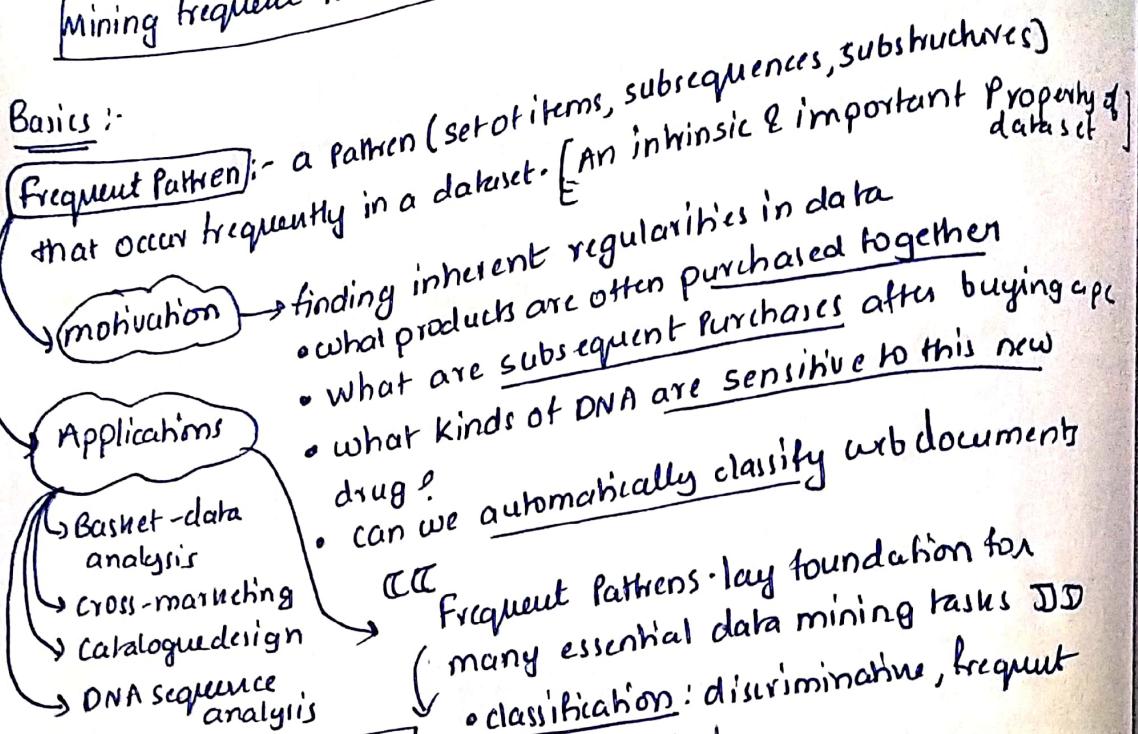
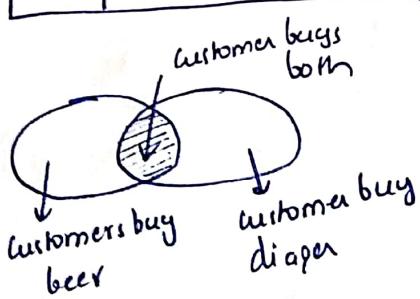


Basics :-

TID	Items Bought
10	Beer, nuts, diaper
20	Beer, coffee, diaper
30	Beer, Diaper, eggs
40	Nuts, eggs, Milk
50	Nuts, coffee, Diaper, eggs, Milk



(CC) An itemset x is frequent if x 's support is no less than minimum Support Threshold DD

Confidence :- c, conditional Probability that a transaction having x also has y $P(y|x)$

Association rules : finding all rules $x \rightarrow y$ with min support & confidence

(absolute) Support / Support count of x : Frequency of occurrence of an itemset x

(relative) Support :- s, is the fraction of transactions that contain x (Probability of transaction Contains x) $P(A \cup B)$

- If a subsequence appears frequently in dataset then it is a frequent subsequence pattern
- If a substructure appears frequently in dataset then it is a frequent substructured pattern :-

$= P(A \cap B) / P(B)$ Structure \rightarrow lattice, subgraphs, substructures, sub-lattices.

Market-Basket Analysis

: customer analysis, buying habits, shopping trends & above all concepts come here

[ARM is used]

Long Patterns problem

A long pattern contains combinatorial numbers of sub patterns

$$\text{eg: } (a_1, a_2, \dots, a_{100})$$

$$\downarrow$$

$$(100!) + (100^2) + \dots$$

$$= 2^{100} - 1$$

$$= 1.27 \times 10^{30} \text{ subpatterns!}$$

Solution:

mine closed patterns & max patterns instead

worst case in generating itemsets

$$M^N$$

- M : distinct items

- N : max length.

why? → Mine information, extract knowledge

Downward closure property of frequent patterns

Any subset of a frequent itemset must be frequent

Mining Association Rules :- Process of finding frequent patterns or associations within dataset or from set of data sets.

How it is done / Scalable Frequent itemset Mining methods

• Apriori (candidate generation & test)

• FP growth (frequent pattern growth)

• Vertical data format (ECLAT)

Apriori Algorithm :- [R. Agrawal, R. Srikant in 1994] [Iterative]

• Apriori Pruning Principle :- If there is any itemset which is infrequent, its superset should not be generated/tested!

• Method :-

1. Initially scan DB for getting frequent 1-itemset

2. Generate length($k+1$) candidate itemsets from length k frequent itemsets

3. Test the candidates against DB

4. Terminate when no frequent in candidate set can be generated.

Closed Pattern

An itemset X is closed, if X is frequent and there exist no super-pattern Y such that

$$Y \supset X$$

and with same support as X

Max Pattern

An itemset X is Max, if X is frequent and there exist no super-pattern Y such that

$$Y \supset X$$

III closed pattern is lossless compression of frequent patterns i.e.

reducing number of patterns & rules

III Association Rule mining is of two types single level & multi-level

An example of Apriori Algorithm

- Support-count : 60%.
- min-confidence : 80%.

TID	Itemset
T1	F, A, D, B
T2	D, A, C, E, B
T3	C, A, B, E
T4	B, A, D

Ques :- Finding the 1-itemset

Items	Count	Support Count %
A	4	$4/4 \times 100 = 100\%$
B	4	$4/4 \times 100 = 100\%$
D	3	$3/4 \times 100 = 75\%$
C	2	$2/4 \times 100 = 50\%$
E	2	$2/4 \times 100 = 50\%$
F	1	$1/4 \times 100 = 25\%$

Finding the 2-itemset

Items	Count	Support Count %
A, D	3	$3/4 \times 100 = 75\%$
A, B	4	$4/4 \times 100 = 100\%$

Finding the 3-itemset

Items	Count	Support Count %
A, D, B	3	$3/4 \times 100\% = 75\%$

Association rules :-

$$A, D \rightarrow B \quad 3/3 \times 100\% = 100\% \quad \checkmark$$

$$B \rightarrow A, D \quad 3/4 \times 100\% = 75\%$$

$$A, B \rightarrow D \quad 3/4 \times 100\% = 75\%$$

$$D \rightarrow A, B \quad 3/3 \times 100\% = 100\% \quad \checkmark$$

$$B, D \rightarrow A \quad \text{INVALID}$$

$$A \rightarrow B, D \quad \text{INVALID}$$

The APRIORI ALGORITHM pseudocode

C_k : candidate itemset of size k

L_k : frequent itemset of size k

L_1 : {frequent itemsets} ;

for ($k=1$; $L_k \neq \emptyset$; $k+k$) do begin
 C_{k+1} = candidates generated from L_k ;
 foreach transaction t in database do
 increment the count of all candidates in C_{k+1}
 that are contained in t

L_{k+1} = candidates in C_{k+1} with min support

end

return $U_k L_k$

(How to generate candidate keys)

Step 1: Self-joining L_k

Step 2: Pruning

why counting supports of a candidate
a problem

→ Total number of candidates can be
very huge

→ One transactions may contain many
candidates

Solution / Hashing itemset count

candidate itemsets are stored
in hash tree

leaf node of hash tree contains
a list of itemsets & count

Interior node contains a
hash table

subset function, finds all
candidates contained in
a transaction

Improving the efficiency of
Apriori Algorithm

eg: $L_3 = \{abc, abd, acd, ace, bcd\}$

self joining $L_3 \times L_3$

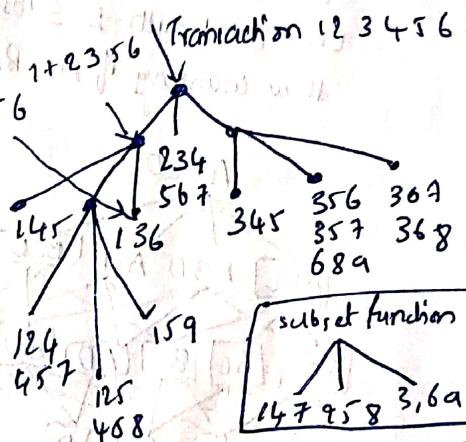
• abcd from abc & abd

• acde from acd & ace

→ pruning

acde is removed, because
ace is not in L_3

$L_4 = \{abcd\}$



Anti-monotonicity

If a set cannot pass a test, all of its
supersets will fail same test as well

1. Partition : scan database only twice

- Any itemset that is potentially frequent in DB must be frequent
in atleast one of partitions of DB

Scan1 : Partition database & find local frequent patterns

Scan2 : consolidate global frequent patterns

2. DHP : Reduce Number of Candidates :-

- A k-itemset whose corresponding hashing bucket count is
below threshold cannot be frequent.

An effective hash-based algorithm for mining association rules

↓
SIGMOD'95

Count	Itemsets
35	{ab, ad, ae}
88	{bd, bc, de}
1	
102	{y2, y3, wt}

- frequent 1-itemset : a, b, d,
- ab is not a candidate
- itemset if sum of count of ae) is below support threshold.

3. Sampling for frequent patterns

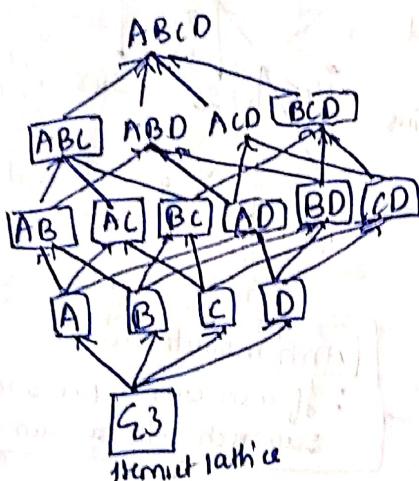
- select a sample of original database, mining frequent patterns within sample using Apriori
- scan database once to verify frequent itemsets found in sample, only borders of closure of frequent patterns are checked

VLDB'96

Sampling
large databases e.g. check abcd instead of ab, ac, - - etc
for Association rules - scan database again to find missing frequent patterns

4. DIC: Reduce number of scans [dynamic itemset counting]

- once both A & D are determined frequent, then counting of AD begins
- once all length-2 subsets of BCD are determined frequent then counting of BCD begins



SIGMOD'97

Pattern growth approach

- Bottleneck of Apriori Approach:

- Breadth first (level-wise) search
- candidate generation & test
- often generates a huge number of candidates

- FP-growth Approach

- Depth first Search
- Avoid explicit candidate generation

(CC) growing long paths from short ones using local frequent items only

e.g.: If d is a local frequent pattern, abc → abcd is frequent pattern.

- compress a large database into a compact, frequent-pattern tree (FP-tree) structure.
 - avoids costly database scans.
 - highly condensed, but complete.

Construction of FP-Tree

- First create the root of tree, labelled with Null
- scan database D a second time.
The items in each transaction are processed in L order (Sorted according to descending support count) and a branch is created for each transaction.
- when considering the branch to be added for a transaction, the count of each node among a common prefix is incremented by 1.

Steps to create FP Tree

- Scan DB once, find frequent 1 itemset (single item pattern)
- Order frequent items in frequency descending order
- Scan DB again, construct FP-Tree

e.g.: T100 : I1, I2, I5

T200 : I2, I4

T300 : I2, I3

T400 : I1, I2, I4

T500 : I1, I3

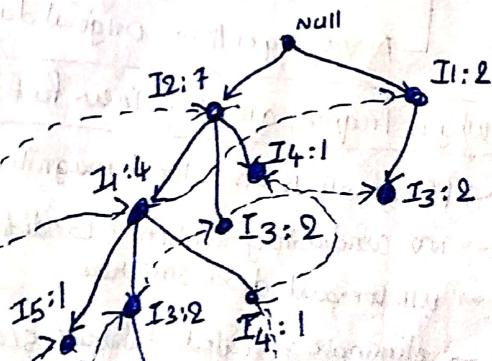
T600 : I2, I3

T700 : I3, I1

T800 : I1, I2, I3, I5

T900 : I1, I2, I3

Item	Supp Count
I1	6
I2	7
I3	6
I4	2
I5	2



In descending order:

Item	Support Count	Node-line
I2	7	-
I1	6	-
I3	6	-
I4	2	-
I5	2	-

Mining Frequent Patterns using FP-Tree

- divide & conquer

- recursively grow frequent pattern path using FP tree

- Method

- For each item, construct its conditional Pattern-base and thin its conditional FP-Tree
- Repeat the process on each newly created Conditional FP-Tree
- until resulting FP-tree is empty (or) it contains only one path,

Item	Conditional Pattern Base	Conditional FP Tree	Frequent Patterns
I ₅	{(I ₂ , I ₁ :1), (I ₂ , I ₁ , I ₃ :1)}	{I ₂ :2, I ₁ :2}	(I ₂ , I ₅ :2) (I ₂ , I ₁ , I ₅ :2)
I ₄	{(I ₂ , I ₁ :1) (I ₂ :1)}	{I ₂ :3}	{I ₂ I ₄ :2}
I ₃	{(I ₂ , I ₁ :2), (I ₂ :2) (I ₁ :2)}	{I ₂ :4, I ₁ :2} {I ₁ :2}	(I ₂ I ₃ :4), (I ₂ , I ₃ :2), (I ₂ I ₁ , I ₃ :2)
I ₁	{(I ₂ :4)}	{I ₂ :4}	(I ₂ I ₁ :4)

Benefits of FP-Tree structure

- Completeness

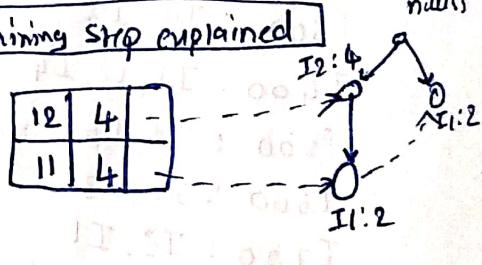
Preserve complete information for frequent pattern mining

- Compactness

Reduce irrelevant information
- infrequent items are gone
- Items in frequency descending order
- Never larger than original database

Association rules can be drawn out same as apriori

Mining Step explained



Why is frequent growth pattern fast?

- FP-growth is an order of magnitude faster than Apriori & also tree projection
- no candidate generation & candidate test
- use compact data structure
- eliminate repeated database scan

ECLAT : Mining By exploring vertical data format

[mining closed patterns]

- Vertical format : $t(AB) = \{T_{11}, T_{21} - \}$

- Deriving frequent patterns based on vertical intersections

$t(x) = t(f) : x \& f \text{ always happen together}$

$t(x) \subset t(y) : \text{transaction having } x \text{ always have } y$

which patterns are interesting? - Pattern evaluation method

- Most Association rule mining algorithms employ a support+confidence framework.
- many of rules generated are still not interesting to user.
- The above statement is true when mining for long patterns or at low Support Thresholds.

Strong Rules are not necessarily interesting SJD

Based on subjective (on)
objective perspectives.

- The interestingness measures are objective statistics.
- Then which Strong association rules are interesting?
- There are several Co-relation measures help us to choose Association Rules.

Lift: The occurrence of itemset A is independent of occurrence of itemset B if $P(A \cup B) = P(A) / P(B)$;
Otherwise itemsets A & B are dependent & correlated events

$$\therefore \text{Lift}(A, B) = \frac{P(A \cup B)}{P(A) P(B)} \quad (\text{or } P(B|A) / P(B))$$

- If $\text{Lift}(A, B) < 1$, then occurrence of A is negatively correlated with occurrence of B (occurrence of one absence of other)
- If $\text{Lift}(A, B) > 1$, then A & B are positively correlated.
(One occurrence implies another occurrence)
- If $\text{Lift}(A, B) = 1$ then A & B are independent & there is no correlation between them.

Eg:- Calculating the chi-square value (χ^2) for the given data

	game	!game	Row
video	4000	3500	7500
!video	2000	500	2500
Col	6000	4000	10000

$$\rightarrow P(\text{game}) = 60\% \quad P(\text{game, video}) / P(\text{game}) + P(\text{video}) = 0.5$$

$$P(\text{video}) = 75\%$$

<1

- negative Co-relation

Customer Purchasing behavior

two independent purchases.

$$\chi^2 = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

$$= \frac{(4000 - 4500)^2}{4500} + \frac{(3500 - 3000)^2}{3000} + \frac{(2000 - 1500)^2}{1500} \\ + \frac{(500 - 100)^2}{100} = 555.6 \quad \chi^2 > 1$$

Lift & χ^2 are not null invariant

A Comparison of pattern evaluation measures

→ If two itemsets are given then

$$\cdot \underline{\text{All Confidence}} : \text{all-cont}(A, B) = \frac{\text{sup}(A \cup B)}{\max[\text{sup}(A), \text{sup}(B)]} \\ = \min[P(A|B), P(B|A)]$$

$$\underline{\text{Max Confidence}} : \text{max-cont}(A, B) = \max[P(A|B), P(B|A)]$$

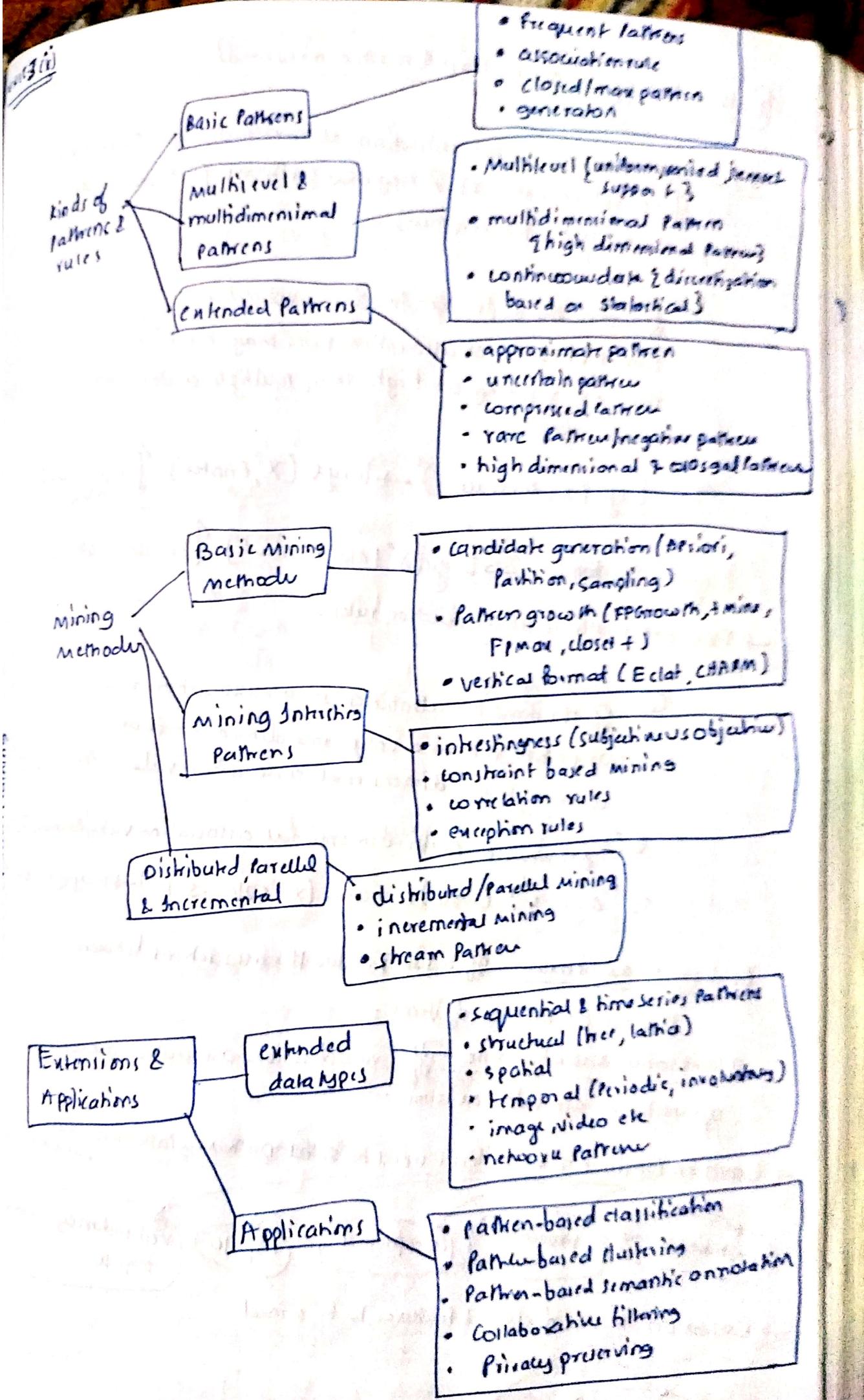
$$\underline{\text{KULCZYNSKI}} : \text{kulc}(A, B) = \frac{1}{2} (P(A|B) + P(B|A))$$

$$\underline{\text{Cosine measure}} : \text{cosine}(A, B) = \frac{P(A \cup B)}{\sqrt{P(A) \times P(B)}} = \frac{\text{sup}(A \cup B)}{\sqrt{\text{sup}(A) \times \text{sup}(B)}} \\ = \sqrt{P(A|B) \times P(B|A)}$$

Multiple-level Association Rules

- It is not always easy to find the strong association
- So we must use multi levels of abstraction

Imbalance Ratio



Pattern mining in Multi-level & multi-dimensional

- Sometimes we also want interesting or rare patterns (occurs rarely, but of critical importance) & negative patterns (patterns with negative correlation between them).

(C) Based on the abstraction levels involved in a pattern. Patterns or association rules may have items that are residing at high, low, multiple abstraction levels. SJD

e.g.: $\text{buy}_r(x, \text{"computer"}) \rightarrow \text{buy}_r(x, \text{"Printer"})$ ↑ high level abstraction

$\text{buy}_r(x, \text{"laptop"}) \rightarrow \text{buy}_r(x, \text{"laser printer"})$ ↑ low level abstraction

→ There are multi-level association rules.

(C) If the items or attributes in an association rule or pattern reference only one dimension then it is a single dimensional association rule/pattern SJD

(C) Otherwise multi-dimensional association rule/pattern MJD

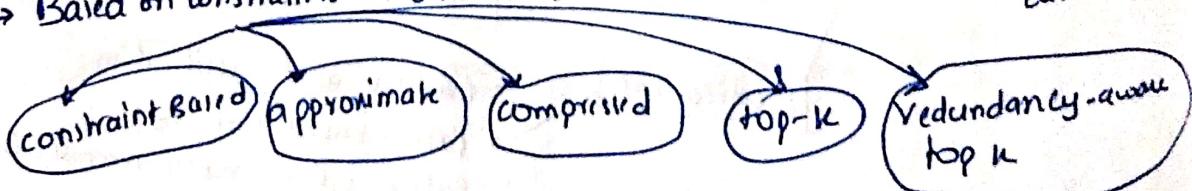
e.g.: $\text{age}(x, \text{"20..29"}) \wedge \text{income}(x, \text{"\$2k .. \$3k"}) \rightarrow \text{buy}_r(x, \text{"ipad"})$

Boolean association rule: If a rule involves the associations between

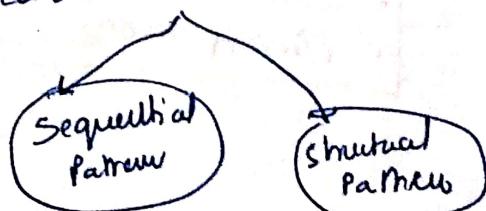
presence or absence of items -

quantitative association rule: If a rule describes association between quantitative attributes on terms -

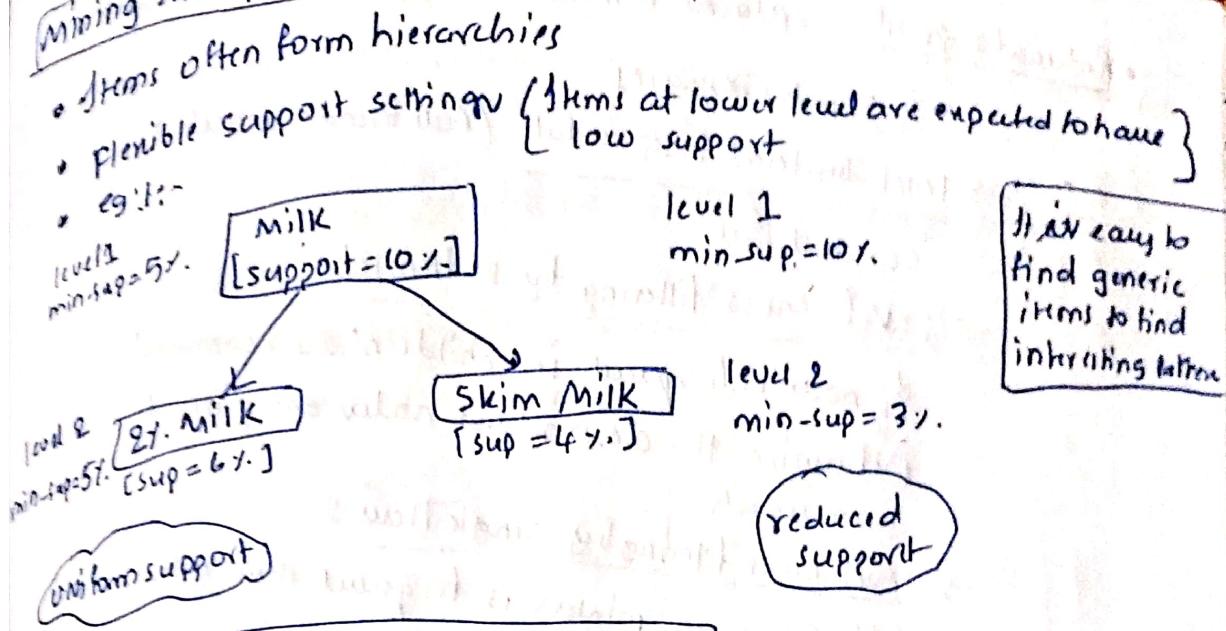
→ Based on constraints or criteria used to select patterns - patterns can be selected based on constraints like:



→ Based on kinds of data & features to be mined



Mining Multiple-level Association rules



Flexible Support & Redundancy filtering

- flexible support: some items are rare but more valuable (less frequent)

→ use non-uniform, group based min support
eg: (diamond, watch, camera) : 0.05%
(milk, bread) → 5%.

- Redundancy filtering: some rules may be redundant due to "ancestor" relationships between items.

eg:- milk → bread [$s = 8\%$, $c = 70\%$] (ancestor)

2% milk → bread [$s = 2\%$, $c = 72\%$]

A rule is redundant if its support is close to the expected value, based on ancestor rule. ↗

Uniform Support vs Reduced Support

- uniform support: same minimum support for all levels

because if ($sup - threshold$)

too high → miss low level association

too low → generate too many high level associations

level-1 eg:-
min-sup = 5%.

Computer [support = 10%]

level 2
min-sup = 5%

Laptop Computer [support = 6%]

Desktop Computer [support = 4%]

- Reduced support: reduced minimum support at lower levels

• 4 search strategies

→ level-by-level independent (full breadth search)

Searched full

→ level-cross filtering by k-itemset

If node is frequent, its children are examined

Otherwise the descendant nodes are pruned from search

→ level-cross filtering by single item :-

If {computer, printer} is frequent then other nodes are examined

→ controlled level-cross filtering by

single item :- (same as above except)

A threshold called level passage threshold can be set up for passing down relatively frequent items to lower levels

Mining Multi-dimensional Association

- Single dimensional rules :-

$\text{buys}(x, \text{"milk"}) \rightarrow \text{buys}(x, \text{"bread"})$

- Multi-dimensional rules (2 or more predicates)

→ Inter-dimension assoc. rules (no repeated predicate)

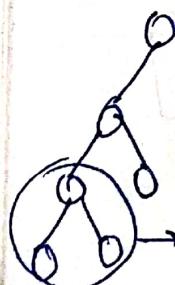
e.g.: $\text{age}(x, \text{"19-25"}) \wedge \text{Occupation}(x, \text{"student"}) \rightarrow \text{buys}(x, \text{"laptop})$

→ hybrid-dimension assoc. rules (repeated predicates)

$\text{age}(x, \text{"19-25"}) \wedge \text{buys}(x, \text{"popcorn"}) \rightarrow \text{buys}(x, \text{"cake})$

- Categorical attributes: finite no. of possible values & no ordering [data cube]

- Quantitative attributes: numeric, implicit ordering among values - discretization, clustering, gradient approaches



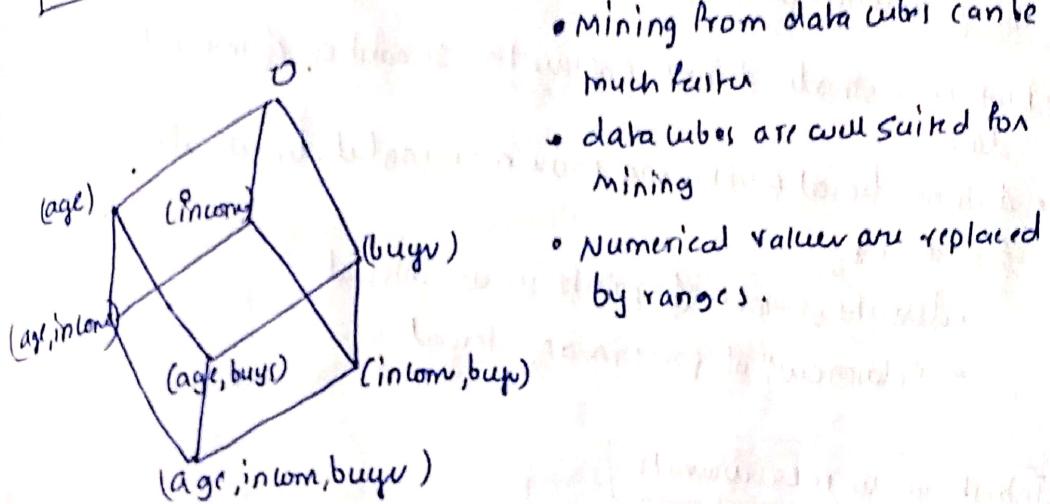
The deeper the abstraction level, the smaller the corresponding threshold

for each group different support, threshold

Mining quantitative associations

- Techniques can be categorized by how numerical attributes are treated
 1. Static discretization, based on predefined concept hierarchies (data cubes method)
 2. Dynamic discretization, based on data distribution (discretized into bins dynamically)
 3. Distance-Based association (clustering)

static discretization



dynamic discretization

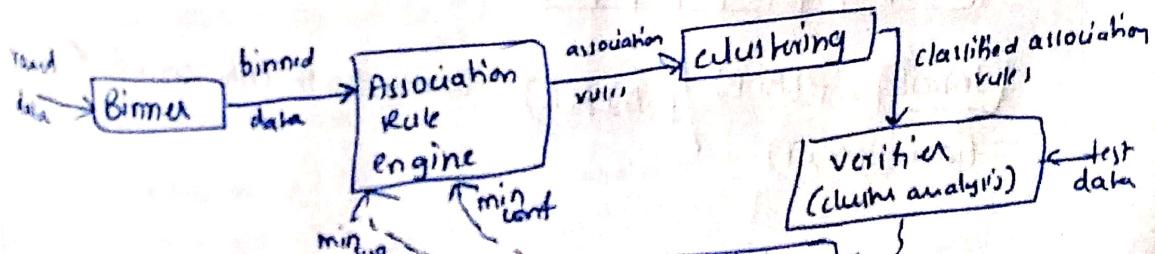
Ex (Sex = female) \Rightarrow wage : mean = \$7/hr (overall mean = \$9)

- LHS : subset of population
- RHS : An extraordinary behaviour of this subset
- This rule is accepted only if a statistical test (z-test) confirms the interestingness with high confidence.

\rightarrow Numeric values are dynamically discretized.

Such that confidence or compactness of rules is maximized

ARCS [Association Rule clustering sys]



clustering the association rule

• $\text{Age}(x, 34) \wedge \text{Income}(x, "31K..40K") \rightarrow \text{buys}(x, "iPhone")$

$\text{Age}(x, 35) \wedge \text{Income}(x, "31K..49K") \rightarrow \text{buys}(x, "iPhone")$

$\text{Age}(x, 34) \wedge \text{Income}(x, "31K..50K") \rightarrow \text{buys}(x, "iPhone")$

↓
They can be clustered & replaced by

$\text{Age}(x, 34..35) \wedge \text{Income}(x, "31K..50K") \rightarrow \text{buys}(x, "iPhone")$

Mining distance base association rules

- Binning methods do not capture the semantics of interval data

- distance-based partitioning more meaningful discretization considering

- density, number of points in an interval
- "closeness" of points in an interval

Interestingness measurements

Objective measures

- support
- confidence

Subjective measure

A rule pattern is interesting if
(unexpected) surprise to user &
& actionable (user can do something with it)

Correlation / lift

$$\text{corr}(A, B) = \frac{P(A \cup B)}{P(A)P(B)} = \frac{P(B|A)}{P(B)}$$

If $\text{corr} < 1$ -ve corr

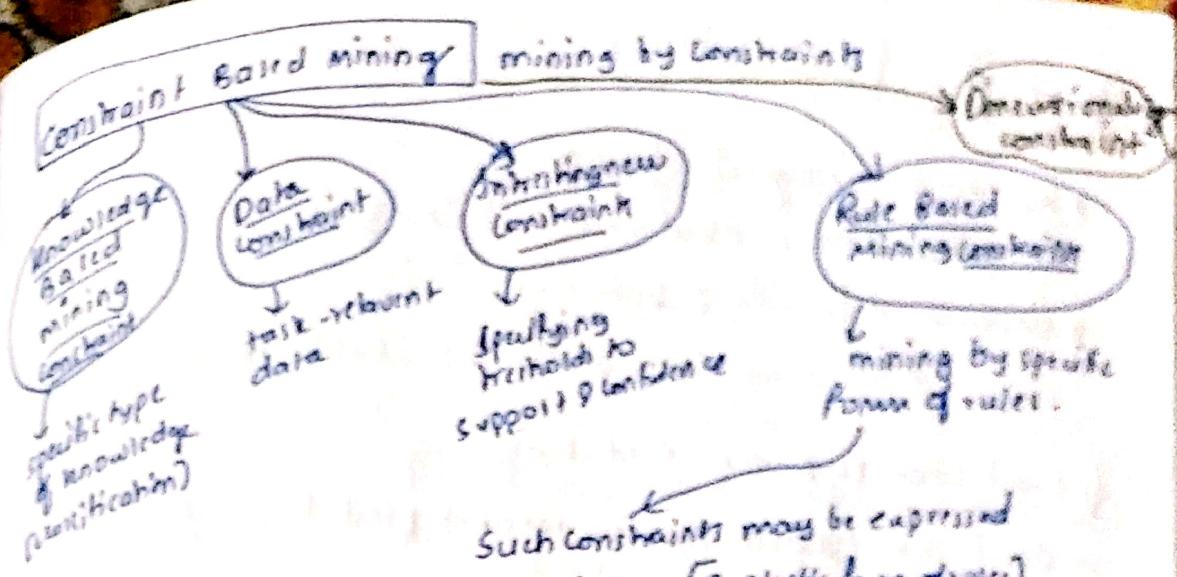
If $\text{corr} > 1$ +ve corr

If $\text{corr} = 1$ independent

e.g:-

X	1	1	1	1	0	0	0	0
Y	1	1	0	0	0	0	0	0
Z	0	1	1	1	1	1	1	1

Interest : $P(A \cap B) / P(A)P(B)$: Taking Both into consideration
(correlation) if



Such constraints may be expressed as metarules [Syntactic form of rules]

or max or min # of predicates that can occur in rule antecedent, consequent or relationship among attributes or in aggregator -

improve efficiency of mining process.

Template of metarule.

$$P_1 \wedge P_2 \wedge P_3 \wedge \dots \wedge P_j \Rightarrow Q_1 \wedge Q_2 \wedge Q_3 \wedge \dots \wedge Q_k$$

e.g.: Datamining can search for rules that match with metarule

age(x, "30..39") \wedge income(x, "41k..60k") \rightarrow buys(x, "TV")

Monotonicity: If a set S satisfies a constraint then any superset of S satisfies a constraint.

e.g.: $\text{sum}(l.\text{Price}) >= 100$

Anti-monotonicity: If a set S violates a constraint then any superset of S also violates constraint.

e.g.: 1. $\text{sum}(s.\text{Price}) \leq V$ \rightarrow is anti-monotonic

e.g.: 2. $\text{Avg}(l.\text{Price}) <= 100$ is not anti-monotonic.

Succinct constraint: only sets that satisfy constraint are enumerated

Convertible constraint: neither monotonic nor Anti-monotonic
But convertible

Relationships among Category of Constraints

