13.23 In our analysis of the wumpus world, we used the fact that each square contains a pit with probability 0.2, independently of the contents of the other squares. Suppose instead that exactly $N/5$ pits are scattered at random among the N squares other than $[1,1]$. Are the variables $P_{i,j}$ and $P_{k,l}$ still independent? What is the joint distribution $P(P_{1,1}, \ldots, P_{4,4})$ now? Redo the calculation for the probabilities of pits in $[1,3]$ and $[2,2]$.

At the beginning $P_{11} = 0$ and all the rest of $P_{ij} = (N/5) / N = 1/5 = 0.2$
But once we discover if a new P_{kl} is a pit or not we need to recalculate P_{ij}

- P_{kl} is a pit
  - Set P_{kl} to one:
    $P_{kl} = 1$
  - Recalculate all unknown P_{ij}:
    $P_{ij} = ((N/5) - 1) / (N - 1) = (N - 5) / (5 \times (N - 1))$
- P_{kl} is not a pit
  - Set P_{kl} zero:
    $P_{kl} = 0$
  - Recalculate all unknown P_{ij}:
    $P_{ij} = (N/5) / (N - 1) \ = \ N / (5 \times (N - 1))$

So knowing that $P_{k,l}$ is or is not a pit affects the possible value of $P_{i,j}$.
We conclude that So $P_{i,j}$ and $P_{k,l}$ are no more independent of each other.

Using product rule:

$$P(P_{1,1}, \ldots, P_{4,4}) = P(P_{1,1}, \ldots, P_{4,3} \mid P_{4,4}) \times P_{4,4}$$

If $P_{4,4}$ is not a pit, then $P_{4,4} = 0$ then $P(P_{1,1}, \ldots, P_{4,4}) = 0$
If $P_{4,4}$ is a pit, then $P_{4,4} = 1$ then $P(P_{1,1}, \ldots, P_{4,4}) = P(P_{1,1}, \ldots, P_{4,3} \mid P_{4,4})$

This is logic. There are a total of N/5 pits. The total number of cases decreased to just the cases where there are present N/5 pits on the world. The number of cases can be calculated with combinatorics.

$$C_k^N = \binom{N}{k} = \frac{N!}{k! \times (N-k)!} = \frac{N!}{(N/5)! \times (N - N/5)!} = \frac{N!}{(N/5)! \times (4N/5)!}$$

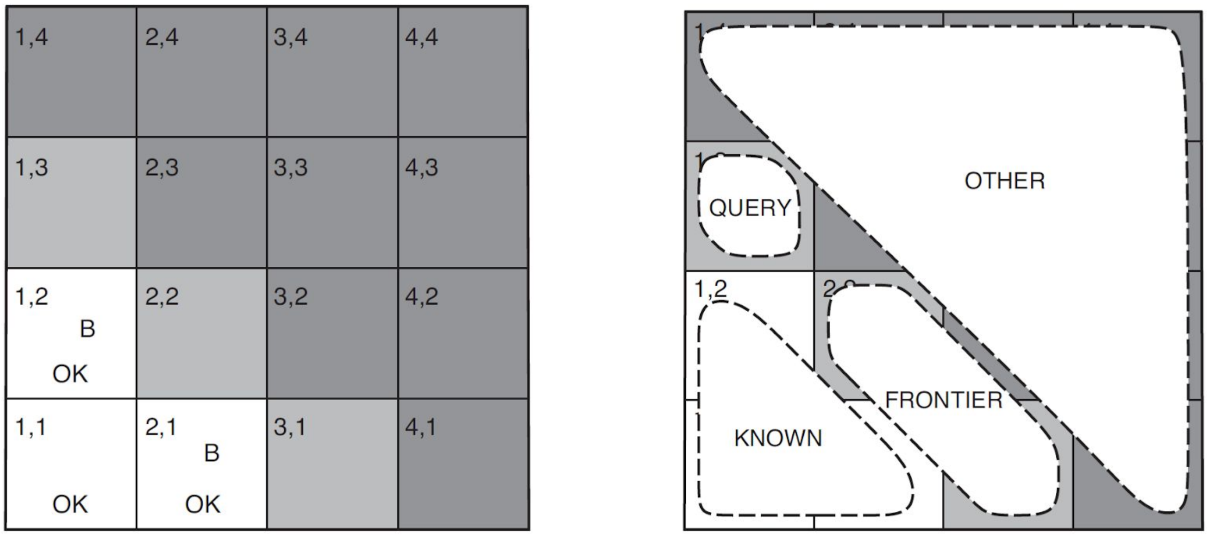
The Wumpus World presents 16 cells where we already know that $P_{11} = 0$. Then: $N = 15$

$$Initial\ Sample\ Space\ Size = \frac{15!}{(15/5)! \times (4 \times 15/5)!} = \frac{15!}{3! \times 12!} = 455$$

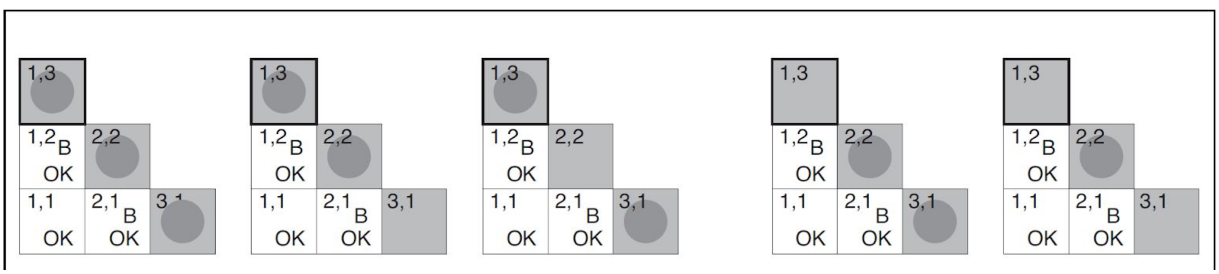
Suppose that after some moves we discover that $P_{1,2} = 0$ and $P_{2,1} = 0$. Then $N = 13$ and the number of pits out there is 3 yet. Then, the sample space size is reduced to:

$$New\ Sample\ Space\ Size = \frac{13!}{3! \times 10!} = 286$$

Now, if $B_{2,1}$ and $B_{1,2}$ are true, there are five consistent models in the frontier that we can use to analyze every frontier cell and select the best one.



(a) (b)



The number of cases for each consistent model could be calculated looking at the number of pits and the total number of cells in the other group of cells. Cases = $C^{number\ of\ cells\ in\ other\ group}_{number\ of\ pits\ in\ other\ group}$

Query = P_{13}**:**

Three consistent models with $P_{13} = True\ (21\ cases)$

- $P_{13} = True,\ P_{22} = True,\ P_{31} = True\ (1\ case)$
- $P_{13} = True,\ P_{22} = False,\ P_{31} = True\ (10\ cases)$
- $P_{13} = True,\ P_{22} = True,\ P_{31} = False\ (10\ cases)$

Two consistent models with $P_{13} = False\ (55\ cases)$

- $P_{13} = False,\ P_{22} = True,\ P_{31} = True\ (10\ cases)$
- $P_{13} = False,\ P_{22} = True,\ P_{31} = False\ (45\ cases)$

$P_{13} = \alpha \times\ < 21,\ 55 >\ =\ < 21/76,\ 55/76 >\ \simeq\ < 0.276,\ 0.724 >$

Query = P_{22} **:**

Four consistent models with $P_{22} = True$ $(66\ cases)$

- $P_{13} = True,\ P_{22} = True,\ P_{31} = True$ $(1\ case)$
- $P_{13} = True,\ P_{22} = True,\ P_{31} = False$ $(10\ cases)$
- $P_{13} = False,\ P_{22} = True,\ P_{31} = True$ $(10\ cases)$
- $P_{13} = False,\ P_{22} = True,\ P_{31} = False$ $(45\ cases)$

One consistent model with $P_{22} = False$ $(10\ cases)$

- $P_{13} = True,\ P_{22} = False,\ P_{31} = True$ $(10\ cases)$

$$P_{22} = \alpha \times \ < 66,\ 10 > \ = \ < 66/76,\ 10/76 > \ \simeq \ < 0.868,\ 0.132 >$$

Query = P_{31} **:**

Three consistent models with $P_{31} = True$ $(21\ cases)$

- $P_{13} = True,\ P_{22} = True,\ P_{31} = True$ $(1\ case)$
- $P_{13} = True,\ P_{22} = False,\ P_{31} = True$ $(10\ cases)$
- $P_{13} = False,\ P_{22} = True,\ P_{31} = True$ $(10\ cases)$

Two consistent models with $P_{13} = False$ $(55\ cases)$

- $P_{13} = False,\ P_{22} = True,\ P_{31} = False$ $(45\ cases)$
- $P_{13} = True,\ P_{22} = True,\ P_{31} = False$ $(10\ cases)$

$$P_{31} = \alpha \times \ < 21,\ 55 > \ = \ < 21/76,\ 55/76 > \ \simeq \ < 0.276,\ 0.724 >$$

Conclusion: the most probable cell with a pit is P_{22}. We could try going to P_{13} or P_{31}.