**13.4 Would it be rational for an agent to hold the three beliefs P(A)=0.4, P(B)=0.3, and P(A ∨ B)=0.5? If so, what range of probabilities would be rational for the agent to hold for A ∧ B? Make up a table like the one in Figure 13.2, and show how it supports your argument about rationality. Then draw another version of the table where P(A ∨ B)=0.7. Explain why it is rational to have this probability, even though the table shows one case that is a loss and three that just break even. (Hint: what is Agent 1 committed to about the probability of each of the four cases, especially the case that is a loss?)**

By the *inclusion-exclusion principle* we can calculate the probability of the two event at the same time:

$$
\begin{aligned}
P(A \vee B) &= P(A) + P(B) - P(A \wedge B) \\
P(A \wedge B) &= P(A) + P(B) - P(A \vee B) \\
P(A \wedge B) &= 0.4 + 0.3 - 0.5 = 0.2
\end{aligned}
$$

It is exactly 0.2, a value between 0 and 1, so the agents presents a rational behaviour.

It is not possible to present a multiple bet to win in all cases:

| Agent 1 | | Agent 2 | | Outcome and payoffs to Agent 1 | | | |
|---|---|---|---|---|---|---|---|
| Proposition | Belief | Bet | Stakes | $A, B$ | $A, \neg B$ | $\neg A, B$ | $\neg A, \neg B$ |
| $A$ | 0.4 | $A$ | 4 to 6 | -6 | -6 | 4 | 4 |
| $B$ | 0.3 | $B$ | 3 to 7 | -7 | 3 | -7 | 3 |
| $A \vee B$ | 0.5 | $\neg(A \vee B)$ | 2 to 2 | 2 | 2 | 2 | -2 |
| | | | | -11 | -1 | -1 | 5 |

If the Agent 2 tries to win on $(\neg A, \neg B)$, for example with a stake of "8 to 8", then it will lose in $(A, \neg B)$ and $(\neg A, B)$.

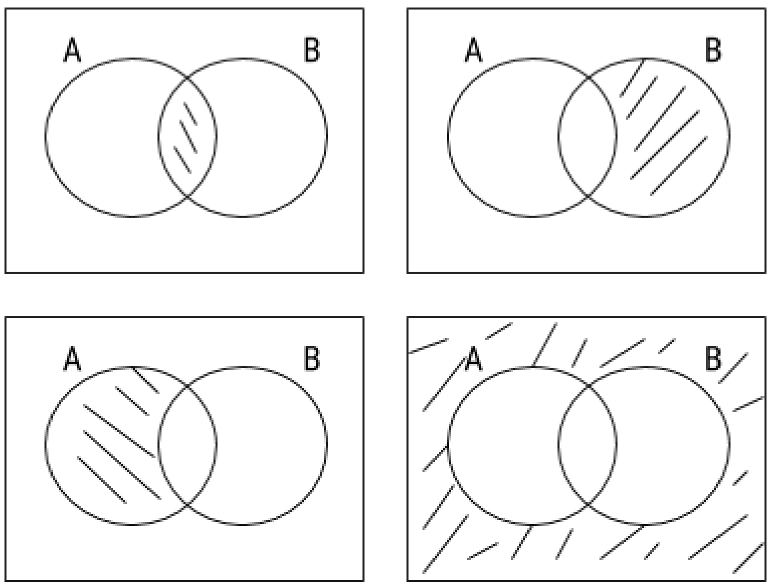
With $P(A \vee B)$ = 0.7 the agent still presents a rational behaviour:
$P(A \wedge B) = 0.4 + 0.3 - 0.7 = 0.0$

| Agent 1 | | Agent 2 | | Outcome and payoffs to Agent 1 | | | |
|---|---|---|---|---|---|---|---|
| Proposition | Belief | Bet | Stakes | $a, b$ | $a, \neg b$ | $\neg a, b$ | $\neg a, \neg b$ |
| $a$ | 0.4 | $a$ | 4 to 6 | -6 | -6 | 4 | 4 |
| $b$ | 0.3 | $b$ | 3 to 7 | -7 | 3 | -7 | 3 |
| $a \vee b$ | 0.7 | $\neg(a \vee b)$ | 7/3 to 1 | 7/3 | 7/3 | 7/3 | -1 |
| | | | | -32/3 | -2/3 | -2/3 | 6 |

I found another example using the atomic events of a world and two prepositions:

| $\cap$ | $A$ | $\neg A$ |
|---|---|---|
| $B$ | $a$ | $b$ |
| $\neg B$ | $c$ | $d$ |

These four events are independent and they can be represented in a Venn's diagram as (please forgive my drawing skills):




I) Here we see the equivalents probabilities of the atomic events:

$P(a) = P(A \wedge B)$
$P(b) = P(\neg A \wedge B)$
$P(c) = P(A \wedge \neg B)$
$P(d) = P(\neg A \wedge \neg B)$

And we can calculate the different atomic event probabilities for second version where $P(A \vee B) = 0.7$ :

II) By hypothesis:

$P(A) = P(a) + P(c) = 0.4$

III) By hypothesis:

$P(B) = P(a) + P(b) = 0.3$

IV) By hypothesis:

$P(A \vee B) = P(a) + P(b) + P(c) = 0.7$

V) By Axioms of Probability Theory:

$\Omega = \{a, b, c, d\}$
$P(\Omega) = P(a \vee b \vee c \vee d) \ = \ P(a) + P(b) + P(c) + P(d) = 1$

VI) By (I) and (V):

$P(d) = 1 - (P(a) + P(b) + P(c)) = 1 - P(A \vee B) = 1 - 0.7 = 0.3$

VII) (IV) - (II)

$P(a) + P(b) + P(c) - P(a) - P(c) = 0.7 - 0.4 \ = \ 0.3$
$P(b) = 0.3$

VIII) By (III) and (VII)

$P(a) = 0$

IX) By complement:

$P(c) = 1 \ - \ (P(a) + P(b) + P(d)) \ = 1 \ - \ 0.6 \ = \ 0.4$

All atomic events presents valid values!