14.14 Consider the Bayes net shown in Figure 14.23.
a. Which of the following are asserted by the network structure?
(i) P(B, I,M) = P(B)P(I)P(M).
(ii) P(J |G) = P(J | G, I).
(iii) P(M | G,B, I) = P(M | G,B, I, J).
b. Calculate the value of P(b, i,¬m, g, j).
c. Calculate the probability that someone goes to jail given that they broke the law, have been indicted, and face a politically motivated prosecutor.
d. A context-specific independence (see page 542) allows a variable to be independent of some of its parents given certain values of others. In addition to the usual conditional independences given by the graph structure, what context-specific independences exist in the Bayes net in Figure 14.23?
e. Suppose we want to add the variable P = PresidentialPardon to the network; draw the new network and briefly explain any links you add.

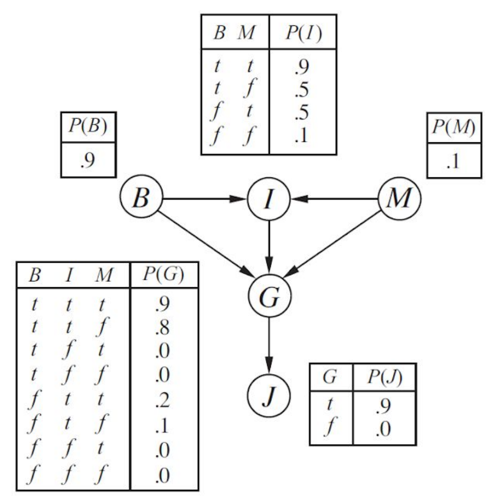

Figure 14.23 A simple Bayes net with Boolean variables $B = BrokeElectionLaw$, $I = Indicted$, $M = PoliticallyMotivatedProsecutor$, $G = FoundGuilty$, $J = Jailed$.

a. Which of the following are asserted by the network structure?
(i) P(B, I,M) = P(B)P(I)P(M).
(ii) P(J |G) = P(J | G, I).
(iii) P(M | G,B, I) = P(M | G,B, I, J).

(i) Is not asserted by network structure
"The topological semantics specifies that each variable is conditionally independent of its non-descendants, given its parents." [AIMA3E, p. 517]

By Chain Rule and AIMA3E Equation 14.3:
P(B, I, M) = P(B | Parents(B)).P(I | Parents(I)).P(M | Parents(M))
P(B, I, M) = P(B).P(I | B, M).P(M)
P(B).P(I | B, M).P(M) != P(B).P(I).P(M)

(ii) Is asserted by network structure
By AIMA3E Equation 14.3:

P(J | B, I, M, G) = P(J | Parents(J))
P(J | B, I, M, G, I) = P(J | Parents(J), I)
P(J | B, I, M, G) = P(J | G, I)

(iii) Is asserted by network structure
"...a node is conditionally independent of all other nodes in the network, given its parents, children, and children's parents—that is, given its Markov blanket." [AIMA3E, p. 517] Proof in Exercise 14.7.

P(M | G, B, I, J) = P(M | Markov_Blanket(M))
P(M | G, B, I, J) = P(M | I, G)
P(M | G, B, I, J, B) = P(M | I, G, B)
P(M | G, B, I, J) = P(M | I, G, B)

b. Calculate the value of P(b, i,¬m, g, j).

By Chain Rule and Equation 14.3:
P(b, i, ¬m, g, j) = P(b).P(¬m).P(i | b, ¬m).P(g | b, i,¬m).P(j | g)
Then we only need to use the Conditional Probabilities Tables of the network:
P(b, i, ¬m, g, j) = 0.9*0.9*0.5*0.8*0.9
P(b, i, ¬m, g, j) = 0.2916

c. Calculate the probability that someone goes to jail given that they broke the law, have been indicted, and face a politically motivated prosecutor.

P(j | b, i, m) = P(j, b, i, m) / P(b, i, m)

Chain Rule on denominator
P(b, i, m) = P(b).P(m).P(i | b, m) = 0.9 * 0.1* 0.9 = 0.081

Marginalization Rule on numerator:
P(j, b, i, m) = P(j, b, i, m, g) + P(j, b, i, m, -g)

Chain Rule on numerator:
P(j, b, i, m) = P(b).P(m).P(i | b, m).P(g | b, i, m).P(j | g) + P(b).P(m).P(i | b, m).P(-g | b, i, m).P(j | -g)
P(j, b, i, m) = P(b).P(m).P(i | b, m).(P(g | b, i, m).P(j | g) + P(-g | b, i, m).P(j | -g))

Using CPTs for numerator:
P(j, b, i, m) = 0.9*0.1*0.9*(0.9*0.9 + 0.1*0) = 0.081 * 0.81

Then we can resolve what we were looking for:
P(j | b, i, m) = 0.081*0.81 / 0.081 = 0.81

d. A context-specific independence (see page 542) allows a variable to be independent of some of its parents given certain values of others. In addition to the usual conditional independences given by the graph structure, what context-specific independences exist in the Bayes net in Figure 14.23?

Variable G is independent of variables M and B when variable I is false.

e. Suppose we want to add the variable P = PresidentialPardon to the network; draw the new network and briefly explain any links you add.

If we look at the network P(J=true | G=true) = 0.9 meaning that there are other hidden vars that allows a person avoid the chail. Then, the PresidentialPardon could be one of these variables.

Then variable P will be parent of J. Then J will be evaluated given G and P: P(J | G, P)
The probability that P=True when G is false is 0 (zero) because the president can not give his or her pardon to an innocent person. So variable G is parent of P. I think until here this could be a simple network that is compliant with requirement. We can complicate things a little. What if Presidential Pardon is related with the kind of crime or its reasons. Then we could add B and M as parents of P too.