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CASOT: Tutorial 11

Q.1. (i). z^2+z

$$f(z) = z^2+z$$
$$= (x+iy)^2 + (x+iy)$$
$$= x^2+2ixy - y^2 + x+iy$$

$$\therefore f(z) = (x^2+x+y^2) + i(2xy+y)$$

$u = x^2+x-y^2$ $\qquad$ $v = 2xy+y$

$\frac{\partial u}{\partial x} = 2x+1$ $\qquad$ $\frac{\partial v}{\partial x} = 2y$

$\frac{\partial u}{\partial y} = -2y$ $\qquad$ $\frac{\partial v}{\partial y} = 2x+1$

$$\therefore \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \& \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$\therefore$ C-R equations are satisfied.
$\therefore f(z)$ is analytic.

$$f(z) = z^2+z$$
$$f(z') = 2z+z. \quad //$$

(ii). $\frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\frac{y}{x}$

$$f(z) = u+iv$$

$$\therefore u = \frac{1}{2}\log(x^2+y^2), \quad v = \tan^{-1}\frac{y}{x}$$

$$u_x = \frac{1}{2}\cdot\frac{2x}{x^2+y^2} = \frac{x}{x^2+y^2}$$

$$u_y = \frac{1}{2}\cdot\frac{2y}{x^2+y^2} = \frac{y}{x^2+y^2}$$

$$V_x = \frac{1}{1+(y/x)^2} \cdot \left(\frac{-y}{x^2}\right) = \frac{x^2}{x^2+y^2}\left(\frac{-y}{x^2}\right) = \frac{-y}{x^2+y^2}$$

$$V_y = \frac{1}{1+(y/x)^2} \cdot \left(\frac{1}{x}\right) = \frac{x^2}{x^2+y^2}\left(\frac{1}{x}\right) = \frac{x}{x^2+y^2}$$

$$V_x = \frac{-y}{x^2+y^2} \quad \& \quad V_y = \frac{x}{x^2+y^2}$$

$u_x = v_y$ & $u_y = -v_x$

$\therefore$ CR equations are satisfied.

$f(z) \Rightarrow$ analytic

$$f(z) = \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\left(\frac{y}{z}\right).$$

$f'(z) = u_x - iu_y$ (by Milne Thomson theorem).

$$f'(z) = \frac{x}{x^2+y} - \frac{iy}{x^2+y^2}$$

Q.2. (i). $f(z) = \bar{z} = x - iy$

$u = x$, $v = -y$

$u_x = 1$, $v_x = 0$

$u_y = 0$, $v_y = -1$

$u_x \neq v_y$

$\therefore$ CR equation is not satisfied.

$\therefore$ $f(z)$ is not analytic.

$\therefore$ $f(z')$ does not exit.

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(ii). $f(z) = z|z|$
$= (x+iy)\sqrt{x^2+y^2}$

$\therefore \quad u = x\sqrt{x^2+y^2}$, $v = y\sqrt{x^2+y^2}$

$u_x = \sqrt{x^2+y^2} + \frac{x \cdot 1}{\cancel{2}\sqrt{x^2+y^2}} \cdot \cancel{2}x = \sqrt{x^2+y^2} + \frac{x^2}{\sqrt{x^2+y^2}}$

$v_x = \frac{xy}{x^2+y^2}$

$u_y = \frac{x2y}{2\sqrt{x^2+y^2}} = \frac{xy}{\sqrt{x^2+y^2}}$

$v_y = \sqrt{x^2+y^2} + \frac{y^2y}{2\sqrt{x^2+y^2}} = \sqrt{x^2+y^2} + \frac{y^2}{\sqrt{x^2+y^2}}$

$u_x \neq v_y$

$\therefore$ CR equation is not satified.
f(z) is not analytic.
f(z') does not exit.

Q3. Find constraints a, b, c, d, e, if,
$f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y)$
is analytic.

$u = ax^3 + bxy^2 + 3x^2 + cy^2 + x$
$v = dx^2y - 2y^3 + exy + y$

$\therefore \quad u_x = 3ax^2 + by^2 + 6x + 1$, $v_x = 2dxy + ey$.

$u_y = 2bxy + 2cy$, $v_y = dx^2 - 6y^2 + ex + 1$

$f(x)$ is analytic. ($\rightarrow$ given).

$\therefore$ $u_x = v_y$ $\qquad$ $u_y = -v_x$

$\therefore$ $3ax^2 + by^2 + 6x + 1 = dx^2 - 6y^2 + cx + 1$

$\therefore$ $e = 6$, $b = -6$, $d = 3a$

Also,

$2bxy + 2cy = -2dxy - ey$

$\therefore$ $c = \frac{-e}{2}$, $b = -d$

$c = -\frac{6}{2}$, $\cancel{b}$ $d = -(-6)$

$c = -3$, $d = 6$

$a = \frac{d}{3} = \frac{6}{3} = 2.$

Hence, $\{a, b, c, d, e\} = \{2, -6, -3, 6, 6\}$ //

Q4. $f(z) = u + iv$ is analytic.

CR equation stays that.

$u_x = v_y$ & $u_y = -v_x$

$$LHS = \begin{vmatrix} \partial u/\partial x & \partial u/\partial y \\ \partial v/\partial x & \partial v/\partial y \end{vmatrix} = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$$

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$= u_x v_y - v_x u_y$
$= v_x v_x - v_x(-v_x)$
$= v_x^2 + v_x^2$

$f'(z) = u_x - i v_x$

$|f'(z)| = \sqrt{(u_x)^2 + (-iv_x)^2}$

$= \sqrt{u_y^2 + i v_x^2} = v_x^2 + i v_x^2 = LHS.$

Hence proven, $\begin{vmatrix} \partial u/\partial x & \partial u/\partial y \\ \partial v/\partial x & \partial v/\partial y \end{vmatrix} = |f(z')|^2$.

Q.5. $u = x^3 - 3y^2x + 3x^2 - 3y^2 + 1$

$u_x = 3x^2 - 3y^2 + 6x$
$u_y = -6xy - 6y$

$f'(z) = u_x - i u_y$
$= (3x^2 - 3y^2 + 6x) + i(6xy + 6y)$

Putting $x = z, y = 0$

$f'(z) = (3z^2 + 6z) + i(0) = 3z^2 + 6z$

$\therefore f(z) = \int f'(z)\, dz$

$\therefore f(z) = z^3 + 3z^2 + c$ //

Q.6. $v = \frac{x}{x^2+y^2} + \cosh x \cos y$

$$v_x = \frac{(x^2+y^2) - x(2x)}{(x^2+y^2)^2} + \sinh x \cos y$$

$$= \frac{y^2 - x^2}{(x^2+y^2)^2} + \sinh x \cos y$$

$$v_y = \frac{-x}{(x^2+y^2)^2} \cdot (2y) - \cosh x \sin y$$

$$= -\left(\frac{2xy}{(x^2+y^2)^2} + \cosh x \sin y \right)$$

$$f'(z) = v_y + i v_x$$

put $x = z$, $y = 0$

$$f'(z) = -\left(\frac{2xy}{(x^2+y^2)^2} + \cosh x \sin y \right) + i \left(\frac{y^2+x^2}{(x^2+y^2)^2} + \sinh y \cos y \right)$$

$$f'(z) = i \left(\frac{-1}{z^2} + \sinh z \right)$$

$$f(z) = \int f'(z)\, dz$$

$$= i \int -\frac{1}{z^2} + \sinh z \; dz$$

$$= i \left(\frac{1}{z} + \cosh z \right) + C$$

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Q.7. Let $\phi = 3x^2 + \sin x + y^2 + 5y + 4$

$\frac{\partial \phi}{\partial x} = 6x + \cos x$, $\frac{\partial^2 \phi}{\partial x^2} = 6 - \cancel{\beta}\sin x$

$\frac{\partial \phi}{\partial y} = 2y + 5$, $\frac{\partial^2 \phi}{\partial y^2} = 2$

$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 6 - \sin x - 2 = 4 - \sin x \neq 0$

$\therefore$ This function is <u>not</u> harmonic.

$\therefore$ It is true that there does not exit an analytic function whose real part is ϕ.

Q8. (i). $v = e^{-x}(x\cos y + y\sin y)$

$\frac{\partial v}{\partial x} = e^{-x}(x\cos y + y\sin y) + e^{-x}(\cos y)$

$\frac{\partial^2 v}{\partial x^2} = -e^{-x}(x\cos y + y\sin y) - e^{-x}(\cos y) - e^{-x}(\cos y)$

$= e^{-x}(x\cos y + y\sin y) - 2e^{-x}\cos y$
$= e^{-x}(x\cos y + y\sin y - 2\cos y)$
$= e^{-x}((x-2)\cos y + y\sin y)$

$\frac{\partial u}{\partial y} = e^{-x}(-x\sin y + y\cos y + \sin y)$

$\frac{\partial^2 u}{\partial y^2} = e^{-x}(-x\cos y + \cos y - y\sin y + \cos y)$

$= e^{-x}((-x+2)\cos y - y\sin y)$

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = e^{-x}(\cancel{(x-2)\cos y} + \cancel{y\sin y}) + e^{-x}(\cancel{(2-x)\cos y} - \cancel{y\sin y})$$

$$= 0$$

$\therefore$ V is harmonic.

Using milne thomson method,

$$f'(z) = u_x - iv_y$$
$$= e^{-x}\left[(-x+1)\sin y + y\cos y\right] + i\left[(-e^{-x})(x\cos y + y\sin y) + e^{-x}\cos y\right].$$

put $x = z$ & $y = 0$.

$$f'(z) = i\left[(-e^{-z}z) + e^{-z}\right]$$
$$= i\,e^{-z}\left[-z+1\right] = e^{-z}(-z+1)\,i$$
$$= i(1-z)e^{-z}$$

$$f(z) = \int f(z)\,dx$$
$$= \int (1-z)e^{-z}\,dz = (1-z)\frac{e^{-z}}{-1} - (-1)e^{-z} + c$$

$$f(z) = e^{-z}(z) + c$$
$$f(z) = e^{-(x+iy)}(x+iy) + c \;//$$

$$= e^{-x}e^{iy}(x+iy)$$
$$= e^{-x}(\cos y - i\sin y)(x+iy)$$
$$= e^{-x}\left[x\cos y + iy\cos y - ix\sin y + y\sin y\right]$$

$$f(z) = e^{-x}\left[x\cos y + y\sin y\right] + ie^{-x}\left[y\cos y - x\sin y\right]$$

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Q.8. (i). $\therefore$ Harmonic conjugate.

$$v = e^{-x}[y\cos y - x\sin y]$$

(ii). $u = (x-1)^3 - 3xy^2 + 3y^2$

$$\frac{\partial u}{\partial x} = 3(x-1)^2 - 3y^2 \qquad \frac{\partial u}{\partial y} = 6xy + 6y$$

$$\frac{\partial^2 u}{\partial x^2} = 6(x-1) \qquad \frac{\partial^2 u}{\partial y^2} = -6x + 6$$

$$= 6x - 6$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x - 6 + (-6x) + 6 = 0$$

$\therefore$ u is harmonic.

using milne thomson method,

$$f'(z) = u_x - iu_y$$
$$= 3(x-1)^2 - 3y^2 - i(-6xy + 6y)$$

putting $x = z$ & $y = 0$

$$\therefore f'(z) = 3(z-1)^2 - i(0)$$
$$= 3(z^2 - 2z + 1)$$
$$= \cancel{3f}\ 3z^2 - 6z + 3.$$

$$\therefore f(z) = \int f'(z)\,dz$$

$$= \int (3z^2 - 6z + 3)\,dz$$

$$= \frac{\cancel{3}z^3}{\cancel{3}} - \frac{\cancel{6}^{3}z^2}{\cancel{2}_{1}} + 3z + C$$

Q.8. (ii). $= (x+iy)^3 - 3(x+iy)^2 + 3x + 3iy + c$

$= x^3 + (iy)^2 + 3x^2(iy) + 3x(iy)^2 - 3x^2 - 3(iy)^2 - 6ixy + 3x + 3iy + c$

$= x^2 - iy^3 + 3x^2y - 3xy^2 - 3x^2 + 3y^2 - 6ixy + 3x + 3iy + c$

$= (x^3 - 3xy^2 - 3x^2 + 3y^2 + 3x) + i(-y + 3x^2y - 6xy + 3y)$

$= (x^3 - 3x^2(1) + 3x(1)^2 - 1) + 1 - 3xy^2 + 3y^2 + i(3x^2y - y^3 - 6xy + 3iy) + c$

$= ((x-1)^3 - 3xy^2 + 3y^2) + i(3x^2y - y^3 - 6xy + 3y)$

$\therefore$ Harmonic conjugate, $v = 3x^2y - 6xy + 3y - y^3$

Q.9. Orthogonal trajectories, $v = c$ are given by $v = c_2$ where v is harmonic conjugate of u.

$u = 3x^2y - y^3$

$u_x = 6x$, $u_y = 3x^2 - 3y^2$

$f'(z) = u_x - iv_y$

$= 6xy - i(3x^2 - 3y^2)$

Putting $x = z$ & $y = 0$.

$f'(z) = -i(3z^2)$

$\therefore f(z) = \int f'(z)\,dz = \int -i(3z^2)\,dz = -iz^3 + c.$

$f(z) = -iz^3 + c$

$= -i(x+iy)^3 + c$

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Q. 9. $= -i\left[x^3 - iy^3 + i3x^2y - 3xy^2\right] + C$

$= -ix^3 + (-1)y^3 - (-1)\,3x^2y + i3xy^2 + C$

$= 3x^2y - y(3) + i(3xy^2 - x^3) + C.$ //

$\therefore$ Orthogonal trajectories are $3xy^2 - x^3 = C_2$. //