Quasi-Two-Dimensional Reservoir Simulation Model

By Graeme C. Hocking[1] and John C. Patterson[2]

Abstract: A quasi-two-dimensional model for the simulation of temperature, salinity, and density in a reservoir is described. The model is based upon the one-dimensional reservoir simulation model DYRESM, and is extended into two dimensions using a Lagrangian formulation of self-contained parcels within the layer structure of the one-dimensional model. These parcels resolve the horizontal motions and gradients by moving vertically or horizontally and splitting and combining to simulate the various physical processes in the reservoir occurring in response to the prevailing climatological conditions. The model is not a full two-dimensional model, because not all horizontal processes are included. Algorithms to simulate inflow, selective withdrawal, and adjustment of horizontal density gradients are included, however, and the model is therefore quite useful for examining some simple scenarios in which these processes are dominant. The results of simulations of Canning Reservoir in Western Australia showing several inflow and intrusion events at different times of the year, and the return of the reservoir temperature field to a one-dimensional structure after the cessation of inflow, are presented. These results are compared with measurements taken in the field, with good agreement. The usefulness of the model in its present form for following the path and estimating the detention time of various parcels of water is discussed.

Introduction

The water enclosed in a storage reservoir or lake in most parts of the world is stratified for most of the year by variations in temperature and salinity. This density stratification inhibits vertical motions within the lake, since any fluid particle displaced vertically will experience a restoring force and eventually return to its original level. In the past, this property has been used to develop one-dimensional models of reservoir dynamics, in which the lake is assumed to vary only in the vertical direction. The assumption upon which these models is based is that the horizontal adjustments of the displaced fluid are instantaneous. Since the horizontal movement of water does not generally alter the vertical structure significantly, processes that lead to such motions, and the motions themselves, are neglected. The development of these one-dimensional models was reviewed by Harleman (1982) and Imberger and Patterson (1989), and the conditions under which the one-dimensional assumption is valid are given in Patterson et al. (1984) and Imberger and Patterson (1989). These models, although very useful tools for reservoir management and for assessing long-term trends within a reservoir, have the limitation that even if the density structure is one dimensional, the horizontal transport of tracers cannot be modeled. Thus, for example, estimates of the rate of spread of toxic spills or contaminated water or calculations of residence time cannot be made.

In this paper a quasi-two-dimensional Lagrangian reservoir-simulation

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Note. Discussion open until March 1, 1992. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on March 28, 1990. This paper is part of the *Journal of Environmental Engineering*, Vol. 117, No. 5, September/October, 1991. ©ASCE, ISSN 0733-9372/91/0005-0595/$1.00 + $.15 per page. Paper No. 26201.

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model is described that accounts for some of the horizontal motions in water impoundments. The structure and formulation of the model are based upon the successful one-dimensional reservoir simulation model DYRESM, which has been extensively described and verified in the literature (Imberger et al. 1978; Hebbert et al. 1979; Imberger and Patterson 1981; Imberger 1982; Ivey and Patterson 1984; Patterson et al. 1984; Hocking et al. 1988). The two-dimensional aspects of the new model are based upon the preliminary work of Jokela and Patterson (1985), with the addition of several new physical algorithms that describe various horizontal transport processes. As in the case of the one-dimensional model, no calibration of the model is required, since all of the algorithms are process based.

The model thus takes a different approach to that of the existing two-dimensional models of reservoir dynamics. Buchak and Edinger (1979) described a laterally averaged, two-dimensional hydrodynamic and water-quality model, and Waldrop et al. (1980) described a similar model. These models use finite differences on a fixed grid to solve the equation that results from the substitution of the longitudinal momentum equation into the vertically integrated continuity equation. These models have been successfully applied to a range of reservoirs (Edinger et al. 1983; Gordon et al. 1981; Kim et al. 1983; Waldrop et al. 1980).

On many occasions the one-dimensional assumption is valid, yet there is some interest in determining the horizontal transport in the impoundment due to inflow and intrusion of streams and withdrawal, without resorting to a full two-dimensional model. The present model allows this to be done, providing some horizontal resolution and tracer monitoring on what is essentially a one-dimensional background. The very inclusion of these processes, however, must introduce some perturbations to this mean field and a degree of two-dimensionality.

Despite the simplicity of the approach described, the results presented are good, and show that under these conditions both underflows and mid-level intrusions can be resolved quite well in terms of both location and timing. The model thus takes advantage of the speed and simplicity of the structure of the one-dimensional models to resolve some horizontal motions.

Clearly, the model will not be appropriate in cases in which a significant two-dimensional structure in the density field is created by reasons other than inflow or if the inflows become very large. Equally, if there are significant lateral variations in the temperature field, this model is inappropriate. In these cases a full two- or three-dimensional model of the kind just described is required.

The model described here does not attempt to simulate wind-driven circulation, differential deepening due to wind sheltering, differential heating due to differing depths in side arms or patches of higher turbidity, boundary mixing, or upwelling, all of which can cause significant horizontal velocities. The local geometry is a significant factor in all of these processes. A review of these processes and a discussion of recent work is presented in Imberger and Patterson (1989).

The model retains the lateral averaging implicit in the one-dimensional approach. Further, the model cannot in its present form simulate local small-scale regions of instability or simulate in detail the internal wave field. It can, however, model the collapse and subsequent intrusion of regions of homogeneity in a stratified reservoir.

Since the new model is based upon the one-dimensional model, we begin with a review of the original model and its physical algorithms. This basis

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is then extended with a description of how the algorithms were upgraded to cater to the two-dimensional structure, and a description of the new algorithms created to model horizontal transport. Data requirements, the range of validity of the model, and possible extensions are also considered. The resulting model is used to simulate the conditions in the Canning Reservoir, in the southwest of Western Australia, over a period of 230 days and the results are compared with the field data for the corresponding period.

ONE-DIMENSIONAL MODEL

The one-dimensional reservoir simulation model DYRESM was developed over a number of years to a point where, given recent advances in computer technology and refinements in the code, it can be run easily and quickly on a personal computer. It uses Lagrangian layers that combine, expand, contract, move vertically, and divide in response to the physical processes within the reservoir, and has been applied successfully on a number of different reservoirs (Imberger et al. 1978; Hebbert et al. 1979; Ivey and Patterson 1984; Patterson et al. 1984). DYRESM may, for example, be used to assess long-term trends in a reservoir when changes are made to the management policy, or it can be used to predict the thermal and salinity character for various climatological scenarios. In addition, several special versions exist that have been used for modeling dissolved oxygen (Patterson et al. 1985), for modeling lakes with an ice cover (Patterson and Hamblin 1988), and modeling the effect of bubble plume destratification devices (Patterson and Imberger 1989).

The averaged background structure upon which the two-dimensional motions occur is provided by the processes resolved by the one-dimensional model, and hence an accurate prediction of this is crucial to the extension of the model into higher dimensions.

Each of the one-dimensional physical processes important in determining the structure within the reservoir has been identified and modeled separately up to the level of current understanding of that process. In this way, events that occur over time scales as small as several minutes or as long as several days can be accurately modeled. As new processes are discovered or our understanding of the current processes is increased the model can be upgraded. The separate description of each physical process means that no calibration of the model is necessary when it is applied to a new lake, because the parameters associated with the processes have been determined from laboratory or field investigations, independent of the modeling exercise.

The use of Lagrangian layers and a variable time step allows the model to make the most efficient use of computer time. Layers are automatically concentrated to give maximum resolution near regions of greatest change, for example at the thermocline. Similarly, the time step is shortened when mixing activity driven by the surface inputs increases, and lengthened at times when there is little activity.

The adjustment of the layer structure is performed subject to a table of volumes and areas computed at every 10 cm depth. Thus layers move up and down within the confines of the reservoir boundaries, increasing their thickness if they happen to move vertically downward to a height where the area is smaller, and decreasing if they move upward. To maintain the desired range of layer sizes there are upper and lower bounds on both the thickness and volume of layers, and layers are split or merged if they exceed these bounds.

The layer structure is ideal for modeling the mixing and transfer of energy in the surface region of a lake using an energy budget formulation. The Lagrangian formulation allows the mixing processes to be modeled by using the available energy to combine layers, thus increasing the stored energy. The algorithms developed to handle these processes are described in detail in Imberger and Patterson (1981) and Imberger (1982).

The physical processes within a reservoir are driven by the atmospheric conditions and the inflow and outflow of water. Daily total values of short- and long-wave radiation and rainfall; and daily averages of air temperature, relative humidity, and wind speed taken at the site of the reservoir are required as inputs to the model. These inputs drive the heating, cooling, and mixing phases of the model. Since the measurements are usually available at a single height above the water level, the usual bulk aerodynamic formulas (TVA: "Heat" 1972; Henderson-Sellers 1986) are used to compute the transfer of heat, momentum, and moisture across the air-water interface.

In addition to the surface-heat transfers, the long-wave radiation emitted from the surface must be computed (TVA: "Heat" 1972), and the long- and short-wave absorption are added. Long-wave exchange occurs in the top few centimeters of the lake, but the shortwave penetrates to greater depths, a fact that is accounted for by the inclusion of a light-attenuation factor measured in the lake under consideration. The long-wave radiation affects only the top layer of the model, but the amount of heat due to shortwave radiation entering each layer must be calculated and added. The changes in surface level due to evaporation and rainfall are computed.

All of the inputs are assumed to be constant throughout the day with the exception of shortwave radiation, which is always zero for the second 12 hr to account for the diurnal light-dark cycle. The geographical effect of variation of the number of daylight hours is accounted for by distributing the total incoming shortwave radiation over the modeled 12 hr period, causing negligible errors since the energy budget for the day is still correct.

The time step is set depending upon the amount of meteorological activity prevailing at the time, and is limited to a range between 15 min and 12 hr.

The surface mixed layer deepening algorithm is based on an integral turbulent kinetic energy (TKE) model that includes the effects of natural convection caused by surface cooling, energy input by wind stirring the surface and seiche-induced shear at the base of the surface layer, and billowing at the interface due to shear instability. Thus a fraction of the energy input from these processes is made available at the base of the surface layer to mix the heavier quiescent water below into the surface layer.

When water from a river enters a lake or reservoir, it may maintain its identity for a long period of time. If it is lighter than the water in the basin it will flow over the surface, but if it is heavier than the surface water it will plunge down until it reaches its level of neutral buoyancy. Once at this level, it will intrude horizontally into the main water body. The intrusion is restricted in its vertical extent by buoyancy forces, and may travel across the whole reservoir in a layer that is only a few meters deep, depending upon the amount of flow and the degree of density stratification.

The underflow algorithm allows the inflow parcels, each corresponding to a single days' inflow from a single source, to remain separate from the main layer structure of the model and yet remain within the body of the reservoir. Fig. 1 shows this algorithm schematically, with the daily inflow parcels numbered to indicate the time (in days) before they will enter the reservoir. During a time step, each parcel is considered in turn, and its path

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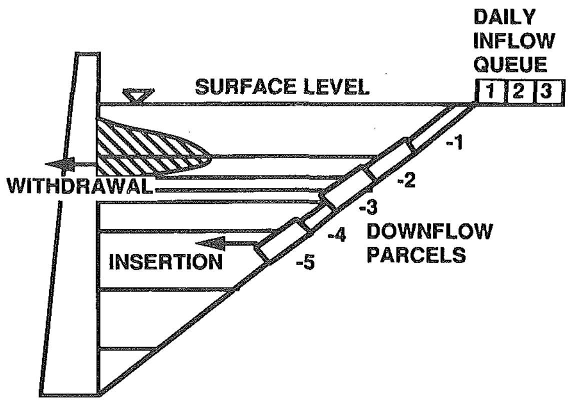

FIG. 1. Schematic Diagram of One-Dimensional Inflow and Withdrawal Algorithms

followed as it travels downward, entraining fluid as it progresses, until the end of the time step or the parcel reaches its level of neutral buoyancy. The algorithms for computing the flow velocity and the entrainment are given in Patterson and Imberger (1981) and Imberger (1982). A parcel that has reached its level of neutral buoyancy is set aside until the downflow and entrainment of all of the other parcels have been considered for that time step; then the vertical extent of the resultant intrusion is calculated from either an inertia-buoyancy or a viscous-buoyancy balance, and the parcel is distributed over the appropriate layers.

The water that remains in the downflow must be stored separately from the main layer structure of the model but still within the confines of the reservoir. This is achieved, as described in Hocking and Patterson (1987), by recalculating the storage versus depth table, taking into account the volume and location of the inflows. All layer calculations are performed using this modified storage table until the next passage of the model through the inflow algorithm. Although this water is included in the body of the storage, it plays no part in the mixing or heating processes. The error in doing this is small if the volume contained within the region is less than about 10% of the total reservoir volume. This inflow algorithm introduces some two-dimensionality to the model, as described in Hocking and Patterson (1987).

In a similar manner to the intrusion of fluid, most of the flow induced when fluid is withdrawn from a stratified water body occurs within a horizontal withdrawal layer centered approximately at the level of the outlet. Hocking et al. (1988), following the assumed form for the velocity within the withdrawal layer used by Spigel and Farrant (1984), integrated back along the streamlines over the time step to compute curves bounding the region of fluid that would be withdrawn for the given flow rate and stratification. An example of such a curve is given in Fig. 1, which shows schematically an envelope of withdrawal (Hocking et al. 1988). In the model, the fluid to be withdrawn is calculated from these envelopes by integrating them across the boundaries of the model layers. The appropriate amount is removed from each layer and the layer structure is allowed to relax according to the constraints of the storage-depth table.

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The modeling of sporadic turbulent mixing events in the deeper part of the reservoir was performed (Imberger et al. 1978; Imberger and Patterson 1981; Imberger 1982) on the assumption that the sum of all of these mixing events may be modeled as a diffusion process. These mixing events may consist of boundary mixing, internal wave interactions, and internal wave-breaking (Garret 1979; Imberger and Patterson 1989). This mixing is characterized by an overturning motion on a scale of at most several meters. Energy introduced into the lake by the wind acting on the surface of the lake, and the plunging of inflows, induces mean motions and internal waves into the lake. These have scales ranging from a few centimeters to the size of the lake basin. While these motions do not induce any direct mixing, their interaction may result in local overturning events given the right conditions. The energy introduced to the lake is distributed by internal wave activity, and hence less energy will be dissipated in the weakly stratified hypolimnion. The model accounts for this by linking the energy dissipation to the stratification as described in Imberger (1982). The determination of the turbulent diffusion coefficient is described in Imberger (1982), and follows the revised formulation of Weinstock (1981).

Turbulent diffusion is the only process in the one-dimensional model that is handled by a finite-difference scheme. An explicit scheme resembling a classical forward-time centered-space (FTCS) scheme is used to solve the diffusion equation with a turbulent diffusion coefficient calculated as indicated earlier.

Many successful simulations have been conducted with this one-dimensional model (Imberger et al. 1978; Hebbert et al. 1979; Imberger and Patterson 1981; Imberger 1982; Ivey and Patterson 1984; Patterson et al. 1984; Patterson and Imberger 1989). A recent example, relevant to the validation of the new model, is the Canning Reservoir in Western Australia. Fig. 2(*a*) shows the thermal structure measured in the main basin of the reservoir over a 400-day period between June 1986 and October 1987. Fig. 2(*b*) shows the simulation results obtained using the one-dimensional model over the corresponding period. All aspects of the thermal stratification are accurately predicted over the full period of the simulation.

Two-Dimensional Model

The success of the one-dimensional model in reproducing the vertical thermal and salinity structure in a reservoir indicates that the process descriptions model the physical behavior correctly. This suggests that these processes may be applied separately over much smaller regions of the lake, rather than uniformly over the entire surface, to give horizontal resolution over the reservoir.

The local application of processes is limited, however, by the availability of data. To model the local deepening of the surface mixed layer due to the effect of the wind, for example, measurements of wind speed at many locations on the lake would be required. Similar sets of data would be required for all of the other meteorological variables. Without this information, the effect of sheltering by the local topography can only be estimated. Such complete sets of data are rarely available, and consequently this model assumes uniformity of the surface inputs, although there is no reason why a more complete data set could not be used.

Two major examples of two-dimensional processes are intrusions caused by inflows from rivers and withdrawal of fluid through the outlets. These

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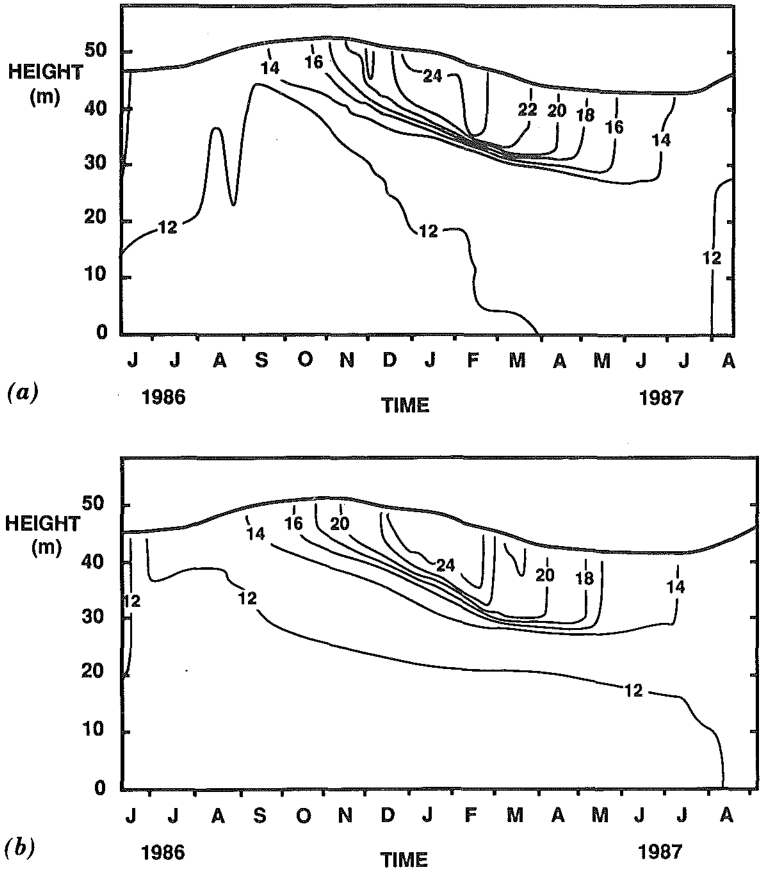

FIG. 2. Comparison of Thermal Structure in Canning Reservoir between June 1986 and August 1987: Contours from (*a*) Field Measurements, and (*b*) Simulation Results

processes have already been considered in relation to the one-dimensional model. The one-dimensional algorithms describing these processes were modified to include horizontal transport, and another algorithm that models density-driven horizontal motions was added.

This density-relaxation algorithm allows the later inclusion of other algorithms that simulate physical processes that result in density-driven motions, such as differential heating due to bathymetric effects and differential deepening due to variations in wind strength, as they become more clearly understood. The model described here is only quasi-two-dimensional, since it does not include all of these effects. Nevertheless, the model is useful in examining some horizontal transport issues, such as the dispersal of inflows.

The concept of Lagrangian elements is retained in the expanded model and horizontal parcels of variable size are included within each of the horizontal layers. The upper and lower boundaries of the parcels correspond to the bounds of the layer within which they fall (see Fig. 3). Each parcel

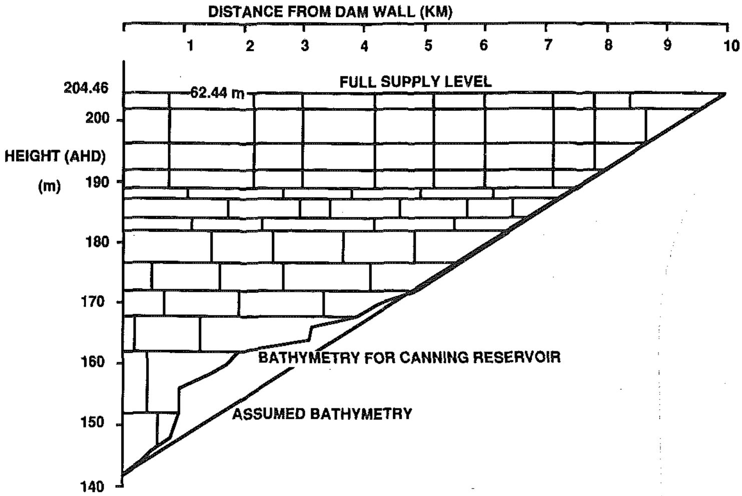

FIG. 3. Layer and Parcel Structure of Two-Dimensional Model and Comparison of Bathymetry of Canning Reservoir with that Assumed by Model

is assumed to extend the full width of the reservoir at that section, implicitly assuming that the reservoir is much longer than it is wide. Since the motions across the reservoir occur on the same time scales as the longitudinal motions, there is no justification for having parcels of length less than the width of the reservoir at any section. The model thus simulates a vertical section up the main reservoir valley, starting at the dam wall.

The volume-depth table used in the two-dimensional model is the same as that used in the one-dimensional model to compute the volumes within each layer. It is envisaged that an equivalent table will eventually be added that computes cumulative volumes in the horizontal direction and enables more accurate calculation of parcel volumes, but in the absence of these data the reservoir is assumed to be triangular in plan, with the apex of the triangle at the upstream end. This approximation is only used to calculate the volume of each parcel within a layer, however, and not the total volume within a layer. The mean slope of the major river valley forms the bottom boundary of the model, although this can easily be modified to include a piecewise linear representation of the bottom bathymetry. Fig. 3 shows a comparison of the assumed shape of the bottom boundary with the actual shape for Canning Reservoir.

As with the layers in the one-dimensional model, there are limits on the size of parcels. The average parcel length is usually around 400 m, depending on the physical shape of the reservoir in question. Parcels that become too large are divided, and those that become too small are combined with their smallest neighbor. This process leads to a small amount of numerical diffusion, since small parcels created when two layers are combined, for example, may be mixed with their smallest neighbor, spreading their combined properties over the new volume. Since this only affects small parcels the error is small.

The fundamental elements of the successful one-dimensional mixing and surface inputs algorithms are retained in the two-dimensional model. Properties are averaged over all of the parcels in a given layer, and the relevant algorithms are performed as before. Thus surface mixing occurs to a uniform depth over the full reservoir.

To retain the horizontal resolution in the mixing processes, algorithms were developed to handle the merging and splitting of those layers containing horizontal parcels. Fig. 4 shows how these two processes are modeled. When two layers are mixed, each parcel is only mixed with that fluid directly above or below it. New parcel boundaries (shown as dashed lines) are created at all of those places where the original parcels overlapped. The successful implementation of these algorithms means that the transition to the two-dimensional model involved minimal alterations to the algorithms concerned with mixing.

Turbulent diffusion is once again modeled by solving the diffusion equation. The turbulent diffusion coefficient, calculated as in the one-dimensional model, is assumed to be independent of the horizontal direction, so that all parcels in each layer are subject to the same rate of turbulent diffusion. Rather than solving the equation on a layer-by-layer basis, however, we must now solve it on a parcel-by-parcel basis. The algorithm scans through each layer parcel by parcel, computing the diffusive flux between overlapping parcels. Only vertical diffusion is modeled at this time, since the large parcel sizes and smaller horizontal-density gradients diminish the effect of horizontal diffusion on the evolution of the density field.

As noted in the description of the one-dimensional model, the downflow and entrainment sections of the inflow are described by following the path of a parcel representing each day's inflow. In this way a degree of two-dimensionality was introduced to the one-dimensional model.

This process is retained in describing the downward motion of an inflowing stream. As it travels down it is successively compared with the parcels on its travel path. If the density is less than the density of the adjacent parcel, the inflow is deemed to have reached its level of neutral buoyancy. The major inflow source is assumed to travel down the main basin, and the minor sources down a path determined by the mean slope of their bottom. Once at their level of neutral buoyancy, the parcels are allowed to intrude horizontally. At this point they are removed from the downflow arrays and added to the main parcel structure of the reservoir.

The horizontal extent of the intrusion is decided as in the one-dimensional model, and new parcels are created in each layer over which the intrusion

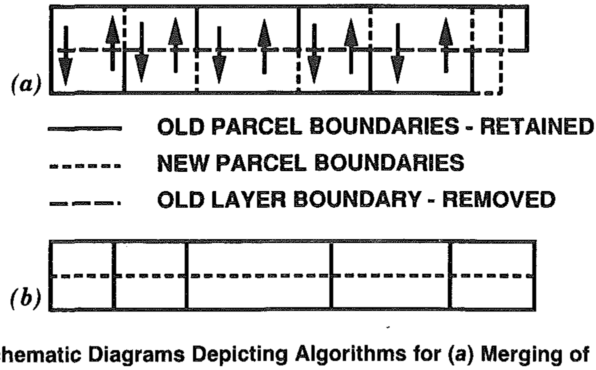

FIG. 4. Schematic Diagrams Depicting Algorithms for (*a*) Merging of Two Layers in Model; and (*b*) Splitting of Two Layers in Model

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occurs. The location of these new parcels is determined by the travel path for each inflow source. Thus intrusions from mid-basin rivers appear as parcels in the middle of the reservoir. If parcels from the previous days' inflows are still at that location, they are pushed along by the arrival of the new parcels.

The basis of the withdrawal algorithm was described in the previous section in relation to the one-dimensional model. The structure chosen for the one-dimensional withdrawal algorithm fits neatly into the two-dimensional model also. The concept of calculating the envelope surrounding the water to be drawn in the next time step is particularly convenient for the parcel structure described.

The envelope of withdrawal calculated from the density gradient in the reservoir and flow rate of withdrawal encompasses a number of parcels over a number of layers. The parcels completely enclosed by the envelope are removed; and the amount to be removed from those that are partially covered must be calculated by integrating the withdrawal curve through that parcel. The properties of the withdrawn water are then calculated by combing properties from the indicated parcels, and the remaining parcels are allowed to adjust to conserve volume. The process is repeated for all of the outlets, and is also required to model overflow if the reservoir surface level rises above the dam spillway.

The intrusion of inflows and motion induced by the withdrawals combined with various mixing processes of the model can lead to several different density fields in which horizontal gradients exist. A major feature of the model is an algorithm that models the relaxation of these gradients.

Gravitational Adjustment Algorithm

Events in which isolated mixing or a river intrusion occurs in a stratified region of fluid lead to the formation of a patch of homogeneous fluid surrounded by the ambient density field. Near the surface, an intrusion into the surface mixed layer followed by surface mixing may lead to two homogeneous, horizontally opposed regions with slightly different densities, as in the lock exchange problem (Brooke-Benjamin 1968).

In the former case, the homogeneous patch of fluid will collapse vertically, causing it to spread horizontally about the level at which it is neutrally buoyant. In the latter case, the lighter of the two regions will flow over the heavier, and the heavier under the lighter.

To resolve these motions completely would require a full solution of Navier-Stokes equation on the irregular grid that resolves the reservoir. This would be a very intensive task numerically, and is unsuitable for the type of model described in this paper, which is designed for rapid modeling of motion in reservoirs. To overcome this problem, the following algorithm was developed.

The fundamental assumption of this algorithm is that the lake is always moving toward a stable, equilibrium state, in which all surfaces of constant density are horizontal. It is designed to handle situations in which any parcel of fluid not at this equilibrium level is part of a larger homogeneous patch that extends vertically to that level. It is not designed to model local, small-scale regions of instability, nor is it designed to simulate the internal wave field, but rather the algorithm is to model the collapse and subsequent intrusion of any region of homogeneity that may form.

It is important to emphasize that this algorithm would be unsuitable for modeling downflowing water from an inflow, since it assumes the velocities of the parcels are small, which will generally not be the case for a plunging inflow. These inflow parcels are treated separately, as described earlier.

Each parcel in the model is considered in turn to determine whether it is at the equilibrium level, defined to be the level at which the density of the parcel coincides with the mean density computed by considering all of the parcels in the layer. It is assumed that each parcel that is not at this neutrally buoyant level is moving vertically toward it and that it is part of a much larger homogeneous patch, with its center at the equilibrium level. The degree of ambient stratification is taken to be the difference between the mean density of the whole layer in which the parcel is situated and the mean density of the layer that has density closest to that of the parcel under consideration.

The vertical velocity of a parcel is therefore calculated by computing the velocity that the parcel would have if it were part of a homogeneous patch within a linearly stratified fluid. It follows from this that the pressure difference between the middle of the homogeneous patch and the ambient fluid, at the equilibrium level, would be approximately

$$\Delta p(t = 0) = -\frac{1}{2}\rho_z g H^2 \quad \text{(1)}$$

where H = vertical distance to the parcel of interest from the neutral level; g = acceleration due to gravity; ρ = density; z = vertical coordinate; and the subscript denotes partial differentiation. Assuming the initial velocity is small, then the u-momentum equation applied between the middle of the homogeneous patch and the ambient fluid at this level gives

$$u_t = \frac{\Delta p}{\rho L} = -\frac{1}{2}\left(\frac{g}{\rho}\rho_z\right)\frac{H^2}{L} = \frac{1}{2}\frac{N^2H^2(t)}{L(t)} \quad \text{(2)}$$

where N = buoyancy frequency, defined as $N = (-g\rho_z/\rho)^{1/2}$. The time to reach steady state for this situation can be shown to be on the order of one day, so for the reservoir model described here, running on quite short time steps, the unsteady balance is appropriate.

Invoking a conservation-of-volume argument, $H_0L_0 = H(t)L(t)$, where H_0 = initial vertical displacement; and L_0 = initial parcel length, it follows that

$$u_t = \frac{1}{2}\frac{N^2H^3(t)}{L_0H_0} \quad \text{(3)}$$

The length of the homogeneous patch at time t is given by

$$L(t) = \int_0^t u(t)dt + L_0 \quad \text{(4)}$$

and thus

$$L_{tt} = \frac{1}{2}\frac{N^2H^3(t)}{H_0L_0} = \frac{1}{2}\frac{N^2H_0^2L_0^2}{L^3(t)} \quad \text{(5)}$$

The solution to this equation is

$$L(t) = \alpha\left(t + \frac{L_0^2}{\alpha^2}\right)^{1/2} \quad \text{(6)}$$

where $\alpha = (2N^2H_0^2L_0^2)^{1/4}$ and solving for $H(t)$ gives

$$H(t) = H_0(\sqrt{2}NAt + 1)^{-1/2} \quad \text{(7)}$$

where A = aspect ratio of the (assumed) initial homogeneous patch, i.e. H_0/L_0.

Thus if H_0 and N are known and if the foregoing assumptions are made, an estimate of the location of the parcel at time t can be made. The parcel under consideration is assumed to begin traveling at the center of the layer in which it begins, and end with its center at $H(t)$. The parcel is assumed to retain its shape as it moves, and consequently may lie over several layers at the end of the time step. If this is the case, it is proportionally divided among them. If all of the parcel lies within the vertical bounds of a single layer, a new parcel is created at that location. If the amount to be added to a given layer is less than the minimum parcel volume, it is simply added to the parcel already at that location. The new parcels are then flagged to prevent them from being moved more than once in a time step.

If the parcels move too far, they will leapfrog over the parcels above or below them, a physically unrealistic situation, and there is consequently a limit on the time step of this algorithm. It therefore must cycle through several time steps within the larger time step of the model. The sub-time step is set using the mean stratification over the full depth of the reservoir to estimate the time taken to move less than the average layer thickness, so that

$$\Delta t < \frac{\text{constant}}{N} \quad \text{(8)}$$

where the constant is chosen to satisfy the aforementioned condition. The parcel is assumed to start from rest each time, since after each time step the whole structure of the reservoir will have changed.

This algorithm thus causes particles (parcels) displaced vertically from their level of neutral buoyancy to move back toward that level, and the horizontal motions occur as a consequence of this vertical motion. In the absence of other inputs, this algorithm would result in the eventual return of the modeled density structure of the reservoir to one of stable equilibrium.

The algorithm was tested in isolation from the remainder of the model in a rectangular domain for the situation of collapse of a homogeneous patch in a stratified fluid. Fig. 5 shows the results for the situation in which a homogeneous patch of fluid 500 m long and at a temperature of 12.5°C was placed at the end of a rectangular region 10 m deep and 5,000 m long containing a linearly stratified fluid ranging from 12°C at the bottom to 13°C at the top. The collapse of the patch was modeled over a 12 hr period and the rate at which the intrusion occurred was compared with the empirically calculated rate of intrusion of $L(t)/L_0 = 1.12(Nt)^{1/2}$ (Manins 1976). The small circles in Fig. 5 indicate the progress of the front of the intrusion computed from the theory. The model follows the theory quite well for the time shown, the error being less than the size of the horizontal grid spacing, indicating that no improvement is possible without changing the grid spacing. In the case of a lock-exchange flow, the tests in isolation indicated that intrusions moved at the correct speed, but the thickness was dependent

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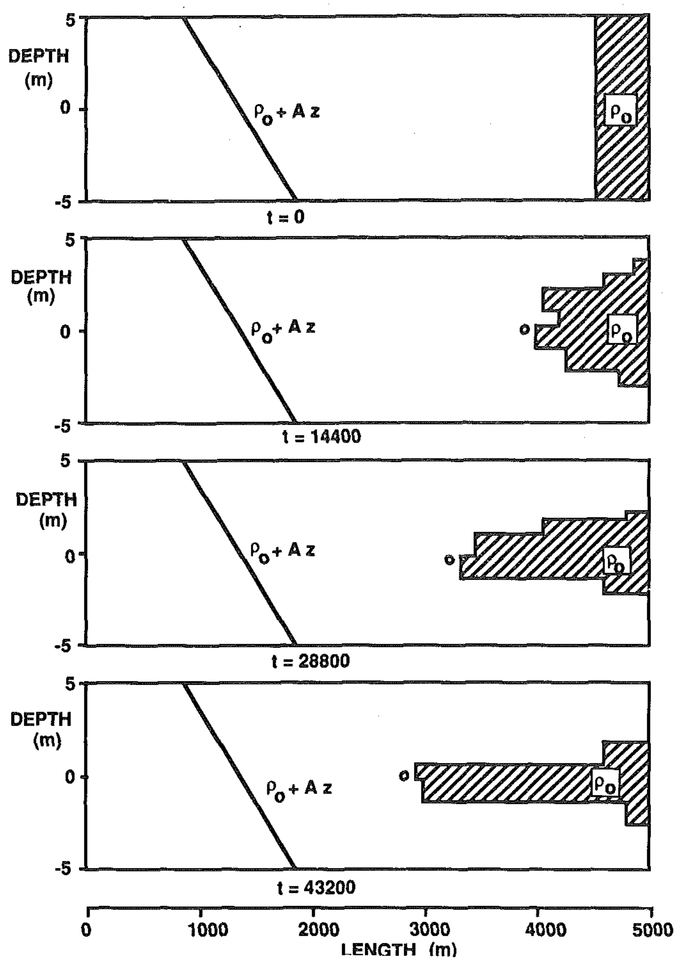

FIG. 5. Collapse of Homogeneous Region into Linearly Stratified Fluid

upon the layer sizes. Work is continuing to improve the algorithm for the second flow type.

Model Verification

The program DYRESM-II was run over a 230 day period, between June 11, 1986 (Julian day 86162) and January 28, 1987 (87028), on the Canning Reservoir. Canning Reservoir is a small domestic supply reservoir situated 50 km southeast of Perth, in Western Australia. At full level it is 62.4 m deep at the dam wall and just 9.8 km long at the longest point up the main inflow source, the Canning River. It is fed by several other minor streams, divided into two inflow sources for the purpose of modeling, and a pump-back from another catchment, which simply runs down the side of the reservoir about 500 m from the dam wall.

The results of the simulations were contoured and compared with the available field data. The data from the field were quite sparse, however, and were contoured using linear interpolation between the four or five available profiles and around the boundaries. The location of these profiles are shown as arrows on the horizontal axes in the contour plots. Two comparisons of temperature contours, and one of salinity contours are shown over the period of simulation.

On day 86231, 70 days after the beginning of the simulation, the field data show an underflow from the Canning River plunging down the drowned river valley [Fig. 6(*a*)]. The simulation results [Fig. 6(*b*)] also show this feature, and the relatively one-dimensional thermal structure of the remainder of the reservoir. The location of the inflow front is well predicted by the model, although the sparseness of the field data prevents this from being a conclusive test. Nevertheless, the results are particularly encour-

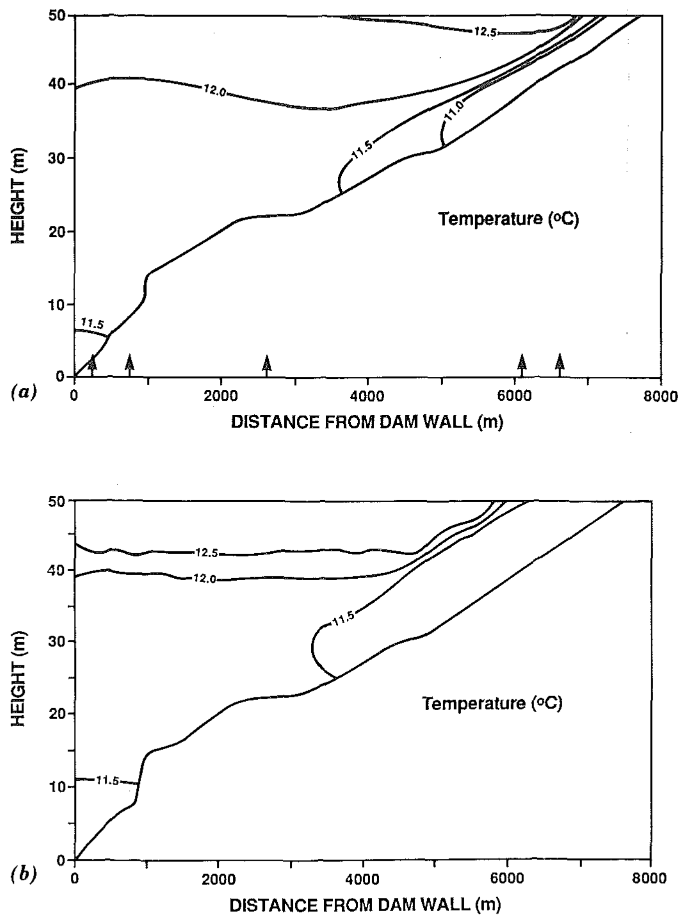

FIG. 6. Comparison on August 19, 1986 (Julian Day 86231) of Temperature Contours from (*a*) Field Measurements; and (*b*) Simulation Results

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aging. The surface temperatures are slightly higher in the model than in the field, but this could depend upon the time of day at which the data were collected. Deeper in the lake, the agreement is good.

More comprehensive data were available for the day 86301, 140 days after the beginning of the simulation. Here, salinity profiles collected at the locations shown mapped the horizontal intrusion shown in Fig. 7(*a*). Salinity levels in the Canning Reservoir rarely reach sufficiently high levels to influence the density of the water, and can be regarded as a tracer for the purposes of this study. The data show the intrusion to be about 10 m down from the surface and to extend to within 2 km of the dam wall.

Continuous salinity measurements for the inflows were not available and salinity inputs to the model were therefore estimated. Since salinity is only a tracer, this procedure is satisfactory. Fig. 7(*b*) shows contours of salinity plotted from the simulation results on the day corresponding to Fig 7(*a*).

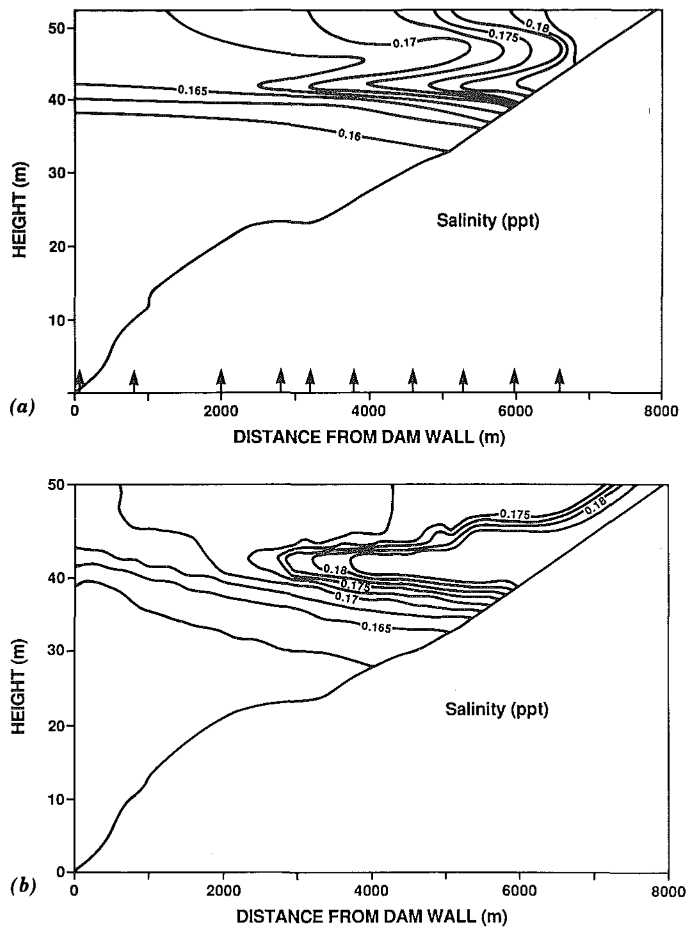

FIG. 7. Comparison on October 28, 1986 (Julian Day 86301) of Salinity Contours from (*a*) Field Measurements; and (*b*) Simulation Results

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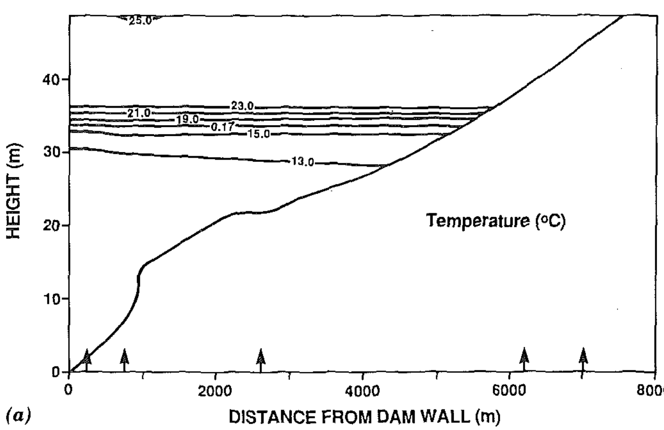
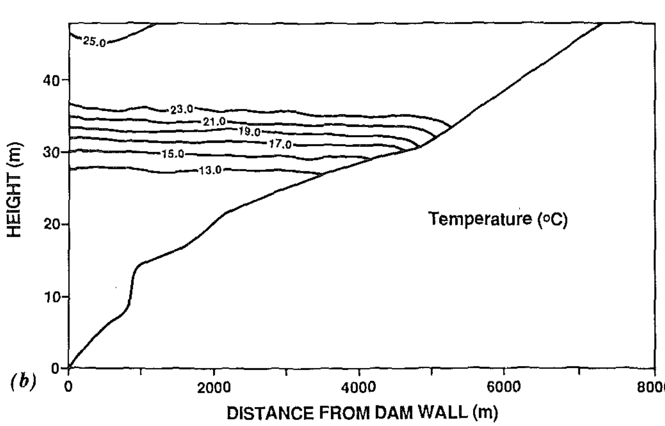
The vertical location of the intrusion is correctly predicted, as is the horizontal extent.

This second figure is a very good test of the model; in effect it is the equivalent of a dye-dispersion test, since the salinity at the levels recorded has minimal influence on the density, yet is a conservative quantity that is carried by the water motion. The mapping of the main intrusion is excellent considering the model had been running for 140 days when this intrusion "occurred." A small surface-density current, apparent in the field data, has been poorly resolved by the model, because the local surface mixing has mixed this inflow with the ambient water. This is because the effect of wind sheltering is not included in the model, yet this part of the reservoir is quite sheltered.

The final comparison is the one obtained on the final day of the simulation, 87028. The reservoir is shown by the field data [Fig. 8(*a*)] to have resumed a one-dimensional character, with a strong thermocline (10°C over 5 m)



(*a*)



(*b*)

FIG. 8. Comparison on January 28, 1987 (Julian Day 87028) of Temperature Contours from (*a*) Field Measurements; and (*b*) Simulation Results

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located about 12 m from the surface. This structure is repeated in the results of the simulation [Fig. 8(*b*)], 230 days from the start. The predicted base of the thermocline is slightly low, but the surface mixed layer depth is correctly predicted. The model shows evidence of a slightly more mixing than the field data, but once again the comparison depends upon the time of day at which the measurements were taken, since the surface temperature may vary by several degrees during the day. Both the model and the data show a region of 25°C water near the dam wall.

Comment

The results indicate that the model captures the essential features of the processes contributing to the vertical and longitudinal density structure of the reservoir. After a 230-day simulation, the error in prediction of the measured data was small, and the model correctly simulated underflow and intrusion events, and subsequent restratification of the reservoir.

The model is based on the original one-dimensional model, which also performs well on this reservoir, and many of the restrictions of the one-dimensional model still apply. Patterson et al. (1984), and more recently Imberger and Patterson (1989), gave conditions under which the one-dimensional assumption is valid; in brief, these restrict application to reservoirs of small to medium size in which the influences of the earth's rotation, lakewide wind-driven circulation, and horizontal advection by inflow and outflow are small in comparison to the effects of the stratification. In its present form, the only two-dimensional effects explicitly treated by the model are those of river inflow and intrusion, and selective withdrawal. More importantly, the model is constructed on the basis of the one-dimensional stratification assumption and, although some relaxation of the inflow and outflow conditions is possible, in general the same restrictions apply.

A feature of the model is its ability to track the horizontal transport of tracers in the reservoir. The foregoing simulations show that the model is able to accurately predict the distribution of salinity in an inflow sequence; in this reservoir salinity makes only a small contribution to density, and may be regarded as a tracer. Other inert tracers may also be followed through the reservoir, with the concentration being modified by the local mixing events affecting that parcel. Simple decay or other nonconservative elements are simply added, and the model has the capacity to predict the subsequent distribution of a tracer or pollutant injected at any longitudinal point. Instantaneous or continuous injection of the tracer are equally simple to incorporate.

As an adjunct to this capability, the model is also configured to evaluate the residence time of each parcel of water in the model of the reservoir. Thus a distribution of water age in the reservoir, with the youngest water identified as the downflow, and the oldest perhaps in the hypolimnion, may be calculated as an aid to evaluation of water quality. The model presently allocates the youngest possible age to a mixture of two parcels of differing age, although other mixing rules may easily be formulated and incorporated. This capability has been used to advise on the siting of recreational use locations; by allocating zero age to water at a potential site, it is possible to determine the age of that water when it reaches some other area of potential use, or the outlet. The sites are chosen to maximize the age, hence minimizing the risk of infection.

These and other water-quality issues are to be addressed elsewhere; in

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the present context it is sufficient to note that the capability of the model to track water through the storage from any given site for an arbitrary time is a powerful tool for the management of water quality in reservoirs.

Acknowledgments

The writers would like to thank Jörg Imberger for valuable discussion, and he and Brad Sherman for their comments on a draft of this paper. The data for this work were collected by the Centre for Water Research and the Water Authority of Western Australia. This work was supported by the Water Authority of Western Australia as part of the Canning Reservoir Dynamics Study and the Australian Water Resources Advisory council.

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Appendix II. Notation

The following symbols are used in this paper:

A = aspect ratio (height/length) of patch of uniform density in reservoir;
g = acceleration due to gravity;
g' = reduced gravity due to density difference, i.e. $\Delta\rho g/\rho$;
H = vertical displacement of particle from its level of neutral buoyancy;
L = length of parcel in reservoir model;
N = buoyancy frequency of stratified fluid;
p = pressure;
t = time;
u = horizontal component of velocity;
u_t = derivative of u with time;
z = coordinate axis in vertical direction;
Δp = pressure differential;
Δt = time step of reservoir model;
ρ = density of water; and
ρ_z = derivative of ρ in z-direction.

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