Tuesday, November 9, 2021 2:17 PM

Prof. S. S. Kadge (SUK)

Inverse Laplace Transform. (ILT)

- To obtain I.L.T

$$X(s) = \frac{N(s)}{D(s)}$$ → Numerator poly, Denomi. poly

$N(s) > D(s)$

$$= \frac{N(s)}{(s-s_0)(s-s_1)(s-s_2)\cdots(s-s_n)}$$

↓ roots: $s_0, s_1, s_2, \ldots s_n$ → poles.

$$= \frac{K_0}{s-s_0} + \frac{K_1}{s-s_1} + \cdots + \frac{K_n}{s-s_n}$$

PFE ↗

$K_0, K_1, K_2, \ldots$ residues.

Eg: $e^{at}u(t) = \frac{1}{s-a}$ ROC: $Re(s) > a$

$-e^{+at}u(-t) = \frac{1}{s-a}$ ROC: $Re(s) < a$

① ILT using PFE

Case I → simple & real roots

Eg $X(s) = \frac{s^2+2s-2}{s(s+2)(s-3)}$ ROC : $Re(s) > 3$

ILT = ?

⇒ $X(s) = \frac{K_0}{s} + \frac{K_1}{s+2} + \frac{K_2}{s-3}$

$$K_0 = s\,X(s)\Big|_{s=0} = \cancel{s}\cdot\frac{s^2-2s-2}{\cancel{s}(s+2)(s-3)} = \frac{1}{3}$$

$$K_1 = (s+2)\,X(s)\Big|_{s=-2} = \frac{s^2-2s-2}{s(s-3)} = \frac{1}{5}$$

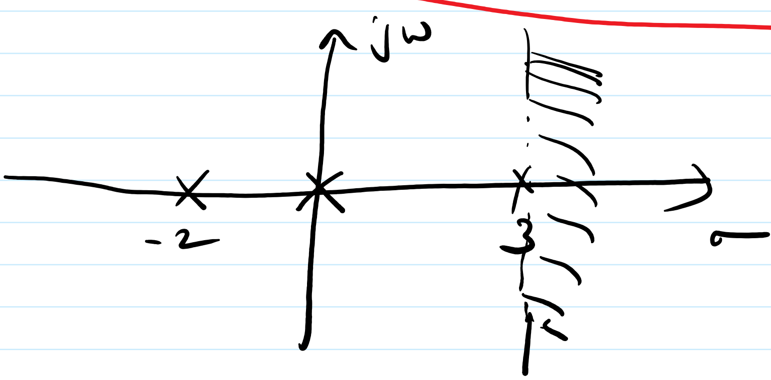
$$K_2 = (s-3)\cdot X(s)\Big|_{s=3} = \frac{s^2-2s-2}{s(s+2)} = \frac{13}{15}$$

$$X(s) = \frac{1/3}{s} + \frac{1/5}{s+2} + \frac{13/15}{s-3}$$

<u>Taking ILT</u>

$$x(t) = \mathcal{L}^{-1}\,X(s)$$

$$\boxed{x(t) = \frac{1}{3}\,u(t) + \frac{1}{5}\,e^{-2t}\,u(t) + \frac{13}{15}\,e^{3t}\,u(t)}$$



Eg $\quad X(s) = \dfrac{1}{s^2+3s+2}\,, \qquad$ RoC : $\;-2 < \mathrm{Re}(s) < -1$

RHS $+u(t)$ $\qquad$ LHS $-u(-t)$

$$\Rightarrow \quad X(s) = \frac{1}{(s+1)(s+2)}$$

$$X(s) = \frac{K_0}{(s+1)} + \frac{K_1}{(s+2)}$$

K_0 & K_1 = ?

$$K_0 = (s+1)\,X(s)\Big|_{s=-1} = \frac{1}{s+2} = \underline{\underline{1}}$$

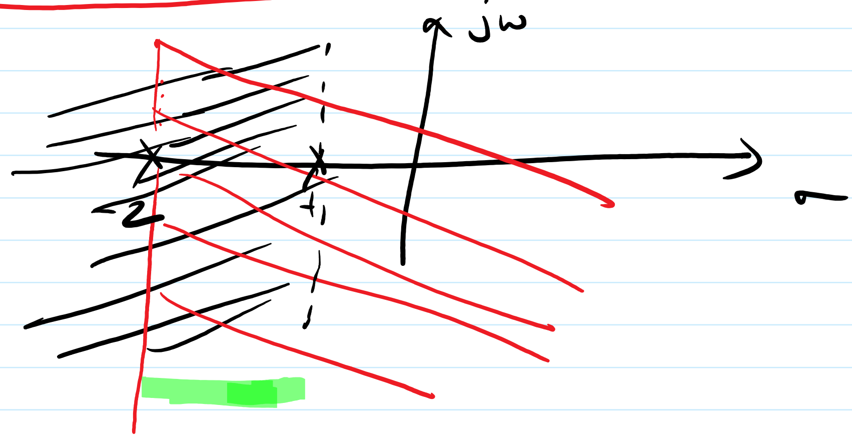
$$K_1 = (s+2)\,X(s)\Big|_{s=-2} = \frac{1}{s+1} = -1$$

$$X(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

$$\mathcal{L}^{-1}\ \frac{1}{s+2} = e^{-2t}\,\underline{\underline{u(t)}} \qquad Re(s) > -2$$

$$\mathcal{L}^{-1}\ \frac{1}{s-1} = -e^{-t}\,u(-t) \qquad \underline{\underline{Re(s) < -1}}$$

$$\boxed{x(t) = -e^{-t}\,u(-t) - e^{-2t}\,u(t)}$$



overlap $\qquad -2 < Re(s) < -1$

Eg: $X(s) = \dfrac{3s+7}{(s^2-2s-3)}$

Obtain ILT for ROC

i) $Re(s) > 3$ ✓

ii) $Re(s) < -1$ ✓

iii) $-1 < Re(s) < 3$ ✓

$\Rightarrow$ $X(s) = \dfrac{3s+7}{(s-3)(s+1)}$

$X(s) = \dfrac{K_0}{s-3} + \dfrac{K_1}{s+1}$

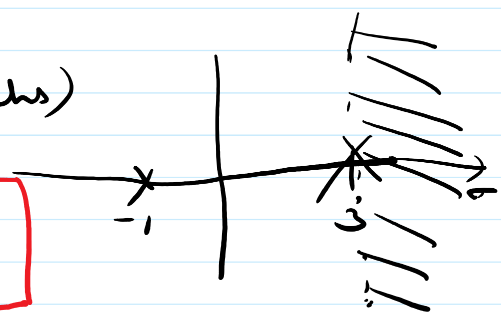
$K_0 = 4$

$K_1 = -1$

$X(s) = \dfrac{4}{s-3} - \dfrac{1}{s+1}$

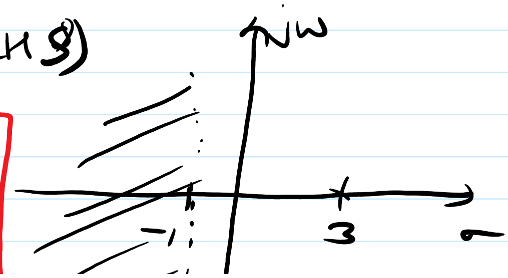
(1) ILT for $Re(s) > 3$ (RHS)

$$\boxed{x(t) = 4 \cdot e^{3t} u(t) - e^{-t} u(t)}$$

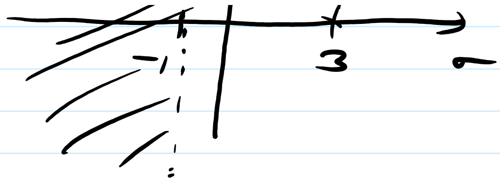


(2) ILT for $Re(s) < -1$ (LHS)

$$\boxed{x(t) = -4e^{3t}u(-t) + e^{-t}u(-t)}$$

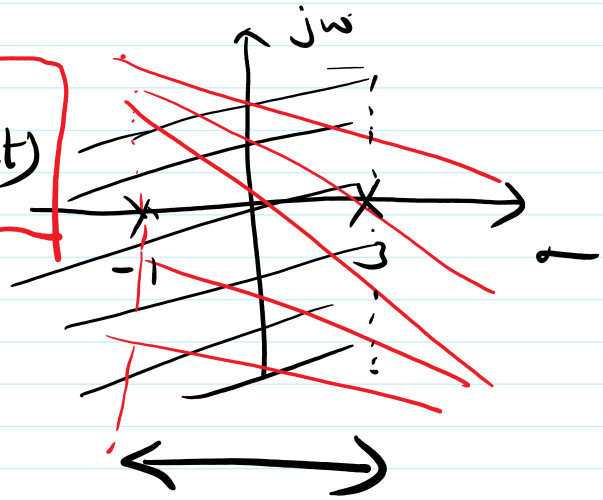


$$\boxed{x(t) = -4e\,u(-t) + e\,u(-t)}$$



③ ILT for $-1 < Re(s) < 3$

$$\boxed{x(t) = -4\cdot e^{3t}u(-t) - e^{-t}u(t)}$$



Eg: Repeated poles:-

$$X(s) = \frac{4}{(s+1)(s+2)^3}$$

$$\Rightarrow \quad X(s) = \frac{K_0}{s+1} + \frac{A_0}{(s+2)^3} + \frac{A_1}{(s+2)^2} + \frac{A_2}{(s+2)}$$

$$K_0 = (s+1)X(s)\Big|_{s=-1} = \frac{4}{(s+2)^3} = \underline{\underline{4}}$$

$$A_0 = (s+2)^3 X(s)\Big|_{s=-2} = \frac{4}{s+1}\Big|_{s=-2} = -4 /\!/$$

$$A_1 = \frac{d}{ds}\frac{4}{s+1}\Big|_{s=-2} = -\frac{4}{(s+1)^2}\Big|_{s=2} = -4$$

$$A_2 = \frac{1}{\;}\frac{d^2}{\;}\left(\frac{4}{\;}\right)\Big| = \frac{4}{\;}\Big| = 4$$

$$A_2 = \frac{1}{2}\frac{d^2}{ds^2}\left(\frac{4}{s+1}\right)\Bigg|_{s=-2} = \frac{4}{(s+1)^3}\Bigg|_{s=-2} = 4$$

$$X(s) = \frac{4}{s+1} - \frac{4}{(s+2)^3} - \frac{4}{(s+2)^2} + \frac{4}{s+2}$$

ILT

$$\boxed{x(t) = 4e^{-t}u(t) - 2t^2e^{-2t}u(t) - 4te^{-2t}u(t) - 4e^{-2t}u(t)}$$

$*$ $\quad t^n u(t) = \dfrac{n!}{s^{n+1}}$

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X(S) = 1/S(S+1)(S+2)(S+3)
FIND ILT

A = 1/6

B= -1/2

C= ½

D = 1/6

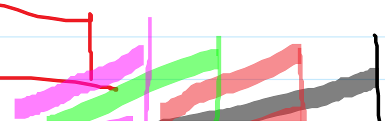
x(S) = A /S +B /(S+1)+C/(S+2) +D/(S+3)

Case 1:

ROC: RE(S) >0

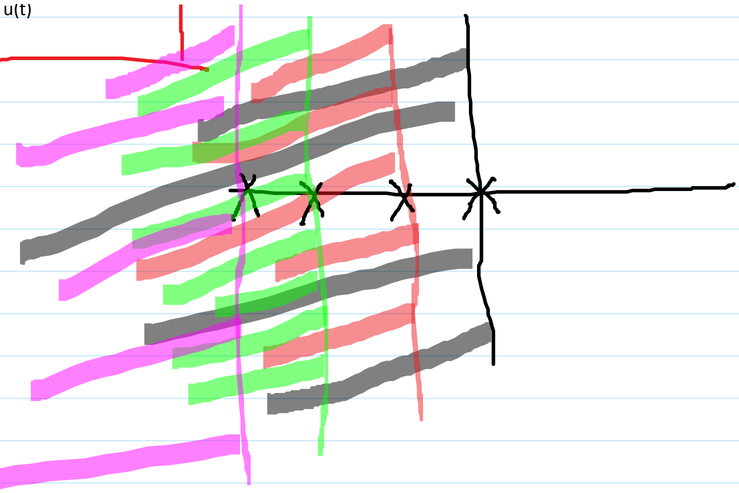
OVERLAP

$$\boxed{x(t) =[A +B e^{-t} +c e^{-2t} +D e^{-3t}] u(t)}$$



$x(t) = [A + B e^{-t} + c e^{-2t} + D e^{-3t}]\, u(t)$

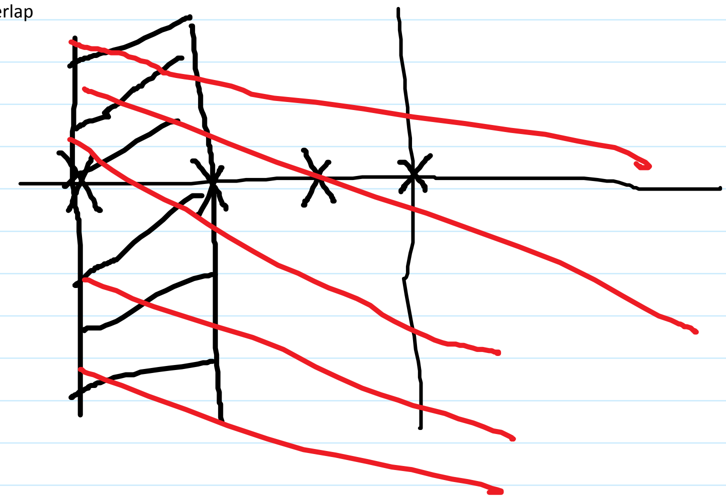
Case 2:



ROC: Re(s) <-3

$x(t) = [-A - B e^{-t} - c e^{-2t} - D e^{-3t}]\, u(-t)$

Overlap



Case 3:

Roc: -3 <Re(s) <-2

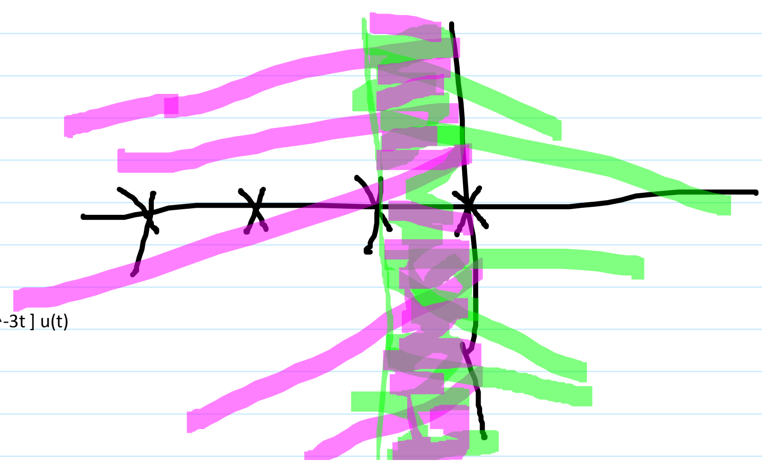
$\boxed{x(t) = [-A - B e^{-t} - c e^{-2t}]\, u(-t) + D e^{-3t} u(t)}$

Case 4:

Overlap

ROC: -2<Re(s)<-1

$x(t) = A\, u(-t) + [B e^{-t} + c e^{-2t} + D e^{-3t}]\, u(t)$



Case 5: ROC: -1< Re(s)<0

$x(t) = -A\, u(-t) + [B e^{-t} + c e^{-2t} + D e^{-3t}]\, u(t)$