Example 7

A tree of Wavelet Decompositions

For a set of dilation matrices this demo illustrates nested spaces build by scaling functions of de la Vallée Poussin type.

```
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```

License

Loading the Library

The MPAWL is located in the parent directory (see `MPAWL.m`) in order to load the library, we add its path to **$Path**.

In[1]:=
```
$Path = Join[$Path, {ParentDirectory[NotebookDirectory[]]}];
SetDirectory[NotebookDirectory[]];(*Set to actual directory*)
Needs["MPAWL`"];
```

Examine different ways to decompose a given sampling of a function by using different factorizations of the initial matrix. This Example is a new possibility of the de la Vallée Poussin means and hence extends the Dirichlet case is these observations by taking Shear matrices. The examples are taken from chapter 4 in [1] and more details to the wavelet transform and its complexity can be found in [2].

Sampling

We take the function

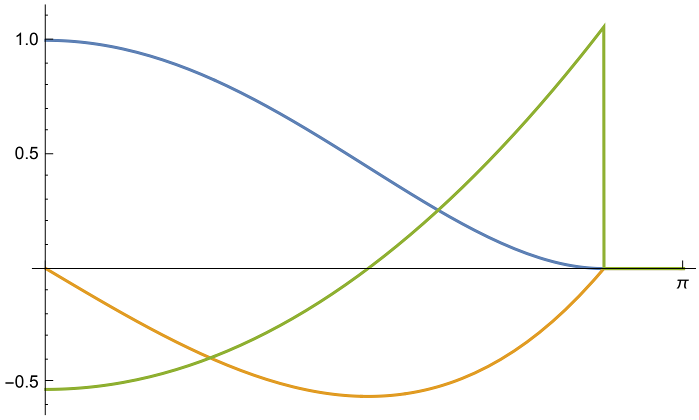
In[4]:=
```
f[x_] := Which[Abs[x] ≤ 1, (x + 1)^2 (x - 1)^2, True, 0]
```

In[5]:=
```
sf[x_] := f[8 / 7 x / π]
```

Which has a discontinuity in its second derivative at

In[6]:=
```
Plot[{sf[x], sf'[x], sf''[x]}, {x, 0, π},
 Ticks → {{0, π}, Automatic}, PlotRange -> All]
```

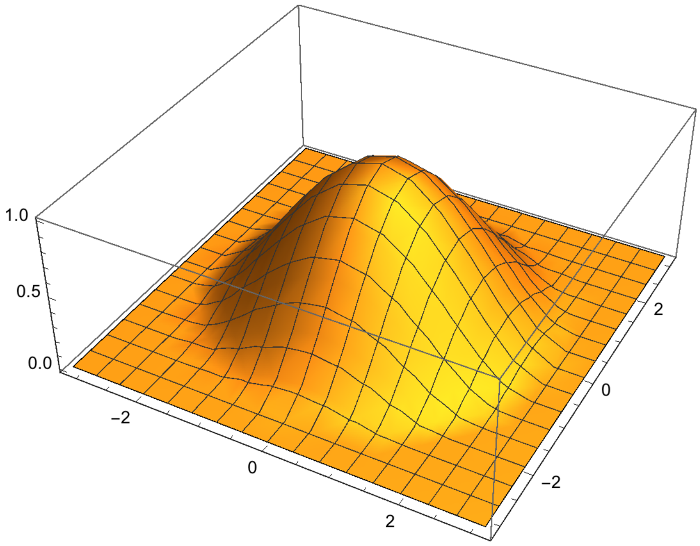
Out[6]=



Hence defining a radial function based on that, we obtain a circle (with radius 7 $\pi/8$, where orthogonal to each tangent we obtain a discontinuity in the directional derivative, i.e. we use

In[7]:=
```
fr[x_] := sf[Norm[x]]
```

In[8]:=
```
Plot3D[fr[{x, y}], {x, -π, π}, {y, -π, π}]
```

Out[8]=


We now perform the same steps as in Example 6, but include one further Debug info to print just the images of the last level.

In[9]:= **? decomposeData2D**

decomposeData2D[g(Vec), JSet(Vec), mM, data]
decomposeData2D[l,g,JSet, mM, data, ckφM]

Decompose the data given as coefficients with respect to ck$\varphi\varphi$ this method decomposes the data on different dilationPaths given by JSet(Vec), where the corresponding de la Vallée Poussin means (and wavelets) are given by g(Vec) (see delaValleePoussinMean for restrictions on g). The dilation matrices are given in terms of the characters used by dilationMatrix2D.

Both g and JSet might be given as vectors. If both are vectors, gVec has to be one element longer than JSetVec. If one is given as a vector, the other is expanded to a vector of corresponding length, having constant entries. This length determines the number of levels of the wavelet decomposition, while g also specifies additionally the starting space of mM.

If none of them is a vector, the second definition is needed to specify a level depth of decomposition.

Options

ImagePrefix → *"Img"* | String
prefix that is used to save the images; the path in terms of the matrices is appended. This String may also contain Folders, that are relative to the Notebook.
ImageSuffix → *".png"* | String
suffix to determine the file format of the images.
DataPrefix → *"Data"* | String
prefix that is used to save the data files; here an S or a W for the two subspaces is added and the decomposition path in terms of the matrices is appended. This String may also contain Folders, that are relative to the Notebook.

Setting any of those String to an empty String disables the Image or Coefficient savings.

PlotResolution → *64* | nonnegative Integer
resolution of the resulting Image generated by discreteFourierSeries
computeWavelet –> → *True* | False
whether to compute the wavelet part of the specific level and produce it's image
computeScale → *False* | True
whether to compute the scale part of the specific level and produce it's image
Validate → *True* | False
whether to perform a check (via **isMatrixValid**) on the matrix mM and mJ(s) Validity of data. If some matrices mN in the decomposition are not valid, the algorithm continues on the other leaves.
Debug → *"None"* | "Text" | "Time" | "Image" | "Leaves"
or any combination of these Words in one String (i.e. concatenated via "&") to produce intermediate results, indicate progress and display computation times. The Term „Leaves" restricts all debug output to just the last level.

and any Option of discretePlotFourierSeries (including Plot and BarLegend), that are passed on and any Option of Export (especially Resolution but not ImageSize) Show (especially ImageSize). By the last two, Fonts are scaled, but a pixel size of the image is not that easy to be computed.

```
In[10]:= mM = 256 * IdentityMatrix[2];

In[11]:= {ckφM, φMBS} = delaValleePoussinMean[1 / 10, mM, Debug → "Text&Time", File →
           {"example7/dlVP-ckS.dat", "example7/dlVP-BS.dat"}, BracketSums → True];

Loading de la Vallée Poussin mean from file “example7/dlVP-ckS.dat”...

failed.

Computing de la Vallée Poussin coefficients...

Creating the de La Vallée Poussin kernel took 11.651846 seconds.

Orthonormalization...

Orthonormalization took 100.442472 seconds.

Loading Bracket sums from file “example7/dlVP-BS.dat”...

In[12]:= data = sampleFunction[mM, fr, Debug → "Text&Time",
           File → "example7/samples.dat", validateMatrix → False];

Loading data from file “example7/samples.dat”...

In[13]:= cdata = changeBasis[mM, N[data], φMBS, Debug → "Text&Time", Input → "Time"];

The Fourier transform of the input took 0.073577 seconds.

The change of basis took 0.010018 seconds.
```

Then, we can look at different directions of the first half of the first quadrant by using different dilation matrices especially including shearing. By precomputing (saving) the coefficients of the involved de la Vallée Poussin means, the computational time can be reduced by roughly 2/3.

```
In[14]:= decomposeData2D[1 / 14, {{"Y", "Y-"}, {"Y", "Y-"}, {"X"}}, mM, cdata,
          ImagePrefix → "", ImageSuffix → "", DataPrefix → "example7/dlVP-",
          Debug → "Text&Time&Image&Leaves", ColorLegend → False]

Computing the decomposition. Only Debug
  output of the leaves of the wavelet tree are printed.
```

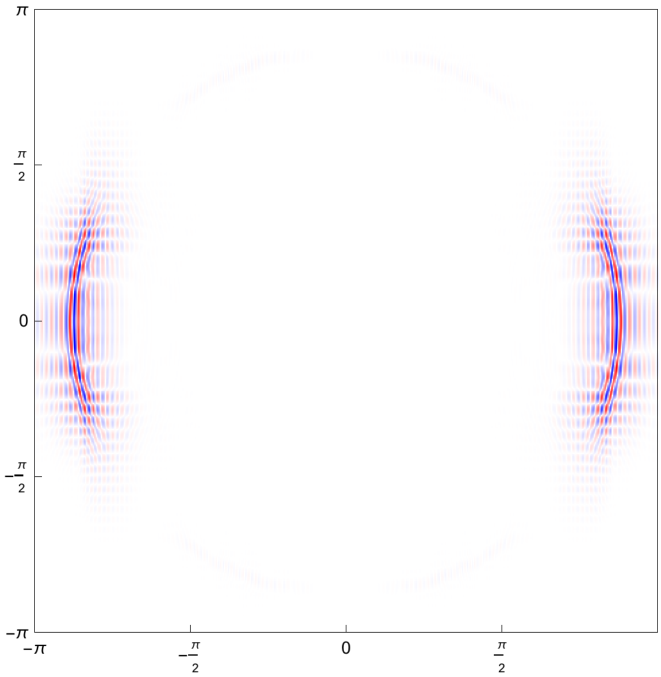
Decompose $\begin{pmatrix} 256 & 0 \\ 0 & 256 \end{pmatrix}$ with Y

Decompose $\begin{pmatrix} 256 & 0 \\ 0 & 128 \end{pmatrix}$ (Y) with Y

Decompose $\begin{pmatrix} 256 & 0 \\ 0 & 64 \end{pmatrix}$ (YY) with X

```
Loading subspace coefficients from “
 example7/dlVP-YYX-S.dat” and “example7/dlVP-YYX-W.dat”...

Performing the Wavelet transform...

Performing the Wavelet transform took 8.412455 seconds.

Padding Data...

Fourier transforming...

Shifting back...

Creating Color Image...
```

The range of Values is from -3.77309×10^{-5} to 3.59854×10^{-5}.



Decompose $\begin{pmatrix} 256 & 0 \\ 0 & 128 \end{pmatrix}$ (Y) with Y-

Decompose $\begin{pmatrix} 256 & 64 \\ 0 & 64 \end{pmatrix}$ (YY-) with X

Loading subspace coefficients from "example7/dlVP-YY-X-S.dat" and "example7/dlVP-YY-X-W.dat"...

Performing the Wavelet transform...

Performing the Wavelet transform took 8.239767 seconds.

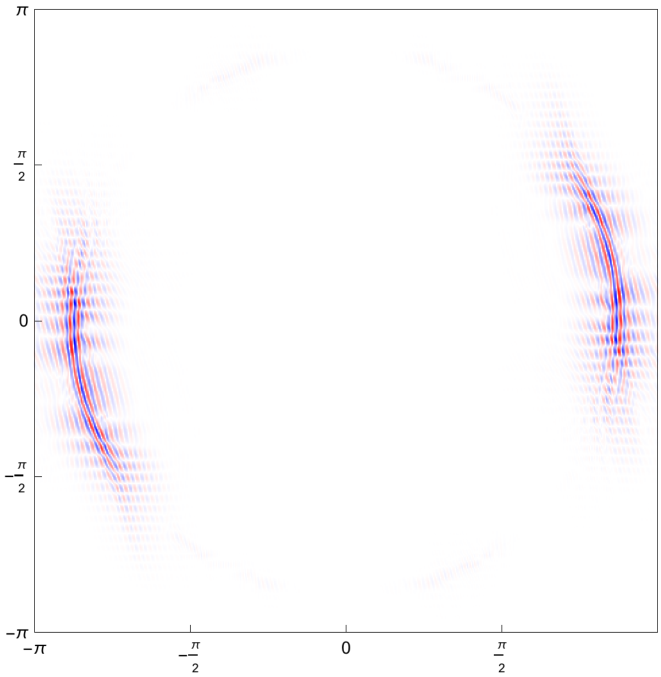
Padding Data...

Fourier transforming...

Shifting back...

Creating Color Image...

The range of Values is from -3.75763×10^{-5} to 3.60189×10^{-5}.



Decompose $\begin{pmatrix} 256 & 0 \\ 0 & 256 \end{pmatrix}$ with Y-

Decompose $\begin{pmatrix} 256 & 128 \\ 0 & 128 \end{pmatrix}$ (Y-) with Y

Decompose $\begin{pmatrix} 256 & 128 \\ 0 & 64 \end{pmatrix}$ (Y-Y) with X

Loading subspace coefficients from "example7/dlVP-Y-YX-S.dat" and "example7/dlVP-Y-YX-W.dat"...

Performing the Wavelet transform...

Performing the Wavelet transform took 8.513381 seconds.

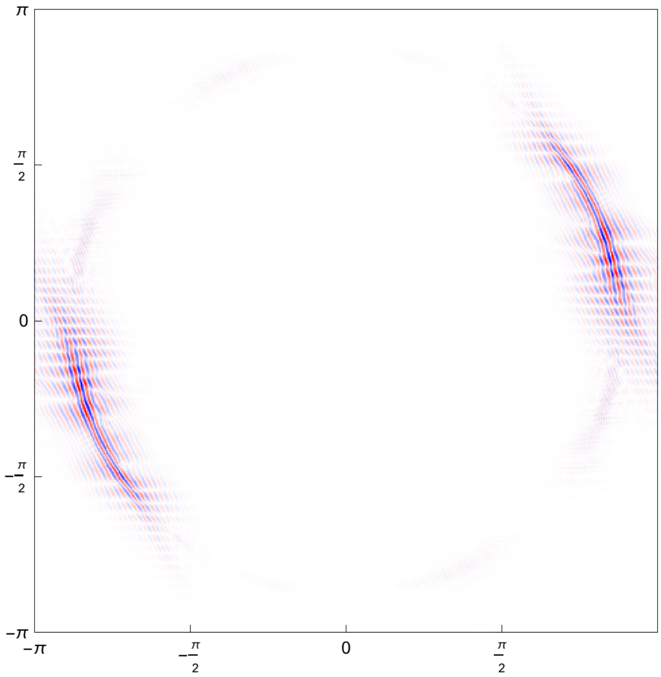
Padding Data...

Fourier transforming...

Shifting back...

Creating Color Image...

The range of Values is from -3.39322×10^{-5} to 3.27859×10^{-5}.



Decompose $\begin{pmatrix} 256 & 128 \\ 0 & 128 \end{pmatrix}$ (Y−) with Y−

Decompose $\begin{pmatrix} 256 & 192 \\ 0 & 64 \end{pmatrix}$ (Y−Y−) with X

Loading subspace coefficients from “
example7/dlVP-Y-Y-X-S.dat” and “example7/dlVP-Y-Y-X-W.dat”...

Performing the Wavelet transform...

Performing the Wavelet transform took 8.435292 seconds.

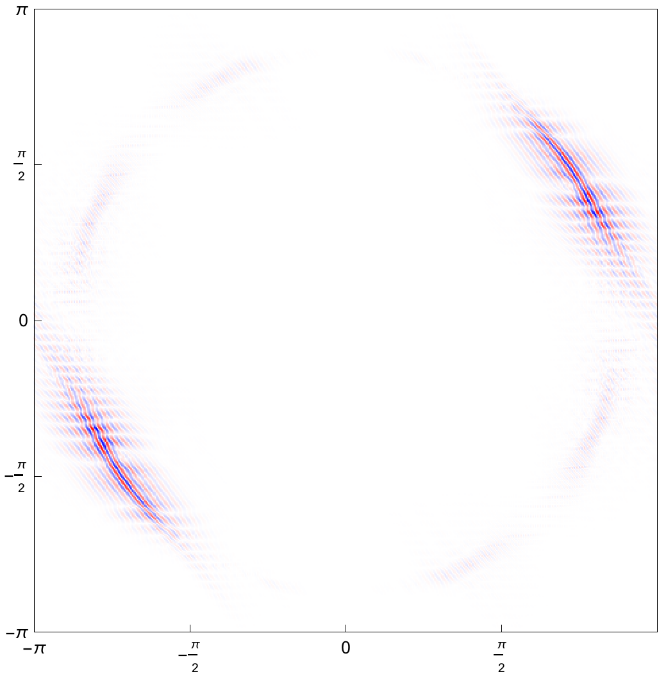
Padding Data...

Fourier transforming...

Shifting back...

Creating Color Image...

The range of Values is from -2.83845×10^{-5} to 2.85568×10^{-5}.



Literature

[1] R. Bergmann, *Translationsinvariante Räume multivariater anisotroper Funktionen auf dem Torus*, Dissertation, University of Lübeck, 2013.

[2] R.Bergmann, *The fast Fourier transform and fast wavelet transform for patterns on the torus*, Appl. Comp. Harmon. Anal. 35 (2013) 39–51, doi: 10.1016/j.acha .2012 .07 .007.

[3] R.Bergmann, J. Prestin, Multivariate periodic wavelets of de la Vallée Poussin type. J. Fourier Anal. Appl. (to appear), doi: 10.1007/s00041-014-9372-z.