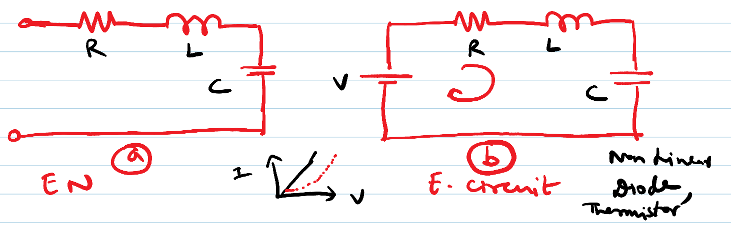
Network & Circuit :-

Linear Element
Eg. RLC



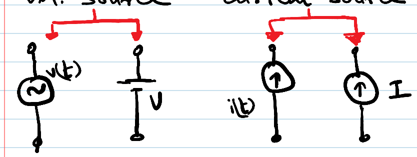
Non Linear
Diode,
Thermistor

Sources

Independant sources

Independant vol. source

Independant current source



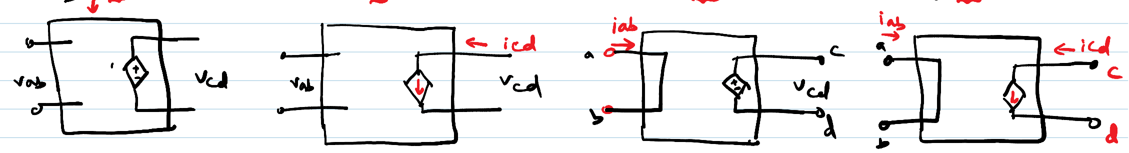
Dependant Sources

vol. controlled vol. source — VCVS

VCCS

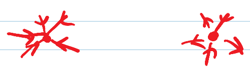
Current controlled vol. source — CCVS

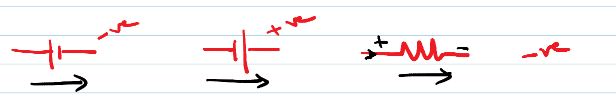
CCCS



$V_{cd} = \mu V_{ab}$ $\quad$ $i_{cd} = g_m V_{ab}$ $\quad$ $V_{cd} = r\, i_{ab}$ $\quad$ $i_{cd} = \beta i_{ab}$

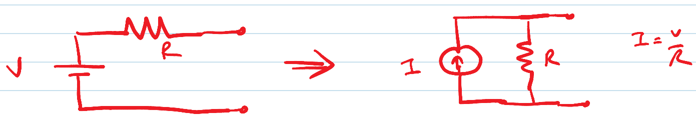
- KCL
- KVL





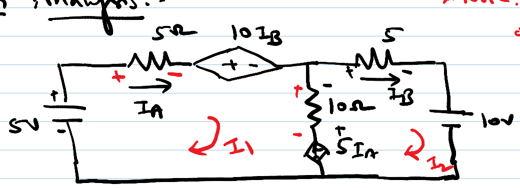
fall in voltage $\quad$ rise in voltage $\quad$ fall in voltage

- Source transformations



$I = \frac{V}{R}$

Mesh Analysis :-



*Note:- more no. of vol. sources → mesh.

→ $I_A = I_1$

$I_B = I_2$

$5 - 5I_1 - 10I_B - 10(I_1 - I_2) - 5I_A = 0 \quad (1)$

$5I_A - 10(I_2 - I_1) - 5I_2 - 10 = 0 \quad (2)$

Rewriting eqn. (1)

$5 - 5I_1 - 10\cancel{I_2} - 10I_1 + 10\cancel{I_2} - 5I_1 = 0$

$-20I_1 = -5 \quad \therefore \boxed{I_1 = \frac{1}{4} = 0.25\text{A}}$ I_A

Sub I_1 in eqn (2)

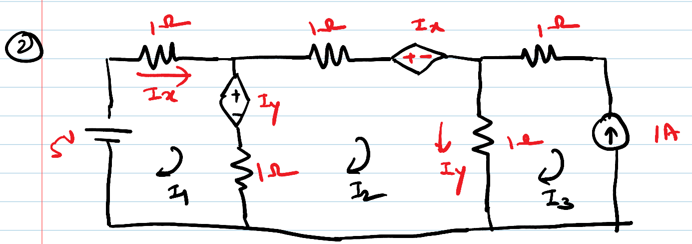
$5I_1 - 10I_2 + 10I_1 - 5I_2 = 10$

$15I_1 - 15I_2 = 10$

$15(0.25) - 15I_2 = 10$

$\boxed{I_2 = -0.416 \text{ A}}$ I_B

(2)



⇒ $I_3 = -1\text{A}$ (figure)

$I_x = I_1$

$I_y = I_2 - I_3 = I_2 + 1\text{A}$ *

Applying KVL to mesh I. & mesh II

$5 - I_1 - \underline{I_y} - (I_1 - I_2) = 0 \quad$ — (1)

$-(I_2 - I_1) + \underline{I_y} - I_2 - I_x - (I_2 - I_3) = 0 \quad$ (2)

$5 - I_1 - (I_2 + 1) - (I_1 - I_2) = 0$

$-I_1 - \cancel{I_2} - I_1 + \cancel{I_2} = -5 + 1$

$-2I_1 = -4$

$\boxed{I_1 = 2 \text{ A}}$

Sub $I_1 = 2\text{A}$ & I_y in eqn (2)

$\boxed{I_2 = 0}$

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02/06/21

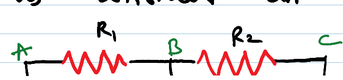
Supermesh Analysis :-

- We know in mesh analysis, analysis consists of writing mesh equation by KVL in terms of unknown mesh currents.

What is supermesh?
How to identify supermesh?

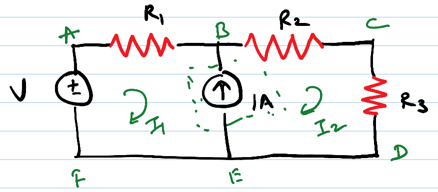
Let us consider an example

STL



Supermesh current

Let us consider an example



STL

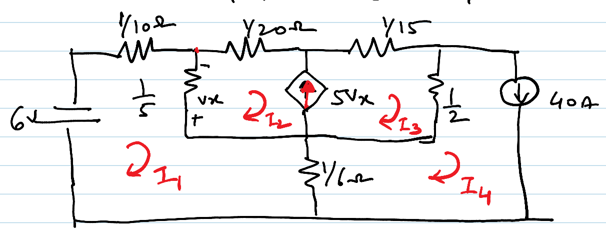
① Supermesh current
$I_2 - I_1 = 1A$

② ABCDEFA → KVL for supermesh

③ If any other loops write KVL eqn

* mesh that shares a current source with other meshes (no other current source in outer loop of mesh) → supermesh

* find currents I_1, I_2, I_3 & I_4 in the network.



⇒ I_2 & I_3 form a Supermesh

$I_3 - I_2 = 5V_x$ → ①

$V_x = \frac{1}{5}(I_2 - I_1)$ or $-V_x = \frac{1}{5}(I_1 - I_2)$

$I_3 - I_2 = \frac{1 \cdot \cancel{5}}{\cancel{5}}(I_2 - I_1)$

$I_3 = 2I_2 - I_1$ → ②

$I_4 = 40\,A$ → ③

KVL to Supermesh

$$-\frac{1}{5}(I_2 - I_1) - \frac{1}{20}I_2 - \frac{1}{15}I_3 - \frac{1}{2}(I_3 - I_4) = 0 \quad ④$$

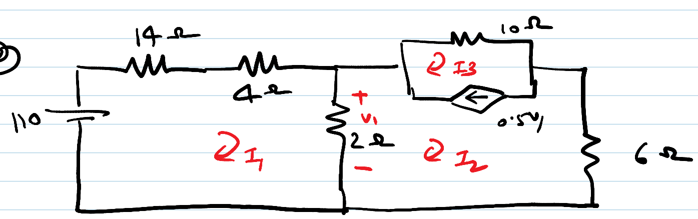
Applying KVL to mesh 1,

$$-6 - \frac{1}{10}I_1 - \frac{1}{5}(I_1 - I_2) - \frac{1}{6}(I_1 - I_4) = 0 \quad ⑤$$

Solve equations 2, 3, 4, & 5,

$$\boxed{\begin{aligned} I_1 &= 10A \\ I_2 &= 20A \\ I_3 &= 30A \\ I_4 &= 40A \end{aligned}}$$

②



⇒ Mesh 2 & 3 forming a supermesh

Current equation for supermesh

$I_3 - I_2 = 0.5V_1$ — (1)

$V_1 = 2(I_1 - I_2)$

$I_3 - \cancel{I_2} = 0.5 \times 2(I_1 - \cancel{I_2})$

$I_3 = I_1$ — (2)

KVL eqn for supermesh

$-2(I_2 - I_1) - 10I_3 - 6I_2 = 0$ → (3)

$-2I_2 + 2I_1 - 10I_1 - 6I_2 = 0$

$-8I_1 - 8I_2 = 0$

$I_1 = -I_2$ — (4)

KVL for mesh I

$110 - 14I_1 - 4I_1 - 2(I_1 - I_2) = 0$

$110 - 20I_1 + 2I_2 = 0$

$110 + 20I_2 + 2I_2 = 0$

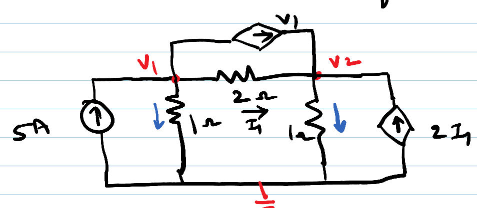
$I_2 = -110/22 = \underline{\underline{-5A}}$

$I_1 = +5A$
$I_2 = -5$
$I_3 = 5A$.

Node Analysis:- (KCL)

- Assign node, then one node → ref.node
- Apply KCL at each node (except ")
- Simultaneous eqn.

(3)



$V_1 = ?$
$V_2 = ?$

→ Applying KCL at node 1 (Assuming unknown branch currents are moving away from node)

$5 = \frac{V_1}{1} + \frac{V_1 - V_2}{2} + V_1$ → (1)

$5 = V_1 + 0.5V_1 - 0.5V_2 + V_1$.

$2.5V_1 - 0.5V_2 = 5$. → (2)

$V_2 = \frac{V_1 - V_2}{2} + V_1 + 2I_1$ → (3)

$I_1 = \frac{V_1 - V_2}{2}$

Sub I_1 in eqn (3)

$V_2 = \frac{V_1 - \cancel{V_2}}{2} + V_1 + V_1 - \cancel{V_2}$

$v_2 = v_1$

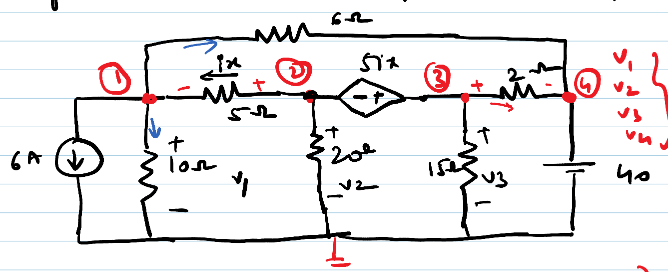
Sub $v_2 = v_1$ in eqn ②

$$\boxed{\begin{aligned} v_1 &= 2.5V \\ v_2 &= 2.5V \end{aligned}}$$

Supernode Analysis :-

Nodes are connected to each other by voltage sources (not to ref. node) → supernode

④



v_1, v_2, v_3, v_4 ?

$\Rightarrow$ $V_4 = 40V \rightarrow ①$

Node 2 & 3 form a supernode

$V_3 - V_2 = 5ix$ ②

As $ix = \frac{v_2 - v_1}{5} \rightarrow ③$

$v_3 - v_2 = v_2 - v_1$

$v_3 = 2v_2 - v_1$ — ④

Apply KCL for the supernode (2 & 3)

$$\frac{v_2 - v_1}{5} + \frac{v_2}{20} + \frac{v_3}{15} + \frac{v_3 - 40}{2} = 0$$

$$\frac{v_2}{2} - \frac{v_1}{5} + \frac{v_2}{20} + \frac{2v_2}{15} - \frac{v_1}{15} + \frac{\cancel{2}v_2}{\cancel{2}} - \frac{v_1}{2} - 20 = 0$$

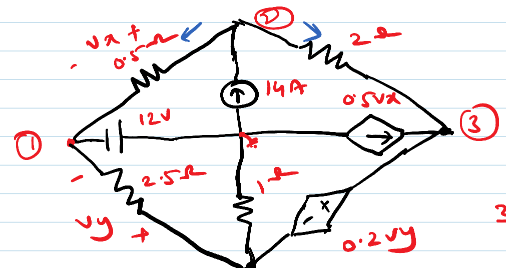
$$-\frac{23}{30} v_1 + \frac{83}{60} v_2 = 20 \quad ⑤$$

Applying KCL at node 1

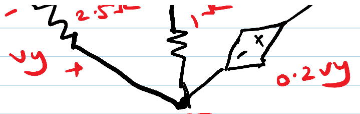
$$6 + \frac{v_1}{10} + \frac{v_1 - 40}{6} + \frac{v_1 - v_2}{5} = 0 \quad ⑤$$

$$\boxed{\begin{aligned} v_1 &= 10V \\ v_2 &= 20V \\ v_3 &= 2v_2 - v_1 = 30V \\ v_4 &= 40V. \end{aligned}}$$

④



3 & 4 supernode



3 4 4 Supernode

$\Rightarrow$ $V_1 = -12V$ (1)

KCL at node 2

$$\frac{V_2 - V_1}{0.5} + \frac{V_2 - V_3}{2} = 14$$

$$-2V_1 + 2.5V_2 - 0.5V_3 = 14 \quad (2)$$

Node 3 & 4 form Supernode

$$V_3 - V_4 = 0.2V_y = 0.2(V_4 - V_1)$$

$$0.2V_1 + V_3 - 1.2V_4 = 0 \rightarrow (3)$$

KCL to the supernode (3 & 4)

$$\frac{V_3 - V_2}{2} - 0.5V_x + \frac{V_4}{1} + \frac{V_4 - V_1}{2.5} = 0$$

$$\frac{V_3 - V_2}{2} - 0.5(V_2 - V_1) + V_4 + \frac{V_4 - V_1}{2.5} = 0$$

$$0.1V_1 - V_2 + 0.5V_3 + 1.4V_4 = 0 \rightarrow (4)$$

$$\boxed{\begin{aligned} V_1 &= -12V \\ V_2 &= -4V \\ V_3 &= 0 \\ V_4 &= -2V \end{aligned}}$$

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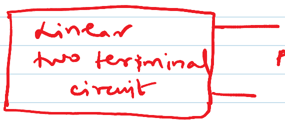
03/06/21

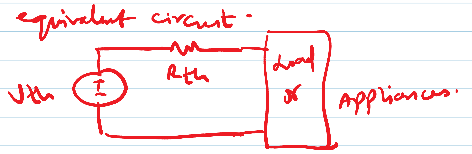
Theorems:-

1) Superposition theorem < indep. / dep. — Vol. source → S.C, I. source
2) Thevenin's theorem
3) Norton's theorem
4) Maximum power transfer theorem
5) Millman's theorem
6) Tellegan's theorem
7) Reciprocity theorem



Appliances :- variable load

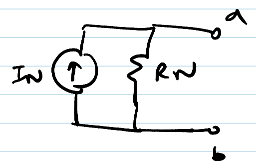
fixed part
↓
equivalent circuit.



$V_{th} = V_{oc}$

$R_{th} =$ equi resistance

Norton theorem:-



$I_N = I_{SC}$

$R_N =$ equi. Resi

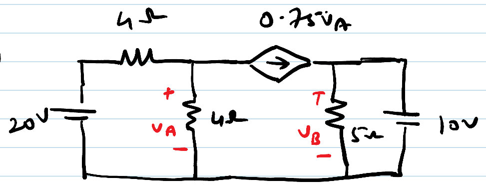
$\boxed{R_N = R_{th}}$

V_{th}, I_N, & R_{th}

find

1) $V_{OC} = V_{th}$
2) $I_{SC} = I_N$
3) $R_{th} = R_N = \dfrac{V_{OC}}{I_{SC}} = \dfrac{V_{th}}{I_N}$

(5)

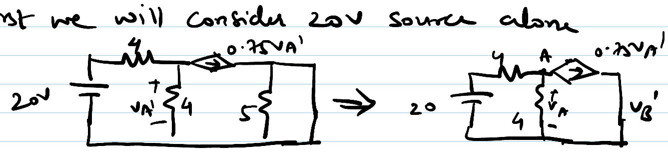


$V_A = ?$

$V_B = ?$

SPT

⇒ First we will consider 20V source alone



$V_B' = 0$

$V_A' = ?$

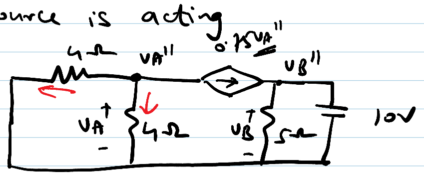
Apply KCL at node A

$$\frac{V_A' - 20}{4} + \frac{V_A'}{4} + 0.75V_A' = 0$$

$$1.25V_A' = 5$$

$$\boxed{\begin{matrix} V_A' = 4V \\ V_B' = 0V \end{matrix}}$$

10V source is acting



$V_B'' = 10V$

Applying KCL at node A

$$\frac{V_A''}{4} + \frac{V_A''}{4} + 0.75V_A'' = 0$$

$$\boxed{\begin{aligned} V_A'' &= 0 \\ V_B'' &= 10V \end{aligned}}$$

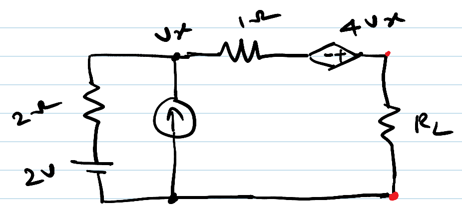
By SPT

$V_A = V_A' + V_A'' = 4V + 0 = 4V$

$V_B = V_B' + V_B'' = 0 + 10V = 10V$

$$\boxed{\begin{aligned} V_A &= 4V \\ V_B &= 10V \end{aligned}}$$

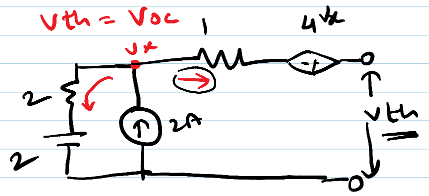
⑥



obtain thevenin's equivalent N/w

⇒ Find Vth & Rth

Vth = Voc



I = 0

Applying KCL at node V_x

$$\frac{V_x - 2}{2} - 2 = 0$$

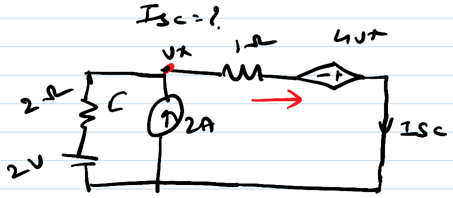
$V_x = 6V$

$V_{th} = V_x + 4V_x$

$V_{th} = \underline{30V}$

$R_{th} = ?$

$R_{th} = \dfrac{V_{th}}{I_{sc} = ?}$



$I_{sc} = \dfrac{V_x + 4V_x}{1} = \dfrac{5V_x}{1}$

Apply KCL at node V_x

$$\frac{V_x - 2}{2} + \frac{V_x + 4V_x}{1} = 2$$

$$\frac{V_x}{2} + 5x = 3$$

$5.5V_x = 3$

$V_x = 3/5.5 = 0.545V$

$$I_{sc} = \frac{5V_t}{1} = 5 \times 0.545 = \underline{2.73\,A}$$

$$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{30}{2.73} = 10.98\,\Omega$$

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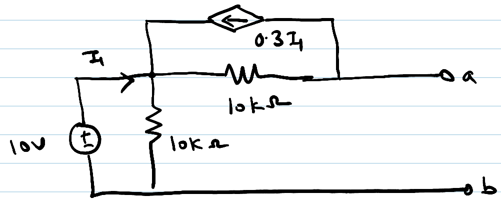
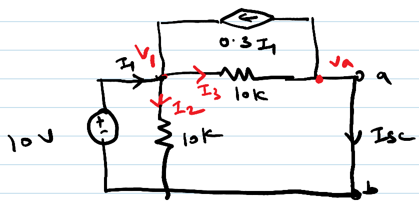
⑦ Find Norton equivalent circuit at terminals a & b



$I_N = ?$

$R_N = ?$

⇒ Short terminals a-b. (I_N)



From the figure $V_1 = \underline{10V}$, $\underline{Va = 0V}$

KCL at node V_1

$$I_1 - I_2 - I_3 + 0.3 I_1 = 0$$

$$I_1 - \frac{V_1}{10k} - \frac{(V_1 - V_a)}{10k} + 0.3 I_1 = 0$$

Sub V_1 & V_a

$$I_1 - \frac{10}{10k} - \frac{10}{10k} + 0.3 I_1 = 0$$

$$1.3 I_1 = \frac{20}{10k}$$

$$\boxed{I_1 = 1.538\,mA}$$

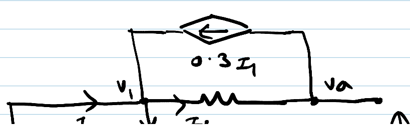
KCL at node a,

$$I_{sc} = I_3 - 0.3 I_1 \qquad \because I_3 = \frac{V_1}{10k} = \underline{1mA}$$

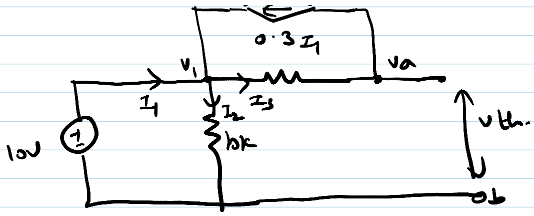
$$I_{sc} = 1mA - 1.538 \times 0.3\,mA.$$

$$\boxed{I_{sc} = 0.538\,mA}$$

$$R_N = R_{th} = \frac{V_{th}}{I_N} = \frac{V_{th}?}{I_{sc}} \checkmark$$



$I_3 = 0.3 I_1$



$I_3 = 0.3 I_1$
$V_1 = 10V$

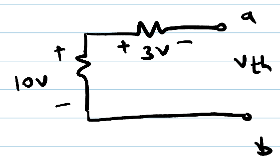
KCL at node 1

$$I_1 + 0.3 I_1 - I_2 - I_3 = 0$$

$$I_2 = \frac{V1}{10k} = 1mA$$

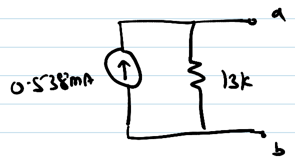
$$I_1 = 1mA$$

$$I_3 = 0.3 I_1 = 0.3mA.$$

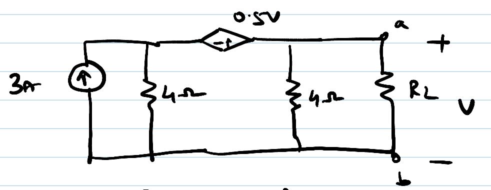


$$\boxed{Vth = 10 - 3 = 7V}$$

$$\boxed{R_N = \frac{Vth}{I_N} = \frac{7}{0.538} = 13k.}$$



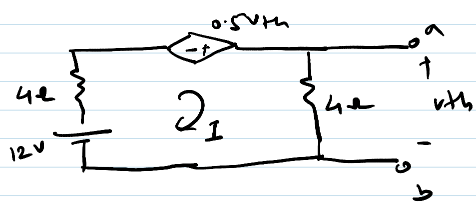
8



$R_L = ?$
max power delivered

⇒ Vth = ? Rth = ?

open the terminals a-b



Vth = 4I

KVL to the mesh

$$12 - 4I + 0.5Vth - 4I = 0$$
$$12 - Vth + 0.5Vth - Vth = 0$$

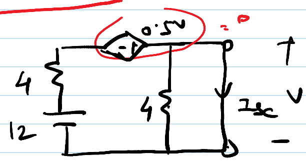
$$12 - 1.5Vth = 0$$

$$\boxed{Vth = \frac{12}{1.5} = 8V}$$

$12 - 1.5V_{th} = 0$

$$\boxed{V_{th} = \frac{12}{1.5} = 8V}$$

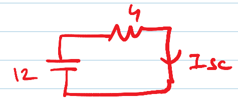
$R_{th} = ?$ $\quad \frac{V_{th} ✓}{I_{sc} ?}$



4Ω = shorted

$V = 0$

$0.5V = 0$

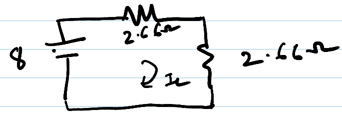


$$\boxed{I_{sc} = \frac{12}{4} = 3A}$$

$$\boxed{R_{th} = \frac{V_{th}}{I_s} = \frac{8V}{3A} = 2.66\Omega}$$

Maximum power delivered to the load,

$R_L = R_{th} = 2.66\Omega$



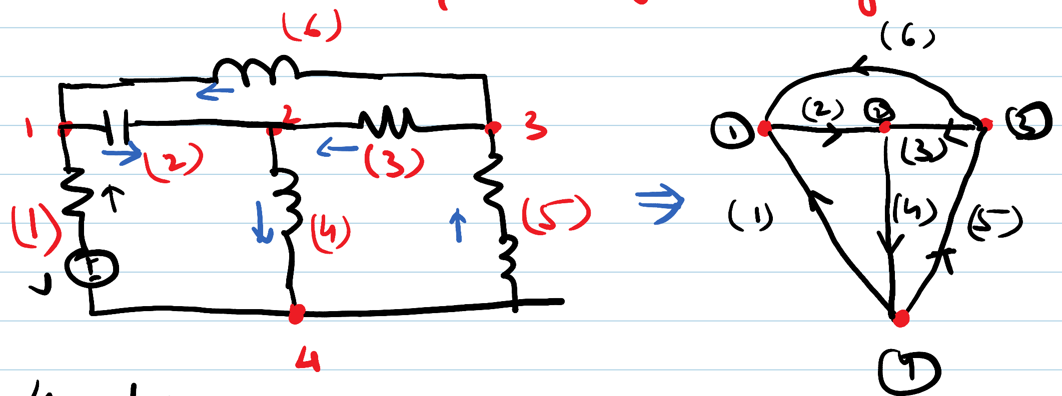
$$\boxed{P_{max} = I_L^2 \times R_L}$$

Prof. Sushma Kadge
Friday, June 4, 2021
10:04 AM

GRAPH THEORY

- KCL
- KVL
- Theorems → large → complicated
- Graph theory → Analysis → V & I

Graph → Collection of nodes & branches → they are joined together.



4 nodes
6 branches

Vol. source → S.C
Current source → O.C

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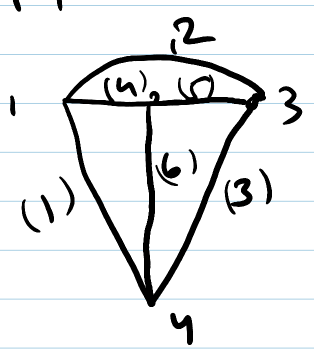
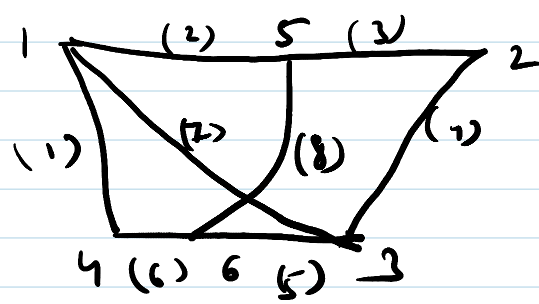
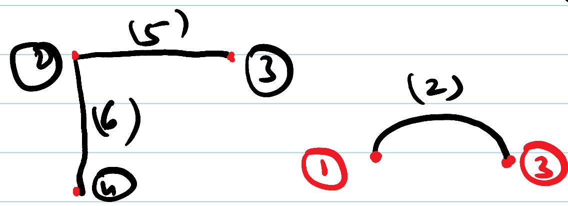


Planer graph

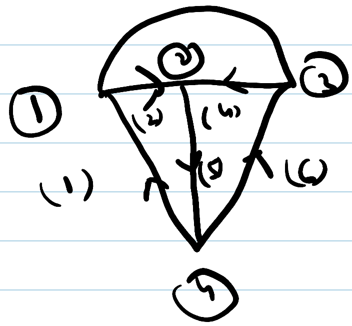


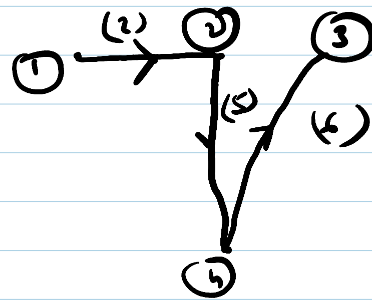
Non-planer graph



Sub graph

Path



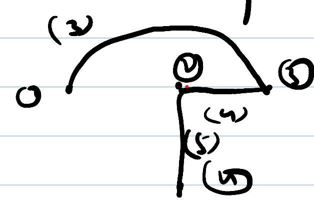


n = 4
b = 6

3 twigs

3 links.

2, 5, 6 → 4 nodes → path



Tree:-
- Connected sub graph
- All the nodes of graph
- No loops

Branches of trees → twigs

Trees → n-1 twigs (n → nodes)

$$\boxed{\text{No. of possible trees} = \det(AA^T)}$$

Branches which are not on tree → links/Chords → cotree

co-tree → b-(n-1) links
= 6 - (4 - 1)
= 3 links

(Complement of tree)

Incident matrix:- It gives us the information about which branch is incident on which node, & what are its orientation w.r.t to nodes.
Information is written in form of matrix which is known as Incident matrix

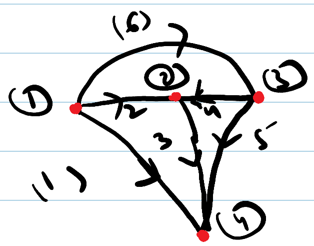
Complete Incident Matrix (A_a)

$a_{nk} = 1$ if branch k^{th} is incident at node 'n^{th}' & its oriented away from the node n.

$= -1$ " " " " " "

towards the node

= 0 Not incident



nodes ↓ / Branches →	1	2	3	4	5	6
1	1	1	0	0	0	1
2	0	-1	1	-1	0	0
3	0	0	0	1	1	-1
4	-1	0	-1	0	-1	0

$$A_a = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ \boxed{0 \quad 0 \quad 0 \quad 1 \quad 1 \quad -1} \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix}$$

ZERO = Sum of each columns

A = ?

Reduced Incidence matrix (A) → Eliminate any one row

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix}$$

H.W No. of possible trees = $\det(\underline{A} \cdot \underline{A}^T) = \underline{16}$

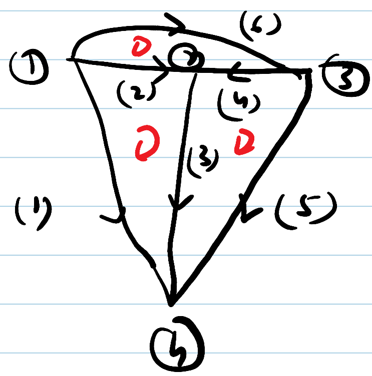
Loop matrix or Circuit matrix

Matrix gives us information about which branches constitute loop or circuit.

b_{nk} = 1 if branch 'k' is in loop 'n' & their orientation coincide

-1 " " " " donot "

0 " " Not in loop



$$
\begin{array}{c|c}
 & \text{Branches} \rightarrow \\
\text{Loops} \downarrow & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array}
\end{array}
$$

Loop 1: 1, 2, 3
Loop 2: 3, 4, 5
3 : 2, 4, 5
4 = 1, 2, 4, 5
5 = 1, 5, 6
6 = 2, 3, 5, 6
7 = 1, 3, 4, 6

$$
\begin{bmatrix}
-1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & -1 & 1 & 0 \\
0 & -1 & 0 & 1 & 0 & 1 \\
-1 & 1 & 0 & -1 & 1 & 0 \\
-1 & 0 & 0 & 0 & 1 & 1 \\
0 & -1 & -1 & 0 & 1 & 1 \\
-1 & 0 & 1 & 1 & 0 & 0
\end{bmatrix}
$$

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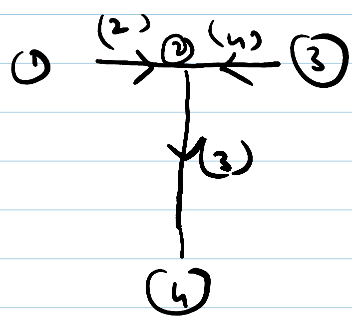
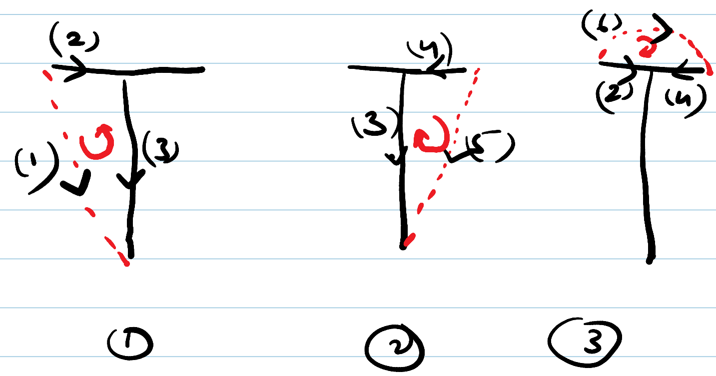
08/06/21

Tieset matrix (Fundament circuit)





tieset 1 : (1, 2, 3)
tieset 5 : (5, 3, 4)
tieset 6 : (6, 2, 4)

No. of tiesets = $b - n + 1$
$= 6 - 4 + 1$
$= 3$

$$
B = \begin{array}{c} \\ 1 \\ 5 \\ 6 \end{array}
\begin{array}{c}
\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{bmatrix}
1 & -1 & -1 & 0 & 0 & 0 \\
0 & 0 & -1 & -1 & 1 & 0 \\
0 & -1 & 0 & 1 & 0 & 1
\end{bmatrix}
\end{array}
$$

Rearrange

$$
B = \begin{array}{c}
\begin{array}{ccc|ccc} \multicolumn{3}{c|}{\text{Twigs}} & \multicolumn{3}{c}{\text{Links}} \\ 2 & 3 & 4 & 1 & 5 & 6 \end{array} \\
\begin{bmatrix}
-1 & -1 & 0 & 1 & 0 & 0 \\
0 & -1 & -1 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

$$B = \begin{bmatrix} -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$B = [B_t : B_l] = [B_t : U]$$

unit matrix

$A_a \rightarrow$ Complete incident matrix

$A \rightarrow$ Reduced " "

$B_a \rightarrow$ Loop matrix

$B \rightarrow$ Tie set matrix

orthogonal

orthogonal

$$\boxed{\begin{array}{ccc} A_a B_a^T = 0 & \text{or} & B_a A_a^T = 0 \\ A B^T = 0 & \text{or} & B^T A = 0 \end{array}}$$

Cutset matrix :-

The removal of set of branches results in cutting the graph into parts.

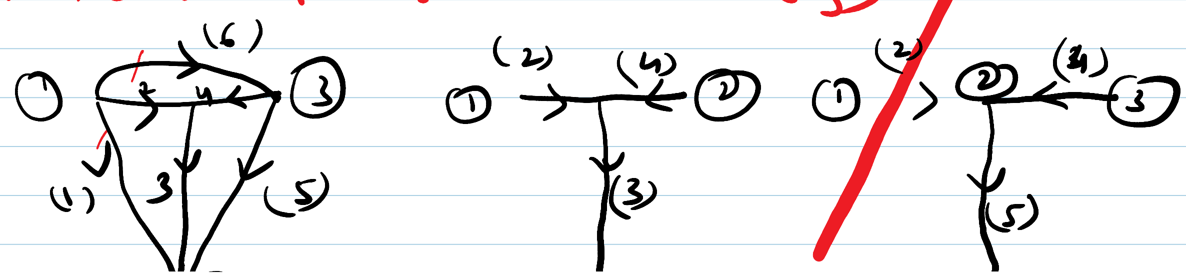
q_{nk} = 1 if branch 'k' is in cutset 'n' & orientations coincide

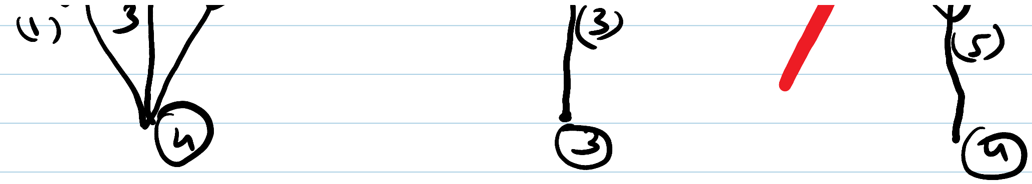
-1 " " " " " " " donot coincide

0 " " " is not in the cutset

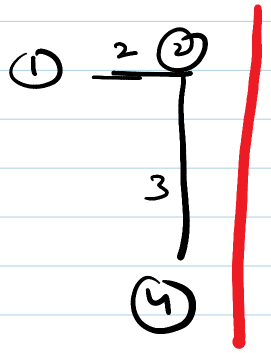
$$Q_a = \begin{bmatrix} \quad \end{bmatrix}$$

Fundamental cutset matrix (Q)





fcutset 3: [3,1,5)



fcutset 2: [2,1,6]

fcutset 4 : [4,5,6]

$$
Q = \begin{array}{c|c}
 & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{array}{c} 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 \end{bmatrix}
\end{array}
$$

$$
Q = [Q_t : Q_l] = [U : Q_l]
$$

Relationship between B & Q matrix

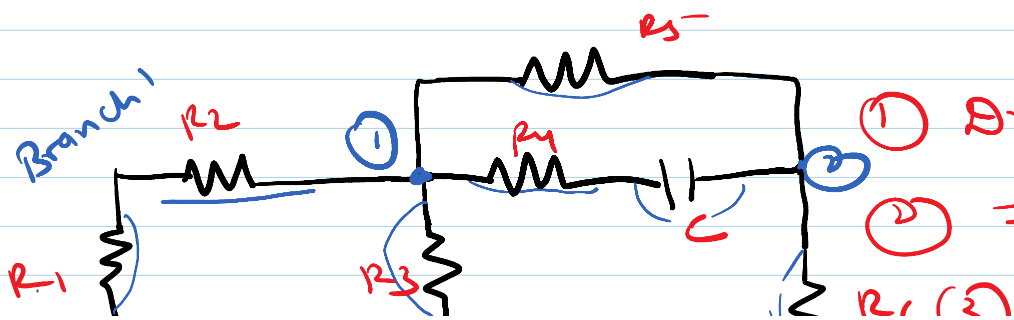
orthogonal

$$
\boxed{\begin{array}{c} Q_a B_a^T = 0 \\ Q B^T = 0 \end{array}}
$$

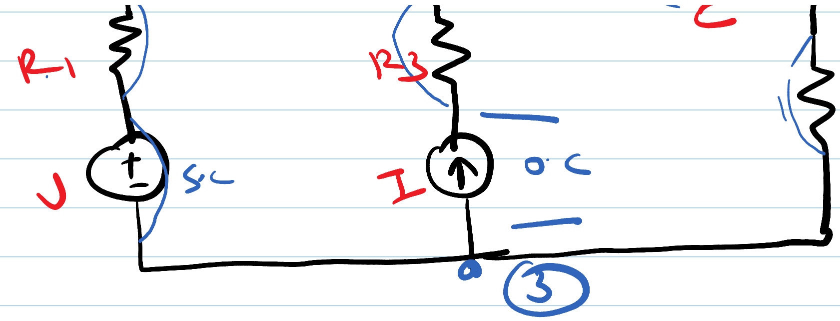
or $B_a Q_a^T = 0$

or $B Q^T = 0$

*



1. Draw oriented graph
2. Incidence matrix
3. Tieset-



(3) Tieset

(4) Cutset

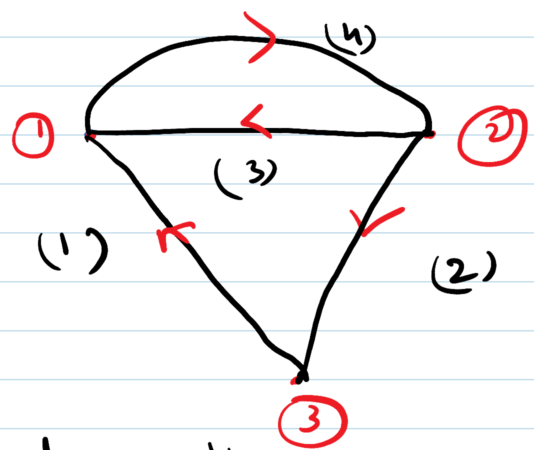
⇒

(1) Replace R, L, C → line segments

(2) " Vol. Source → S.C
I. Source → O.C

(3) Assume directions of branch current

(4) All the nodes & branches → Number



$b = 4$
$n = 3$

(2) Complete Incidence matrix

$$A_a = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} -1 & 0 & -1 & 1 \\ 0 & +1 & +1 & -1 \\ 1 & -1 & 0 & 0 \end{bmatrix} \end{array}$$

ZERO

Reduced Incidence matrix

$$A = \begin{bmatrix} -1 & 0 & -1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

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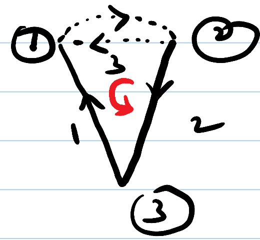
09/06/21(10:30am)

(3) Tieset matrix :-

1) select any Tree

1) select any Tree



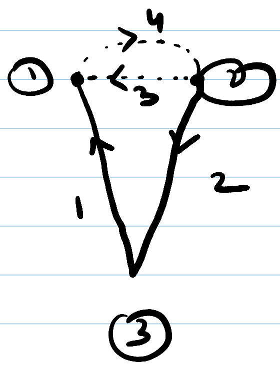
Twigs → 1, 2
links → 3, 4

Tieset 3 → 3, 1, 2
4 → 4, 1, 2

$$B = \begin{array}{c} \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} -1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

B_t : U
B_t : B_l

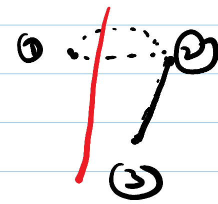
(4) f-cutset matrix (Q):

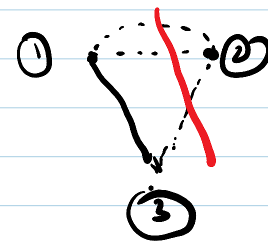


f-cutset 1 : [1, 3, 4]

f-cutset 2 : [2, 3, 4]

$$Q = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 1 & -1 \end{bmatrix} \end{array}$$





* $A = \begin{bmatrix} 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}$ graph = ?

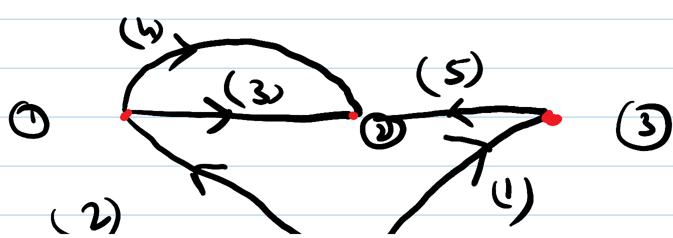
→ Complete incidence matrix

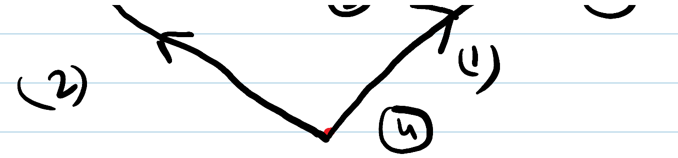
$$A_a = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 \\ 1 & +1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

← Branches.

↑ nodes

ZERO





Duality :- (planer n/w)

Two n/ws → dual of each other

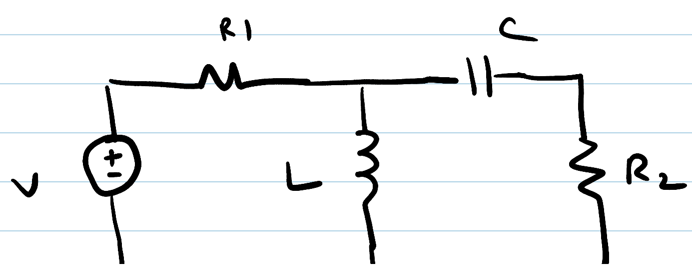
Two graphs → "

Conversion for Dual circuits

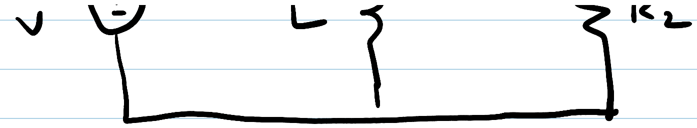
	Loop basis	Node basis
1)	Current	voltage
2)	Resistance	conductance
3)	Inductance	capacitance
4)	Branch current	Branch voltage
5)	Mesh	Node
6)	S.C	O.C
7)	Parallel path	Series path

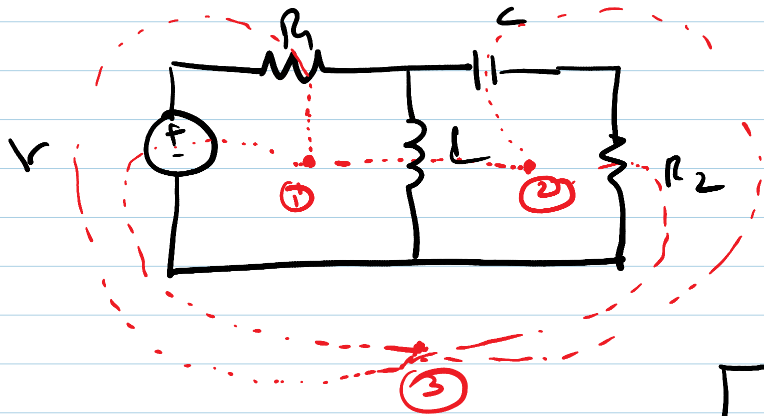
Construction of a dual N/w :-

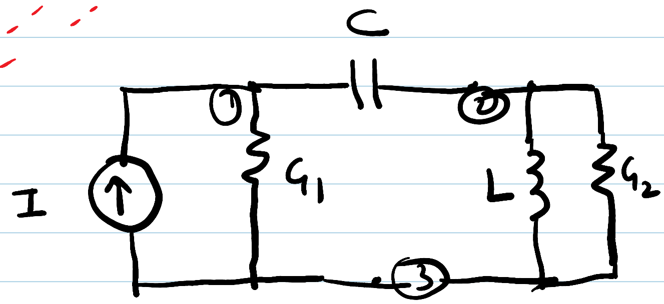
1) place a dot in each mesh of a N/W
2) " " " outside N/W → reference node
3) connect all internal dot in adjacent
4) " " " " to the external dot
5) Clockwise current → +ve polarity
6) Voltage rise in dir^n of a clockwise current correspond current flowing towards the dual indep. node.

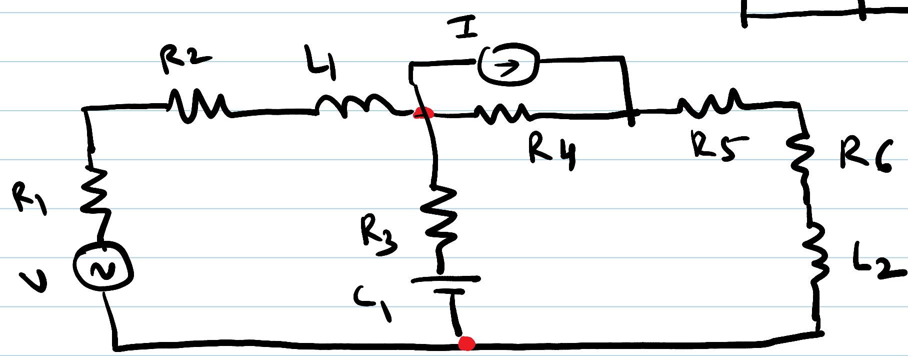


Dual N/w ?

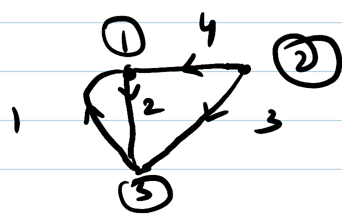


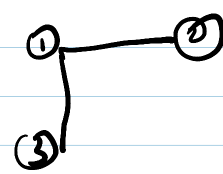






1. Oriented graph
2. Incidence matrix
3. Tieset matrix
4. Cutset matrix





Complete incidence matrix (Aa):

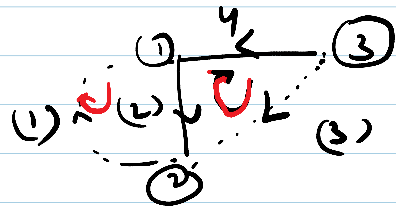
$$
\begin{array}{cc}
 & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
Aa = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \left|\begin{array}{cccc} -1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & -1 & 0 \end{array}\right|
\end{array}
$$

ZERO

Incidence matrix :-

$$A = \begin{bmatrix} -1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

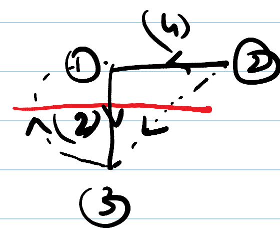
Tieset matrix.

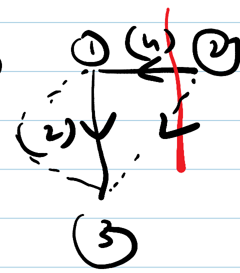


$$B = \begin{array}{c} \\ 1 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 \end{bmatrix} \end{array}$$

Cutset matrix Q =

$$Q = \begin{array}{c} \\ 2 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$





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09/06/21 (2:15pm)

* KVL, KCL, Network equilibrium equation

KVL,

$$\sum v_k = 0 . \qquad k^{th} \rightarrow \text{branch.}$$

$$\sum_{k=1}^{b} b_{hk} \, v_k = 0$$

$$\boxed{B V_b = 0}$$

$V_b \rightarrow$ column vector

KCL :-

$$\sum i_k = 0$$

b

$$\sum_{k=1} a_{hk} i_k = 0$$

$\downarrow$

$$A_a I_b = 0$$

$I_b \rightarrow$ column vector

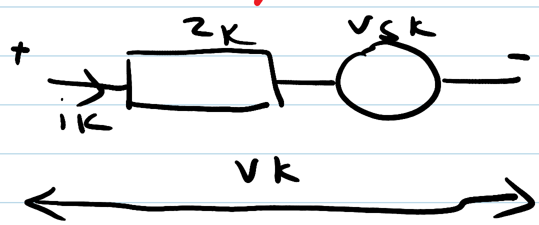
$$A I_b = 0$$

$$\boxed{Q I_b = 0}$$

$$\boxed{\begin{aligned} V_b &= Q^T v_t \\ I_b &= B^T I_l \end{aligned}}$$

Relation between

Network Equilibrium equation (KVL)



$$v_k = z_k i_k - v_{sk} \qquad k = 1, 2, 3 \cdots b$$

$$V_b = z_b i_b - V_s$$

$$B V_b = 0 \qquad \text{KVL}$$

$\downarrow$

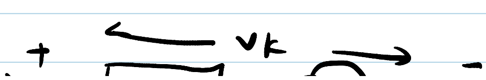
$$B[z_b i_b - V_s] = 0$$

$$B z_b i_b = B V_s$$

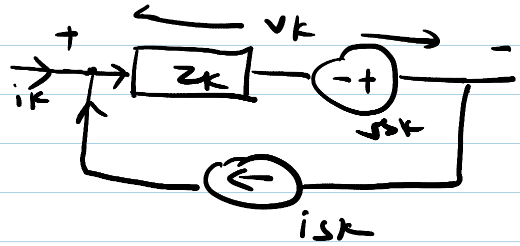
sub $I_b = B^T I_l$

$$\boxed{B z_b B^T I_l = B V_s}$$

$$Z = B z_b B^T$$



(1)



$$i_k = \frac{v_k + v_{sk}}{z_k} - i_{sk}$$

$$v_k = z_k i_k + z_k i_{sk} - v_{sk} \qquad k = 1, 2, 3 .. b$$

$$V_b = Z_b I_b + Z_b I_s - V_s$$

$$B V_b = 0 \qquad \text{KVL}$$

$$B\left[Z_b I_b + Z_b I_s - V_s \right] = 0$$

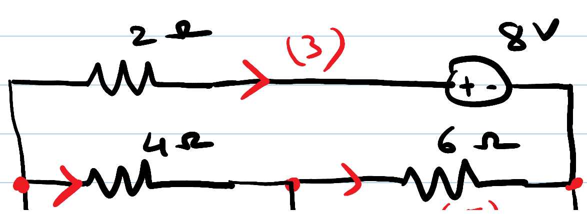
$$B Z_b I_b = B V_s - B Z_b I_s$$

$$\boxed{B Z_b B^T I_l = B V_s - B Z_b I_s} \qquad \text{KVL}$$

Generalised KVL equation in matrix form

$$\boxed{Q Y_b Q^T V_t = Q I_s - Q Y_b V_s} \qquad \text{KCL}$$

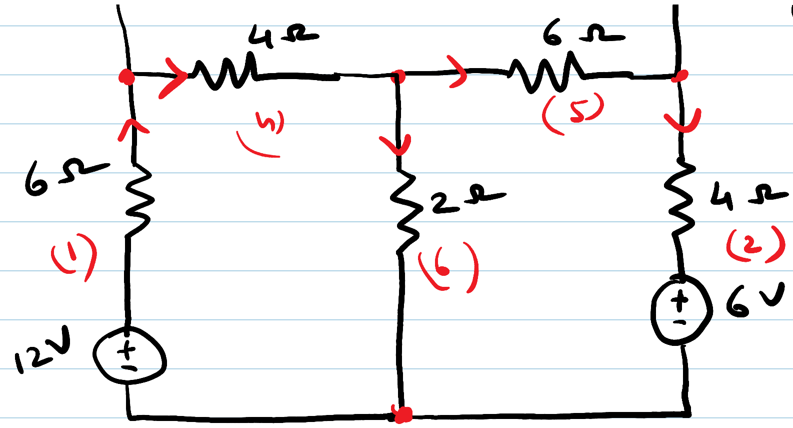
*

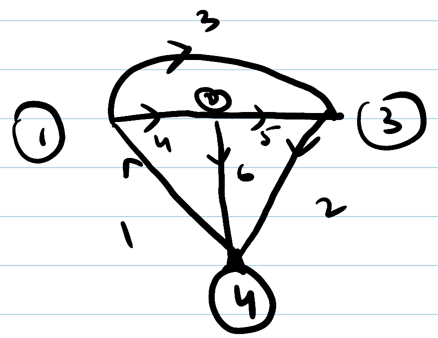


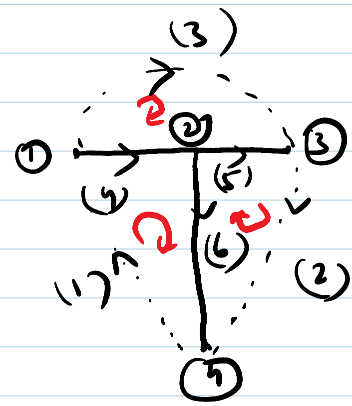
Obtain N/W equilibrium eqn in matrix form

Obtain N/W equilibrium eqn in matrix form using KVL.

calculate loop currents







$B = ?$

$Z_b = ?$

$$B = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{bmatrix} \end{array}$$

$$Z_b = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \end{array}$$

KVL,

$$B Z_b B^T I_l = B V_s - B Z_b I_s \qquad I_s = 0$$

$$\underline{\underline{B V_s}} = B Z_b B^T I_l$$

$$\underset{3\times 6}{\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{bmatrix}} \begin{bmatrix} 12 \\ -6 \\ -8 \\ 0 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} \cdots \\ 0 & 0 & 1 & -1 & -1 & 0 \end{bmatrix}_{3\times 6} \begin{bmatrix} \vdots \\ -8 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \vdots \\ -6 \\ -8 \end{bmatrix}$$

$B \cdot V_S$

$$BZ_b = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 6 & & & & & \\ & 4 & & & & \\ & & 2 & & & \\ & & & 4 & & \\ & & & & 6 & \\ & & & & & 2 \end{bmatrix}_{6\times 6}$$

$$= \begin{bmatrix} 6 & 0 & 0 & 4 & 0 & 2 \\ 0 & 4 & 0 & 0 & 6 & -2 \\ 0 & 0 & 2 & -4 & -6 & 0 \end{bmatrix}$$

$$\underline{B \cdot Z_b} \cdot B^T = \underbrace{\begin{bmatrix} 6 & 0 & 0 & 4 & 0 & 2 \\ 0 & 4 & 0 & 0 & 6 & -2 \\ 0 & 0 & 2 & -4 & -6 & 0 \end{bmatrix}}_{B \cdot Z_b} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}}_{B^T}$$

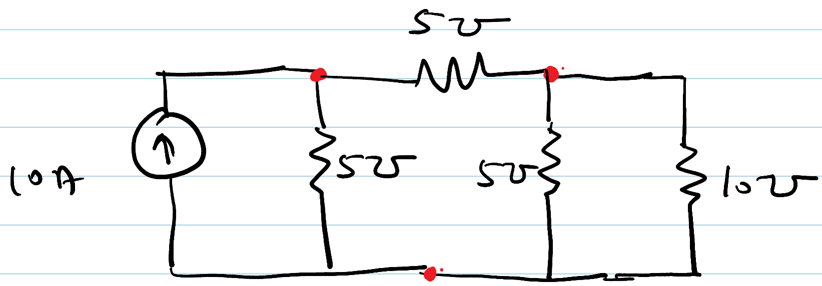
$$BZ_bB^T = \begin{bmatrix} 12 & -2 & -4 \\ -2 & 12 & -6 \\ -4 & -6 & 12 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 12 \\ -6 \\ -8 \end{bmatrix} = \begin{bmatrix} 12 & -2 & -4 \\ -2 & 12 & -6 \\ -4 & -6 & 12 \end{bmatrix} \begin{bmatrix} I_{l_1} \\ I_{l_2} \\ I_{l_3} \end{bmatrix}}$$ KVL

Solve this matrix

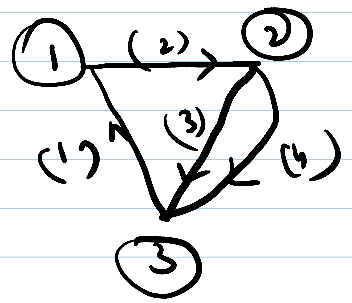
$\boxed{I_{l_1} = 0.557A}$

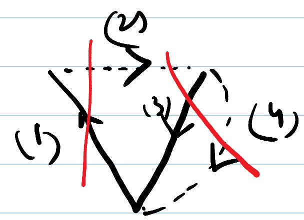
$$\boxed{\begin{aligned} I_{l_1} &= 0.55A \\ I_{l_2} &= -0.866 \\ I_{l_3} &= -0.916A \end{aligned}}$$

* 

Obtain equilibrium eqn on node basis (KCL)

⇒





$Q = ?$

$y_b = ?$

$$Q = \begin{array}{c} \\ 1 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \end{array}$$

KCL

$$\underline{Q Y_b} Q^T v_t = Q I_s - \boxed{Q Y_b V_s} = 0 \qquad v_s = 0$$

?

$$Q \cdot I_s = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -10 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \end{bmatrix}$$

$$Q \cdot Y_b = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$$\begin{matrix} 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 10 \end{matrix}\Bigg]$$

$$= \begin{bmatrix} 5 & -5 & 0 & 0 \\ 0 & -5 & 5 & 10 \end{bmatrix}$$

$$g \cdot y_b \cdot g^T = \underbrace{\begin{bmatrix} 5 & -5 & 0 & 0 \\ 0 & -5 & +5 & 10 \end{bmatrix}}_{g\,y_b} \underbrace{\begin{bmatrix} 1 & 0 \\ -1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}}_{g^T}$$

$$= \begin{bmatrix} 10 & 5 \\ 5 & 20 \end{bmatrix}$$

<u>KCL</u>

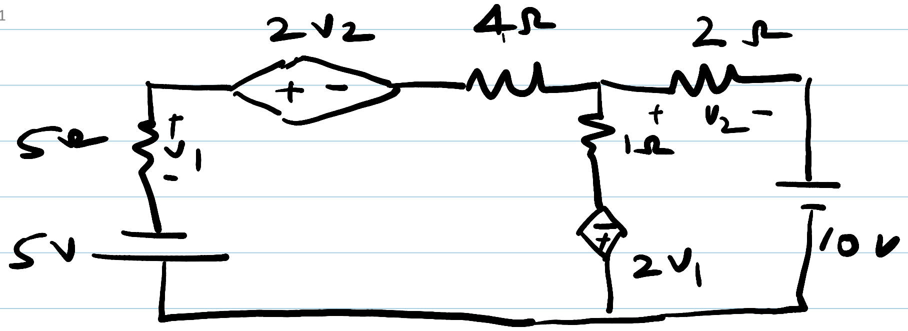
$$\begin{bmatrix} 10 & 5 \\ 5 & 20 \end{bmatrix} \begin{bmatrix} v_{t1} \\ v_{t2} \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \end{bmatrix}$$

Solve v_{t1} & v_{t2}

$$\boxed{\begin{aligned} v_{t1} &= -8/7 \text{ V} \\ v_{t2} &= 2/7 \text{ V} \end{aligned}}$$

TUTORIAL 1

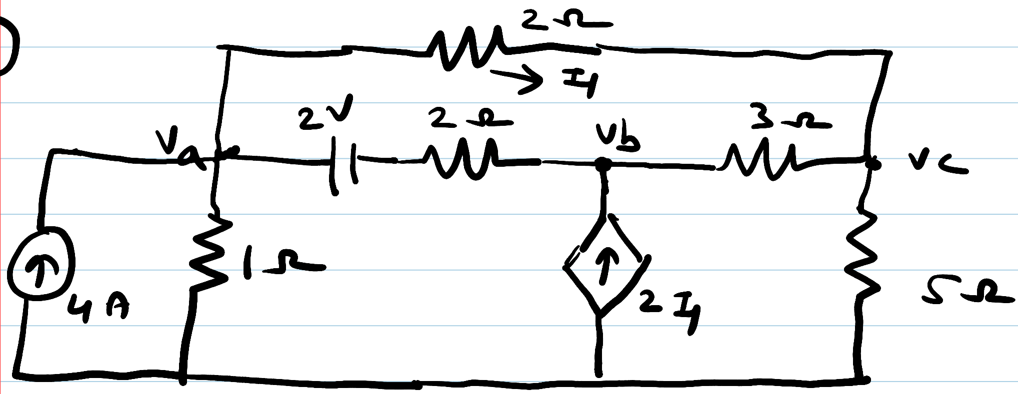
1.



I_1 & I_2 ?

Mesh

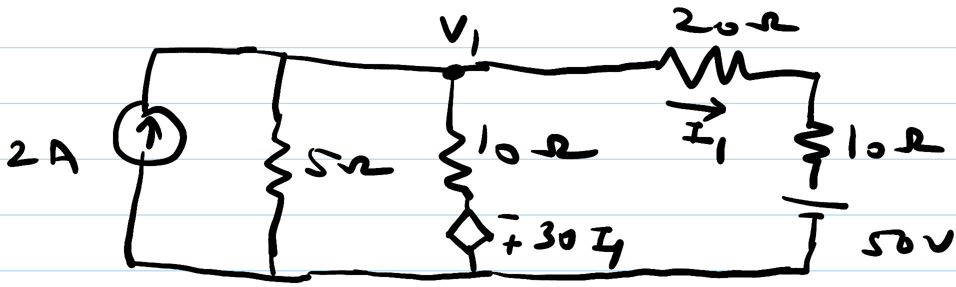
2.



Node voltages

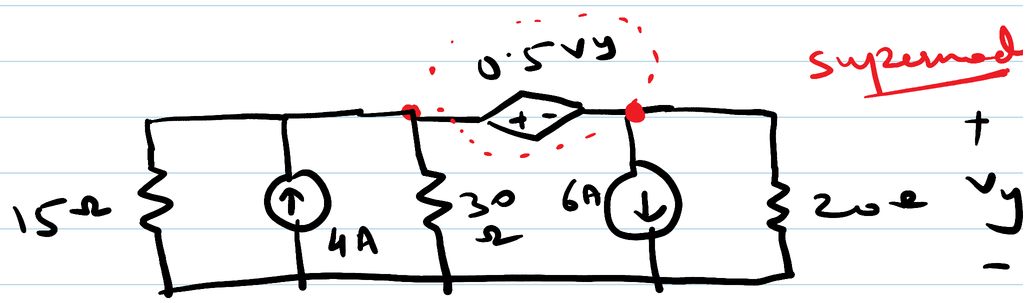
Va, Vb, Vc ?

3.



Find voltage across 5Ω ?

4.



supernode

Find V_y?