

13.19 In this exercise, you will complete the normalization calculation for the meningitis example. First, make up a suitable value for $P(s | \neg m)$, and use it to calculate unnormalized values for $P(m | s)$ and $P(\neg m | s)$ (i.e., ignoring the $P(s)$ term in the Bayes' rule expression, Equation (13.14)). Now normalize these values so that they add to 1.

$$P(s | m) = 0.7$$

$$P(m) = 1/50,000 = 0.00002$$

$$P(s) = 0.01$$

$$P(\neg s | m) = 1 - P(s | m) = 0.3$$

$$P(\neg m) = 1 - P(m) = 0.99998$$

We already saw in the book the values of $P(m | s)$ and $P(\neg m | s)$:

$$P(m | s) = P(s | m) \times P(m) / P(s) = 0.7 \times 0.00002 / 0.01 = 0.0014$$

$$P(\neg m | s) = 1 - P(m | s) = 0.9986$$

We need $P(s | \neg m)$ because it will be useful to estimate requested unnormalized values:

$$P(s | \neg m) = P(\neg m | s) \times P(s) / P(\neg m) = 0.9986 \times 0.01 / 0.99998 = 0.009986 / 0.99998$$

To calculate previous $P(s | \neg m)$ value I used $P(\neg m | s)$ and that is weird because the exercise says that we need to find it (getting in first place the no normalized values).

Now we must suppose we do not know the value of $P(s)$ to make use of normalization. First we calculate the unnormalized values of $P(M | s)$:

$$P(M | s) = \delta \times \langle P(m | s); P(\neg m | s) \rangle$$

$$P(M | s) = \delta \times \langle P(s | m) \times P(m); P(s | \neg m) \times P(\neg m) \rangle$$

$$P(M | s) = \delta \times \langle 0.7 \times 0.00002; (0.009986 / 0.99998) \times 0.99998 \rangle$$

$$P(M | s) = \delta \times \langle 0.000014; 0.009986 \rangle$$

Now we calculate delta:

$$\delta \times (0.000014 + 0.009986) = 1$$

$$\delta \times 0.01 = 1$$

$$\delta = 100$$

And now we can normalize the values:

$$P(M | s) = 100 \times \langle 0.000014; 0.009986 \rangle$$

$$P(M | s) = \langle 0.0014; 0.9986 \rangle$$

We can check that $P(m | s) + P(\neg m | s) = 0.0014 + 0.9986 = 1$