

## Tutorial-12: Complex Integration

### TYPE-I LINE INTEGRAL

1. Evaluate  $\int_0^{3+i} z^2 dz$ 
  - (i) along the real axis from 0 to 3 and then vertically to  $3 + i$ .
  - (ii) along the imaginary axis from 0 to  $i$  and then horizontally to  $3 + i$
  - (iii) along the parabola  $x = 3y^2$Is the line integral independent of the path? Explain?
2. Show that  $\int_C \log z dz = 2\pi i$ , where  $C$  is the unit circle in the  $z$  – plane.
3. Evaluate  $\int_C (z^2 + 3z^{-4})dz$ , where  $C$  is upper half of the unit circle from  $(1,0)$  to  $(-1,0)$
4. Evaluate  $\int f(z)dz$  along the square whose vertices are  $(1,1), (2,1), (2,2), (1,2)$  in anti – clockwise direction where  $f(z) = x - 2iy$

### TYPE-II CAUCHY'S INTEGRAL THEOREM, CAUCHY'S INTEGRAL FORMULA

5. Evaluate  $\int_C \frac{1}{z} \cdot \cos z dz$  where  $C$  is the ellipse  $9x^2 + 4y^2 = 1$
6. Evaluate  $\int_C \frac{e^{3z}}{z-i} dz$  where  $C$  is the curve  $|z - 2| + |z + 2| = 6$
7. Evaluate  $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ , where  $C$  is the circle  $|z| = 3$
8. If  $f(z) = z^3 + iz^2 - 4z - 4i$  evaluate  $\int_C \frac{f'(z)}{f(z)} dz$   
where  $C$  is a simple closed curve enclosing zeros of  $f(z)$
9. Evaluate  $\int_C \frac{\sin^6 z}{(z-\pi/6)^3} dz$  where  $C$  is  $|z| = 1$
10. Evaluate  $\int_C \frac{z-1}{(z+1)^2(z-2)} dz$  where  $C$  is  $|z - i| = 2$
11. Evaluate  $\int_C \frac{ze^{2z}}{(z-1)^3} dz$  where  $C$  is  $|z + i| = 2$
12. If  $f(\zeta) = \int_C \frac{4z^2 + z + 4}{z - \zeta} dz$  where  $C$  is the ellipse  $4x^2 + 9y^2 = 36$  find the values of
  - (i)  $f(4)$  (ii)  $f(1)$  (iii)  $f(i)$  (iv)  $f'(-1)$  (v)  $f''(-i)$