13.15 After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is 99% accurate (i.e., the probability of testing positive when you do have the disease is 0.99, as is the probability of testing negative when you don't have the disease). The good news is that this is a rare disease, striking only 1 in 10,000 people of your age. Why is it good news that the disease is rare? What are the chances that you actually have the disease?

$P(T \mid D) = 0.99$
$P(\neg T \mid \neg D) = 0.99$
$P(D) = 0.0001$
$P(\neg T \mid D) = 1 - P(T \mid D) = 0.01$
$P(T \mid \neg D) = 1 - P(\neg T \mid \neg D) = 0.01$
$P(\neg D) = 1 - P(D) = 0.9999$

We are interested in finding P(D | T)
$P(D \mid T) = P(T \mid D) \times P(D) / P(T) = 0.99 \times 0.0001 / P(T)$

We can see that the value is directly related to P(D) and because P(D) is a rare disease (1/10,000) then it will help to find good news. To find the value of P(T) we will need to resolve other probability:
$P(\neg D \mid T) = P(T \mid \neg D) \times P(\neg D) / P(T) = 0.01 \times 0.9999 / P(T)$

And knowing that we can go ahead:
$P(D \mid T) + P(\neg D \mid T) = 1$
$P(D \mid T) + P(\neg D \mid T) = 0.99 \times 0.0001 / P(T) + 0.01 \times 0.9999 / P(T) = 1$
$P(T) = 0.000099 + 0.009999 = 0.010098$

Just an observation: knowing that the Test will result positive in near 1% of the population (100/10,000) and that the disease will be present in 0.01% of the population (1/10,000), we can see that the test will present a lot of false positives in the entire population!.

And then we can arrive to the chances that you we actually have the disease:
$P(D \mid T) = P(T \mid D) \times P(D) / P(T) = 0.000099 / 0.010098 \approx 0.0098 < 1\%$

Chances are less than 1 in 100 (99 in 10098)