Example 2

The Fourier transform

This example demonstrates, that the Fourier transform can be used fortwo different kinds of data sets.


```
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Created:        15.08.2013
Last Changed:   15.08.2013
```


License

Loading the Library

The MPAWL is located in the parent directory (see `MPAWL.m`) in order to load the library, we add its path to **$Path**.

In[1]:=
```
$Path = Join[$Path, {ParentDirectory[NotebookDirectory[]]}];
SetDirectory[NotebookDirectory[]];(*Set to actual directory*)
Needs["MPAWL`"];
```

The data set as a matrix

Let' s look at a matrix having more than one cycle (in contrast to the matrix from Example 1). For

In[4]:=
```
mM = {{16, 4}, {0, 16}}; MatrixForm[mM]
```

Out[4]//MatrixForm=

$$\begin{pmatrix} 16 & 4 \\ 0 & 16 \end{pmatrix}$$

We have

In[5]:=
```
patternDimension[mM]
```

Out[5]= 2

and

In[6]:= $\{v_1, v_2\}$ = patternBasis[mM]

Out[6]= $\left\{\left\{0, \frac{1}{4}\right\}, \left\{\frac{1}{64}, -\frac{1}{16}\right\}\right\}$

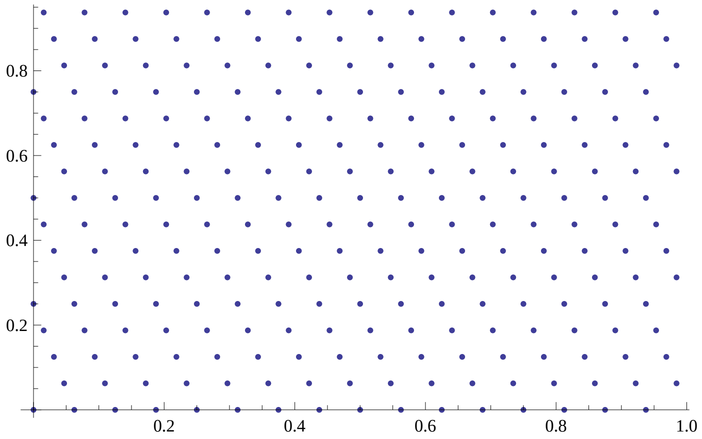
where the elementary divisors are

In[7]:= $\{\epsilon_1, \epsilon_2\}$ = Diagonal[IntegerSmithForm[mM, ExtendedForm → False]]

Out[7]= {4, 64}

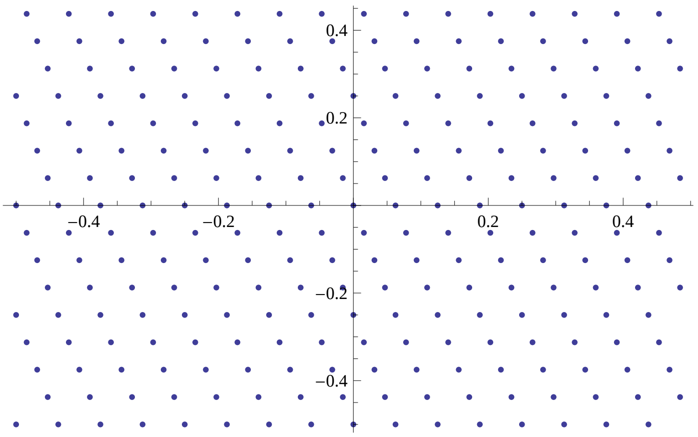
and hence

In[8]:=
```
ListPlot[pattern[getPatternNormalform[mM]]]
```

Out[8]=


is the same as (where **Flatten[...,1]** is used to get a vector of points instead of a matrix)

In[9]:=
```
ListPlot[
 Flatten[Table[modM[k1 * v1 + k2 v2, IdentityMatrix[2], Target → "Symmetric"],
   {k1, 0, ε1 - 1}, {k2, 0, ε2 - 1}], 1]]
```

Out[9]=


If we think of these points as sampling points and scale them onto the torus $[-\pi, \pi)^2$,
i.e. by multiplying them with 2π,
we obtain a matrix of dimension $\epsilon_1 \times \epsilon_2 = 4 \times 64$ of sampling values, for example

In[10]:=
```
b = Table[If[(k1 == 0) && (k2 == 0), 1, 0], {k1, 0, ε1 - 1}, {k2, 0, ε2 - 1}]
```

Out[10]=
```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}
```

We obtain its Fourier Transform with respect to mM by

In[11]:=
```
? FourierTransformTorus
```

FourierTransformTorus[mM, b]

Perform the Fourier transform on the pattern with respect to mM. b is either a vector of length m=|Det[mM]| or adressing the points with respect to the basis of the pattern, i.e. the cycles having the length of the elementary divisors.

Options

Validate → True | False
  whether to perform a check (via isMatrixValid[mM]) on the matrix mM and the check, whether the Origin is in Range.

Compute → “Numeric” | “Exact”
  Providing numerical data, the Fourier method is used to perform the transform using FFT techniques. If all entries of mM and b are given exact, the “Exact” computation can be used to obtain the exact transform

In[12]:=
```
hatb = FourierTransformTorus[mM, b]
```

Out[12]=
```
{{0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625},
 {0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625},
 {0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625},
 {0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
  0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625}}
```

which can also be switched to exact computations

In[13]:=
```
hatb = FourierTransformTorus[mM, b, Compute → "Exact"]
```

Out[13]=

$$\begin{gathered}
\Big\{\Big\{\frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \\
\frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \\
\frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \\
\frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}\Big\}, \\
\Big\{\frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \\
\frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \\
\frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \\
\frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}\Big\}, \\
\Big\{\frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \\
\frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \\
\frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \\
\frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}\Big\}, \\
\Big\{\frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \\
\frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \\
\frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \\
\frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}\Big\}\Big\}
\end{gathered}$$

Of course now, the values correspond to the same order used above with respect to the basis of the generating set

In[14]:=
```
generatingSetBasis[Transpose[mM]]
```

Out[14]=
```
{{4, 1}, {1, 16}}
```

Due to

In[15]:=
```
Abs[Det[mM]]
```

Out[15]=
```
256
```

this is of course the unitary version of the Fourier transform and hence

In[16]:=
```
FourierTransformTorus[mM, hatb, Compute → "Exact"]
```

Out[16]=
```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}
```

But we could also reshape the data to be a vector by ordering the indices **{0, 0}, ..., {3, 63}** lexicographically, i.e. **{0, 0}, {0, 1}, ..., {1, 0}, {1, 1}, ... {3, 62}, {3, 63}**. For details see [1].

In[17]:=
```
? reshapeData
```

reshapeData[M,data,direction]

Perform a reshape of data, where direction denotes
True: From vector to matrix
False: The other way around

Options

Validate → True | False
whether to perform a check (via isMatrixValid[mM]) on the matrix mM
and the check, whether the Origin is in Range.

In[18]:=
```
b2 = reshapeData[mM, b, False]
```

Out[18]=
```
{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

In[19]:=
```
hatb2 = FourierTransformTorus[mM, b2]
```

Out[19]=
```
{0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625,
 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625, 0.0625}
```

again using the above ordering of the indices but now with respect to the generating set of **Transpose[mM]**. Of course the inverse transform yields again

In[20]:=
```
FourierTransformTorus[mM, hatb2]
```

Out[20]=
```
{1., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.}
```

but of course this works if and only if the ordering of the elements in the vector is as just explained. Notice that while the input shape is analyzed automatically, i.e. whether its a vector or a matrix of data, the usual computation is always done numerically and the exact computation must be activated by the option Compute-> . Also the exact computation yields $Failed if any numeric number is given as does the numeric one if any element is not a number.

Literature

[1] R.Bergmann, *The fast Fourier transform and fast wavelet transform for patterns on the torus*,

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