13.6 Prove Equation (13.4) from Equations (13.1) and (13.2).

Equation 13.1:
Being Ω the sample space (conformed by all the possible worlds):

$$0 \le P(\omega) \le 1 \text{ for every } \omega \text{ and } \sum_{\omega \in \Omega} P(\omega) = 1$$

Equation 13.2:
For any proposition φ:

$$P(\varphi) = \sum_{\omega \in \varphi} P(\omega)$$

Let's see for Equation 13.4:

$$P(a \vee b) = \sum_{\omega \in (a \vee b)} P(\omega) = \sum_{\omega \in (a-b)} P(\omega) + \sum_{\omega \in (b-a)} P(\omega) + \sum_{\omega \in (a \wedge b)} P(\omega)$$

$$P(a \vee b) = \sum_{\omega \in (a-b)} P(\omega) + \sum_{\omega \in (a \wedge b)} P(\omega) + \sum_{\omega \in (b-a)} P(\omega) + \sum_{\omega \in (a \wedge b)} P(\omega) - \sum_{\omega \in (a \wedge b)} P(\omega)$$

$$P(a \vee b) = \left(\sum_{\omega \in (a-b)} P(\omega) + \sum_{\omega \in (a \wedge b)} P(\omega)\right) + \left(\sum_{\omega \in (b-a)} P(\omega) + \sum_{\omega \in (a \wedge b)} P(\omega)\right) - \sum_{\omega \in (a \wedge b)} P(\omega)$$

$$P(a \vee b) = \sum_{\omega \in a} P(\omega) + \sum_{\omega \in b} P(\omega) - \sum_{\omega \in (a \wedge b)} P(\omega)$$

$$P(a \vee b) = P(a) + P(b) - P(a \wedge b)$$

And that is *the exclusion-exclusion principle*.

Q.E.D.