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SOMAIYA
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Experiment / assignment / tutorial No. 10

Grade: Signature of the Faculty with date

Q1. Find z transform of $3^k + 5^k$, $k \geq 0$.

Z transformed is defined as,
$X(z) = \sum_{k=0}^{\infty} x(k) z^{-k}$

Z transform of 3^k, $X_1(z) = \sum_{k=0}^{\infty} 3^k z^{-k}$.

Using formula for sum of geometric series, $X_1(z) = \sum_{k=0}^{\infty} (3z^{-1})^k$

$= \frac{1}{1-3/z}$

Similarly, z transform of 5^k is $\frac{1}{1-5/z}$

Now using linearity property,
$X(z) = X_1(z) + X_2(z) = \frac{1}{1-3/z} + \frac{1}{1-5/z}$

$= \frac{1-5/z + 1-3/z}{1+15/z^2 - 8/z} = \frac{2-8/z}{15/z^2 - 8/z + 1}$

(multiplying by z^2) $= \frac{2z^2 - 8z}{15 - 8z + z^2}$

$= \frac{2z^2 - 8z}{(z-3)(z-5)}$ //.

Q.2. Find z transform of $e^{-3k} \sin 2k$.

We know, $z\{\sin bk\} = \frac{z \sin b}{z^2 - 2z\cos b + 1}$

$\therefore z\{\sin 2k\} = \frac{z \sin 2}{z^2 - 2z\cos 2 + 1}$

$\therefore z\{e^{-3k} \sin 2k\} = \frac{(e^3 z) \sin(2)}{(e^3 z)^2 - 2(e^3 z)\cos(2) + 1}$

$= \frac{e^{-3} z \sin(2)}{z^2 - 2e^{-3} \cdot z\cos(2) + e^{-6}}$ //

Q.3. Find z transform of $k^2 a^{k-1}, k \geq 0$.

We know that $Z\{f(k-n)\} = z^{-n} \cdot F(n)$

$\therefore Z\{a^{k-1}\} = z^{-1}F(z)$ where $F(z) = Z(a^k) = \frac{z}{z-a}$

$\therefore Z\{a^{k-1}\} = z^{-1} \cdot \frac{z}{z-a} = \frac{1}{z-a}$

$\therefore Z\{k \cdot a^{k-1}\} = -z \frac{d}{dz} \cdot \left(\frac{1}{z-a}\right)$

$= -z \cdot \frac{(-1)}{(z-a)^2} = \frac{z}{(z-a)^2}$

$\therefore Z\{k^2 \cdot a^{k-1}\} = Z\{k \cdot (k a^{k-1})\}$

$= -z \frac{d}{dz}\left[\frac{z}{(z-a)^2}\right] = -z\left[\frac{(z-a)^2 \cdot 1 - z \cdot 2(z-a) \cdot 1}{(z-a)^4}\right]$

$= -z\left[\frac{(z-a) - 2z}{(z-a)^3}\right]$

$= \frac{z(z+a)}{(z-a)^3}, \quad |z| > |a|$ //

Q.4. Find inverse z transform of $F(z) = \frac{1}{z-2}$ when

(i). $|z| < 2$ (ii). $|z| > 2$.

(i). When $|z| < 2$,
if $|z| < |2|$ i.e $|z/2| < 1$, we take '2' outside,

$F(z) = Z\{f(k)\} = \frac{1}{z-2} = \frac{1}{2[z/2 - 1]} = \frac{-1}{2} \cdot \frac{1}{1-(z/2)}$

$= \frac{-1}{2}\left(1 - \frac{z}{2}\right)^{-1}$

$F(z) = -\frac{1}{2}\left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \dots + \frac{z^k}{2^k} + \dots\right]$

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$$= -\left[\frac{1}{\cancel{\alpha}2} + \frac{z}{\cancel{\alpha}^2 2} + \frac{z^3}{\cancel{\alpha}^3 2} + \dots + \frac{z^k}{\cancel{\alpha}^{k+1} 2}\right]$$

$$= -\left[2^{-1} + 2^{-2}z + 2^{-3}z^2 + \dots + 2^{-k-1}z^k + \dots\right]$$

$\therefore$ Coefficient of $z^k = -\cancel{\alpha}\, 2^{-k-1}$, $k \geq 0$

$\therefore$ Coefficient of $z^{-k} = -2^{k-1}$, $k \leq 0$

$\therefore Z^{-1}[F(z)] = \{f(k)\} = \{-2^{k-1}\}$, $k \leq 0$

(ii). When $|z| > |2|$,
if $|z| > |2|$, $\left|\frac{z}{2}\right| > 1$ i.e $\left|\frac{2}{z}\right| < 1$, we take larger term '$z$' outside,

$$F(z) = Z[f(k)] = \frac{1}{z-2} = \frac{1}{z[1-2/z]} = \frac{1}{z}\left(1-\frac{2}{z}\right)^{-1}$$

$$= \frac{1}{z}\left(1 + \frac{2}{z} + \frac{2^2}{z^2} + \dots + \frac{2^{k-1}}{z^{k-1}} + \dots\right)$$

$$= \frac{1}{z} + \frac{\cancel{\alpha}2}{z^2} + \frac{2^2}{z^3} + \dots + \frac{2^{k-1}}{z^k} \dots$$

$\therefore$ Coefficients of $z^{-k} = 2^{k-1}$, $k \geq 1$

$\therefore Z^{-1}[F(z)] = \{f(k)\} = \{2^{k-1}\}$, $k \geq 1$

Q.5. Find inverse z transform of $F(z) = \frac{z+2}{z^2-2z+1}$, $|z| > 1$.

We have, $F(z) = \frac{z+2}{z^2-2z+1} = \frac{z+2}{(z-1)^2} = \frac{3}{(z-1)^2} + \frac{1}{z-1}$

Since, $|z| > 1$, $\frac{1}{|z|} < 1$, We take z out.

$$\therefore F(z) = \frac{3}{z^2[1-(1/z)]^2} + \frac{1}{z[1-(1/z)]} = \frac{3}{z^2}\left(1-\frac{1}{z}\right)^{-2} + \frac{1}{z}\left(1-\frac{1}{z}\right)^{-1}$$

$$= \frac{3}{z}\left(1 - (-2)\cdot\frac{1}{z} + \frac{(-2)(-3)}{2!}\cdot\frac{1}{z^2} - \frac{(-2)(-3)(-4)}{3!}\cdot\frac{1}{z^3} + \cdots\right) + \frac{1}{z}\left(1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots\right)$$

$$= \frac{3}{z^2}\left(1 + \frac{2}{z} + \frac{3}{z^2} + \frac{4}{z^3} + \cdots + \frac{k-1}{z^{k-2}}\right) + \frac{1}{z}\left(1 + \frac{1}{z} + \frac{1}{z^2} + \cdots + \frac{1}{z^{k-1}}\right)$$

$$= 3\left(\frac{1}{z^2} + \frac{2}{z^3} + \cdots + \frac{k-1}{z^k}\right) + \left(\frac{1}{z} + \frac{1}{z^2} + \cdots + \frac{1}{z^k}\right)$$

$$= \frac{1}{z} + \frac{3+1}{z^2} + \cdots + \frac{3k-3+1}{z^k} + \cdots$$

$$\therefore F(z) = \frac{1}{z} + \frac{4}{z^2} + \frac{7}{z^3} + \cdots + \frac{3k-2}{z^k}$$

$\therefore$ coefficients of $z^{-k} = 3k-2,\ k \geq 1$

$$\therefore z^{-1}[F(z)] = \{3k-2\},\ k \geq 1$$

Q.6. Find inverse z transform of $F(z) = \frac{z^2}{(z-3)(z-2)}$ using convolution.

By convolution theorem $z^{-1}\{F(z) \times G(z)\} = z^{-1}\{F(z)\} z^{-1}\{G(z)\}$.

$$z^{-1}\left\{\frac{z^2}{(z-2)(z-3)}\right\} = z^{-1}\left\{\frac{z}{(z-3)} \cdot \frac{z}{(z-2)}\right\} = z^{-1}\left\{\frac{z}{z-3}\right\} z^{-1}\left\{\frac{z}{z-2}\right\}$$

$$= (3)^n \times (2)^n$$

$$= \sum_{k=0}^{n} f(k)\, g(n-k) = \sum_{k=0}^{n} (3)^n \times (2)^{n-k} = (2)^n \sum_{k=0}^{n} \left[\frac{3}{2}\right]^k$$

$$= (2)^n\left[1 + \left(\frac{3}{2}\right)^1 + \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^3 + \cdots + \left(\frac{3}{2}\right)^n\right]$$

$$= (2)^n\left[\frac{1-(3/2)^{n+1}}{1-3/2}\right] \quad \left(\text{since } a + ar + ar^2 \ldots = \frac{1-r^{n+1}}{1-r}\right)$$

$$= (2)^n \cdot \frac{\cancel{2}^{1/3}}{\cancel{2}\cancel{3}}\left[1 - \left(\frac{\cancel{3}^{1/3}}{\cancel{2}}\right)^{n+1}\right] = \frac{1}{3}\left[(\cancel{3}^{2})^{n+1} - 1\right]$$

$$\therefore z^{-1}\left\{\frac{z^2}{(z-3)(z-2)}\right\} = \frac{1}{3}\left[2^{n+1} - 1\right]$$