13.7 Consider the set of all possible five-card poker hands dealt fairly from a standard deck of fifty-two cards.

a. How many atomic events are there in the joint probability distribution (i.e., how many five-card hands are there)?

Combinatorial of n elements taken in groups of k elements where the order is not important: $\frac{n!}{k!\times(n-k)!}$

$52! / (5! \times 47!) = (52 \times 51 \times 50 \times 49 \times 48) / 120 = 52 \times 51 \times 10 \times 49 \times 2 = 2,598,960$

There are 2,598,960 atomic events.

b. What is the probability of each atomic event?

With a fair dealer the probability of every atomic event is assumed to be the same, and we know that the sum of all atomic events must be one by the axioms of probability.

Being Ω the sample space (conformed by all the possible worlds):

$0 \le P(\omega) \le 1 \; for\ every\ \omega\ and \sum_{\omega\in\Omega} P(\omega) = 1$

And there are 2,598,960 atomic events (possible worlds in the sample space) with the same probability.

$\sum_{\omega\in\Omega} P(\omega) = 1 = 2,598,960\ x\ P(atomic\ event)$

Now it is easy to calculate the probability of an atomic event:

$P(atomic\ event) = 1/2,598,960$

c. What is the probability of being dealt a royal straight flush? Four of a kind?

P(straight flush)

A royal straight flush of a suit is equivalent to an atomic event (10♠, J♠, Q♠, K♠, A♠). There are only 4 suits, so there are 4 hands or atomic events that represent straight flushes.

$P(straight\,flush) = 4 \times P(atomic\ event) = 4/2,598,960 = 1/649,740$

P(four of a kind)

$P(four\ of\ a\ kind) = P(poker)$

There are 13 kinds: $\{A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K\}$

$P(four\ of\ a\ kind) = 13 \times P("AAAA?")$

In a hand with 4 of a kind there are one more card than can be anyone of the other 48 cards.

$P("AAAA?") = 48/2,598,960$

Now we can calculate the final answer:

$P(four\ of\ a\ kind) = 13 \times 48\ /\ 2,598,960 = 1/4,165 \simeq 0.024\%$