 Evaporating sphere

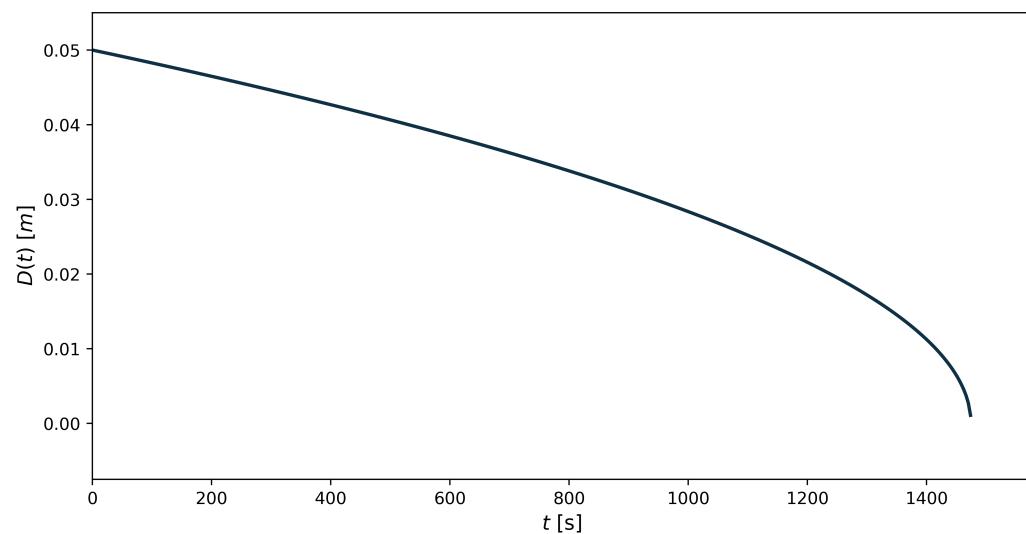


Figure 3: Evaporating sphere diameter history.

6 One-dimensional binary diffusion

7 The Stefan problem

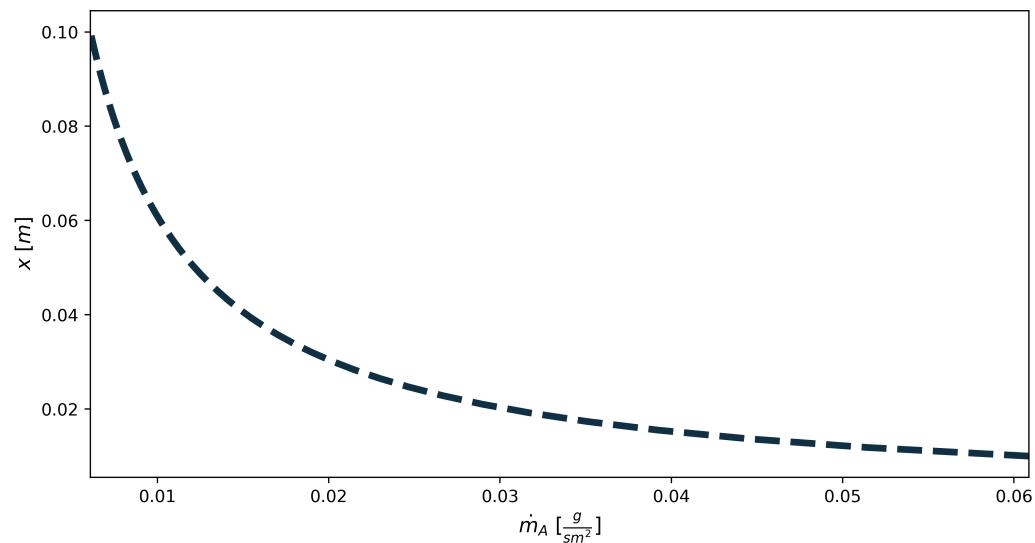


Figure 4: Stefan problem for water evaporating from the 0.1m tube.