Regression: Evaluation and interpretation

Herman Kamper

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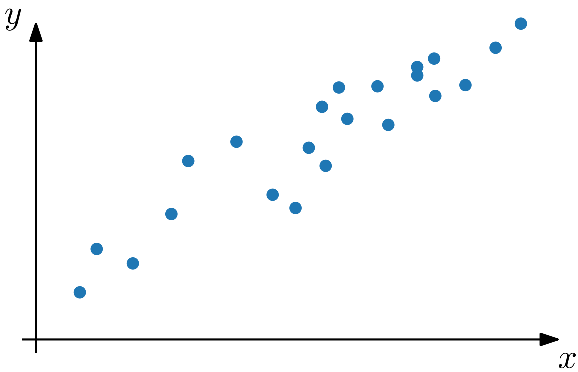
Interpretation of linear regression weights

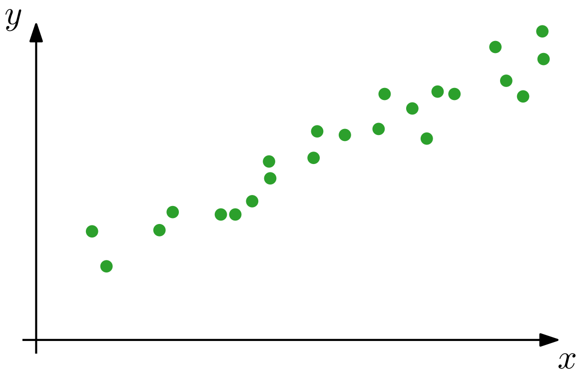
What does w_1 tell us in this case?

$$f(x; \mathbf{w}) = w_0 + w_1 x$$



What does w_1 tell us in this case?

$$f(x; \mathbf{w}) = w_0 + w_1 x$$



Could potentially solve this problem by using multiple linear regression:

$$f(\mathbf{x}; \mathbf{w}) = w_0 + w_1 x_1 + w_2 x_2$$

This might still not solve all your problems and you will still not know whether temperature causes ice cream sales or the other way around.

Linear regression and causation

Example

Let us use L_1 regularisation to pick the five most meaningful features in a problem where $\mathbf{x} \in \mathbb{R}^{100}$.

We vary λ until we only have five non-zero w's.

Are these five features the things that most "cause" the output y?

No! They are the best five features for predicting y, given several assumptions (e.g. that we are using a linear model).

Takeaway

Be careful about making statements about linear regression coefficients.

Linear models can be very useful since they are often more interpretable than more complex models.

But they won't always give a complete picture—they are often most useful in conjunction with some other model or hypothesis of the real world (domain knowledge).

Sometimes the most you are able to say is: "These features are the most useful for predicting the output given that we are using a linear model." (And maybe that is useful enough.)

Further reading and watching

- D. Mackenzie and J. Pearl, *The Book of Why*, 2018.
- https://www.tylervigen.com/spurious-correlations
- Why Brad Pitt's eating matters

Regression evaluation metrics

We want one number to summarise performance.

Squared loss:

$$J = \sum_{n=1}^{N} \left(y^{(n)} - f(\mathbf{x}^{(n)}; \mathbf{w})\right)^2$$

Mean squared error (MSE):

$$\mathrm{MSE} = \frac{1}{N} \sum_{n=1}^{N} \left(y^{(n)} - f(\mathbf{x}^{(n)}; \mathbf{w})\right)^2$$

Root-mean-square error (RMSE):

$$\mathrm{RMSE} = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (y^{(n)} - f(\mathbf{x}^{(n)}; \mathbf{w}))^2}$$

Videos covered in this note

- Linear regression 6: Evaluation and interpretation (11 min)

Reading

- ISLR 2.2.1
- ISLR 3.2.1