Tutorial-12: Complex Integration

TYPE-I LINE INTEGRAL

1. Evaluate $\int_0^{3+i} z^2 dz$

(i) along the real axis from 0 to 3 and then vertically to $3 + i$.

(ii) along the imaginary axis from 0 to i and then horizontally to $3 + i$

(iii) along the parabola $x = 3y^2$

Is the line integral independent of the path? Explain?

2. Show that $\int_c \log z \, dz = 2\pi i$, where C is the unit circle in the z – plane.

3. Evaluate $\int_c (z^2 + 3z^{-4})dz$, where C is upper half of the unit circle from $(1,0) to\ (-1,0)$

4. Evaluate $\int f(z)dz$ along the square whose vertices are $(1,1), (2,1), (2,2), (1,2)$ in anti – clockwise direction where $f(z) = x - 2iy$

TYPE-II CAUCHY'S INTEGRAL THEOREM, CAUCHY'S INTEGRAL FORMULA

5. Evaluate $\int_c \frac{1}{z}.\cos z \, dz$ where C is the ellipse $9x^2 + 4y^2 = 1$

6. Evaluate $\int_c \frac{e^{3z}}{z-i} dz$ where C is the curve $|z-2| + |z+2| = 6$

7. Evaluate $\int_c \frac{e^{2z}}{(z-1)(z-2)} dz$, where C is the circle $|z| = 3$

8. If $f(z) = z^3 + iz^2 - 4z - 4i$ evaluate $\int_c \frac{f'(z)}{f(z)} dz$

where C is a simple closed curve enclosing zeros of $f(z)$

9. Evaluate $\int_c \frac{sin^6 z}{(z-\pi/6)^3} dz$ where C is $|z| = 1$

10. Evaluate $\int_c \frac{z-1}{(z+1)^2(z-2)} dz$ where C is $|z - i| = 2$

11. Evaluate $\int_c \frac{ze^{2z}}{(z-1)^3} dz$ where C is $|z + i| = 2$

12. If $f(\zeta) = \int_c \frac{4z^2+z+4}{z-\zeta} dz$ where C is the ellipse $4x^2 + 9y^2 = 36$ find the values of

(i) $f(4)$ **(ii)** $f(1)$ **(iii)** $f(i)$ **(iv)** $f'(-1)$ **(v)** $f''(-i)$