

13.8 Given the full joint distribution shown in Figure 13.3, calculate the following:

- P(toothache) .**
- P(Cavity) .**
- P(Toothache | cavity) .**
- P(Cavity | toothache \vee catch) .**

	<i>toothache</i>		\neg <i>toothache</i>	
	<i>catch</i>	\neg <i>catch</i>	<i>catch</i>	\neg <i>catch</i>
<i>cavity</i>	0.108	0.012	0.072	0.008
\neg <i>cavity</i>	0.016	0.064	0.144	0.576

Figure 13.3 A full joint distribution for the *Toothache*, *Cavity*, *Catch* world.

a. P(toothache)

Probability of a literal (observe that it is being used a lowercase first letter in *toothache*).

$$P(\text{toothache}) = 0.108 + 0.012 + 0.016 + 0.064 = 0.120 + 0.080 = 0.200$$

b. P(Cavity)

Probability of a boolean variable (observe that it is being used an uppercase first letter in *Cavity*).

$$P(\text{Cavity}) = \langle P(\text{cavity}), P(\neg \text{cavity}) \rangle$$

$$P(\text{cavity}) = 0.108 + 0.012 + 0.072 + 0.008 = 0.120 + 0.080 = 0.2$$

$$P(\neg \text{cavity}) = 1 - P(\text{cavity}) = 0.8$$

$$P(\text{Cavity}) = \langle 0.2, 0.8 \rangle$$

c. P(Toothache | cavity)

Probability of a boolean variable given some event.

$$P(\text{Toothache} | \text{cavity}) = \langle P(\text{toothache} | \text{cavity}), P(\neg \text{toothache} | \text{cavity}) \rangle$$

$$P(\text{toothache} | \text{cavity}) = P(\text{toothache}, \text{cavity}) / P(\text{cavity}) = (0.108 + 0.012) / 0.2 = 0.120 / 0.2 = 0.6$$

$$P(\neg \text{toothache} | \text{cavity}) = 1 - P(\text{toothache} | \text{cavity}) = 0.4$$

$$P(\text{Toothache} | \text{cavity}) = \langle 0.6, 0.4 \rangle$$

d. P(Cavity | toothache \vee catch)

l) Probability of a boolean variable given some event:

$$P(\text{Cavity} | \text{toothache} \vee \text{catch}) = \langle P(\text{cavity} | \text{toothache} \vee \text{catch}), P(\neg \text{cavity} | \text{toothache} \vee \text{catch}) \rangle$$

$$\text{II) } P(\text{toothache} \vee \text{catch}) = 0.108 + 0.016 + 0.012 + 0.064 + 0.072 + 0.144 = 0.416$$

$$\text{III) } P(\text{cavity} \mid \text{toothache} \vee \text{catch}) = (0.108 + 0.012 + 0.072) / 0.416 = 0.4615$$

$$\text{IV) } P(\neg \text{cavity} \mid \text{toothache} \vee \text{catch}) = 1 - P(\text{cavity} \mid \text{toothache} \vee \text{catch}) = 0.5385$$

V) Using (III) and (IV) in (I)

$$P(\text{Cavity} \mid \text{toothache} \vee \text{catch}) = \langle 0.4615, 0.5385 \rangle$$