13.11 We wish to transmit an *n*-bit message to a receiving agent. The bits in the message are independently corrupted (flipped) during transmission with ε probability each. With an extra parity bit sent along with the original information, a message can be corrected by the receiver if at most one bit in the entire message (including the parity bit) has been corrupted. Suppose we want to ensure that the correct message is received with probability at least $1-\delta$. What is the maximum feasible value of *n*? Calculate this value for the case $\varepsilon = 0.001$, $\delta = 0.01$

For an n-bit message plus an extra parity bit we can estimate the probabilities:

$$P(no\, flipped\, bits) \; = (1-\varepsilon)^{n+1}$$
$$P(just\, one\, flipped\, bit) \; = (n+1) \times (\varepsilon \times (1-\varepsilon)^{n})$$
$$P(message\, transmitted\, correctly) \; = P(no\, flipped\, bits) + P(just\, one\, flipped\, bit) \; \geq 1-\delta$$
$$P(message\, transmitted\, correctly) \; = (1-\varepsilon)^{n+1} + \varepsilon \times (n+1) \times (1-\varepsilon)^{n} \geq 1-\delta$$

Now let find the the answer with some python help:

```
epsilon = 0.001
delta = 0.01
n = 0
parity = 1
condition = True
while condition:
    n += 1
    probability_of_correctly = (1-epsilon)**(n+parity) + (n+parity)*epsilon**(n+parity-1)
    condition = probability_of_correctly >= 1-delta
    print('n:{}: {} >= {} is {}'.format(n, probability_of_correctly, 1-delta, condition))
```

OUTPUT

```
n:1: 1.000001 >= 0.99 is True
n:2: 0.997005999 >= 0.99 is True
n:3: 0.996006000001 >= 0.99 is True
n:4: 0.995009990009999 >= 0.99 is True
n:5: 0.994014980015 >= 0.99 is True
n:6: 0.993020965034979 >= 0.99 is True
n:7: 0.992027944069944 >= 0.99 is True
n:8: 0.991035916125874 >= 0.99 is True
n:9: 0.9900448802097482 >= 0.99 is True
n:10: 0.9890548353295384 >= 0.99 is False
```

The maximum feasible value of n is 9 for correctly transmitted messages with a probability >= 0.99