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~~Experiment / assignment~~ / tutorial No. 9
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Q.1. Prove that $\bar{F} = (z^2+2x+3y)i + (3x+2y+z)j + (y+2zx)k$ is irrotational and find scalar potential function ϕ such that $\bar{F} = \nabla\phi$ and $\phi(1,1,0) = 4$. Also find the work done in moving a particle from $A(0,0,0)$ & $B(\phi, 1, 2, 3)$.

→ Given, $\bar{F} = (z^2+2x+3y)i + (3x+2y+z)j + (y+2xz)k$

$$\text{curl } \bar{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z^2+2x+3y & 3x+2y+z & y+2xz \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(y+2xz) - \frac{\partial}{\partial z}(3x+2y+z)\right] - j\left[\frac{\partial}{\partial x}(y+2xz) - \frac{\partial}{\partial z}(3y+2x+z^2)\right]$$
$$+ k\left[\frac{\partial}{\partial x}(3x+2y+z) - \frac{\partial}{\partial y}(z^2+2x+3y)\right]$$

$$= i(1-1) - j(2z-2z) + k(3-3)$$
$$= 0$$

As curl $\bar{F} = 0$, F is irrotational //.

Now, $F = \nabla\phi$

$$(z^2+2x+3y)i + (3x+2y+z)j + (y+2xz)k = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$$

Comparing both sides,

$$\frac{\partial\phi}{\partial x} = z^2+2x+3y \implies \int\frac{\partial\phi}{\partial x} = \int z^2+2x+3y\,dx = z^2x + x^2 + 3xy + f_1(y,z)$$

$$\frac{\partial\phi}{\partial y} = 3x+2y+z \implies \int\frac{\partial\phi}{\partial y} = \int 3x+2y+z\,dy = 3xy + \frac{\cancel{2}y^2}{\cancel{2}} + yz + f_2(x,z)$$

$$\frac{\partial\phi}{\partial z} = y+2xz \implies \int\frac{\partial\phi}{\partial z} = \int y+2xz\,dz = yz + \frac{\cancel{2}xz^2}{\cancel{2}} + f_3(x,y)$$

$\therefore \phi = x^2 + y^2 + xz^2 + 3xy + yz + c$, where c is a constant.

$\phi(1,1,0) = 4 \implies 4 = 1+1+0+3+0+c \implies c = -1$

Hence, $\phi = x^2 + y^2 + xz^2 + 3xy + yz$ //.

We have, $\int_c \bar{F}\cdot d\bar{r} = \int_c (z^2+2x+3y)\,dx + (3x+2y+z)\,dy + (y+2zx)\,dz$

equation of line : $\dfrac{x-x_1}{x_2-x_1} = \dfrac{y-y_1}{y_2-y_1} = \dfrac{z-z_1}{z_2-z_1}$

$\therefore$ equation of line from $A(0,0,0)$ to $B(1,2,3)$, $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3} = t$

$\therefore\ x = t\ ,\quad y = 2t\ ,\quad z = 3t$

$dx = dt\ ,\quad dy = 2dt\ ,\quad dz = 3dt$

At $t=0$, $(x,y,z) = (0,0,0)$ & at $t=1$, $(x,y,z) = (1,2,3)$

$\therefore \int_0^1 (9t^2+2t+6t)\,dt + (3t+4t+3t)(2dt) + (2t+6t^2)(3dt)$

$= \int_0^1 9t^2 + 2t + 6t + 10t(2) + 6t + 18t^2\,dt$

$= \left[\dfrac{27t^3}{3} + \dfrac{34t^2}{2}\right]_0^1$

$= 26$ //

Q.2. Evaluate $\int 3xy\,dx - y^2\,dy$ along the parabola $y = 2x^2$ from $A(0,0)$ to $B(1,2)$. What is the value of this integral if the point is the straight line joing from A & B?

→ Equation of parabola : $y = 2x^2$

$\therefore\ dy = 4x\,dx$ and x varies from 0 to 1.

Putting $y=2x^2$ in main integral and $dy = 4x\,dx$, we get,

$\int_A^B 3xy\,dx - y^2\,dy = \int_0^1 3x(2x^2)\,dx - (2x^2)^2(4x\,dx)$

$= \int_0^1 (6x^3 - 16x^5)\,dx = \left[\dfrac{6x^4}{4} - \dfrac{16x^6}{6}\right]_0^1$

$\therefore \int_A^B 3xy\,dx - y^2\,dy = -\dfrac{7}{6}$ //

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considering straight line passes through (0,0) & (1,2), we have, eq^n of line, $\frac{x}{1} = \frac{y}{2} = t$

$\therefore \quad x = t \quad , \quad y = 2t$
$\quad dx = dt \quad , \quad dy = 2dt$

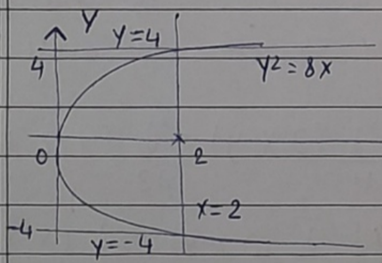
When $t=0$, $(x,y) = (0,0)$ & at $t=1$, $(x,y) = (1,2)$

$$\therefore \int_0^1 3t(2t)\,dt - (2t)^2(2dt) = \int_0^1 -2t^2\,dt$$

$$= \left[\cancel{+}\frac{2t^3}{3} \right]_1^0 = \frac{-2}{3}$$

$$\therefore \int_A^B 3xy\,dx - y^2 dy = \frac{-2}{3}$$

Q.3. verify greens theorem for $\oint_c (x^2-2xy)dx + (x^2y+3)dy$ where c is the boundary of the region defined by $x=2$ & $y^2 = 8x$.

→ 

$y^2 = 8x \quad , \quad x=2$
$\therefore \quad y^2 = 8(2) = 16 \implies y = \pm 4$

We have, $\bar{F}\cdot d\bar{r}$
$= (x^2-2xy)dx + (x^2y+3)dy$

By greens theorem,

$$\int_c (Pdx + Qdy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy$$

here, $P = x^2 - 2xy$ & $Q = x^2y + 3$

$\therefore \frac{\partial Q}{\partial x} = 2xy$ and $\frac{\partial P}{\partial y} = -2x$

$$\therefore \int_c (Pdx + Qdy) = \int_0^2 \int_{-\sqrt{8x}}^{\sqrt{8x}} (2xy + 2x)\,dy\,dx$$

$$= \int_0^2 \left[xy^2 + 2xy \right]_{-\sqrt{8x}}^{\sqrt{8x}} dx = \int_0^2 \left[4x^2\sqrt{8x} \right] dx$$

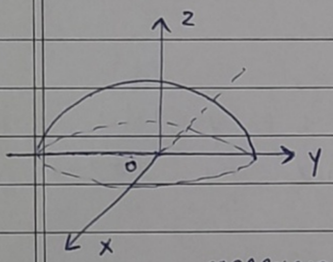
$$= 8\sqrt{2}\int_0^2 x^{3/2}\,dx$$

$$= 8\sqrt{2}\left[\frac{x^{5/2}}{5/2}\right]_0^2 = \frac{8\sqrt{2}\cdot 2^{5/2}}{5/2} = \frac{2^7}{5}$$

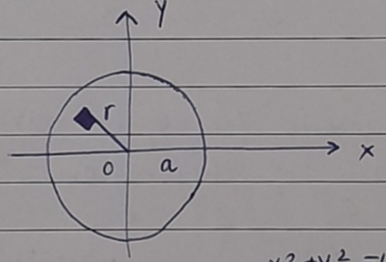
$$\therefore \int_c (P\,dx + Q\,dy) = \frac{2^7}{5}$$

Q.4. Evaluate $\int_c \bar{F}\cdot d\bar{r}$ by Stoke's theorem for $\bar{F} = (x-y-z)i + (y-z-x)j + (z-x-y)k$ over paraboloid $x^2+y^2 = 4-z$, $z \geq 0$.

$\rightarrow$ $\int_c \bar{F}\cdot d\bar{r} = \iint_S \bar{N}\cdot\nabla\times\bar{F}\,ds \longrightarrow$ Stokes theorem.



PARABOLOID



$x^2+y^2 = 4 = a^2$

the given surface, a paraboloid with its vertex at $(0,0,4)$ opens downwards & meets the xy plane, where $z=0$, in circle $x^2+y^2=4$.

Now, $\nabla\times\bar{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x-y-z & y-z-x & z-x-y \end{vmatrix} = i\left[\frac{\partial}{\partial y}(z-x-y) - \frac{\partial}{\partial z}(y-z-x)\right]$

$$-j\left[\frac{\partial}{\partial x}(z-x-y) - \frac{\partial}{\partial z}(x-y-z)\right]$$

$$+k\left[\frac{\partial}{\partial x}(y-z-x) - \frac{\partial}{\partial y}(x-y-z)\right]$$

$$= i[1-1] - j[-1+1] + k[-1+1]$$

$$= \bar{0}$$

$\therefore$ Since the given surface is in the xy plane, $\bar{N} = k$

$\therefore$ $\bar{N}\cdot(\nabla\times\bar{F}) = 0$

$$\therefore \int_c \bar{F}\cdot d\bar{r} = 0$$