13.14 Suppose you are given a coin that lands heads with probability x and tails with probability 1 – x. Are the outcomes of successive flips of the coin independent of each other given that you know the value of x? Are the outcomes of successive flips of the coin independent of each other if you do not know the value of x? Justify your answer.

If we know x, the next probabilities values are always the same:
$P(H) = x$
$P(T) = 1 - x$

For example, for two consecutives flips, we can see that the flips of the coins are independent of each other.
$P(H_2 | H_1) = x \text{ then } P(H_2 | H_1) = P(H_2)$

And this happens when events are independents.

So, successive flips will present the same probability. They do not require knowledge about its history.
$P(H_1 H_2) = P(H_1) \times P(H_2) = x \times x$

What happens when we don't know the value of x

We need to estimate the value of x. And we can do that using the experience, the history. The next table shows how the probability of flipping head change based on the history (it lets us build our knowledge at time n to make predictions for time n+1). The first time we know nothing about the probability of Head vs the probability of Tail, but we can assume that they are the same.

History	time = n	$P(H_{n+1}) = x$	$P(T_{n+1}) = 1 - x$
No data	0	1/2	1/2
H	1	(1/2 + 1)/2 = 3/4	(1/2 + 0)/2 = 1/4
T	1	(1/2 + 0)/2 = 1/4	(1/2 + 1)/2 = 3/4
HH	2	(1/2 + 1 + 1)/3= 5/6	(1/2 + 0 + 0)/3= 1/6
HT	2	(1/2 + 1 + 0)/3= 1/2	(1/2 + 0 + 1)/3= 1/2
TH	2	(1/2 + 0 + 1)/3= 1/2	(1/2 + 1 + 0)/3= 1/2
TT	2	(1/2 + 0 + 0)/3= 1/6	(1/2 + 1 + 1)/3= 5/6

The table above just presents values for 3 possible moments (n=0, n=1, and n=2). The formula to estimate the total number of history cases in the time range [0, k] is $2^k - 1$. At moment n=k we will be located in one of be 2^{k-1} possible histories. In example, at time n=2 there are 4 possible histories that generate 3 possible values for x. At time n=2 there are three possible and differents values for $P(H_3)$ and these are 1/6, 1/2 and 5/6 but it is interesting to observe that if we already are arrive at time n=1, then we will know the value of x at this time and we will know what could be the next value of x after flipping the coin (two new possible values). So it is easy to see that the value of x is dependent. Every new data we collect (new flipping of the coin) the value of x is recalculated.