TWO PORT NETWORKS

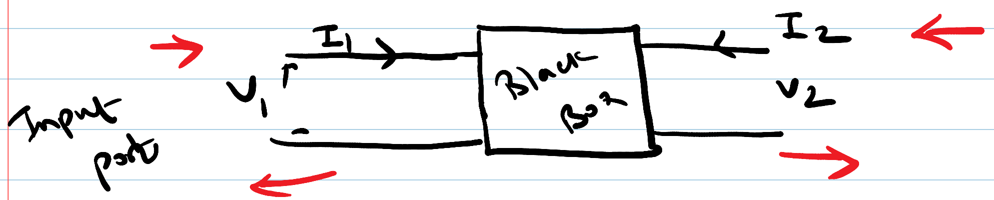
Monday, July 5, 2021
11:25 AM

Introduction :-

Eg :- BJT, MOSFETS, opamps.

Two port → Two pair of terminals

Two port N/w → is a electrical N/w with two pair of terminals to connect to external circuits.



Need:-

2 port N/w model → ① Mathematical circuit analysis technique,

② isolate portions of larger circuits

Eg → filters, transmission lines, transformers.

4 Variables → V_1, V_2, I_1, I_2. $\quad {}^4C_2 = \frac{n!}{r!(n-r)!} = \underline{6}$

2 variables $\xrightarrow{\text{expressed}}$ 2 variables

	Parameter	Express	in terms of	Equation
1)	Z	V_1, V_2	I_1, I_2	$V_1 = Z_{11}I_1 + Z_{12}I_2$ $V_2 = Z_{21}I_1 + Z_{22}I_2$
2)	Y	I_1, I_2	V_1, V_2	
3)	ABCD	V_1, I_1	V_2, I_2	

3)	ABCD	V_1, I_1	V_2, I_2
4)	A'B'C'D	V_2, I_2	V_1, I_1
5)	Hybrid	V_1, I_2	I_1, V_2
6)	Inverse hybrid	I_1, V_2	V_1, I_2

1 Z - parameters (open circuit impedance)

$$(V_1, V_2) = f(I_1, I_2)$$

zero

$$V_1 = Z_{11} I_1 + Z_{12} \underline{I_2}$$ zero

$$V_2 = Z_{21} I_1 + Z_{22} \underline{I_2}$$ zero

zero

$$V = Z I$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Case 1:- $I_2 = 0$ (O.C o/p port)

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0}$$

Z_{11} = O.C input impedance or driving point impedance with $I_2 = 0$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0}$$

Z_{21} = O.C forward transfer impedance

Case 2:- $I_1 = 0$ (I/p port is o.c)

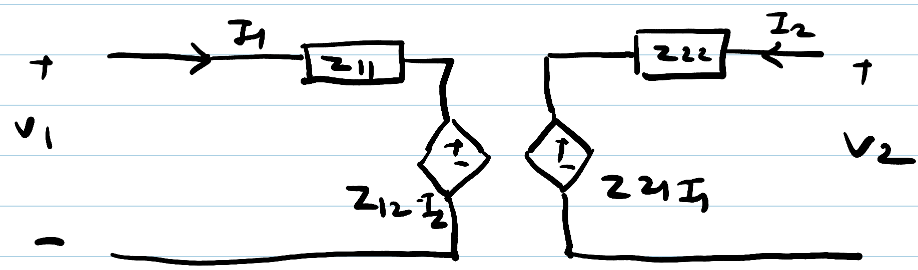
$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

Z_{12} = OC reverse transfer impedance.

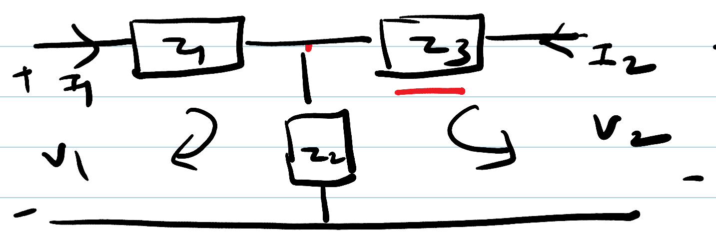
$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Z_{22} = output impedance.

Equivalent circuit of 2 port N/w



Ex. 1



$I_2 = 0$

Z parameters ?

⇒ 1st method → mesh analysis

2nd method → 2 port N/w

$I_2 = 0$

$$V_1 = (Z_1 + Z_2) I_1 \quad \text{--- (1)}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \boxed{Z_1 + Z_2}$$

$$z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = \boxed{z_1+z_2}$$

$$z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$$

$$V_2 = z_2 I_1 \quad (2)$$

$$\therefore \boxed{z_{21} = z_2}$$

$I_1 = 0$

$$V_2 = (z_2+z_3) I_2 \rightarrow (3)$$

$$V_1 = z_2 I_2 \rightarrow (4)$$

$$\boxed{z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = z_2 \; z_3}$$

$$\boxed{z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} = z_2}$$

Z - parameters are

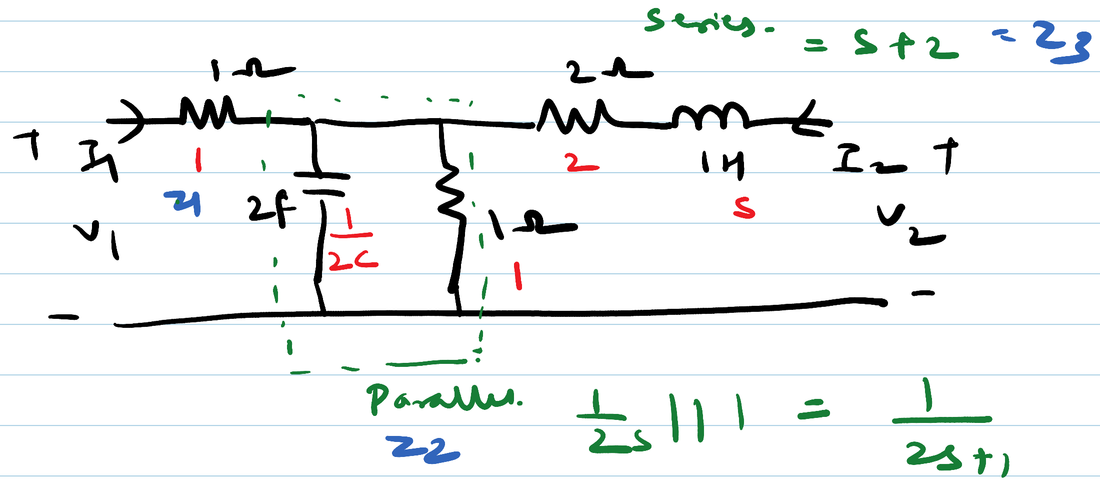
$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} z_1+z_2 & z_2 \\ z_2 & z_2+z_3 \end{bmatrix}$$

Note:- Problems are based on R, L, C.

Transformed N/W $\rightarrow$ R, LS, $\frac{1}{CS}$

Series.

Ex 2 :-



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = z_1 + z_2 = 1 + \frac{1}{2s+1}$$

$$\boxed{z_{11} = \frac{2s+2}{2s+1}}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = z_2 = \frac{1}{2s+1} \,//$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = z_2 = \frac{1}{2s+1} \,//$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = z_2 + z_3 = \frac{1}{2s+1} + s + 2$$

$$\boxed{z_{22} = \frac{2s^2 + 5s + 3}{2s+1}}$$

12/07/21
11:30 am

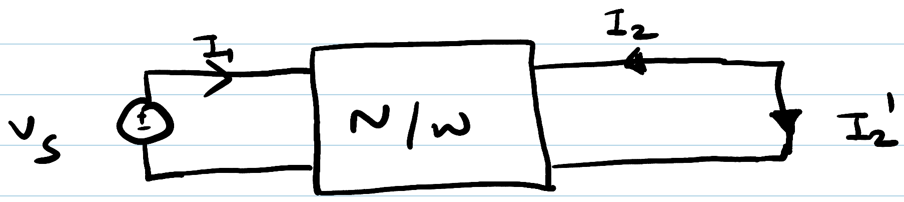
Condition of Reciprocity :-

- A N/W is said to be reciprocal if

ratio of excitation at one port to response at other port is same if excitation & response are interchanged.

Case 1 :-

$V_1 = V_S$; $V_2 = 0$ $I_2 = -I_2'$ *



$\frac{V_S}{I_2'}$?

From z parameter equations

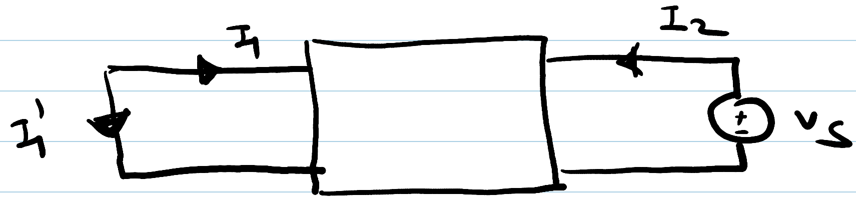
$$V_S = Z_{11} I_1 - Z_{12} I_2'$$
$$0 = Z_{21} I_1 - Z_{22} I_2'$$

$$I_1 = \frac{Z_{22}}{Z_{21}} I_2'$$

$$V_S = Z_{11} \frac{Z_{22}}{Z_{21}} I_2' - Z_{12} I_2'$$

$$\boxed{\frac{V_S}{I_2'} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}}$$

Case 2 :- $V_2 = V_S$; $V_1 = 0$ $I_1 = -I_1'$



$\frac{V_S}{I_1'} = ?$

From z parameter equations

$$0 = -z_{11} I_1' + z_{12} I_2$$

$$V_s = -z_{21} I_1' + z_{22} I_2$$

$$I_2 = \frac{z_{11}}{z_{12}} I_1'$$

$$V_s = -z_{21} I_1' + z_{22} \frac{z_{11}}{z_{12}} I_1'$$

$$\boxed{\frac{V_s}{I_1'} = \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{12}}}$$

Networks to be reciprocal

$$\frac{V_s}{I_1'} = \frac{V_s}{I_2'}$$

$$\boxed{z_{12} = z_{21}}$$

Condition for symmetry :-

The $\frac{V}{I}$ ratio at one port should be the same as $\frac{V}{I}$ ratio at other port with one of the ports o.c.

Case 1:- $I_2 = 0$ (o/p port is o.c)

$$V_S = Z_{11} I_1$$

$$\boxed{\frac{V_S}{I_1} = Z_{11}}$$

Case 2 :- $I_1 = 0$

$$V_S = Z_{22} I_2$$

$$\boxed{\frac{V_S}{I_2} = Z_{22}}$$

N/w to be Symmetrical

$$\boxed{Z_{11} = Z_{22}}$$

Monday, July 12, 2021
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Short Circuit Admittance Parameters (Y)

$$(I_1, I_2) = f(V_1, V_2)$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad V_2 = \text{zero}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad V_1 = \text{zero.}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][V]$$

Case 1:- $V_2 = 0$ (o/p port s.c)

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$ s.c input admittance or driving pt ,,

lly $$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$ s.c forward transfer admittance

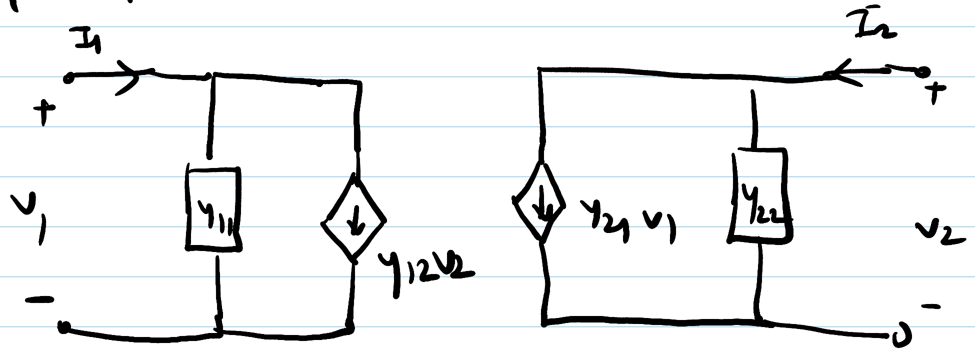
case 2:- $V_1 = 0$ (i/p port s.c)

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$ Sc reverse transfer admittance

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$ s.c o/p admittance

The equivalent circuit of 2 port N/w in terms of

The equivalent circuit of 2 port N/w in terms of Y parameter is



H.W

Condition of reciprocity

$$\frac{I_2'}{V_S} = \frac{I_1'}{V_S}$$

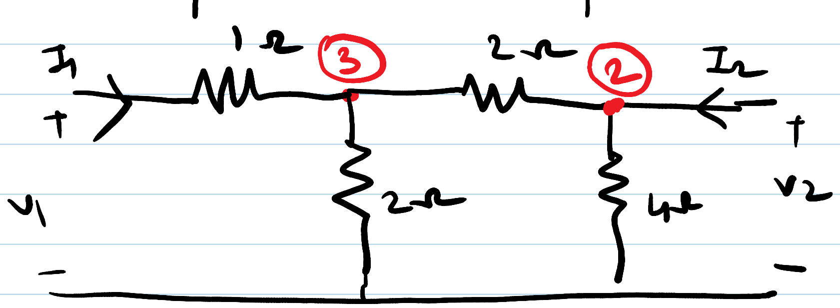
$$\boxed{Y_{12} = Y_{21}}$$

Condition of Symmetry

$$\frac{V_S}{I_1} = \frac{V_S}{I_2}$$

$$\boxed{Y_{11} = Y_{22}}$$

* Determine Y parameters & also findout whether the N/w is reciprocal & symmetrical



→ KCL at node 3 & 2

$$I_1 = \frac{V_3}{2} + \frac{V_3 - V_2}{2}$$

$$I_1 = V_3 - \frac{V_2}{2} \quad (1)$$

y para?

$I_1 = v_1, v_2$

$I_2 = v_1\ v_2$

but

$$I_2 = \frac{V_2}{4} + \frac{V_2 - V_3}{2}$$

$$I_2 = \frac{3}{4}V_2 - \frac{V_3}{2} \quad (2)$$

$$I_1 = V_1 - V_3 \quad (3)$$

Sub eqn (3) in (1)

$$V_1 - V_3 = V_3 - \frac{V_2}{2}$$

$$V_3 = \frac{V_1}{2} + \frac{V_2}{4} \quad (4)$$

Sub eqn (4) in (1) & (2)

$$I_1 = \underline{\frac{V_1}{2} + \frac{V_2}{4}} - \frac{V_2}{2}$$

$$I_1 = \underbrace{\frac{V_1}{2}}_{y_{11}} - \underbrace{\frac{V_2}{4}}_{y_{12}} \rightarrow (5)$$

✓ $I_1 = v_1, v_2$

$$\boxed{y_{11} = \frac{1}{2} \qquad y_{12} = \frac{-1}{4}}$$

$$I_2 = \frac{3}{4}V_2 - \frac{1}{2}\left(\frac{V_1}{2} + \frac{V_2}{4}\right)$$

$$I_2 = \frac{3}{4}V_2 - \frac{1}{2}\left(\frac{V_1}{2} + \frac{V_2}{4}\right)$$

$$I_2 = -\frac{V_1}{4} + \frac{5}{8}V_2$$

$$\quad\quad\quad y_{21} \quad\quad\quad y_{22}$$

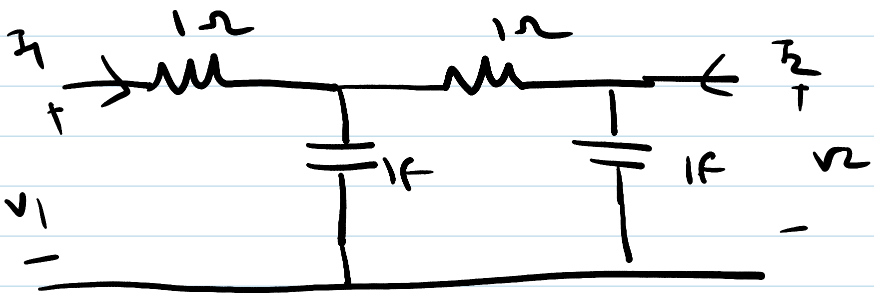
$$\boxed{y_{21} = \frac{-1}{4} \qquad y_{22} = \frac{5}{8}}$$

$$\boxed{y = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{5}{8} \end{bmatrix}}$$

$y_{11} \neq y_{22}$, Not symmetrical

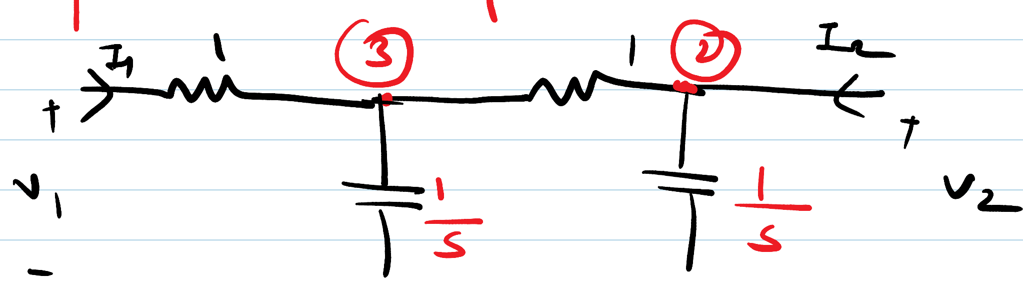
$y_{12} = y_{21}$, N/w is reciprocal

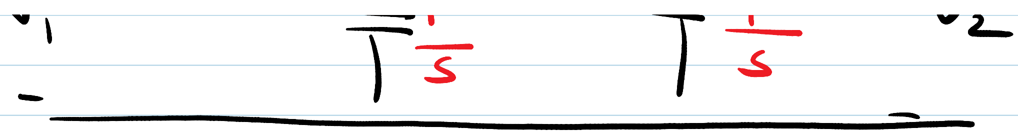
*



Y?

⇒ Transform the N/w





Apply KCL at node (3) & (2)

$$I_1 = sV_3 + V_3 - V_2$$

$$I_1 = (s+1)V_3 - V_2 \quad \text{— (1)}$$

$$I_2 = sV_2 + V_2 - V_3$$

$$= (s+1)V_2 - V_3 \quad \text{— (2)}$$

$$I_1 = V_1 - V_3 \quad \text{— (3)}$$

Sub. I_1 in eqn (1)

$$V_1 - V_3 = (s+1)V_3 - V_2$$

$$\therefore V_3 = \frac{V_1}{s+2} + \frac{V_2}{s+2} \quad (4)$$

Sub (4) in (1) & (2)

$$\boxed{Y = \begin{bmatrix} \frac{s+1}{s+2} & -\frac{1}{s+2} \\ -\frac{1}{s+2} & \frac{s^2+3s+1}{s+2} \end{bmatrix}}$$

13/07/2021
Tuesday, July 13, 2021
2:11 PM

Transmission parameters (ABCD) / chain

* $V_1, V_2 = f(I_1, I_2) \rightarrow Z \rightarrow O.C \rightarrow I = 0$
* $I_1, I_2 = f(V_1, V_2) \rightarrow Y \rightarrow S.C \rightarrow V = 0$
* $V_1, I_1 = f(V_2, -I_2)$

$$V_1 = AV_2 - BI_2 \quad I_2 = 0 \quad V_2 = 0$$

$$I_1 = CV_2 - DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$\downarrow$

transmission matrix

$\rightarrow V_2 = 0$
$I_2 = 0$

Case 1:- when o/p port is O.C $I_2 = 0$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0} \quad \text{reverse vol. gain}$$

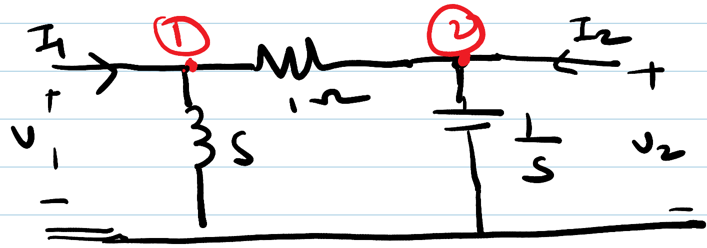
$$C = \left.\frac{I_1}{V_2}\right|_{I_2 = 0} \quad \text{transfer admittance}$$

Case 2:- When o/p port is S.C ie $V_2 = 0$

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2 = 0} \quad \text{transfer impedance}$$

$$D = -\frac{I_1}{I_2}\Big|_{V_2=0} \quad \text{reverse current gain}$$

* Determine ABCD parameters for the n/w given



$V_1 = V_2, I_2$

$I_1 = V_2, I_2$

Apply KCL at node 1 & 2

$$I_1 = \frac{V_1}{s} + V_1 - V_2$$

$V_1 = V_2, -I_2$

$I_1 = V_2, -I_2$

$$I_1 = \left(\frac{s+1}{s}\right)V_1 - V_2 \rightarrow (1)$$

$$I_2 = sV_2 + V_2 - V_1$$

$$I_2 = (s+1)V_2 - V_1 \rightarrow (2)$$

Rearrange

$$V_1 = (s+1)V_2 - I_2 \rightarrow (3)$$

Sub. eqn (3) in eqn (1)

$$I_1 = \frac{s+1}{s}\left[(s+1)V_2 - I_2\right] - V_2$$

$$I_1 = \left[\frac{(s+1)^2}{s} - 1\right]V_2 - \frac{s+1}{s}I_2 \rightarrow (4)$$

$$\boxed{\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} s+1 & +1 \\ \dfrac{s^2+s+1}{s} & \dfrac{+s+1}{s} \end{bmatrix}}$$

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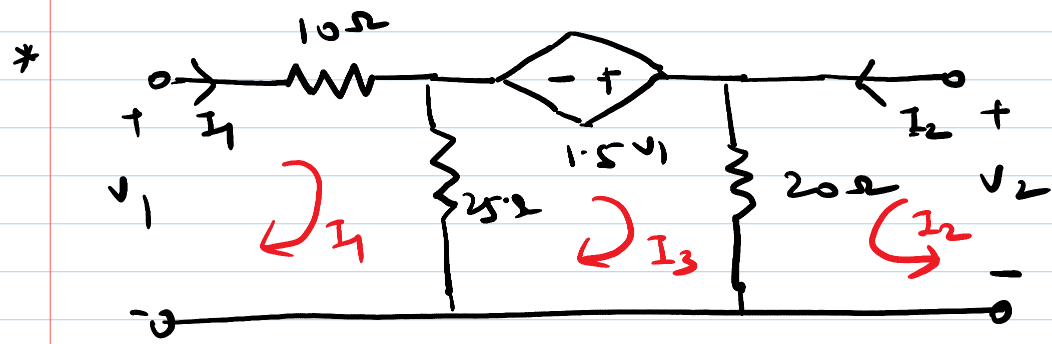


⇒ Applying KVL 1, 2, & 3.

$$V_1 = 35 I_1 - 25 I_3 \quad (1)$$

$$V_2 = 20 I_2 + 20 I_3 \quad (2)$$

$V_1 = V_2, I_2$

$I_1 = V_2, -I_2$

$$-25(I_3 - I_1) + 1.5V_1 - 20(I_2 + I_3) = 0$$

$$-25 I_3 + 25 I_1 + 1.5(35 I_1 - 25 I_3) - 20 I_2 - 20 I_3 = 0$$

$$-25 I_3 + 25 I_1 + 52.5 I_1 - 37.5 I_3 - 20 I_2 - 20 I_3 = 0$$

$$82.5 I_3 = 77.5 I_1 - 20 I_2$$

$$I_3 = 0.94 I_1 - 0.24 I_2 \quad — (3)$$

Sub eqn 3 → eqn (1)

$$V_1 = 35 I_1 - 25(0.94 I_1 - 0.24 I_2)$$

$$V_1 = 11.5 I_1 + 6 I_2 \longrightarrow (4)$$

Z

$$V_1 = 11.5 I_1 + 6 I_2 \longrightarrow (4) \quad (2)$$

Sub eqn 3 → eqn (2)

$$V_2 = 20 I_2 + 20(0.94 I_1 - 0.24 I_2) \quad (2)$$

$$V_2 = 18.8 I_1 + 15.2 I_2 \longrightarrow (5)$$

Rearrange eqn (5)

$$I_1 = 0.053 V_2 - 0.81 I_2 \quad \checkmark \; C, D$$

From eqn (4)

$$V_1 = 11.5(\underbrace{0.053 V_2 - 0.81 I_2}_{I_1}) + 6 I_2 \rightarrow (6)$$

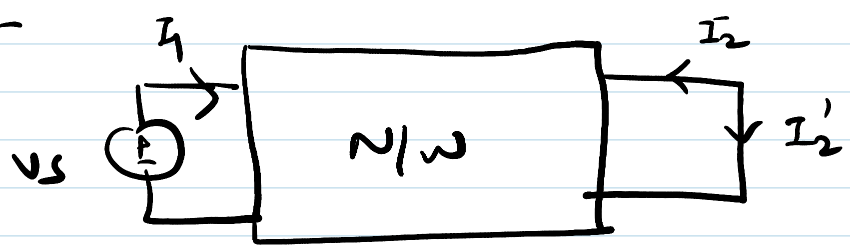
$$V_1 = f(V_2, I_2) \quad \checkmark \quad A, B.$$

$$V_1 = 0.61 V_2 - 3.3 I_2 \quad \checkmark \; A, B$$

$$\boxed{\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.61 & 3.32 \\ 0.053 & 0.81 \end{bmatrix}}$$

* Condition for Reciprocity

Case 1:-



$V_1 = V_S$

$V_2 = 0$

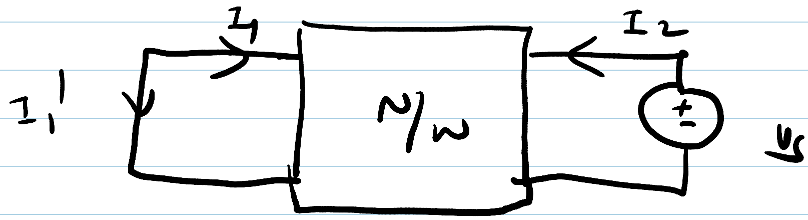
$I_2' = -I_2$

Sub. in Standard ABCD eqn.

$$V_S = B I_2'$$

$$\frac{V_S}{I_2'} = B \qquad (1)$$

Case 2 :-



$V_2 = V_S$

$V_1 = 0$

$I_1' = -I_1$

Sub in standard ABCD eqn.

$$0 = A V_S - B I_2$$

$$-I_1' = C V_S - D I_2$$

$$I_2 = \frac{A}{B} V_S$$

$$-I_1' = C V_S - \frac{AD}{B} V_S$$

$$\frac{V_S}{I_1'} = \frac{B}{AD - BC} \rightarrow (2)$$

N/w to be reciprocal,

$$\frac{V_S}{I_1'} = \frac{V_S}{I_2'}$$

$$\frac{\cancel{B}}{AD - BC} = \cancel{B}$$

$$\boxed{AD - BC = 1} \quad *$$

Condition for Symmetry :-

Condition for symmetry :-

Case 1 :- $I_2 = 0$

$$V_S = A V_2$$

$$I_1 = C V_2$$

$$\frac{V_S}{I_1} = \frac{A}{C} \quad (1)$$

Case 2 :- $I_1 = 0$

$$C V_S = D I_2 \quad \rightarrow (2)$$

$$\therefore \frac{V_S}{I_2} = \frac{D}{C}$$

N/w to be symmetrical

$$\frac{V_S}{I_1} = \frac{V_S}{I_2}$$

$$\boxed{A = D}$$

14/07/21

Wednesday, July 14, 2021
10:04 AM

Hybrid Parameters (h parameters)

$(V_1, V_2) = f(I_1, I_2) \rightarrow z$

$(I_1, I_2) = f(V_1, V_2) \rightarrow y$

$(V_1, I_1) = f(V_2, -I_2) \rightarrow ABCD$

Hybrid parameters

$(V_1, I_2) = f(I_1, V_2) \rightarrow h$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$\rightarrow$ zero
$\rightarrow$ zero.

$h_{11} = ?$ $h_{12} = ?$ ....

Case 1:- $V_2 = 0$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0}$$ S.C i/p impedance

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0}$$ S.C forward current gain

Case 2 :- $I_1 = 0$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0}$$ O.C reverse vol. gain

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} \quad \text{O.C o/p admittance}$$

Equivalent circuit :-

Condition of reciprocity :-

$$\frac{V_S}{I_2'} = \frac{V_S}{I_1'}$$

$$\boxed{h_{21} = -h_{12}}$$

Condition for symmetry ·

$$\boxed{h_{11}h_{22} - h_{12}h_{21} = 1}$$

Parameter	Condition for reciprocity	Condition for Symmetry
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
ABCD	$AD - BC = 1$	$A = D$

ABCD	$AD - BC = 1$	$A = D$
h	$h_{12} = -h_{21}$	$h_{11}h_{22} - h_{21}h_{12} = 1$

* Compute h parameters if

(a) $V_1 = 25V$, $I_1 = 1A$, $I_2 = 2A \rightarrow$ o/p port s.c

(b) $V_1 = 10V$ $V_2 = 50V$ $I_2 = 2A \rightarrow$ i/p port o.c

$$\Rightarrow \quad V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

(a) o/p port is s.c $V_2 = 0$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \underline{\underline{25\,\Omega}}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \underline{\underline{2}}$$

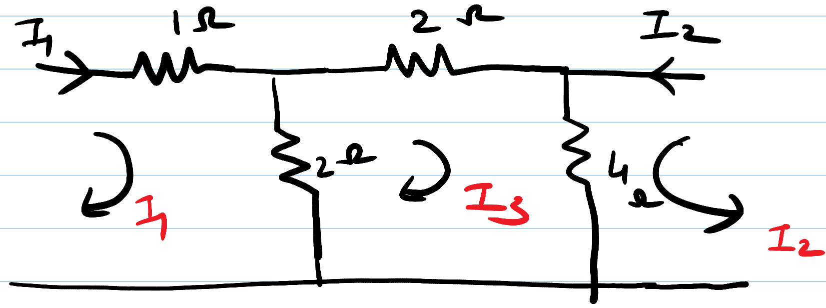
(b) i/p port is o.c $I_1 = 0$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{10}{50} = \underline{\underline{0.2}}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{2}{50} = \underline{\underline{0.04\,℧}}$$

$$\boxed{\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 25 & 0.2 \\ 2 & 0.04 \end{bmatrix}}$$

* For the given n/w determine whether n/w is reciprocal.



⇒ N/w to be reciprocal

$$\boxed{h_{12} = -h_{21}}$$

$V_1 = f(I_1, V_2)$

$I_2 = f(I_1, V_2)$

Apply KVL to mesh 1, 2 & 3

$$V_1 = 3I_1 - 2I_3 \rightarrow (1)$$

$$V_2 = 4I_2 + 4I_3 \rightarrow (2)$$

$$-2(I_3 - I_1) - 2I_3 - 4(I_3 + I_2) = 0$$

$$-8I_3 + 2I_1 - 4I_2 = 0$$

$$I_3 = \frac{I_1}{4} - \frac{I_2}{2} \rightarrow (3)$$

Sub eqn 3 in (1) & (2)

$$V_1 = 3I_1 - 2\left(\frac{I_1}{4} - \frac{I_2}{2}\right)$$

$$V_1 = \frac{5}{2}I_1 + I_2 \rightarrow (4)$$

$$V_2 = 4I_2 + \cancel{4}\left(\frac{I_1}{\cancel{4}} - \frac{I_2}{2}\right)$$

$$V_2 = I_1 + 2I_2 \rightarrow (5)$$

Rearrange eqn (5)

$$I_2 = \underbrace{-\frac{I_1}{2}}_{h_{21}} + \underbrace{\frac{1}{2}}_{h_{22}} V_2 \rightarrow (6)$$

Sub. I_2 in eqn (4)

$$V_1 = \frac{5}{2} I_1 - \frac{I_1}{2} + \frac{1}{2} V_2$$

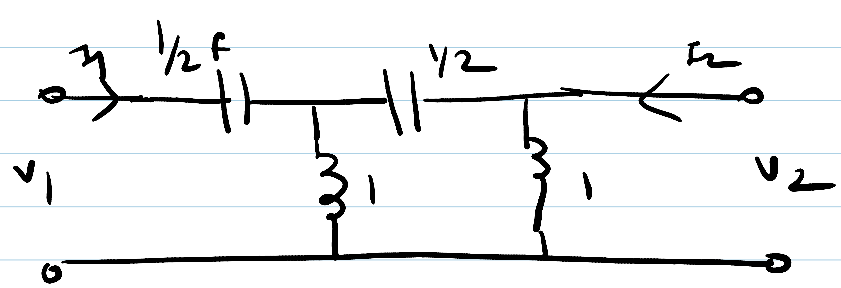
$$V_1 = \underbrace{2}_{h_{11}} I_1 + \underbrace{\frac{1}{2}}_{h_{12}} V_2 \rightarrow (6)$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

Yes, N/w is reciprocal

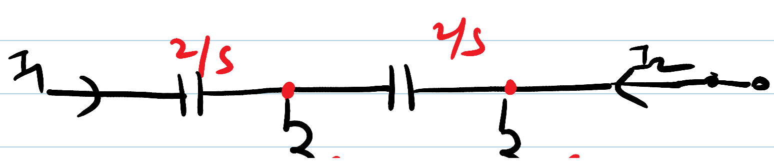
$$\boxed{h_{12} = -h_{12}}$$ Answer.

*

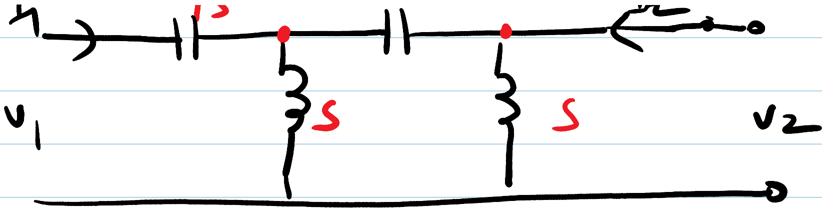


$h = ?$

$\Rightarrow$



Transformed N/w



$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{4(s^2+1)}{s(s^2+2)} & \frac{s^2}{s^2+2} \\ -\frac{s^2}{s^2+2} & \frac{2(s^2+1)}{s(s^2+2)} \end{bmatrix}$$

14/07/21 | 2:15pm

Wednesday, July 14, 2021
1:06 PM

Interrelationship between the parameters

* Z parameters in terms of other parameters

1) Z parameters in terms of Y parameters

⇒ Y parameter $(I_1, I_2) = f(V_1, V_2)$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$Z = ?$ $(V_1, V_2) = f(I_1, I_2)$

$V_1 = ?$
$V_2 = ?$

Cramer's rule

$$V_1 = \frac{\begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \frac{Y_{22} I_1 - Y_{12} I_2}{\underbrace{Y_{11} Y_{22} - Y_{12} Y_{21}}_{\Delta Y}}$$

$$V_1 = \underbrace{\frac{Y_{22}}{\Delta Y}}_{Z_{11}} I_1 - \underbrace{\frac{Y_{12}}{\Delta Y}}_{Z_{12}} I_2$$

$$\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$$

$$\boxed{Z_{11} = \frac{Y_{22}}{\Delta Y} \qquad Z_{12} = \frac{Y_{12}}{\Delta Y}}$$

$$V_2 = \frac{\begin{vmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \frac{Y_{11} I_2}{\Delta Y} - \frac{Y_{21} I_1}{\Delta Y}$$

$$V_2 = \underbrace{\frac{-Y_{21}}{\Delta Y}}_{Z_{21}} I_1 + \underbrace{\frac{Y_{11}}{\Delta Y}}_{Z_{22}} I_2$$

$$\boxed{Z_{21} = \frac{-Y_{21}}{\Delta Y} \qquad Z_{22} = \frac{Y_{11}}{\Delta Y}}$$

* **Z parameters in terms of ABCD parameters**

$\Rightarrow$ $Z = ?$

ABCD parameters

$$V_1 = AV_2 - BI_2 \quad (1)$$

$$I_1 = CV_2 - DI_2 \quad (2)$$

Z

$V_1 = I_1 \; I_2$

$V_2 = I_1 \; I_2$

Rewrite the 2nd eqn

$$V_2 = \underbrace{\frac{I_1}{C}}_{Z_{21}} + \underbrace{\frac{D}{C}}_{Z_{22}} I_2 \quad (3)$$

$$\boxed{z_{21} = \frac{1}{C} \qquad\qquad z_{22} = \frac{D}{C}}$$

$V_1 = I_1, I_2$

Sub eqn 3 in eqn 1

$$V_1 = A\left[\frac{1}{C} I_1 + \frac{D}{C} I_2\right] - B I_2$$

$$V_1 = \underbrace{\frac{A}{C}}_{z_{11}} I_1 \quad \underbrace{\frac{AD - BC}{C}}_{z_{12}} I_2$$

$$\boxed{z_{11} = \frac{A}{C} \qquad\qquad z_{12} = \frac{AD - BC}{C}}$$

3) z - parameters in terms of h parameters

$z = ?$

$V_1 = I_1, I_2$

$V_2 = I_1, I_2$

⇒ h parameters

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad (2)$$

Rewrite the 2nd eqn

$$V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \rightarrow (3)$$

$$\boxed{z_{21} = -\frac{h_{21}}{h_{22}} \qquad z_{22} = \frac{1}{h_{22}}}$$

Sub eqn 3 in eqn 1

$$V_1 = h_{11} I_1 + h_{12}\left[\frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2\right]$$

$$V_1 = h_{11} I_1 - \frac{h_{12} h_{21}}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$V_1 = \left(\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}}\right) I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$h_{11} h_{22} - h_{12} h_{21} = \Delta h$$

$$V_1 = \frac{\Delta h}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$\boxed{z_{11} = \frac{\Delta h}{h_{22}} \qquad z_{12} = \frac{h_{12}}{h_{22}}}$$

* → Y parameter → Z eqn Cramers rule

ABCD → Rewrite 1st eqn *

Sub 3 in 2

h → Rewrite 1st eqn *

Sub 3 in 2

* ABCD → Z → Rewrite 2^{nd} eqn → Sub 1^{st} eqn

Y → " " "

h → " " → "

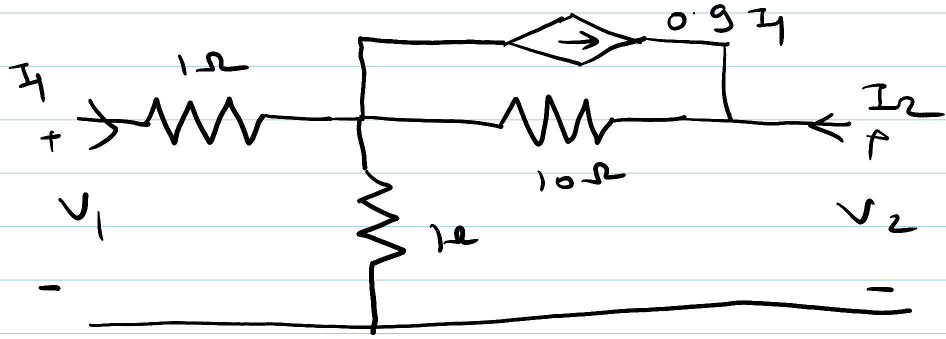
* h parameters → Z → Rewrite 2^{nd} eqn → Sub

Y → Rewrite 1^{st} eqn → Sub *

ABCD → Rewrite 2^{nd} eqn → Sub

	Y	
Z_{11} Z_{12}	$\frac{Y_{22}}{\Delta Y}$ $-\frac{Y_{12}}{\Delta Y}$	
Z_{21} Z_{22}	$\frac{-Y_{21}}{\Delta Y}$ $\frac{Y_{11}}{\Delta Y}$	
	Y_{11} Y_{12}	
	Y_{21} Y_{22}	

$*$ Find z parameters & then find Y & h parameters



$\Rightarrow$

$$V_1 = 2 I_1 + I_2 \quad — (1)$$

$$V_2 = 10 I_1 + 11 I_2 \rightarrow (2)$$

$$\boxed{\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 10 & 11 \end{bmatrix}}$$

$$\boxed{\Delta z = 12}$$

$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{11}{12}\ \mho$$

$$y_{12} = \frac{z_{12}}{\Delta z} = \frac{1}{12}\ \mho$$

$$y_{21} = \frac{-z_{21}}{\Delta z} = -5/6\ \mho$$

$$y_{22} = \frac{z_{11}}{\Delta z} = 1/6\ \mho$$

$$\boxed{\begin{matrix} \frac{11}{12} & \frac{1}{12} \\ -\frac{5}{6} & \frac{1}{6} \end{matrix}}$$

$$h_{11} = \frac{\Delta z}{z_{22}} = \frac{12}{11}\ \Omega$$

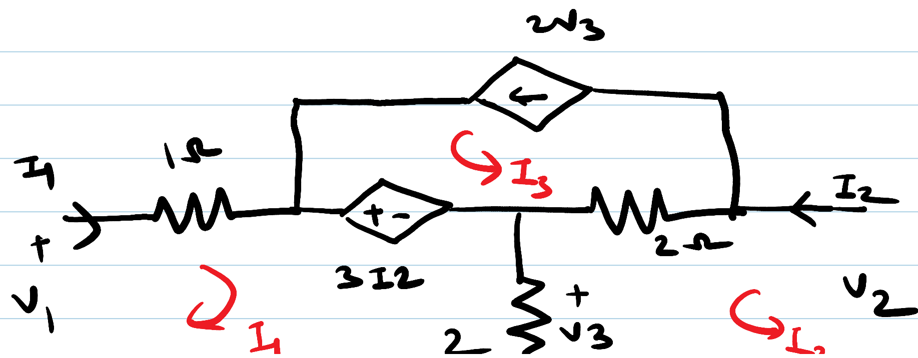
$$h_{12} = \frac{z_{12}}{z_{22}} = \frac{1}{11}$$

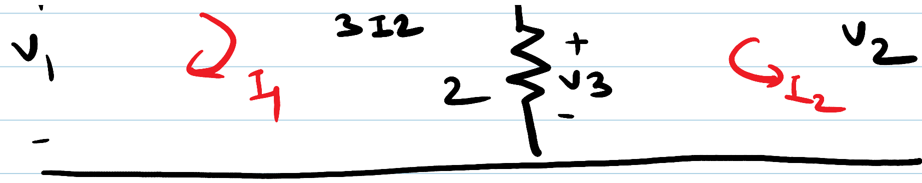
$$h_{21} = \frac{-z_{21}}{z_{22}} = -\frac{10}{11}$$

$$h_{22} = \frac{1}{z_{22}} = \frac{1}{11}\ \mho$$

$$\boxed{\begin{matrix} \frac{12}{11} & \frac{1}{11} \\ -\frac{10}{11} & \frac{1}{11} \end{matrix}}$$

*



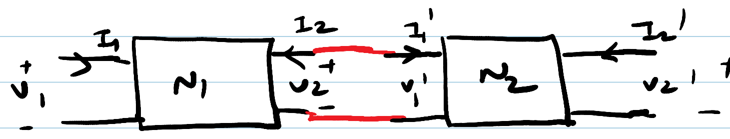


$$Z = \begin{bmatrix} 3 & 5 \\ -6 & -4 \end{bmatrix}$$

Interconnection of two port networks

- Cascade
- Parallel
- series
- series - parallel
- parallel - series

* Cascade Connection :-



$$I_1' = -I_2$$

Let A_1, B_1, C_1 & D_1 be transmission parameters of the network N_1

Let A_2, B_2, C_2 & D_2 " " N_2

N_1,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

N_2,

$$\begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$$

$V_1' = V_2$ & $I_1' = -I_2$

$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$$

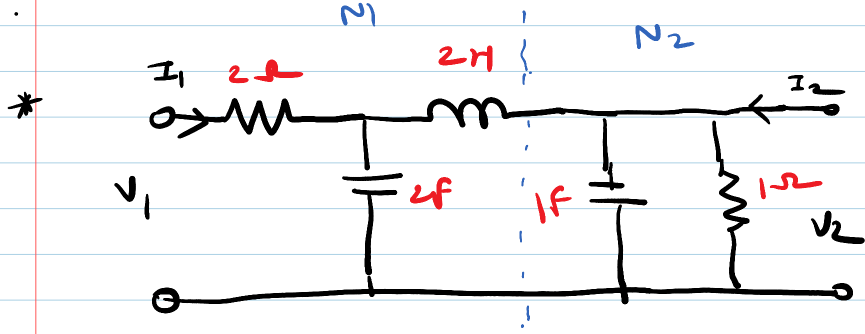
Combine.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$$

$$\begin{bmatrix} A & B \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

product of individual ABCD matrices.

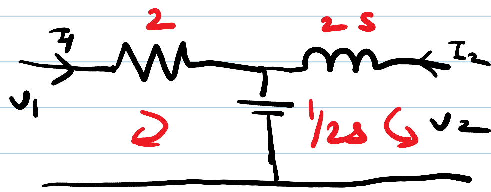


ABCD = ?

→ Cascade connection of two N/w N_1 & N_1

ABCD

Let N_1 Network
(Transformed N/w)



Apply KVL to mesh (1) & (2)

$$V_1 = \left(2 + \frac{1}{2s}\right) I_1 + \frac{1}{2s} I_2 \quad (1)$$

$$V_2 = \frac{1}{2s} I_1 + \left(2s + \frac{1}{2s}\right) I_2 \quad (2)$$

ABCD → $(V_1, I_1) \rightarrow f(V_2, -I_2)$

$V_1 = V_2, -I_2$
$I_1 = V_2, -I_2$

Rearrange eqn (2)

$$I_1 = \underbrace{2s}_{C} V_2 - \underbrace{(4s^2+1)}_{D} I_2 \rightarrow (3)$$

subs. eqn (3) in eqn (1)

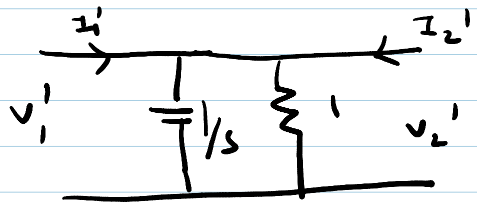
$$V_1 = \left(2 + \frac{1}{2s}\right)\left[2sV_2 - (4s^2+1) I_2\right] + \frac{1}{2s} I_2$$

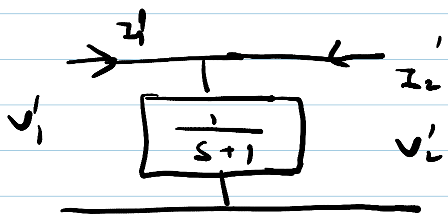
$$V_1 = \underbrace{(4s+1)}_{A} V_2 - \underbrace{(8s^2+2s+2)}_{B} I_2 \rightarrow (4)$$

$$V_1 = \underbrace{(4s+1)}_{A} V_2 - \underbrace{(8s^2+2s+2)}_{B} I_2 \rightarrow (1)$$

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 4s+1 & 8s^2+2s+2 \\ 2s & 4s^2+1 \end{bmatrix}$$

Let Network $N_2 \rightarrow A_2 B_2 C_2 D_2$





Apply KVL to mesh (1) & (2)

$$V_1' = \frac{1}{s+1} I_1' + \frac{1}{s+1} I_2' \quad (1)$$

$$V_2' = \frac{1}{s+1} I_1' + \frac{1}{s+1} I_2' \quad (2)$$

$$I_1' = (s+1) V_2' - I_2' \rightarrow (3)$$

$$V_1' = V_2' \rightarrow (4)$$

$$\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix}$$

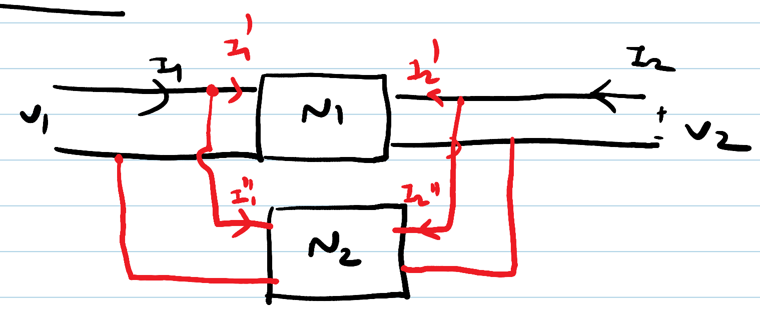
Overall ABCD

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 4s+1 & 8s^2+2s+2 \\ 2s & 4s^2+1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

$\begin{bmatrix} \ & \ \\ C & D \end{bmatrix} = \begin{bmatrix} \ \\ \ \end{bmatrix}$

Parallel connection:-



Y parameters -

$N_1 \rightarrow \quad y_{11}', \; y_{12}', \; y_{21}', \; y_{22}'$

$N_2 \rightarrow \quad y_{11}'', \; y_{12}'', \; y_{21}'', \; y_{22}''$

$N_1 \rightarrow$

$$\begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} y_{11}' & y_{12}' \\ y_{21}' & y_{22}' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

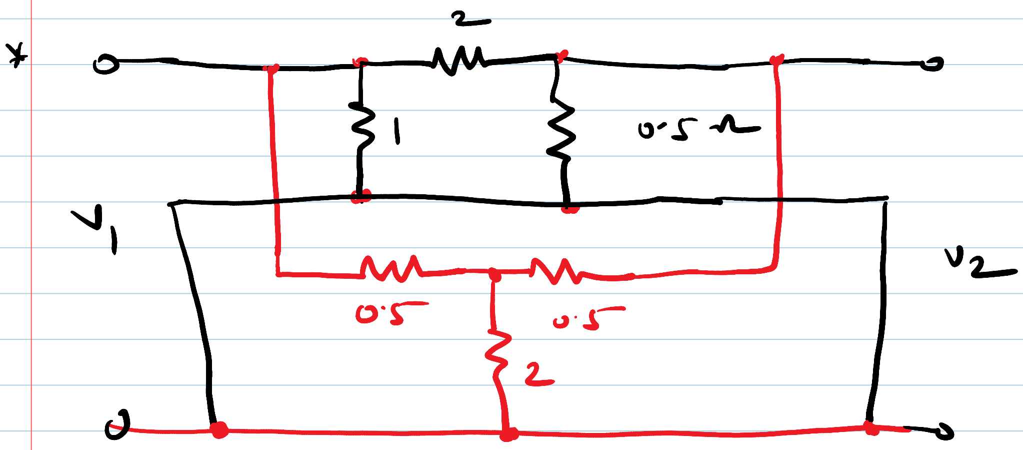
$N_2 \rightarrow$

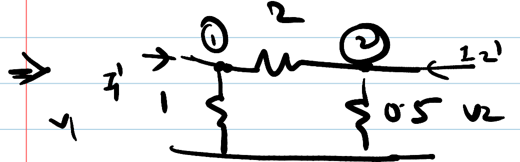
$$\begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix} = \begin{bmatrix} y_{11}'' & y_{12}'' \\ y_{21}'' & y_{22}'' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$I_1 = I_1' + I_1''$ & $I_2 = I_2' + I_2''$

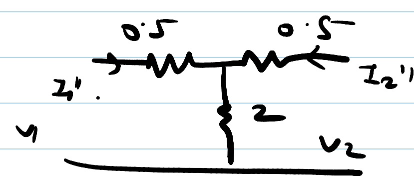
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11}' + y_{11}'' & y_{12}' + y_{12}'' \\ y_{21}' + y_{21}'' & y_{22}' + y_{22}'' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} y_{11}' + y_{11}'' & y_{12}' + y_{12}'' \\ y_{21}' + y_{21}'' & y_{22}' + y_{22}'' \end{bmatrix}$$





N1



N2

Apply KCL

$$I_1' = \frac{3}{2}V_1 - \frac{1}{2}V_2$$

$$I_2' = \frac{V_2}{0.5} + \frac{V_2 - V_1}{2}$$

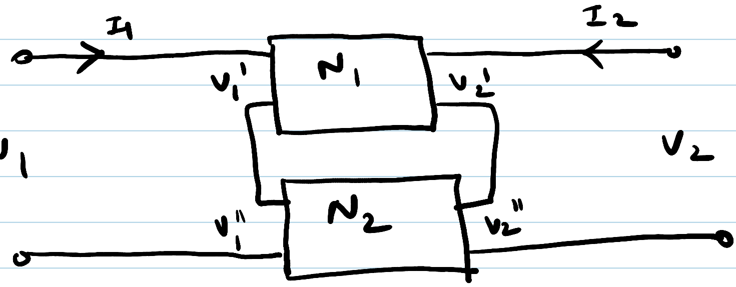
$$I_2' = -\frac{1}{2}V_1 + \frac{5}{2}V_2$$

$$[y'] = \begin{bmatrix} 3/2 & 1/2 \\ -1/2 & 5/2 \end{bmatrix} \qquad [y''] = \begin{bmatrix} 10/9 & -8/9 \\ -8/9 & 10/9 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} 3/2 + 10/9 & 1/2 - 8/9 \\ -1/2 - 8/9 & 5/2 + 10/9 \end{bmatrix} = \begin{bmatrix} 47/18 & -\frac{25}{18} \\ -25/18 & -\frac{65}{18} \end{bmatrix}$$

16/07/21

Series Connection :-



currents are same

Let z_{11}', z_{12}', z_{21}' & z_{22}' $\rightarrow$ N_1 N/w

$|||^{ly}$ z_{11}'', z_{12}'', z_{21}'' & z_{22}'' $\rightarrow$ N_2 N/w

$$\begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} z_{11}' & z_{12}' \\ z_{21}' & z_{22}' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

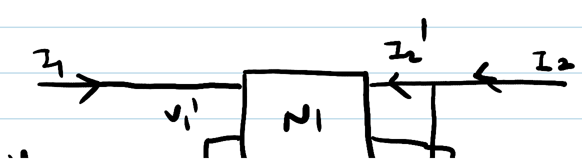
$$\begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix} = \begin{bmatrix} z_{11}'' & z_{12}'' \\ z_{21}'' & z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$V_1 = V_1' + V_1''$ $\qquad$ $V_2 = V_2' + V_2''$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11}' + z_{11}'' & z_{12}' + z_{12}'' \\ z_{21}' + z_{21}'' & z_{22}' + z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

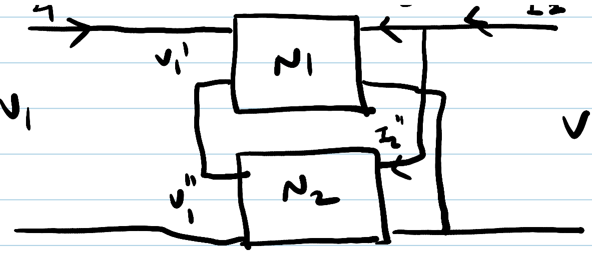
$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \text{Sum of } z \text{ matrices.}$$

Series - Parallel Connection :-



V_1, I_2

(V_1, I_2)



h-parameters. $\rightarrow$ $h_{11}', h_{12}', h_{21}', h_{22}' \rightarrow N_1$
$h_{11}'', h_{12}'', h_{21}'', h_{22}'' \rightarrow N_2$

$V_1 = V_1' + V_1''$ $\qquad I_2 = I_2' + I_2''$

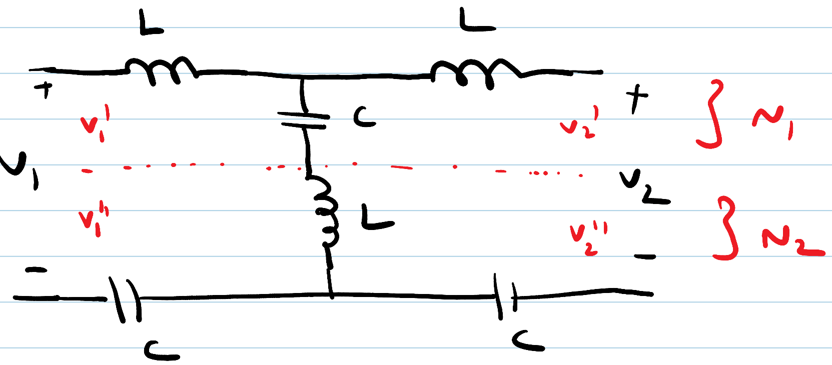
hence

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} h_{11}' + h_{11}'' & h_{12}' + h_{12}'' \\ h_{21}' + h_{21}'' & h_{22}' + h_{22}'' \end{bmatrix}}_{h \text{ parameters.}} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

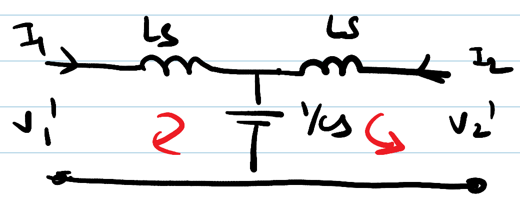
Summary :-

* Cascade connection $\rightarrow$ ABCD $\rightarrow$ product of matrices
* Parallel connection $\rightarrow$ Y $\rightarrow$ sum of matrices
* series " $\rightarrow$ Z $\rightarrow$ "
* Series - Parallel $\rightarrow$ h $\rightarrow$ "

*



$\Rightarrow$



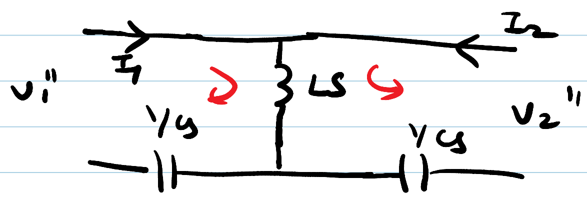
$$V_1' = \left(Ls + \frac{1}{Cs}\right) I_1 + \frac{1}{Cs} I_2 \qquad \rightarrow (1)$$

$$\left(\ldots\frac{1}{Cs}\right) I_1 + \frac{1}{Cs} I_2 \to (1)$$

$$V_2' = \left(\frac{1}{Cs}\right) I_1 + \left(Ls + \frac{1}{Cs}\right) I_2 \to (2)$$

$$z' = \begin{bmatrix} Ls + \frac{1}{Cs} & \frac{1}{Cs} \\ \frac{1}{Cs} & Ls + \frac{1}{Cs} \end{bmatrix}$$

N_2



KVL

$$V_1'' = \left(Ls + \frac{1}{Cs}\right) I_1 + Ls I_2 \to (3)$$

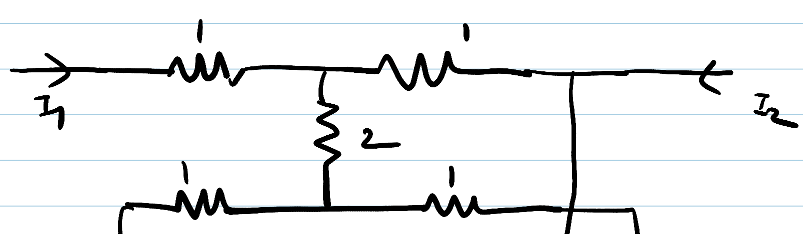
$$V_2'' = Ls I_1 + \left(Ls + \frac{1}{Cs}\right) I_2 \to (4)$$

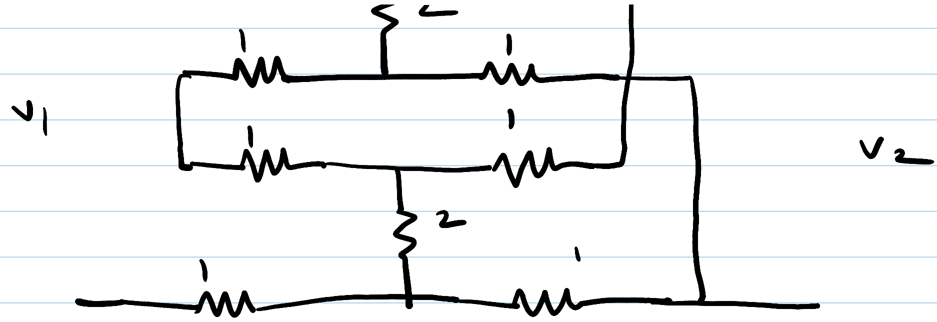
$$z'' = \begin{bmatrix} Ls + \frac{1}{Cs} & Ls \\ Ls & Ls + \frac{1}{Cs} \end{bmatrix}$$

$$z = z' + z''$$

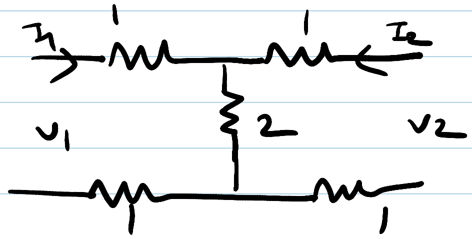
$$\boxed{\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} 2Ls + 2/Cs & Ls + 1/Cs \\ Ls + 1/Cs & 2Ls + 2/Cs \end{bmatrix} = \left(Ls + \frac{1}{Cs}\right)\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}}$$

*





$\Rightarrow N_1 \Rightarrow$



$$V_1 = 4I_1 + 2I_2 \rightarrow (1)$$

$$V_2 = 2I_1 + 4I_2 \rightarrow (2)$$

Rewrite

$$I_2 = -\frac{1}{2} I_1 + \frac{1}{4} V_2 \quad (3)$$

Sub (3) in (1)

$$V_1 = 4I_1 + 2\left(-\frac{1}{2} I_1 + \frac{1}{4} V_2\right)$$

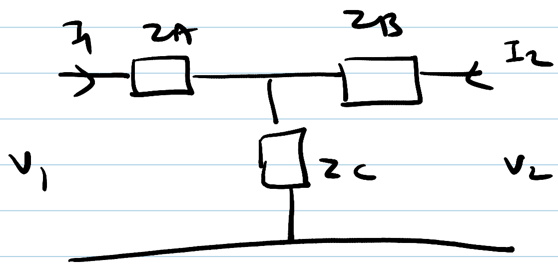
$$V_1 = 3I_1 + \frac{1}{2} V_2 \rightarrow (4)$$

$$h' = \begin{bmatrix} 3 & 1/2 \\ -1/2 & 1/4 \end{bmatrix}$$

$$h'' = \begin{bmatrix} 3 & 1/2 \\ -1/2 & 1/4 \end{bmatrix}$$

$$\boxed{h = h' + h'' = \begin{bmatrix} 6 & 1 \\ -1 & 1/2 \end{bmatrix}}$$

* T → Network :-



Apply KVL

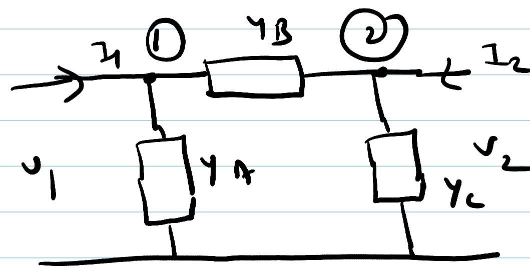
$$V_1 = (Z_A + Z_C) I_1 + Z_C I_2$$

$$V_2 = Z_C I_1 + (Z_B + Z_C) I_2$$

$$Z_{11} = Z_A + Z_C$$
$$Z_{12} = Z_C$$
$$Z_{21} = Z_C$$
$$Z_{22} = Z_B + Z_C$$

$$\boxed{\begin{aligned} Z_A &= Z_{11} - Z_{12} = Z_{11} - Z_{21} \\ Z_B &= Z_{22} - Z_{21} = Z_{22} - Z_{12} \\ Z_C &= Z_{12} = Z_{21} \end{aligned}}$$

* Pi (π) Network



Apply KCL

$$I_1 = (Y_A + Y_B) V_1 - Y_B V_2 \quad (1)$$

$$I_2 = -Y_B V_1 + (Y_B + Y_C) V_2 \rightarrow (2)$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_A + Y_B & -Y_B \\ -Y_B & Y_B + Y_C \end{bmatrix}$$

$Y_A = ?$ $\quad Y_B = ?$ $\quad Y_C = ?$

$$Y_B = -Y_{12} = -Y_{21}$$

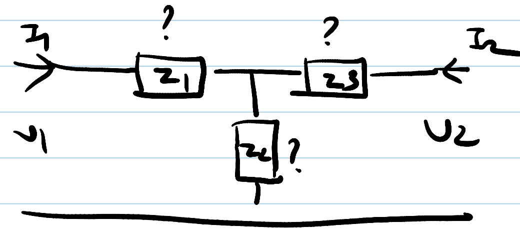
$$Y_A = Y_{11} + Y_{12} = Y_{11} + Y_{21}$$

$$Y_A = Y_{11} + Y_{12} = Y_{11} + Y_{21}$$

$$Y_C = Y_{22} + Y_{12} = Y_{22} + Y_{21}$$

* $z_{11} = 10\Omega$, $z_{12} = z_{21} = 5$, $z_{22} = 20\Omega$ find equivalen T N/w

⇒ Apply KVL or :



$z_{11} = z_1 + z_2 = 10$

$z_{12} = z_2 = 5$

$z_{21} = z_2 = 5$

$z_{22} = z_2 + z_3 = 20$

$z_1 = ?$ $z_2 = ?$ $z_3 = ?$

Solve

$z_1 = 5\Omega$

$z_2 = 5\Omega$

$z_3 = 15\Omega$