Q.1. Determine whether the following functions are analytic and if so find their derivatives

(i) $z^2 + z$

(ii) $\frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\frac{y}{x}$

Q.2. Show that the functions (i) $f(z) = \bar{z}$ (ii) $f(z) = z|z|$ are not analytic

Q.3. Find the constants a, b, c, d, e if

$f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y)$ is analytic

Q.4. If $f(z) = u + iv$ is analytic in R show that $\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = |f'(z)|^2$

Q.5 Construct an analytic function whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$

Q.6 Find an analytic function whose imaginary part is $\frac{x}{x^2+y^2} + \cos hx \cos y$

Q.7 Show that there does not exist an analytic function whose real part is

$3x^2 + \sin x + y^2 + 5y + 4$

Q.8. Show that the following functions are harmonic and find the corresponding analytic function $f(z) = u + iv$ And the harmonic conjugate.

(i) $v = e^{-x}(x \cos y + y \sin y)$

(ii) $u = (x - 1)^3 - 3xy^2 + 3y^2$

Q.9 Find the orthogonally trajectories of the family of the curves $3x^2y - y^3 = c$