13.13 Consider two medical tests, A and B, for a virus. Test A is 95% effective at recognizing the virus when it is present, but has a 10% false positive rate (indicating that the virus is present, when it is not). Test B is 90% effective at recognizing the virus, but has a 5% false positive rate. The two tests use independent methods of identifying the virus. The virus is carried by 1% of all people. Say that a person is tested for the virus using only one of the tests, and that test comes back positive for carrying the virus. Which test returning positive is more indicative of someone really carrying the virus? Justify your answer mathematically.

$P(A \mid V) = 0.95$
$P(A \mid \neg V) = 0.10$
$P(B \mid V) = 0.90$
$P(B \mid \neg V) = 0.05$
$P(V) = 0.01$
$P(\neg V) = 1 - P(V) = 0.99$

We are interested in comparing $P(V \mid A)$ against $P(V \mid B)$

We know that:
$P(V \mid A) + P(\neg V \mid A) = 1$

And that its components are:
$P(V \mid A) = P(A \mid V) \times P(V) / P(A) = 0.95 \times 0.01 / P(A)$
$P(\neg V \mid A) = P(A \mid \neg V) \times P(\neg V) / P(A) = 0.10 \times 0.99 / P(A)$

Then, summing the equations:
$P(V \mid A) + P(\neg V \mid A) = (0.95 \times 0.01 + 0.10 \times 0.99) / P(A) = 1$
$(0.0095 + 0.099) = P(A)$
$P(A) = 0.1085$

Now we can use this information to calculate:
$P(V \mid A) = P(A \mid V) \times P(V) / P(A) = 0.95 \times 0.01 / 0.1085 = 95 / 1085 \approxeq 0.088$

Using the same mechanism for test B, we know that:
$P(V \mid B) + P(\neg V \mid B) = 1$

And that its components are:
$P(V \mid B) = P(B \mid V) \times P(V) / P(B) = 0.90 \times 0.01 / P(B)$
$P(\neg V \mid B) = P(B \mid \neg V) \times P(\neg V) / P(B) = 0.05 \times 0.99 / P(B)$

Then, summing the equations:
$P(V \mid B) + P(\neg V \mid B) = (0.90 \times 0.01 + 0.05 \times 0.99) / P(B) = 1$
$(0.0090 + 0.0495) = P(B)$
$P(B) = 0.0.0585$

Again, we can use previous values to calculate:
$P(V \mid B) = P(B \mid V) \times P(V) / P(B) = 0.90 \times 0.01 / 0.0585 = 90 / 585 \approxeq 0.154$

Then the test B is more indicative of someone really carrying the virus (because $0.154 > 0.088$)