26|10|21
Tuesday, October 26, 2021 10:20 AM

Prof. S.S. Kadge (SUK)

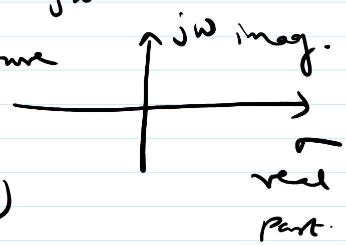
Laplace Transform (LT)

* LT → powerful mathematical tool
* used. — solve the diff. eqn.

  convert " " " into algebraic eqn.

* Simple & systematic method → complete solution.
* s domain (solution) . $s = \sigma + j\omega$

  ↳ indep. vari → complex in nature



L.T of $x(t)$ is defined as $X(s)$

$X(s) \longleftrightarrow x(t)$ Transform pair

$$X(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st}\, dt.$$

Two sided.
Bilateral.
double side

$$X(s) = \int_{0}^{\infty} x(t)\, e^{-st}\, dt \quad (1)$$

one sided.
unilateral.

ILT,

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(s)\, e^{st}\, ds \quad (2)$$

* Relationship between F.T & L.T.

(3) F.T, $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

① diff. eqn.
② doesnt many sig.
③ analyse unstable

(4) L.T $X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$

L.T. $\longrightarrow$ alternative F.T. (conv. factor σ along $j\omega$)

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-(\sigma + j\omega)t}\, dt .$$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-\sigma t} \cdot e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \underline{\left\{ x(t)\, e^{-\sigma t} \right\}} \cdot e^{-j\omega t}\, dt$$

$\sigma = 0$

L.T of $x(t)$ is basically F.T of $x(t)\, e^{-\sigma t}$

$s = j\omega$ ie $\underline{\sigma = 0}$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt .$$

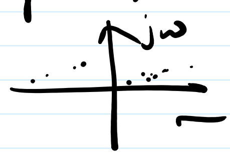
$$\boxed{X(\omega) = X(s)\Big|_{s = j\omega}}$$

* F.T is a special case of L.T.

Concept of Region of Convergence :- (ROC)

Given $x(t)$ → s/g. may not converge all values of complex var.

Most of functions → used in engg practice → L.T.



The range of values of σ for which

L·T converges is called **ROC**

* Existence of L·T

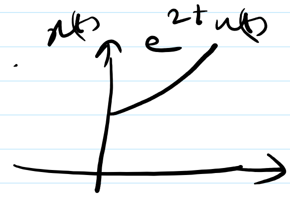
$x(t)\, e^{-\sigma t}$ must be assoulteny integrable

$X(S)$ exist only if

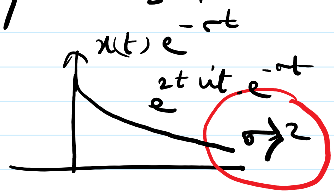
$$\int_{-\infty}^{\infty} \left| x(t) \cdot e^{-\sigma t} \right| dt < \infty$$

* Fns which are not F·T able may be L·T able.

Eg.



$\Rightarrow$



$e^{(2-\sigma)t}$

F·T → doesnot exist

L·T. exists.

Conclude:- L·T exists for s/g's for which F·T does not exist.

Advantage:-

1) s/g which are not convergent in F·T are convergent L·T
2) Convolution in time domain can be obtainted by multiplication in freq domain.
3) LTI systems can be analyzed easily L·T
   → integro diff. eqn $\xrightarrow[\text{converted}]{}$ simple algebraic eqn

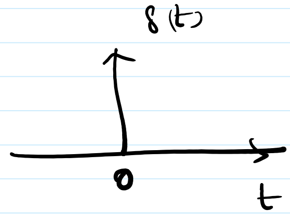
Limitations:-

1) Frequency response cannot be estimated but pole-zero plot can be drawn.

2) $s = j\omega \rightarrow$ used only sinusoidal steady state analysis.

Example

① Impulse signal



$$\delta(t) = 1 \quad t = 0$$
$$= 0 \quad t \neq 0$$

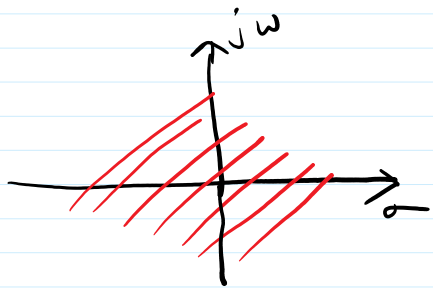
$\mathcal{L}.T = ?$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt = \int \delta(t)\, e^{-st}\, dt$$

$$= e^{-st}\Big|_{t=0} = 1$$

$$\boxed{\delta(t) \xleftrightarrow{\mathcal{L}.T} 1}$$ for all s

ROC is entire s plane.

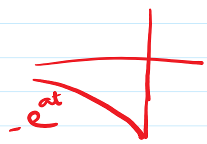


Eg 2

Determine the $\mathcal{L}.T$ & ROC for the sig

$$x(t) = -e^{at}\, u(-t)$$

$$\Rightarrow \quad X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$



$$= \int_{-\infty}^{0} -e^{-at}\, e^{-st}\, dt$$

$$= -\int_{-\infty}^{0} e^{-(s-a)t}\, dt$$

$$= e^{-(s-a)t}\Big|^{0}$$

$$= \left. \frac{e^{-(s-a)t}}{s-a} \right|_{-\infty}^{0}$$

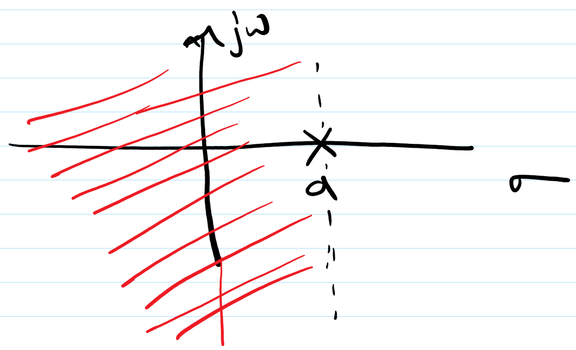
$-ve$

$$= \lim_{t \to 0} \frac{e^{-(s-a)t}}{s-a} - \lim_{t \to -\infty} \frac{e^{-(s-a)t}}{s-a}$$

$$= \frac{1}{s-a} - \frac{e^{-(s-a)(-\infty)}}{s-a}$$

$$= \frac{1}{s-a} - \frac{0}{s-a} \qquad 0 \quad (s-a) < 0$$

$$\boxed{X(s) = \frac{1}{s-a}} \quad \text{for } s-a<0$$
$$\text{or } \underline{s<a}$$
$$\text{or } \sigma < a$$



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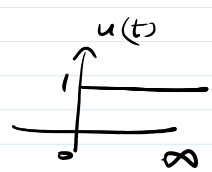
2:15 pm Prof. S. S. Kadge (SUK)

(3) <u>unit step</u> :- $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & o.w \end{cases}$



$\Rightarrow$ $X(s) = ?$
ROC $= ?$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int^{\infty} 1 \cdot e^{-st}\, dt$$

$$= \int_0^\infty 1 \cdot e^{-st}\, dt$$

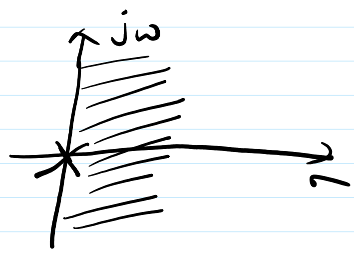
$$= \left. \frac{e^{-st}}{-s} \right|_0^\infty$$

$$= \frac{1}{s} \left[e^0 - e^{-s\infty} \right]$$

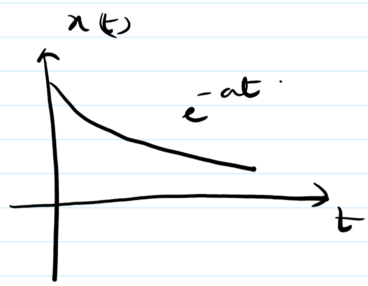
$$\boxed{X(s) = \frac{1}{s}} \qquad \boxed{u(t) \Longleftrightarrow \frac{1}{s}}$$

The ROC is

pole is at origin (x)



(h) $x(t) = e^{-at}\, u(t)$



$$\Rightarrow \quad X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_0^\infty e^{-at} \cdot e^{-st}\, dt$$

$$= \int_0^\infty e^{-(s+a)t}\, dt$$

$$= \left. \frac{e^{-(s+a)t}}{-(s+a)} \right|_0^\infty$$

$$= \frac{1}{\quad} \left[e^0 - e^{-(s+a)\infty} \right]$$

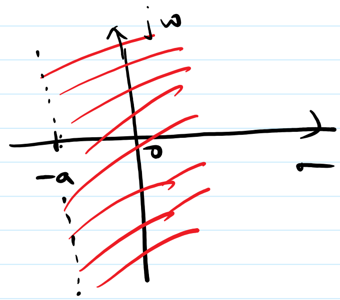
$$= \frac{1}{s+a}\left[e^{0} - e^{-(s+a)\infty}\right]$$

$$\boxed{X(s) = \frac{1}{s+a}}$$

$$e^{-at}u(t) \longleftrightarrow \frac{1}{s+a}$$

$s + a > 0 \qquad s > -a$

$$\sigma > -a$$



$*$ $x(t) = A \sin\omega t \; u(t)$

Eg $x(t) = A \cos\omega t \; u(t)$

$\Rightarrow$

$$\sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$\frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_{0}^{\infty} \left(A \cdot \left[\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right)\right) \cdot e^{-st}\, dt$$

$$= \frac{A}{2j}\int_{0}^{\infty} e^{j\omega t}\cdot e^{-st} - e^{-j\omega t}\cdot e^{-st}\, dt$$

$$= \frac{A}{2j}\left[\int e^{-(s-j\omega)t}\, dt - \int e^{-(s+j\omega)t}\, dt\right.$$

$$= \frac{A}{2j}\left[\frac{e^{-(s-j\omega)t}}{-(s-j\omega)}\Bigg|_0^\infty + \frac{e^{-(s+j\omega)t}}{-(s+j\omega)}\Bigg|_0^\infty\right]$$

$$= \frac{A}{2j}\left[\frac{e^0}{s-j\omega} - \frac{e^{-(s-j\omega)\infty}}{s-j\omega} + \left[\frac{e^0}{s+j\omega} - \frac{e^{-(s+j\omega)\infty}}{s+j\omega}\right]\right]$$

$$= \frac{A}{2j}\left[\frac{1}{s-j\omega} - 0 - \frac{1}{s+j\omega} + 0\right]$$

$$\frac{A}{2}\left[\frac{1}{s-j\omega} + \frac{1}{s+j\omega}\right]$$

$$\frac{A}{\cancel{2}}\left[\frac{\cancel{2}s}{s^2+\omega^2}\right]$$

$$= \frac{A}{\cancel{2j}}\left[\frac{\cancel{2j}\omega}{s^2+\omega^2}\right]$$

$$\boxed{X(s) = \frac{A\omega}{s^2+\omega^2}}$$

$$\boxed{X(s) = \frac{s}{s^2+\omega^2}}$$

ROC $\quad s - j\omega > 0$

$\therefore\ s > j\omega$

or

$\sigma > j\omega$

$s + j\omega > 0$

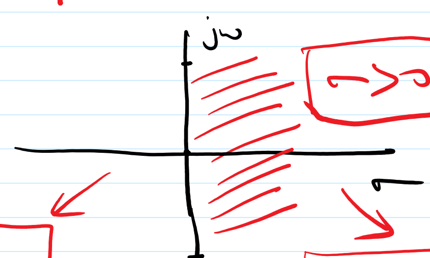
$s > -j\omega$

or

$\sigma > -j\omega$

$\sigma > (0 + j\omega)$

$\sigma > (0 - j\omega)$



Thus the combined $\boxed{\text{ROC}: \sigma > 0}$

$$\boxed{A \sin\omega t\, u(t) \longleftrightarrow \frac{A\omega}{s^2+\omega^2}}$$

$$\boxed{\cos\omega t\, u(t) \longleftrightarrow \frac{A\, s}{s^2+\omega^2}}$$

~~Eg~~ Damped sine wave :-

$$e^{-at} \sin \omega t .$$

$$\Rightarrow \quad X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-at} \left[\frac{e^{j\omega t} - e^{-j\omega t}}{2j} \right] . e^{-st}\, dt .$$

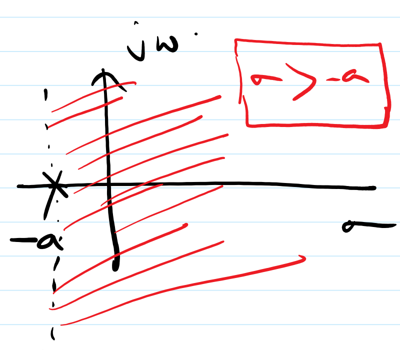
$$= \frac{1}{2j} \int_{-\infty}^{\infty} e^{-(s+a-j\omega)t}\, dt - \int_{-\infty}^{\infty} e^{-(s+a+j\omega)t}\, dt$$

$$= \frac{1}{2j} \left[\left. \frac{e^{-(s+a-j\omega)t}}{-(s+a-j\omega)} \right|_{-\infty}^{\infty} - \left. \frac{e^{-(s+a+j\omega)t}}{-(s+a+j\omega)} \right|_{-\infty}^{\infty} \right]$$

$$= \frac{1}{2j} \left[\frac{1}{s+(a-j\omega)} - \frac{1}{s+a+j\omega} \right]$$

$$X(s) = \frac{1}{2j} \frac{2j\omega}{(s+a)^2+\omega^2}$$

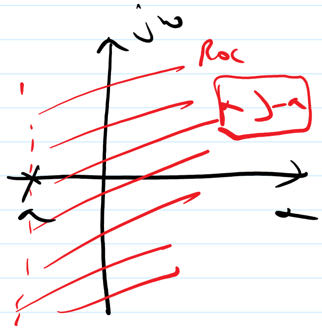
$$\boxed{X(s) = \frac{\omega}{(s+a)^2+\omega^2}}$$



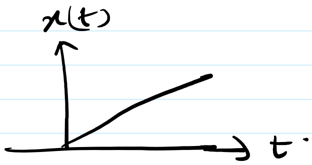
Damped Cosine wave

$$x(t) = e^{-at}\cos\omega t$$

$$\boxed{X(s) = \frac{s+a}{(s+a)^2+\omega^2}}$$



eg: **Ramp:-**



$r(t) = t$

$$\Rightarrow \quad X(s) = \int_{-\infty}^{\infty} x(t)\cdot e^{-st}\,dt$$

$$= \int_0^{\infty} \underset{u}{t}\cdot \underset{v}{e^{-st}}\,dt$$

$$X(s) = \left[\frac{e^{-st}}{s^2}(-st-1)\right]_0^{\infty}$$

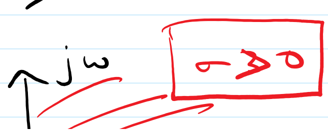
$$= -\frac{e^{-st}}{s^2}(st+1)\Big|_0^{\infty}$$

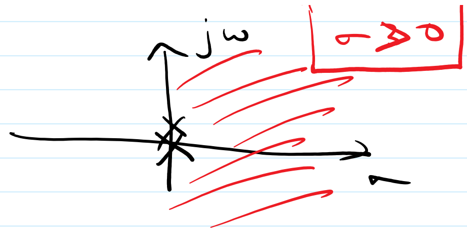
$$= -\frac{1}{s^2}\left[e^{-st}\cdot st\Big|_0^{\infty} + e^{-st}\Big|_0^{\infty}\right]$$

$$\boxed{X(s) = \frac{1}{s^2}}$$

$s > 0$

Roc $\sigma > 0$




jω
σ>0
σ

Thursday, October 28, 2021 11:26 AM

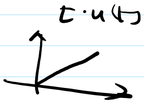
Prof. S.S. Kadge (SUK) Properties of LT.

Amplitude scaling	$A\,x(t)$	$A\,X(s)$
Linearity	$a_1x_1(t) \pm a_2\,x_2(t)$	$a_1X_1(s) \pm a_2X_2(s)$
Time differentiation	$\frac{d}{dt}x(t)$	$s\,X(s) - x(0)$
	$\frac{d^n}{dt^n}x(t)$ where n = 1, 2, 3	$s^n X(s) - \sum_{K=1}^{n} s^{n-K} \left.\frac{d^{(K-1)}x(t)}{dt^{K-1}}\right\vert_{t=0}$
Time integration	$\int x(t)\,dt$	$\frac{X(s)}{s} + \frac{\left[\int x(t)\,dt\right]_{t=0}}{s}$
	$\int .....\int x(t)\,(dt)^n$ where n = 1, 2, 3	$\frac{X(s)}{s^n} + \sum_{K=1}^{n} \frac{1}{s^{n-K+1}} \left[\int ....\int x(t)\,(dt)^k\right]_{t=0}$
Frequency shifting	$e^{\pm at}x(t)$	$X(s \mp a)$
Time shifting	$x(t \pm \alpha)$	$e^{\pm \alpha s}X(s)$
Frequency differentiation	$t\,x(t)$	$-\frac{dX(s)}{ds}$
time t.	$t^n x(t)$ where n = 1, 2, 3	$(-1)^n \frac{d^n}{ds^n}X(s)$
Frequency integration	$\frac{1}{t}x(t)$	$\int_s^{\infty} X(s)\,ds$
Time scaling	$x(at)$	$\frac{1}{\vert a\vert}X\left(\frac{s}{a}\right)$
Periodicity	$x(t + nT)$	$\frac{1}{1-e^{-sT}}\int_0^T x_1(t)\,e^{-st}\,dt$ where, $x_1(t)$ is one period of $x(t)$.
Initial value theorem	$\underset{t \to 0}{Lt}\; x(t) = x(0)$	$\underset{s \to \infty}{Lt}\; s\,X(s)$
Final value theorem	$\underset{t \to \infty}{Lt}\; x(t) = x(\infty)$	$\underset{s \to 0}{Lt}\; s\,X(s)$
Convolution theorem	$x_1(t) * x_2(t) = \int_{-\infty}^{+\infty} x_1(\lambda)\,x_2(t-\lambda)\,d\lambda$	$X_1(s)\,X_2(s)$

x(t)	X(s)	ROC
$\delta(t)$	1	Entire s-plane
$u(t)$	$\frac{1}{s}$	$\sigma > 0$
$t\,u(t)$	$\frac{1}{s^2}$	$\sigma > 0$
$\frac{t^{n-1}}{(n-1)!}u(t)$ where, n = 1, 2, 3, ...	$\frac{1}{s^n}$	$\sigma > 0$
$e^{-at}u(t)$	$\frac{1}{s+a}$	$\sigma > -a$
$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\sigma < -a$
$t^n u(t)$ where, n = 1, 2, 3	$\frac{n!}{s^{n+1}}$	$\sigma > 0$
$t\,e^{-at}u(t)$	$\frac{1}{(s+a)^2}$	$\sigma > -a$
$\frac{1}{(n-1)!}t^{n-1}e^{-at}u(t)$ where, n = 1, 2, 3, ...	$\frac{1}{(s+a)^n}$	$\sigma > -a$
$t^n e^{-at}u(t)$ where, n = 1, 2, 3, ...	$\frac{n!}{(s+a)^{n+1}}$	$\sigma > -a$
$\sin\Omega_0 t\,u(t)$	$\frac{\Omega_0}{s^2+\Omega_0^2}$	$\sigma > 0$
$\cos\Omega_0 t\,u(t)$	$\frac{s}{s^2+\Omega_0^2}$	$\sigma > 0$
$\sinh\Omega_0 t\,u(t)$	$\frac{\Omega_0}{s^2-\Omega_0^2}$	$\sigma > \Omega_0$
$\cosh\Omega_0 t\,u(t)$	$\frac{s}{s^2-\Omega_0^2}$	$\sigma > \Omega_0$
$e^{-at}\sin\Omega_0 t\,u(t)$	$\frac{\Omega_0}{(s+a)^2+\Omega_0^2}$	$\sigma > -a$
$e^{-at}\cos\Omega_0 t\,u(t)$	$\frac{s+a}{(s+a)^2+\Omega_0^2}$	$\sigma > -a$

Eg1. Find the Laplace transform of $x(t) = t \cdot e^{-3t}u(t)$ ① ②

$t \cdot u(t)$.



⇒ ① Let $x_1(t) = t \cdot u(t) = \frac{1}{s^2}$

freq. shifting property.

$e^{at}\,x(t) = \frac{1}{X(s \mp a)}$

$x(t) = x_1(t) \cdot e^{-3t} =$

$$\boxed{X(s) = \frac{1}{(s+3)^2}}$$

② Let $x_1(t) = e^{-3t}u(t)$

$x_1(s) \quad \frac{1}{s+3}$ ✓

$x(t) = t \cdot x_1(t)$.

using freq diff or times t property.

$t \cdot x(t) \longleftrightarrow \frac{d}{ds} x(s)$ ✓

$$X(s) = \frac{d}{ds} \cdot \frac{1}{s+3}$$

$$\boxed{X(s) = \frac{1}{(s+3)^2}}$$

Eg2:- Find L.T of $e^{-3t} \cos(2t) u(t)$.

method-1

$\Rightarrow$ 1 $\cos(2t) u(t) = \frac{s}{s^2+4}$ ✓

$x(t) = e^{-3t} x_1(t)$

freq. shifting property → mention ✓

$$\boxed{X(s) = \frac{s+3}{(s+3)^2+4}}$$ ✓

method 2

$x_1(t) = e^{-3t} u(t) = \frac{1}{s+3}$

$x(t) = \cos(2t) \cdot x_1(t)$

Time - cos property → $\cos(\alpha t)\, x(t) = 0.5\left[x(s+j\alpha) + x(s-j\alpha)\right]$

$$X(s) = 0.5\left[\frac{1}{s+j2+3} + \frac{1}{s-j2+3}\right]$$

$$\boxed{X(s) = \frac{s+3}{(s+3)^2+4}}$$

Eg3: $t^2 u(t-1)$

$\Rightarrow$ $u(t) \longleftrightarrow \frac{1}{s}$ ✓

$u(t)$ shifted 1 unit.

Time shifting property $x(t-a) = e^{-as} X(s)$ ✓

let $x_1(t) \leftarrow u(t-1) \longleftrightarrow \frac{e^{-s}}{s}$ ✓

$$t^2 \cdot x_1(t) = \frac{d^2}{ds^2} X_1(s)$$

$$= \frac{d^2}{ds} \frac{e^{-s}}{s} = \frac{d}{ds}\left(\frac{-e^{-s}}{s^2} - \frac{e^{-s}}{s}\right)$$

$$\boxed{X(s) = e^{-s}\left(\frac{2}{s^3} - \frac{2}{s^2} + \frac{1}{s}\right)}$$

Eg 4:→

$x_1(t) = u(t+5)$ $\quad x_2(t) = \delta(t-7)$

use conv. thm of L.T

$\Rightarrow$ Conv. therm $x_1(t) \underset{\text{conv.}}{*} x_2(t) = X_1(s) \underset{\text{multipⁿ}}{\cdot} X_2(s)$

$X_1(s) = \frac{e^{+5s}}{s}$ $\quad X_2(s) = 1 \cdot e^{-7s}$

$$X_1(s) \cdot X_2(s) = \frac{e^{5s}}{s} \cdot e^{-7s} = \frac{e^{-2s}}{s}$$

$$\boxed{X_1(s) \cdot X_2(s) = \frac{e^{-2s}}{s}}$$

$$\boxed{x_1(t) * x_2(t) = \mathcal{L}^{-1} \frac{e^{-2s}}{s} = u(t-2)}$$

Eg:- Determine the initial value & final value of the fn given

$$X(s) = \frac{5s + 50}{s(s+5)}$$

$s(s+5)$

$\Rightarrow$ $x(0)$ & $x(\infty)$?

$$\lim_{t\to 0} x(t) = x(0) = \lim_{s\to\infty} s\cdot X(s)$$

$$= \lim_{s\to\infty} \cancel{s}\cdot\frac{5s+50}{\cancel{s}(s+5)}$$

$$= \lim_{s\to\infty} \frac{5+\underbrace{\frac{50}{s}}_{\text{zero}}}{1+\underbrace{5/s}_{\text{zero}}}$$

$$= \frac{5+0}{1+0}$$

$$\boxed{x(0) = 5}$$

Final value. $x(\infty)$

$$\lim_{t\to\infty} x(t) = x(\infty) = \lim_{s\to 0} s\cdot X(s)$$

$$= \lim_{s\to 0} \cancel{s}\cdot\frac{5s+50}{\cancel{s}(s+5)}$$

$$= \frac{50}{5} = 10$$

$$\boxed{x(\infty) = 10}$$

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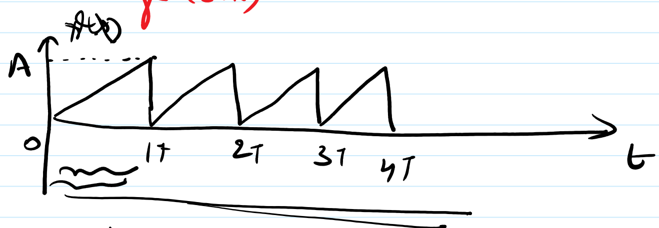
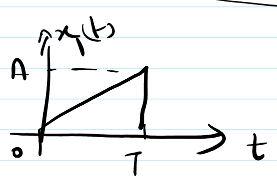
Eg.



$\Rightarrow$

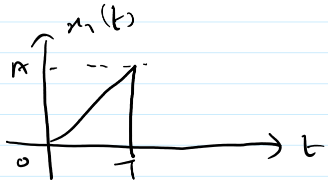


Apply periodicity property

$$x(t+nT) \longleftrightarrow \frac{1}{1-e^{-sT}}\left[X_1(s)\right]_{\text{over one period}}$$

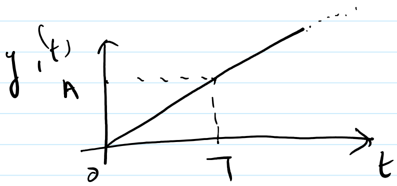
$$x(t+nT) \leftrightarrow \frac{1}{1-e^{-sT}}\left[X_1(s)\right]_{\text{over one period}}$$

$$x_1(t) \leftrightarrow X_1(s) = ?$$

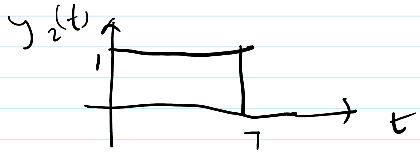


ramp fn

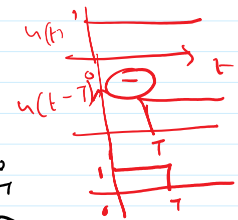
slope $\frac{A}{T}$



$$y_1(t) = \frac{A}{T} \cdot t$$



$$y_2(t) = u(t) - u(t-T)$$



$$x_1(t) = y_1(t) \cdot y_2(t) =$$

$$\mathcal{L}(x_1(t)) = \frac{A}{T} \cdot t\left[u(t) - u(t-T)\right]$$

imp pt.

$$= \mathcal{L}\left[\frac{A}{T} \cdot t\, u(t)\right] - \mathcal{L}\left[\frac{A}{T} \cdot t \cdot u(t-T)\right]$$

Rearrange

$$= \frac{A}{T} \cdot \frac{1}{s^2} - \mathcal{L}\left[\frac{A}{T}(t-T+T) \cdot u(t-T)\right]$$

$$= \text{—}'' \text{—} - \mathcal{L}\left[\frac{A}{T}(t-T)u(t-T)\right] - \mathcal{L}\left[\frac{A}{\cancel{T}}\cancel{T} \cdot u(t-T)\right]$$

no rearrangement.

$$X_1(s) = \;'' \; - \frac{A}{T} \cdot \frac{1}{s^2}e^{-sT} - A \cdot \frac{1}{s}e^{-sT}$$

$$\boxed{X_1(s) = \frac{A}{T}\frac{1}{s^2}\left[1 - e^{-sT} - T \cdot s\, e^{-sT}\right]}$$

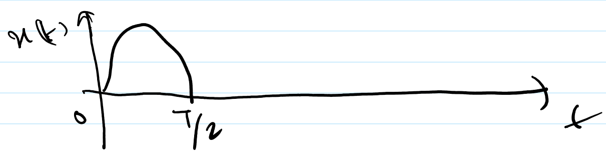
$$X(s) = \frac{1}{1-e^{-sT}} \cdot X_1(s)$$

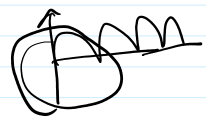
$$x(s) = \frac{1}{1-e^{-sT}} \cdot x_1(s)$$

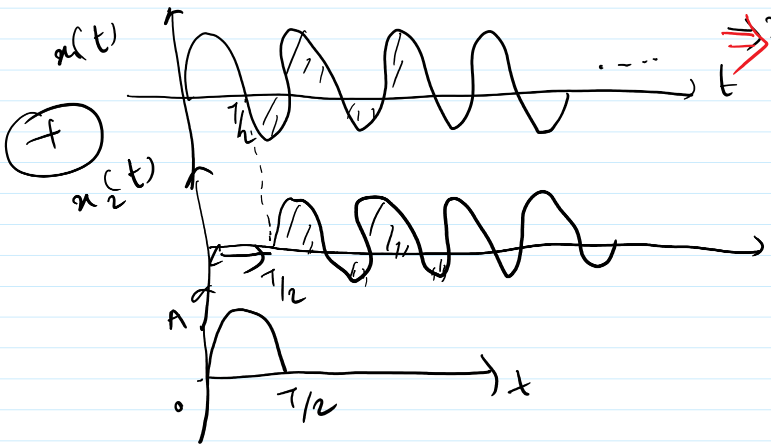
$$\boxed{X(s) = \frac{1}{1-e^{-sT}}\left[\frac{A}{T s^2}\left(1-e^{-sT} - T \cdot s e^{-sT}\right)\right]}$$

Eg: Find out L.T of half sine pulse.

$\Rightarrow$







$\Rightarrow x_1(t) = \sin \omega t \; u(t)$

or $\sin \frac{2\pi t}{T} u(t)$

$$= \frac{\omega}{s^2+\omega^2}$$

$\rightarrow x_2(t)$

$$= \sin \omega\left(t-\frac{T}{2}\right) u\left(t-\frac{T}{2}\right)$$

$$= \frac{\omega}{s^2+\omega^2} \cdot e^{-sT/2}$$

$$\mathcal{L}(x(t)) = \mathcal{L}(x_1(t)) + \mathcal{L}(x_2(t))$$

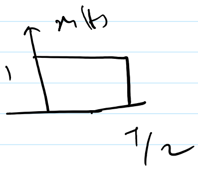
$$X(s) = \frac{A \cdot \omega}{s^2+\omega^2} + \frac{A \cdot \omega \, e^{-sT/2}}{s^2+\omega^2}$$

$$\boxed{X(s) = \frac{A\omega\left(1-e^{-sT/2}\right)}{s^2+\omega^2}}$$

full wave rectifier Conducted.

Eg:- H.W

$\Rightarrow$



$$= u(t) - u\left(t-\frac{T}{2}\right)$$

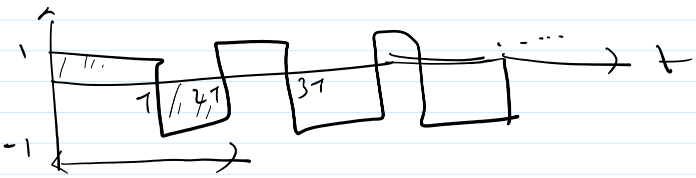
$$= \frac{1}{s} - \frac{e^{-sT/2}}{s}$$

$1 - \qquad \overline{s} \qquad \overline{s}$

$$X(s) = \frac{1}{1-e^{-sT}} \cdot X_1(s)$$

$$= \frac{1-e^{-sT/2}}{1-e^{-sT}} \cdot \frac{1}{s}$$

Eg:



1 method. periodicity property

2nd method * $x(t) = u(t) - 2u(t-T) + 2u(t-2T) - \dots$

$$= \frac{1}{s} - \frac{2}{s} e^{-sT} + \frac{2}{s} e^{-2sT} \dots$$

$$\vdots$$

$$= \frac{1}{s} \tanh\left(\frac{Ts}{2}\right)$$

3rd method $\displaystyle\int_0^T 1 \cdot e^{-sT}\,dt \int_T^{2T} (-1)\, e^{-st}\,dt$

Eg: Four functions are given to you.

↓ plot these functions

diff. each other Find out L.T

1) $f(t-t_0)$

2) $f(t-t_0)\,u(t)$

3) $f(t) \cdot u(t-t_0)$

4) $f(t-t_0)\; u(t-t_0)$

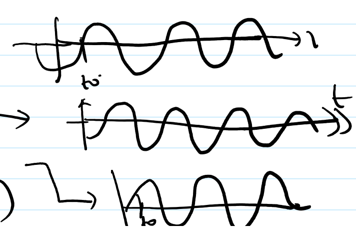
⇒ Let us take $f(t) = \sin \omega t$ then

1) $\sin\omega(t-t_0)$ ✓

2) $\sin\omega(t-t_0) \cdot u(t)$ ✓ →

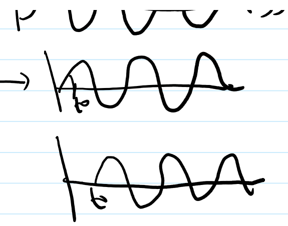
3) $\sin\omega t\; u(t-t_0)$ *

4) $\sin\omega(t-t_0)\; u(t-t_0)$ →



3) $\sin \omega t \; u(t - t_0)$ *

4) $\sin \omega (t - t_0) \; u(t - t_0)$



$\frac{\omega}{s^2+\omega^2} \cdot e^{-st_0}$

① $\sin \omega t = \frac{\omega}{s^2+\omega^2}$

$\sin \omega (t - t_0) = \, ?$

$= \mathcal{L}\left(\sin\omega t \cos\omega t_0 - \cos\omega t \sin\omega t_0 \right)$

$= \frac{\omega}{s^2+\omega^2} \cdot \cos\omega t_0 - \frac{s}{s^2+\omega^2} \cdot \sin\omega t_0$

$$\boxed{\sin \omega (t - t_0) = \frac{\omega \cos\omega t_0 - s \sin\omega t_0}{s^2+\omega^2}}$$

2) Answer is same above

3) $\sin\omega t \, u(t - t_0) = \int_{-\infty}^{\infty} \sin \omega t \; u(t - t_0) \, e^{-st} \, dt$

$= \int_{t_0}^{\infty} \sin\omega t \; e^{-st} \, dt$

$= \int_{t_0}^{\infty} \frac{e^{j\omega} - e^{-j\omega}}{2j} \cdot e^{-st} \, dt$

$= e^{-t_0 s} \left[\frac{\omega \cos\omega t_0 + \sin\omega t_0}{s^2+\omega^2} \right]$

Prof. S. S. Kadge (SUK)

Inverse Laplace Transform. (ILT)

- To obtain I.L.T

$$X(s) = \frac{N(s)}{D(s)} \rightarrow \begin{matrix}\text{Numerator poly}\\ \text{Deno. poly}\end{matrix}$$

$N(s) > D(s)$

$$= \frac{N(s)}{(s-s_0)(s-s_1)(s-s_2)\cdots(s-s_n)}$$

roots: $s_0, s_1, s_2, \dots s_n \rightarrow$ poles.

$$= \frac{K_0}{s-s_0} + \frac{K_1}{s-s_1} + \cdots + \frac{K_n}{s-s_n}$$

PFE $\nearrow$ $K_0, K_1, K_2 \dots$ residues.

Eg: $e^{at}u(t) = \frac{1}{s-a}$ ROC: $Re(s) > a$

$-e^{+at}u(-t) = \frac{1}{s-a}$ ROC: $Re(s) < a$

① ILT using PFE

Case I $\rightarrow$ simple & real roots

Eg $X(s) = \frac{s^2+2s-2}{s(s+2)(s-3)}$ ROC : $Re(s) > 3$

ILT = ?

$\Rightarrow$ $X(s) = \frac{K_0}{s} + \frac{K_1}{s+2} + \frac{K_2}{s-3}$

$$K_0 = sX(s)\Big|_{s=0} = \cancel{s}\cdot\frac{s^2-2s-2}{\cancel{s}(s+2)(s-3)} = \frac{1}{3}$$

$$K_1 = (s+2)X(s)\Big|_{s=-2} = \frac{s^2-2s-2}{s(s-3)} = \frac{1}{5}$$

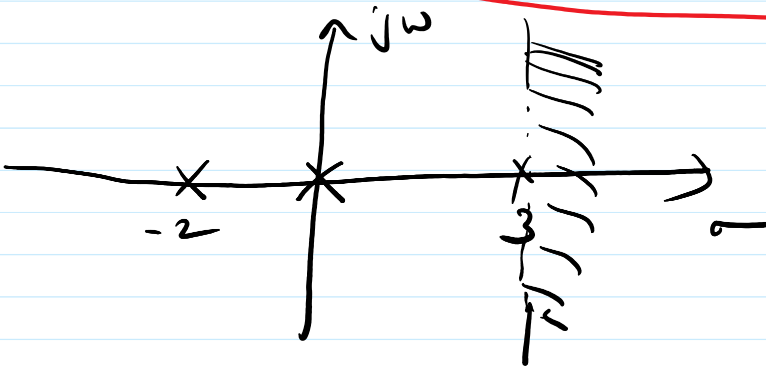
$$K_2 = (s-3)\cdot X(s)\Big|_{s=3} = \frac{s^2-2s-2}{s(s+2)} = \frac{13}{15}$$

$$X(s) = \frac{1/3}{s} + \frac{1/5}{s+2} + \frac{13/15}{s-3}$$

<u>Taking ILT</u>

$$x(t) = \mathcal{L}^{-1} X(s)$$

$$\boxed{x(t) = \frac{1}{3}u(t) + \frac{1}{5}e^{-2t}u(t) + \frac{13}{15}e^{3t}u(t)}$$



Eg $X(s) = \dfrac{1}{s^2+3s+2}$, ROC : $-2 < Re(s) < -1$

RHS $+u(t)$ LHS $-u(-t)$

$$\Rightarrow X(s) = \frac{1}{(s+1)(s+2)}$$

$$X(s) = \frac{K_0}{(s+1)} + \frac{K_1}{(s+2)}$$

K_0 & K_1 = ?

$$K_0 = (s+1)\,X(s)\Big|_{s=-1} = \frac{1}{s+2} - \underline{\underline{1}}$$

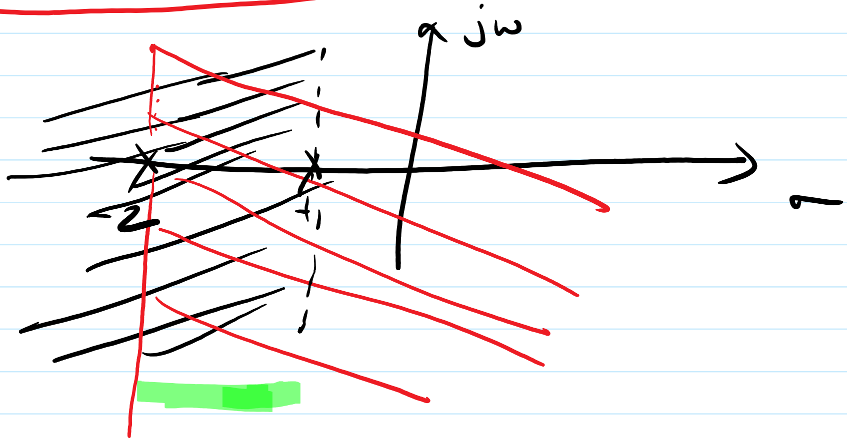
$$K_1 = (s+2)\,X(s)\Big|_{s=-2} = \frac{1}{s+1} = -1$$

$$X(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

$$\mathcal{L}^{-1}\ \frac{1}{s+2} = e^{-2t}\,\underline{\underline{u(t)}} \qquad Re(s) > -2$$

$$\mathcal{L}^{-1}\ \frac{1}{s-1} = -e^{-t}\,u(-t) \qquad \underline{\underline{Re(s) < -1}}$$

$$\boxed{x(t) = -e^{-t}\,u(-t) - e^{-2t}\,u(t)}$$



overlap

$$-2 < Re(s) < -1$$

Eg:

$$X(s) = \frac{3s+7}{(s^2-2s-3)}$$

Obtain ILT for ROC

i) $Re(s) > 3$ ✓

ii) $Re(s) < -1$ ✓

iii) $-1 < Re(s) < 3$ ✓

$$\Rightarrow \quad X(s) = \frac{3s+7}{(s-3)(s+1)}$$

$$X(s) = \frac{K_0}{s-3} + \frac{K_1}{s+1}$$

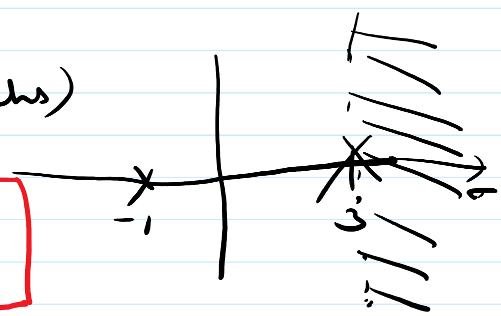
$$K_0 = 4$$

$$K_1 = -1$$

$$X(s) = \frac{4}{s-3} - \frac{1}{s+1}$$

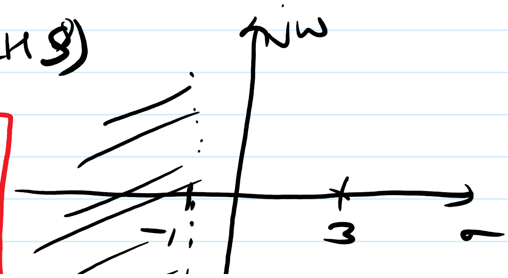
(1) ILT for $Re(s) > 3$ (RHS)

$$\boxed{x(t) = 4 \cdot e^{3t} u(t) - e^{-t} u(t)}$$

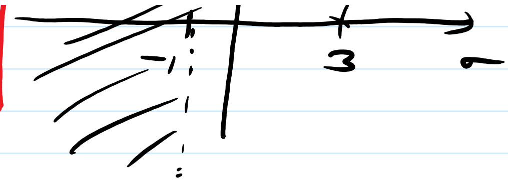


(2) ILT for $Re(s) < -1$ (LHS)

$$\boxed{x(t) = \underset{\approx}{-4} e^{3t} \underset{\sim}{u(-t)} \underset{\sim}{+} e^{-t} \underset{\sim}{u(-t)}}$$

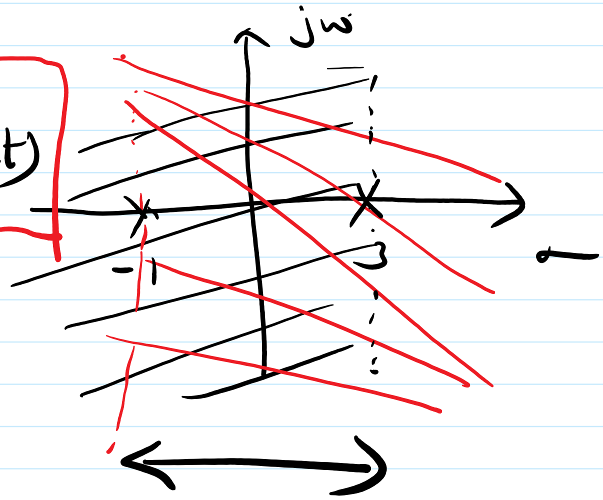


$$\boxed{x(t) = -4\,e\,u(-t) + e\,u(-t)}$$



③ ILT for $-1 < Re(s) < 3$

$$\boxed{x(t) = -4\cdot e^{3t}u(-t) - e^{-t}u(t)}$$



Eg: Repeated poles:-

$$X(s) = \frac{4}{(s+1)(s+2)^3}$$

$$\Rightarrow \quad X(s) = \frac{K_0}{s+1} + \frac{A_0}{(s+2)^3} + \frac{A_1}{(s+2)^2} + \frac{A_2}{(s+2)}$$

$$K_0 = (s+1)X(s)\Big|_{s=-1} = \frac{4}{(s+2)^3} = \underline{\underline{4}}$$

$$A_0 = (s+2)^3 X(s)\Big|_{s=-2} = \frac{4}{s+1}\Big|_{s=-2} = -4 \,//$$

$$A_1 = \frac{d}{ds}\frac{4}{s+1}\Big|_{s=-2} = -\frac{4}{(s+1)^2}\Big|_{s=-2} = -4$$

$$A_2 = \frac{1}{\cdot}\frac{d^2}{\cdot}\left(\frac{4}{\cdot}\right)\Big| = \frac{4}{\cdot}\Big| = 4$$

$$A_2 = \frac{1}{2}\frac{d^2}{ds^2}\left(\frac{4}{s+1}\right)\Bigg|_{s=-2} = \frac{4}{(s+1)^3}\Bigg|_{s=-2} = 4$$

$$X(s) = \frac{4}{s+1} - \frac{4}{(s+2)^3} - \frac{4}{(s+2)^2} + \frac{4}{s+2}$$

ILT

$$\boxed{x(t) = 4e^{-t}u(t) - 2t^2e^{-2t}u(t) - 4te^{-2t}u(t) - 4e^{-2t}u(t)}$$

$$* \quad t^n u(t) = \frac{n!}{s^{n+1}}$$

prof. S.S. Kadge (SUK)

X(S) = 1/S(S+1)(S+2)(S+3)
FIND ILT

A = 1/6

B= -1/2

C= ½

D = 1/6

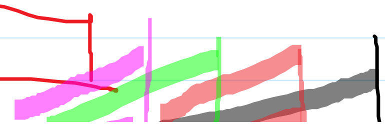
x(S) = A /S +B /(S+1)+C/(S+2) +D/(S+3)

Case 1:

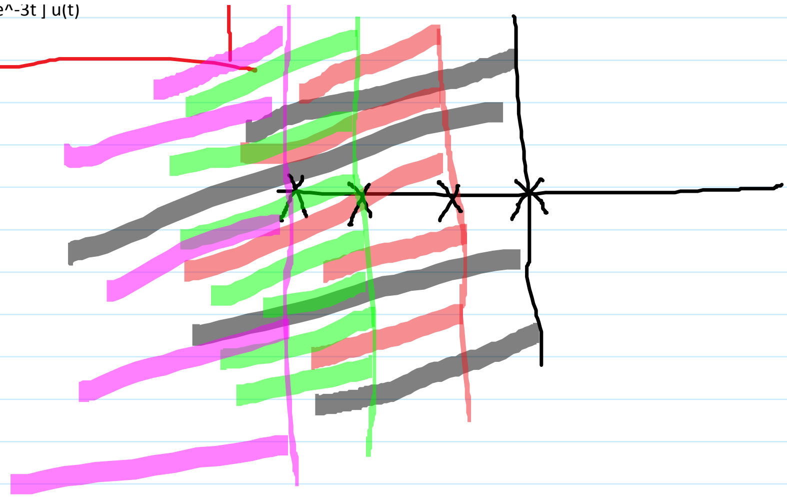
ROC: RE(S) >0

OVERLAP

$\boxed{\text{x(t) =[A +B e^-t +c e^-2t +D e^-3t] u(t)}}$



x(t) =[A +B e^-t +c e^-2t +D e^-3t] u(t)

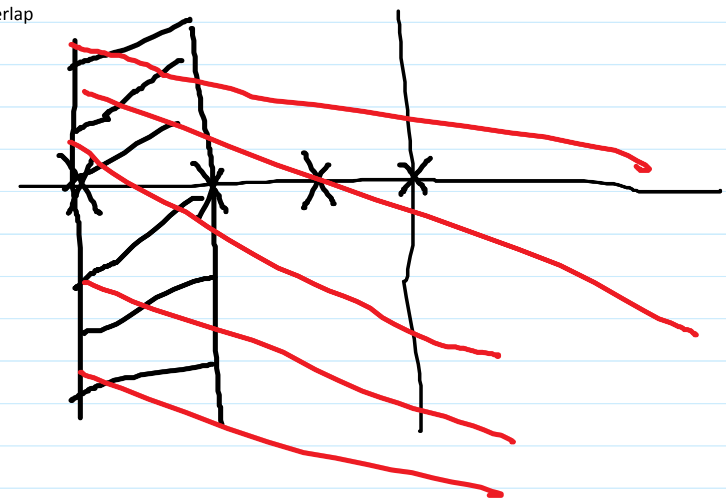


Case 2:

ROC: Re(s) <-3

x(t) =[-A -B e^-t -c e^-2t -D e^-3t] u(-t)

Overlap



Case 3:

Roc: -3 <Re(s) <-2

x(t) =[-A - B e^-t - c e^-2t] u(-t)+D e^-3tu(t)

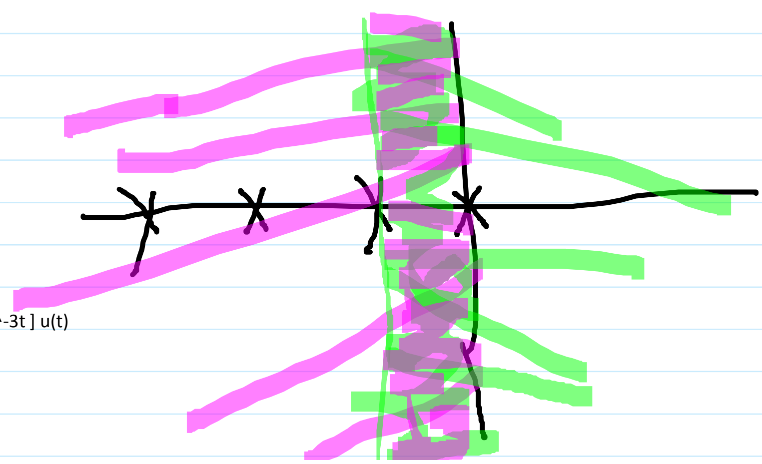
Case 4:

Overlap

ROC: -2<Re(s)<-1

x(t) =A u(-t)+[B e^-t +c e^-2t +D e^-3t] u(t)

Case 5: ROC: -1< Re(s)<0



x(t) =- A u(-t) +[B e^-t +c e^-2t +D e^-3t] u(t)