DISCRETE MATHEMATICS

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What is discrete maths

- **Discrete Mathematics** is a branch of mathematics involving discrete elements that uses algebra and arithmetic.
- Tool for improving reasoning and problem-solving capabilities.
- Sets, Relations and Functions, Mathematical Logic, Group theory, Counting Theory, Probability, Mathematical Induction and Recurrence Relations, Graph Theory, Trees and Boolean Algebra

Application of discrete maths

Everyday applications of discrete mathematics

Computers run software and store files. The software and files are both stored as huge strings of 1s and 0s. Binary math is discrete mathematics.

Scheduling problems---like deciding which nurses should work which shifts, or which airline pilots should be flying which routes, or scheduling rooms for an event, or deciding timeslots for committee meetings, or which chemicals can be stored in which parts of a warehouse---are solved either using graph coloring or using combinatorial optimization, both parts of discrete mathematics. One example is scheduling games for a professional sports league.

Networks are, at base, discrete structures. The routers that run the internet are connected by long cables. People are connected to each other by social media ("following" on Twitter, "friending" on Facebook, etc.). The US highway system connects cities with roads.

An **analog clock** has gears inside, and the sizes/teeth needed for correct timekeeping are determined using discrete math.

Encryption and decryption are part of **cryptography**, which is part of discrete mathematics. For example, secure internet shopping uses public-key cryptography.

Doing web searches in multiple languages at once, and returning a summary, uses linear algebra.

Google Maps uses discrete mathematics to determine fastest driving routes and times. There is a simpler version that works with small maps and technicalities involved in adapting to large maps.

Wiring a computer network using the least amount of cable is a minimum-weight spanning tree problem.

Area codes: How do we know when we need more area codes to cover the phone numbers in a region? This is a basic combinatorics problem.

Designing **password criteria** is a counting problem: Is the space of passwords chosen large enough that a hacker can't break into accounts just by trying all the possibilities? How long do passwords need to be in order to resist such attacks? (find out here!)

Recommended Books

1. Bernard **Kolman**,Busby,” Discrete Mathematical Structures”,PHI.

2. Kenneth H. **Rosen**. ”Discrete Mathematics and its Applications”, Tata McGraw-Hill.

3. Seymour Lipschutz, Marc Lipson “Schaum's Outline of Discrete Mathematics”, Revised Third Edition Tata McGraw-Hill.

4. D. S. Malik and M. K. Sen, “Discrete Mathematical Structures”, Thompson.

5. C. L. Liu, D. P. Mohapatra, “Elements of Discrete Mathematics” Tata McGrawHill.

6. J. P. Trembley, R. Manohar “Discrete Mathematical Structures with Applications to Computer Science”, TataMcgraw-Hill.

7. Y N Singh, “Discrete Mathematical Structures”, Wiley-India.

SET THEORY

- **Set Theory 03 CO1**
- **1.1** Sets, Venn diagrams, Operations on Sets
- **1.2** Laws of set theory, Power set and Products
- **1.3** Partitions of sets, The Principle of Inclusion and Exclusion
-

SET THEORY

- SETS
- NOTATION
- SPECIAL SETS
- DEFINITION
  - SUBSET
  - EQUAL SETS
  - PROPER SUBSET
  - UNIVERSAL SET
  - NULL/EMPY SET
  - SINGLETON SET
  - SUPER SET
  - FINITE /INFINITE SET
  - DISJOINT SET
  - CARDINALITY OF FINITE SET
- SET PROPERTIES
- VENN DIAGRAMS
- SET OPERATIONS
- LAWS OF SET THEORY
- PARTITION OF SETS
- POWER SET

Sets

- A *set* is an **unordered collection of objects.**
  - the students in this class
  - the chairs in this room
- The objects in a set are called the *elements*, or *members* of the set. A set is said to *contain* its elements.
- The notation $a \in A$ denotes that a is an element of the set A.
- If a is not a member of A, write $a \notin A$

Describing a Set: **Roster Method**

- $S = \{a,b,c,d\}$
- Order not important

  $$S = \{a,b,c,d\} = \{b,c,a,d\}$$

- Each distinct object is either a member or not; listing more than once does not change the set.

  $$S = \{a,b,c,d\} = \{a,b,c,b,c,d\}$$

- Elipses (…) may be used to describe a set without listing all of the members when the pattern is clear.

  $$S = \{a,b,c,d, \ldots\ldots,z\}$$

Roster Method

- Set of all vowels in the English alphabet:

$$V = \{a,e,i,o,u\}$$

- Set of all odd positive integers less than 10:

$$O = \{1,3,5,7,9\}$$

- Set of all positive integers less than 100:

$$S = \{1,2,3,........,99\}$$

- Set of all integers less than 0:

$$S = \{...., -3,-2,-1\}$$

Some **Important** Sets

$\mathbf{N}$ = *natural numbers* = $\{1,2,3....\}$

$\mathbf{Z}$ = *integers* = $\{...,-3,-2,-1,0,1,2,3,...\}$

$\mathbf{Z}^{+}$ = *positive integers* = $\{1,2,3,.....\}$

$\mathbf{R}$ = set of *real numbers*

$\mathbf{R}^{+}$ = set of *positive real numbers*

$\mathbf{C}$ = set of *complex numbers.*

$\mathbf{Q}$ = set of rational numbers

SET-BUILDER Notation

- Specify the property or properties that all members must satisfy:

$$S = \{x \mid x \text{ is a positive integer less than } 100\}$$

$$O = \{x \mid x \text{ is an odd positive integer less than } 10\}$$

$$O = \{x \in \mathbf{Z}^{+} \mid x \text{ is odd and } x < 10\}$$

- A predicate may be used: $S = \{x \mid \mathrm{P}(x)\}$
- Example: $S = \{x \mid \mathrm{Prime}(x)\}$

Universal Set and Empty Set

- The *universal set U* is the set containing everything **currently under consideration**.
  - Sometimes implicit/explicit/Contents depend on the context.
- Empty set is the set with no elements. Symbolized $\emptyset$, but $\{\ \}$ also used , but $\{\emptyset\}$???

"A null set is a subset of every set"

Some things to remember

- Sets can be elements of sets.

  $\{\{1,2,3\}, a, \{b,c\}\}$

  $\{\mathbf{N},\mathbf{Z},\mathbf{Q},\mathbf{R}\}$

- The empty set is different from a set containing the empty set.

  $\emptyset \neq \{\emptyset\}$

Set Equality

Definition: Two sets are *equal* if and only if they have the same elements.

- Therefore if A and B are sets, then A and B are equal if and only if **A ⊆ B and B ⊆ A the A=B**
- We write $A = B$ if A and B are equal sets.

$$\{1,3,5\} = \{3, 5, 1\}$$

$$\{1,5,5,5,3,3,1\} = \{1,3,5\}$$

Subsets

Definition: The set A is a *subset* of B, if and only if every element of A is also an element of B.

- The notation $A \subseteq B$ is used to indicate that A is a subset of the set B.

Eg: If A={1,2,3}, B={1,2,3,4,5}, C=(3,2,1}

- $\nsubseteq$ - not a subset

"Every set is a subset of itself"

Proper Subsets

Definition: **If $A \subseteq B$, but $A \neq B$**, then we say A is a *proper subset* of B, denoted by $A \subset B$.

A= {x ,y } , B= {x, y ,z}

then is $A \subset B$ ؟

A= {1,3}

B={1,2,3}

C={1,3,2}

A ⊂ C ? , B ⊂ C ?

Set Cardinality

Definition: The *cardinality* of a finite set *A*, denoted by |*A*|, is the **number of (distinct) elements of *A*.**

Examples:

1. $|\varnothing| = 0$
2. Let S be the letters of the English alphabet. Then |*S*| $= 26$
3. $|\{1,2,3\}| = 3$
4. $|\{\varnothing\}| = 1$
5. The set of integers is infinite.

Superset and Disjoint Set

- If A is the subset of B then **B is the SUPERSET of A**
- Two sets are said to be disjoint if they have **no elements in common**

SET PROPERTIES

- Every Set A is a subset of the Universal set U

$$\emptyset \subseteq A \subseteq U$$

- Every set A is a subset of itself

$$A \subseteq A$$

- Transitivity

$$A \subseteq B,\ B \subseteq C, \text{ then } A \subseteq C$$

- If $A \subseteq B$ and $B \subseteq A$ then $A=B$; converse also holds true

VENN DIAGRAMS & SET OPERATIONS

Union

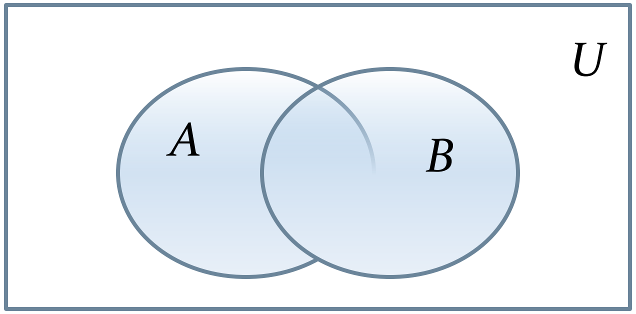
- **Definition**: Let *A* and *B* be sets. The *union* of the sets *A* and *B*, denoted by $A \cup B$, is the set:

$$\{x | x \in A \vee x \in B\}$$

- **Example**: What is $\{1,2,3\} \cup \{3, 4, 5\}$?

Solution: $\{1,2,3,4,5\}$

Venn Diagram for $A \cup B$



Intersection

- **Definition:** The *intersection* of sets *A* and *B*, denoted by ***A* ∩ *B***, is
$$\{x \mid x \in A \land x \in B\}$$
- Note if the intersection is empty, then *A* and *B* are said to be *disjoint* *(i.e. no common element)*.
- **Example:** What is? $\{1,2,3\} \cap \{3,4,5\}$?

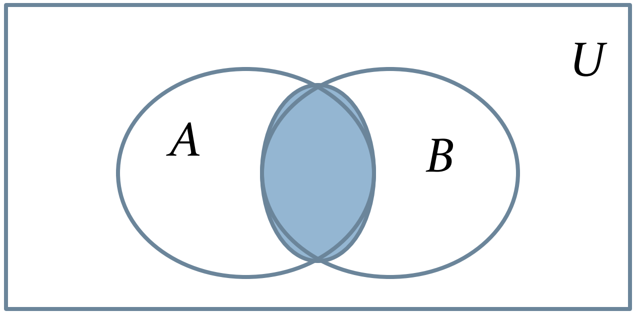
  Solution: $\{3\}$

- **Example:** What is?

  $\{1,2,3\} \cap \{4,5,6\}$?

  Solution: $\emptyset$

Venn Diagram for $A \cap B$



Complement

Definition: If A is a set, then the complement of the A (with respect to U), denoted by $\bar{A}$ is the set $U - A$

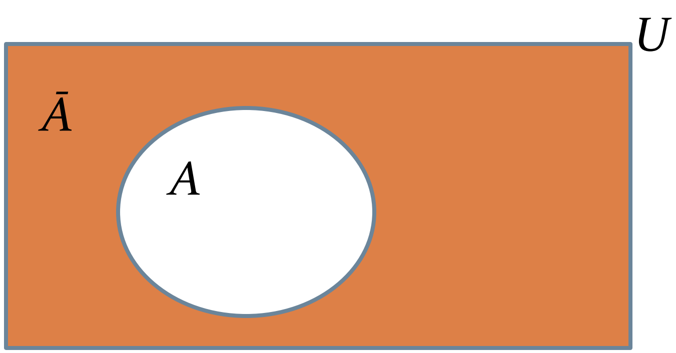
$$\bar{A} = \{x \in U \mid x \notin A\}$$

(The complement of A is sometimes denoted by A^c .)

Example: If U is the positive integers less than 100, what is the complement of $\{x \mid x > 70\}$

Solution: $\{x \mid x \leq 70\}$

Venn Diagram for Complement



Difference

- **Definition:** Let A and B be sets. The *difference* of A and B, denoted by $A - B$, is the set containing the elements of A that are not in B.

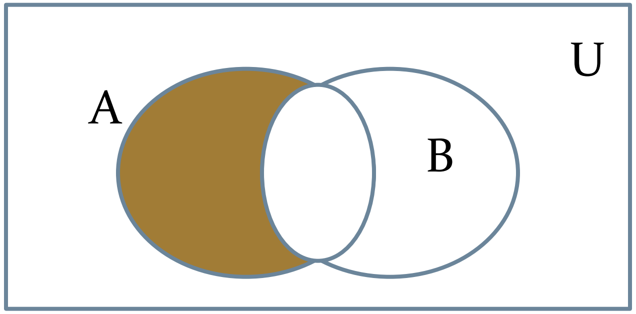
$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

A={a , b , c } B = {b , c , d ,e }

A – B = ? **Solution:** {a}

B – A = ? **Solution:** {d, e}

Venn Diagram for $A - B$



Symmetric Difference

Definition: The *symmetric difference* of **A** and **B**, denoted by $A \oplus B$ is the set

$$(A - B) \cup (B - A)$$

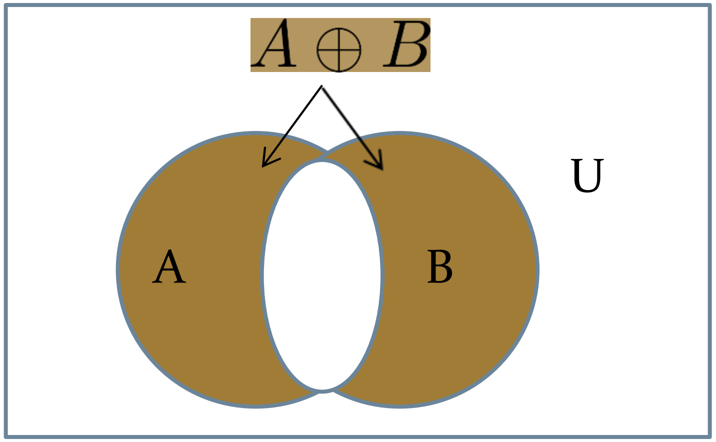
Example:

$U = \{0,1,2,3,4,5,6,7,8,9,10\}$

$A = \{1,2,3,4,5\}$ $B = \{4,5,6,7,8\}$

What is $A \oplus B$: ?

Solution: $\{1,2,3,6,7,8\}$



Venn Diagram

Cartesian Product

Definition: The *Cartesian Product* of two sets A and B, denoted by $A \times B$ is the set of ordered pairs (a,b) where $a \in A$ and $b \in B$ (first element from A and second element from B)

Example:
$$A \times B = \{(a,b) | a \in A \wedge b \in B\}$$

$A = \{a,b\}$ $B = \{1,2,3\}$

$A \times B = \{(a,1),(a,2),(a,3), (b,1),(b,2),(b,3)\}$

Review Questions

Example: $U = \{0,1,2,3,4,5,6,7,8,9,10\}$ $A = \{1,2,3,4,5\}$, $B = \{4,5,6,7,8\}$

1. $A \cup B$

   Solution: $\{1,2,3,4,5,6,7,8\}$

2. $A \cap B$

   Solution: $\{4,5\}$

3. $\bar{A}$

   Solution: $\{0,6,7,8,9,10\}$

4. $\bar{B}$

   Solution: $\{0,1,2,3,9,10\}$

5. $A - B$

   Solution: $\{1,2,3\}$

6. $B - A$

   Solution: $\{6,7,8\}$

LAWS OF SET THEORY

- Commutative laws

$$A \cup B = B \cup A \qquad A \cap B = B \cap A$$

- Associative laws

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

- Distributive laws

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Continued on next slide →

LAWS OF SET THEORY

Identity laws

$$A \cup \emptyset = A \qquad A \cap U = A$$

Properties of Universal Set

A U U= U A ∩ U=A

Properties of Empty Set

A U {} = A A ∩{} = {}

Idempotent laws

$$A \cup A = A \qquad A \cap A = A$$

Complementation law

$$\overline{(\overline{A})} = A$$

Continued on next slide →

LAWS OF SET THEORY

- De Morgan's laws

$$\overline{A \cup B} = \overline{A} \cap \overline{B} \qquad \overline{A \cap B} = \overline{A} \cup \overline{B}$$

- Absorption laws

$$A \cup (A \cap B) = A \quad A \cap (A \cup B) = A$$

- Properties of complement law

$$A \cup \overline{A} = U \qquad A \cap \overline{A} = \emptyset$$

Membership Table

Example: Construct a membership table to show that the distributive law holds. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution:

A	B	C	$B \cap C$	$A \cup (B \cap C)$	$A \cup B$	$A \cup C$	$(A \cup B) \cap (A \cup C)$
1	1	1					
1	1	0					
1	0	1					
1	0	0					
0	1	1					
0	1	0					
0	0	1					
0	0	0					

Membership Table

Example: Construct a membership table to show that the distributive law holds. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution:

A	B	C	$B \cap C$	$A \cup (B \cap C)$	$A \cup B$	$A \cup C$	$(A \cup B) \cap (A \cup C)$
1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1
0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0

Power Sets

Definition: **The set of all subsets of a set *A***, denoted $\mathcal{P}(A)$, is called the *power set* of *A*.

Example: If $A = \{a,b\}$ then

$$\mathcal{P}(A) = \{\emptyset, \{a\},\{b\},\{a,b\}\}$$

- If a set has *n* elements, then the 'cardinality' (number of (distinct) elements of a set) of the power set is 2^n.

Example: If $B = \{1,2, 3\}$ then find $\mathcal{P}(B)$.

$\mathcal{P}(B)$ will consist of following subsets of B

$\{\ \}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1,3\}\ \{2, 3\}\ \{1,2,3\}$.

So, $\mathcal{P}(B)$ has $2^3 = 8$ elements.

Partition of Sets

- If A is a set, a partition of A is any set of non empty subset $A_1, A_2, A_3 \ldots\ldots$ Of A such that

$$A_1 \cup A_2 \cup A_3 \ldots\ldots\ldots = A$$

&

$$A_i \cap A_j = \emptyset$$ for i = j (subsets are mutually disjoint)

Example A = { a , b , c}

Verify whether { { a } , { b , c } } is a partition of A or not

Partition of Sets

Example A = { a , b , c}

Verify whether { { a } , { b , c } } is a partition of A or not?

$A_1 = \{a\}$ $A_2 = \{b,c\}$

1. $A_1 \cup A_2 = \{a, b, c\} = A$
2. $A_1 \cap A_2 = \{\ \} = \varnothing$

Hence it is satisfying for both the required conditions.

So, { { a } , { b , c } } is a **partition of A.**

PROBLEMS/ EXERCISE TO SOLVE

Let A={1 , 2 ,3 }.Determine the power set of A

Let A={a , b , c , d }. Determine the power set of A

P(A)={Ø,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c,d},{a,b,c},{a,b,d},{b,c,d},{a,c,d}}

PROBLEMS/ EXERCISE TO SOLVE

1. $S=\{1,2,3,4,5,6,7,8,9\}$.Determine whether each is a partition or not
   (i) $\{\{1,3,5\},\{2,6\},\{4,8,9,\}\}$
   (ii) $\{\{1,3,5\},\{2,4,6,8\},\{5,7,9\}\}$
   (iii) $\{\{1,3,5\},\{2,4,6,8\},\{7,9\}\}$
2. Let $A=\{a,b,c,d,e,f,g,h\}$.Consider the following subsets of A

$A1=\{a,b,c,d\}$ $A3=\{a,c,e,g\}$

$A2=\{a,c,e,g,h\}$ $A4=\{b,d\}$ $A5=\{f,h\}$

Determine whether following is a partition of A or not. Justify

(i) $\{A1,A2\}$ (ii) $\{A1, A5\}$ (iii) $\{A3,A4,A5\}$

THEOREMS

ADDITION PRINCIPLE

$|A \cup B| = |A| + |B|$….if set A and B are mutually disjoint

$|A-B| = |A| - |A \cap B|$

PRINCIPLE OF MUTUAL INCLUSION EXCLUSION

$|A \cup B| = |A| + |B| - |A \cap B|$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

Ex. 1 : How many elements are in $A_1 \cup A_2$ if there are 12 elements in A_1 and 18 elements in A_2 and

(i) $A_1 \cap A_2 = \phi$ (ii) $|A_1 \cap A_2| = 1$

(iii) $|A_1 \cap A_2| = 6$ (iv) $A_1 \subseteq A_2$

Soln. :

Given : $|A_1| = 12$ and $|A_2| = 18$.

(i) $A_1 \cap A_2 = \phi$

$\therefore \; |A_1 \cap A_2| = 0$

By principle of Inclusion -Exclusion

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$
$$= 12 + 18 - 0 = 30$$
$$|A_1 \cup A_2| = 30$$

(ii) Given : $|A_1 \cap A_2| = 1$

By principle of Inclusion-Exclusion

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$
$$= 12 + 18 - 1 = 29$$

(iii) Given : $|A_1 \cap A_2| = 6$

By principle of Inclusion-Exclusion

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$
$$= 12 + 18 - 6 = 24$$
$$|A_1 \cup A_2| = 24$$

(iv) Given : $A_1 \subseteq A_2$

$\therefore \; |A_1 \cap A_2| = 12$

By principle of Inclusion-Exclusion

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$
$$= 12 + 18 - 12 = 18$$
$$|A_1 \cup A_2| = 18$$

Ex.2) A computer company must hire 25 programmers to handle system programming jobs and 40 programmers for application programming of those hired 10 will be expected to perform jobs of both types. How many programmers must be hired ?

Soln. : Let, A be the set of system programmers hired.

B be the set of applications programmers hired.

We want to find $|A \cup B|$

We have, $|A| = 25$,

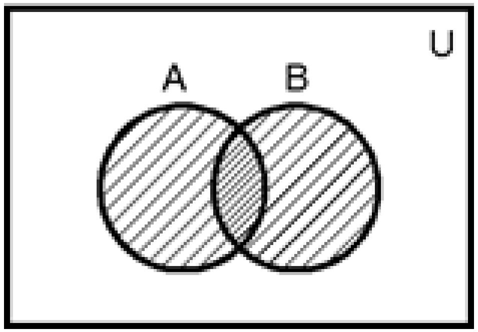
$|B| = 40$

Using the Principle of Inclusion and Exclusion,

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$= 25 + 40 - 10 = 55$$

$A \cup B = 55$



Ex. 3) A sample of 80 people have revealed that 24 like cinema and 62 like television programmes. Find the number of people who like both cinema and television programmes.

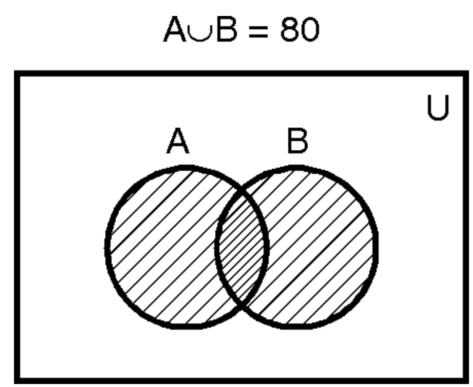
Ans: Let $A =$ Set of people who like cinema

$B =$ Set of people who like television programs

Then, we have

$|A| = 24, \qquad |B| = 62.$

Using principle of Inclusion and Exclusion



$$|A \cup B| = |A| + |B| - |A \cap B|$$

We obtain

$$80 = 24 + 62 - |A \cap B|$$

Therefore,

$$|A \cap B| = 6$$

Thus 6 people like both cinema and television programmes.

Ex. 4 : In a survey of 260 college students, the following data were obtained.

64 had taken a Mathematics course.

94 had taken a computer science course.

58 had taken a business course.

28 had taken both mathematics and business course.

26 had taken both a mathematics and computer science course.

22 had taken both a computer science and Business course.

14 had taken all three types of courses.

(i)How many students were surveyed who had taken none of the three types of courses ?

(ii)Of the students surveyed, how many had taken only a computer science course ?

Ans: Let A be the set of students taken Mathematics course.

B be the set of students taken Computer Science course.

C be the set of students taken Business course.

We have $| A |=64$, $| B | = 94$, $| C | = 58$,

$| A \cap C |=28$, taken mathematics and business course

$| A \cap B |=26$, taken mathematics and computer science course

$| B \cap C |=22$, taken computer science and business course

$| A \cap B \cap C |=14$, taken mathematics, computer science and business course.

(i) Using Principle of Inclusion and Exclusion

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$= 64 + 94 + 58 - 28 - 26 - 22 + 14$$

$$= 154$$

Students who have not taken either of subjects,

$$|\overline{A \cup B \cup C}| = |U| - |A \cup B \cup C|$$

$$= 260 - 154 = 106$$

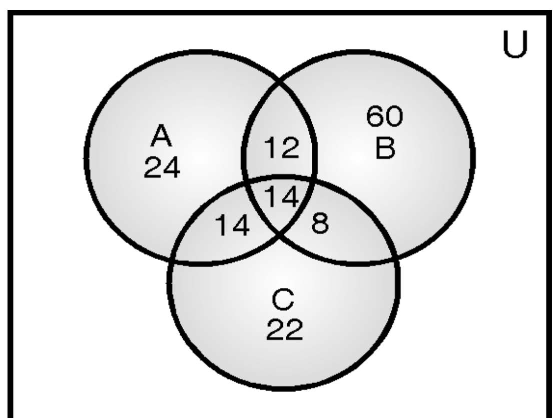
Thus 106 students had taken none of the three types of course.

(ii)Students who have taken only computer science course

$$= |B| - |B \cap A| - |B \cap C| + |A \cap B \cap C|$$

$$= 94 - 26 - 22 + 14 = 60$$

60 students had taken only computer science course.



Ex. 5 : In a survey of 60 people, it was found that 25 read Newsweek magazine, 26 read Time and 26 read Fortune. Also 9 read both Newsweek and Fortune, 11 read both Newsweek and Time, 8 read both Time and Fortune, 8 read no magazine at all.

(a)Find the number of people who read all three magazines.

(b)Fill in the correct number of people in each of eight regions of Venn diagram. Here N, T and F denote the set of people who read Newsweek, Time and Fortune respectively.

(c)Determine the number of people who read exactly one magazine. Refer Fig. 1.18.

Ans: $|N| = 25$, $|T| = 26$, $|F|$ 26,

And $|N \cap T| = 11$, $|N \cap F| = 9$, $|T \cap F| = 8$.

8 read no magazine at all.

$$|N \cup T \cup F| = 60 - 8 = 52$$

By the principle of inclusion and exclusion we have,

$$|N \cup T \cup F| = |N| + |T| + |F| - |N \cap T| - |T \cap F| - |N \cap F| + |N \cap T \cap F|$$

$$52 = 25 + 26 + 26 - 11 - 9 - 8 + |N \cap T \cap F|$$

$$|N \cap T \cap F| = 3$$

Hence, 3 people read all three magazines.

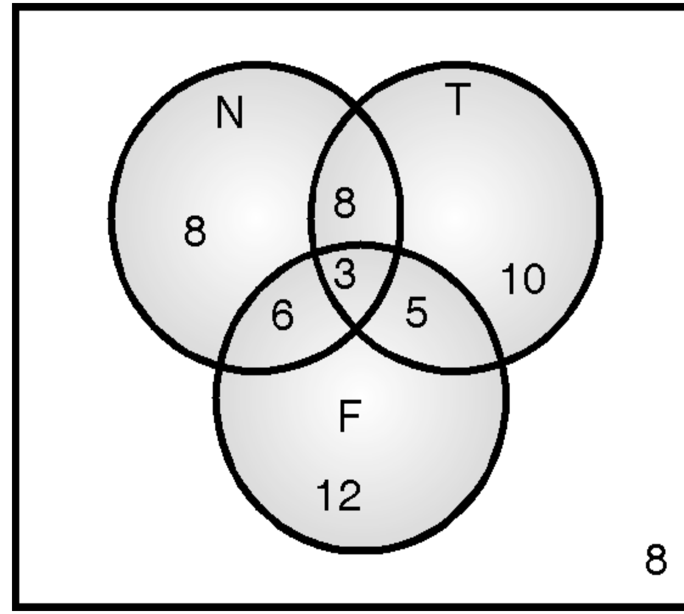
(b)To draw Venn Diagram,

3 read all three magazines

$|N \cap T| - |N \cap T \cap F| = 11 - 3 = 8$ read Newsweek and Time but not all three magazines.

$|N \cap F| - |N \cap T \cap F| = 9 - 3 = 6$ read Newsweek and Fortune but not all three magazines.

$|T \cap F| - |N \cap T \cap F| = 8 - 3 = 5$ read Time and Fortune but not all three magazines.



$|N| - |N \cap T| - |N \cap F| + |N \cap T \cap F| = 25 - 11 - 9 + 3 = 8$ read only Newsweek

$|T| - |T \cap F| - |N \cap T| + |N \cap T \cap F| = 26 - 8 - 11 + 3 = 10$ read only Time

$|F| - |T \cap F| - |N \cap F| + |N \cap T \cap F| = 26 - 8 - 9 + 3 = 12$ read only Fortune

(c)People who read only one magazine is $8 + 10 + 12 = 30$

Ex. 6 : Find how many integers between 1 and 60 are not divisible by 2 nor by 3 and nor by 5 ?

Soln. : Let A_1, A_2 and A_3 be the set of integers between 1 and 60 divisible by 2, 3 and 5 respectively.

$$\therefore \quad |A_1| = \left|\frac{60}{2}\right| = 30$$

$$|A_2| = \left|\frac{60}{3}\right| = 20$$

$$|A_3| = \left|\frac{60}{5}\right| = 12$$

and $|A_1 \cap A_2| = \left|\frac{60}{2 \times 3}\right| = 10$

$$|A_1 \cap A_3| = \left|\frac{60}{2 \times 5}\right| = 6$$

$$|A_2 \cap A_3| = \left|\frac{60}{3 \times 5}\right| = 4$$

and $|A_1 \cap A_2 \cap A_3| = \left|\frac{60}{2 \times 3 \times 5}\right| = 2$

Number of integers between 1 and 60 which are divisible by 2, 3 or 5 are

$$= |A_1 \cup A_2 \cup A_3|$$

$$= |A_1| + |A_2| + |A_3| - |A_1 \cap A_2|$$

$$- |A_1 \cap A_3| - |A_2 \cap A_3| + |A_1 \cap A_2 \cap A_3|$$

$$= 30 + 20 + 12 - 10 - 6 - 4 + 2 = 44$$

Hence the number of integers between 1 and 60 are not divisible by 2, 3 or 5 = 60 – 44 = 16

Ex. : Consider the sets :

$A = \{ x : x^2 - 4x + 3 = 0 \}$

$B = \{ x : x^2 - 3x + 2 = 0 \}$

$C = \{ x : x \in N, x < 3 \}$

$D = \{ x : x \in N, x \text{ is odd}, x < 5 \}$

$E = \{ 1, 2 \}$

$F = \{ 1, 2, 1 \}$

$G = \{ 3, 1 \}$ which of the given sets are equal.

Soln. :

$$
\begin{aligned}
A &= \{x : x^2 - 4x + 3 = 0\} \\
&= \{x : (x - 3)(x - 1) = 0\} \\
&= \{x : x = 3, x = 1\} \\
&= \{3, 1\} \\
B &= \{x : x^2 - 3x + 2 = 0\} \\
&= \{x : (x - 2)(x - 1) = 0\} \\
&= \{x : x = 2, x = 1\} \\
&= \{1, 2\} \\
C &= \{x : x \in N, x < 3\} \\
&= \{1, 2\} \\
D &= \{x : x \in N, x \text{ is odd } x < 5\} \\
&= \{1, 3\}
\end{aligned}
$$

$\therefore$ $A = D = G$ Set A, D and G are equal, $B = C = E = F$ Sets B, C, F and E are equal.

Ex. : Let A, B and C are subset of U (universal set) prove that

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

Soln. :

Let $(x, y) \in A \times (B \cup C)$

$\Rightarrow \quad x \in A$ **and** $y \in (B \cup C)$

$\Rightarrow \quad x \in A$ **and** $y \in B$ **or** $y \in C$

$\Rightarrow \quad (x \in A$ **and** $y \in B)$ **or** $(x \in A$ **and** $y \in C)$

$\Rightarrow \quad (x, y) \in (A \times B)$ **or** $(x, y) \in (A \times C)$

$\Rightarrow (x, y) \in (A \times B) \cup (A \times C)$

$\therefore$ $\mathbf{A \times (B \cup C) = (A \times B) \cup (A \times C)}$

Ex. : Show that (using laws of logic)

(a) $A \cup (A^C \cap B) = A \cup B.$

(b) $A \cap (A^C \cup B) = A \cap B.$

> **Note :** A^C means complement of A. This is another notation.

(a) $$\begin{aligned} \text{L.H.S} &= A \cup (A^c \cap B) \\ &= (A \cup A^c) \cap (A \cup B) && \text{...Distributive law} \\ &= \cup \cap (A \cup B) && \text{...Complement laws} \\ &= A \cup B \end{aligned}$$

Hence $A \cup (A^c \cap B) = A \cup B$

(b) $A \cap (A^c \cup B) = A \cap B$

$$\begin{aligned} \text{L.H.S} &= A \cap (A^c \cup B) \\ &= (A \cap A^c) \cup (A \cap B) \\ & \qquad \text{...Distributive law} \\ &= \phi \cup (A \cap B) \\ & \qquad \text{...Complement law} \\ &= A \cap B \end{aligned}$$

Hence $A \cap (A^c \cup B) = A \cap B$

Ex. : (i) Given that $A \cup B = A \cup C$, is it necessary that $B = C$?

(ii) Given that $A \cap B = A \cap C$, is it necessary that $B = C$?

Soln. :

(i) Let $A = \{1, 2, 3\}, B = \{1\}, C = \{3\}$

$A \cup B = \{1, 2, 3\} = A \cup C$

But $B \neq C$

(ii) Let $A = \{1, 2\}$

$B = \{2, 3, 4, 5\}$

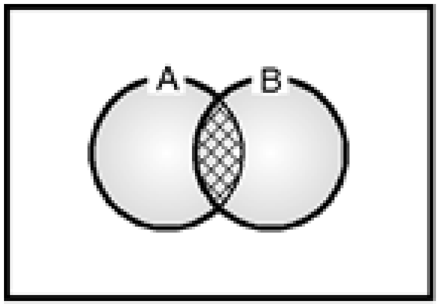
$C = \{2, 6, 7\}$

then $A \cap B = \{2\} = A \cap C$

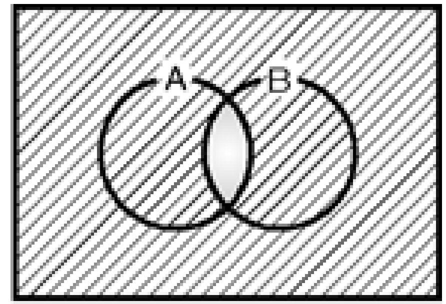
But $B \neq C$

Ex. : Use Venn diagram to illustrate De Morgan's law for sets, viz.

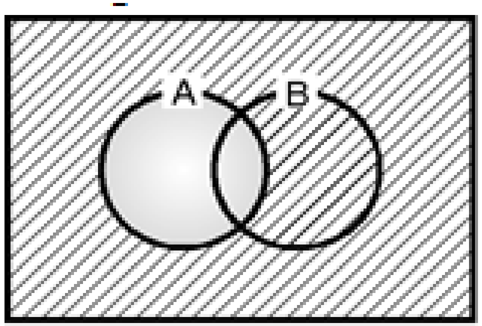
$$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$$



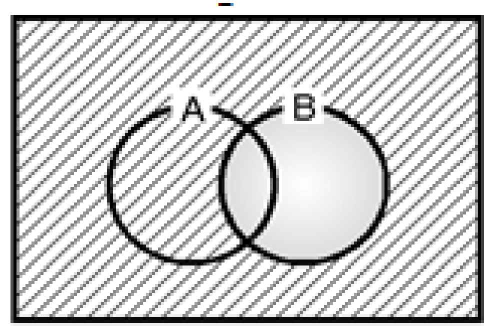
$A \cap B$



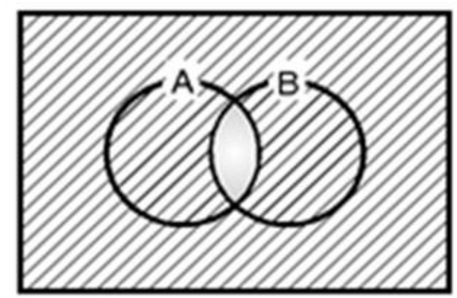
$\overline{A \cap B}$



$\bar{A}$



$\bar{B}$



$\bar{A} \cup \bar{B}$