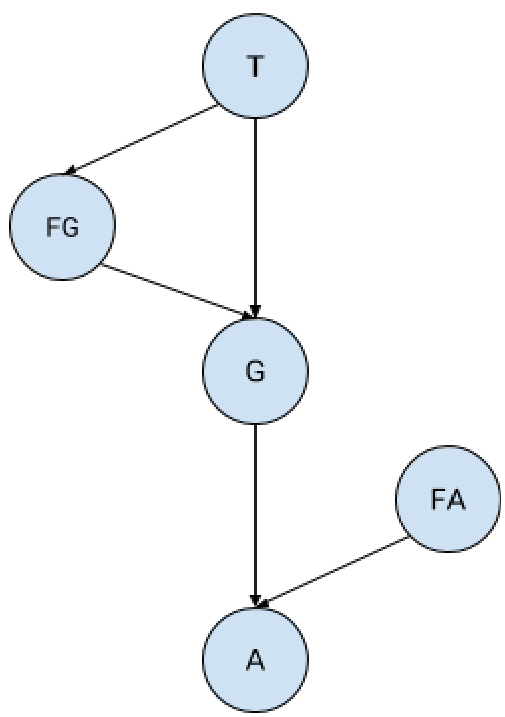
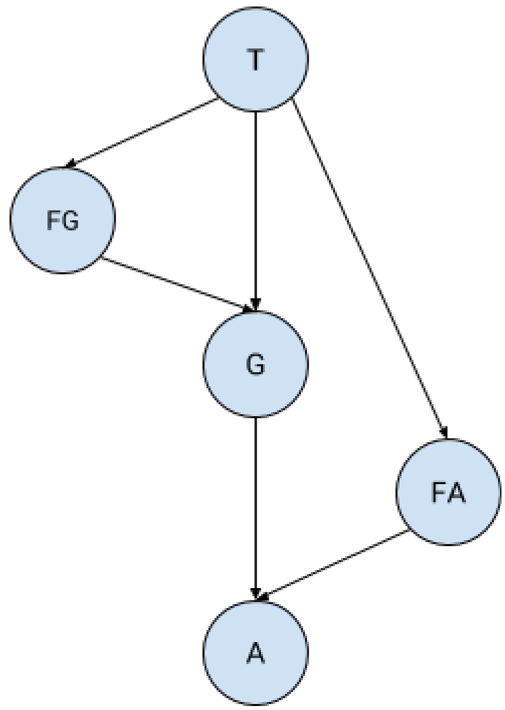
14.11 In your local nuclear power station, there is an alarm that senses when a temperature gauge exceeds a given threshold. The gauge measures the temperature of the core. Consider the Boolean variables A (alarm sounds), FA (alarm is faulty), and FG (gauge is faulty) and the multivalued nodes G (gauge reading) and T (actual core temperature).
a. Draw a Bayesian network for this domain, given that the gauge is more likely to fail when the core temperature gets too high.
b. Is your network a polytree? Why or why not?
c. Suppose there are just two possible actual and measured temperatures, normal and high; the probability that the gauge gives the correct temperature is x when it is working, but y when it is faulty. Give the conditional probability table associated with G.
d. Suppose the alarm works correctly unless it is faulty, in which case it never sounds. Give the conditional probability table associated with A.
e. Suppose the alarm and gauge are working and the alarm sounds. Calculate an expression for the probability that the temperature of the core is too high, in terms of the various conditional probabilities in the network.

a. Draw a Bayesian network for this domain, given that the gauge is more likely to fail when the core temperature gets too high.

- F_G is more likely to be True when T gets to high. Then F_G could depends directly on T .
- G does not work if it is faulty. Then G depends on F_G
- G reads T , then we could say that G depends on T .
- A could be True if the alarm works properly (it is not faulty), then A depends on F_A .
- A is True when T exceeds a given threshold, but A can only access to T throught G (an instruments that allows A to have an idea about the core temperature). Then A depends on G .



One could think too that the alarm will stop working if it is roasted. Then it becomes faulty. Then we could suspect that F_A depends on T .



b. Is your network a polytree? Why or why not?

A polytree (also known as oriented tree or singly connected network) is a directed acyclic graph whose underlying undirected graph is a tree. In other words, if we replace its directed edges with undirected edges, we obtain an undirected graph that is both connected and acyclic.

In both network above we can observe cyclic connections, this means that both networks are not directed acyclic graph. In example, there are more than one path that let us go from T to A. Then none of the networks is a polytree.

c. Suppose there are just two possible actual and measured temperatures, normal and high; the probability that the gauge gives the correct temperature is x when it is working, but y when it is faulty. Give the conditional probability table associated with G.

	T=normal		T=high	
	FG=True	FG=False	FG=True	FG=False
G=normal	P(G=n | T=n, FG=true)	P(G=n | T=n, FG=false)	P(G=n | T=h, FG=true)	P(G=n | T=h, FG=false)
G=high	P(G=h | T=n, FG=true)	P(G=h | T=n, FG=false)	P(G=h | T=h, FG=true)	P(G=h | T=h, FG=false)

1) By hypothesis:
P(G=normal | T=normal, FG=False) = P(G=high | T=high, FG=False) = x

2) By hypothesis:
P(G=normal | T=normal, FG=True) = P(G=high | T=high, FG=True) = y

3) By (1) and probability axiom:
P(G=normal | T=high, FG=False) = P(G=high, T=normal | FG=False) = 1 - x

4) By (2) and probability axiom:
P(G=normal, T=high | FG=True) = P(G=high, T=normal | FG=True) = 1 - y

	T=normal		T=high	
	FG=True	FG=False	FG=True	FG=False
G=normal	y	x	1-y	1-x
G=high	1-y	1-x	y	x

d. Suppose the alarm works correctly unless it is faulty, in which case it never sounds. Give the conditional probability table associated with A.

	G=normal		G=high	
	FA=True	FA=False	FA=True	FA=False
A=True	P(A=True | G=n, FA=True)	P(A=True | G=n, FA=False)	P(A=True | G=h, FA=True)	P(A=True | G=h, FA=False)
A=False	P(A=False | G=n, FA=True)	P(A=False | G=n, FA=False)	P(A=False | G=n, FA=True)	P(A=False | G=h, FA=False)

By hypothesis the alarm will sound only when G is high and alarm is not faulty.

	G=normal		G=high	
	FA=True	FA=False	FA=True	FA=False
A=True	0	0	0	1
A=False	1	1	1	0

e. Suppose the alarm and gauge are working and the alarm sounds. Calculate an expression for the probability that the temperature of the core is too high, in terms of the various conditional probabilities in the network.

Alarm does not sound if G is normal, then G is forced to be high.
Alarm does not sound if FG is True, then it is forced to be False.
Alarm does not sound if FA is True, then it is forced to be False.
Alarm does not sound if A is False, then it is true.

All variables can be used as as boolean literals:

T : *core temperature is high*, $\neg T$: *core temperature is normal*

G : *gauge reads high*, $\neg G$: *gauge reads normal*

A : *alarm sound*, $\neg A$: *alarm not sound*

F_G : *gauge is faulty*, $\neg F_G$: *gauge is not faulty*

F_A : *alarm is faulty*, $\neg F_A$: *alarm is not faulty*

I) By product rule:

$$P(T \mid \neg F_G, \neg F_A, G, A) = \frac{P(T, \neg F_G, \neg F_A, G, A)}{P(\neg F_G, \neg F_A, G, A)}$$

II) By AIMA Equation 14.2:

$$P(T, \neg F_G, \neg F_A, G, A) = P(T).P(\neg F_G \mid T).P(\neg F_A).P(G \mid T, \neg F_G).P(A \mid G, \neg F_A)$$

III) By marginal rule:

$$P(\neg F_G, \neg F_A, G, A) = P(T, \neg F_G, \neg F_A, G, A) + P(\neg T, \neg F_G, \neg F_A, G, A)$$

IV) By AIMA Equation 14.2:

$$P(\neg T, \neg F_G, \neg F_A, G, A) = P(\neg T).P(\neg F_G \mid \neg T).P(\neg F_A).P(G \mid \neg T, \neg F_G).P(A \mid G, \neg F_A)$$

V) Sum of (II) and (IV)

$$P(\neg F_A).P(A \mid G, \neg F_A).[P(T).P(\neg F_G \mid T).P(G \mid T, \neg F_G) + P(\neg T).P(\neg F_G \mid \neg T).P(G \mid \neg T, \neg F_G)]$$

IV) Replacing (II) and (V) in (I) denominator:

$$P(T \mid \neg F_G, \neg F_A, G, A) = \frac{P(T).P(\neg F_G \mid T).P(\neg F_A).P(G \mid T, \neg F_G).P(A \mid G, \neg F_A)}{P(\neg F_A).P(A \mid G, \neg F_A).[P(T).P(\neg F_G \mid T).P(G \mid T, \neg F_G) + P(\neg T).P(\neg F_G \mid \neg T).P(G \mid \neg T, \neg F_G)]}$$

$$P(T \mid \neg F_G, \neg F_A, G, A) = \frac{P(T).P(\neg F_G \mid T)P(G \mid T, \neg F_G)}{P(T).P(\neg F_G \mid T).P(G \mid T, \neg F_G) + P(\neg T).P(\neg F_G \mid \neg T).P(G \mid \neg T, \neg F_G)}$$

So, we only need to visit the CPTs to calculate the probability required:

$P(G \mid T, \neg F_G) = x$

$P(G \mid \neg T, \neg F_G) = 1 - x$

Next are invented:

$P(T) = p$

$P(\neg T) = 1 - P(T) = 1 - p$

$P(\neg F_G | \neg T) = q$

$P(\neg F_G | T) = r$

$$P(T \mid \neg F_G, \neg F_A, G, A) = \frac{p.r.x}{p.r.x + (1-p).q.(1-x)}$$