**13.1 Show from first principles that P(a | b ∧ a) = 1.**

A fully specified **probability model** associates a numerical probability P(ω) with each possible world. The basic axioms of probability theory say that every possible world has a probability between 0 and 1 and that the total probability of the set of possible worlds is 1:

Being Ω the sample space (conformed by all the possible worlds):

$$0 \le P(\omega) \le 1 \text{ for every } \omega \text{ and } \sum_{\omega \in \Omega} P(\omega) = 1$$

For any proposition φ:

$$P(\varphi) = \sum_{\omega \in \varphi} P(\omega)$$

Conditional Probabilities:

$$P(a|b) = \frac{P(a \wedge b)}{P(b)}$$

**Proof**

Using the conditional probability definition:

$$P(a \mid b \wedge a) = \frac{P(a \wedge b \wedge a)}{P(b \wedge a)}$$

By commutative law of intersection $(a \wedge b = b \wedge a)$

$$\frac{P(a \wedge b \wedge a)}{P(b \wedge a)} = \frac{P(a \wedge a \wedge b)}{P(a \wedge b)}$$

By associative law of intersection $(a \wedge a \wedge b = (a \wedge a) \wedge b = a \wedge (a \wedge b))$

$$\frac{P(a \wedge a \wedge b)}{P(a \wedge b)} = \frac{P((a \wedge a) \wedge b)}{P(a \wedge b)} =$$

By indempotent law of intersection $(a \wedge a = a)$

$$\frac{P((a \wedge a) \wedge b)}{P(a \wedge b)} = \frac{P(a \wedge b)}{P(a \wedge b)} = 1$$

So, we can say that:

$$P(a \mid b \wedge a) = 1$$

$P(b \wedge a)$ is required to be not zero and this is given by hypothesis

Q.E.D.