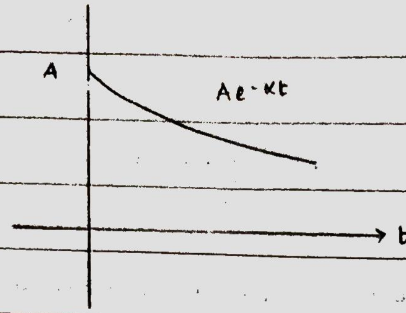
* identify if Ae^{-at} is a power or power signal.

→



it is a non-periodic signal.

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$\therefore E = \int_{0}^{\infty} (Ae^{-\alpha t})^2 dt$$

$$\therefore E = \frac{A^2}{-\alpha}\left[e^{-\alpha t}\right]_0^{\infty} \frac{1}{2} = -\frac{A^2}{2\alpha}\left[0-1\right] = \frac{A^2}{2\alpha}$$

- answer

note: since, the energy is finite; power becomes 0.

↳ energy signal

convolution (integral):

- mathematical equation

- $y(t) = \int_{-\infty}^{\infty} x(z)\, h(t-z)\, dz$

z is a dummy variable

$\therefore y(t) = x(t) * h(t)$

↑ convolution

integration is performed by taking z as a variable and t as a constant.

* convolution yields the zero state response of an LTI systems

$x(t) \longrightarrow$ LTI systems $\longrightarrow y(t)$

excitation | impulse response | response

$\therefore x(t) * h(t) = y(t)$

steps to do graphical Method:

1. replace 't' by 'τ' ⟶ obtain $x(\tau)$ $h(\tau)$

2. fold either $h(\tau)$ or $x(\tau)$

3. shift the folded signal ⟶ $h(t-\tau)$

~~4. multiply the folded signal~~

4. multiply $x(\tau)$ with $h(t-\tau)$

5. integrate product terms $x(\tau)\,h(t-\tau)$.

q: perform the convolution integral in time domain graphically of a unit step's signal with itself.

⟶ we have, $x(t) = h(t) = u(t)$

convolution integral:

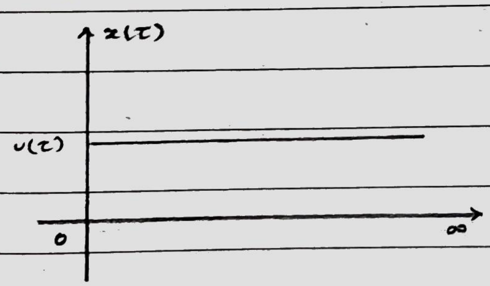
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

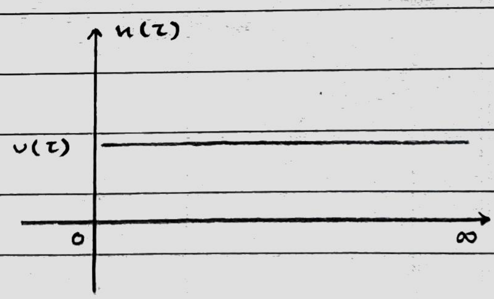
$x(t) = u(t)$ and $h(t) = u(t)$

replacing t by τ, we get,

$x(\tau) = u(\tau)$ and $h(\tau) = u(\tau)$

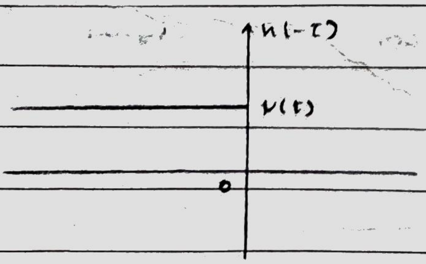
∴ we have,



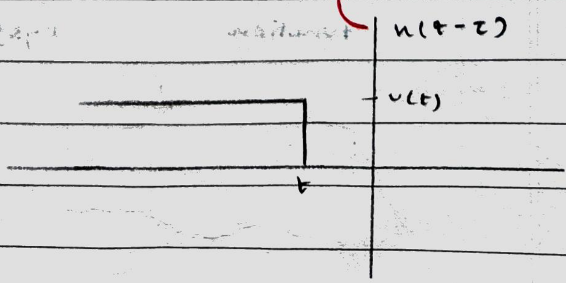


∴ cases ⟶ (0, ∞)

flipping h(τ),

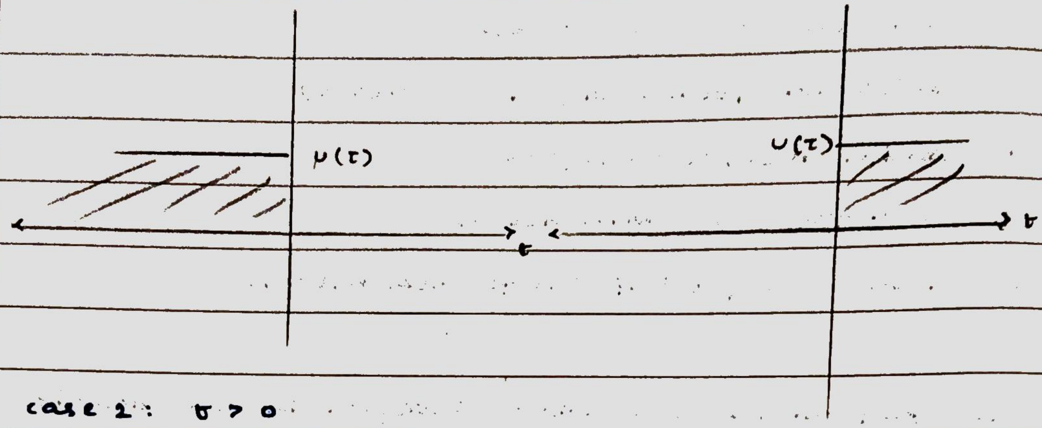


do not draw

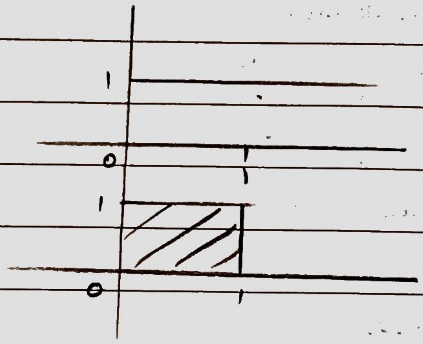


case 1 : $t < 0$

$y(t) = 0$



case 2 : $t > 0$

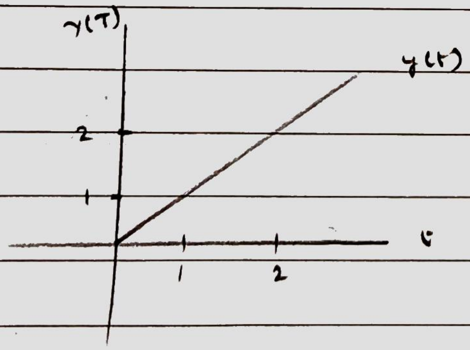


$$y(t) = \int_0^t (1)(1)\,d\tau$$

$\therefore\ y(t) = t$

final answer:

$$y(t) = \begin{cases} 0 \ , & t < 0 \\ t \ , & t > 0 \end{cases}$$



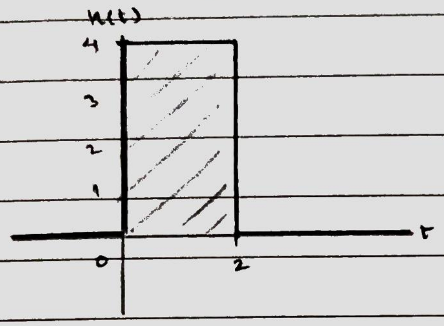
unit step function $\longrightarrow$ LTI system $\longrightarrow$ ramp signal q: perform ~~calculation of~~ convolution of :

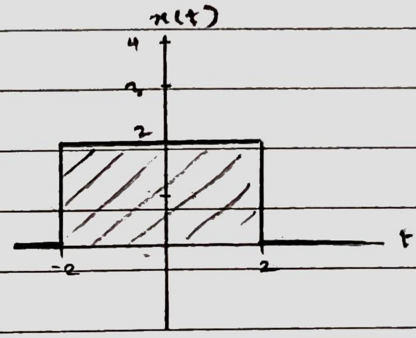
$$x(t) = \begin{cases} 2, & -2 \le t \le 2 \\ 0, & \text{elsewhere} \end{cases}$$ and

$$h(t) = \begin{cases} 4, & 0 \le t \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

→ convolution integral:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

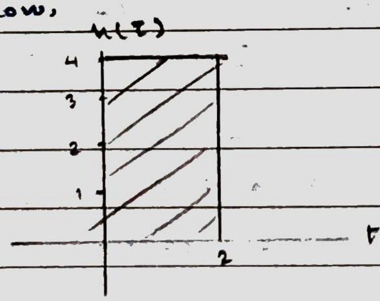


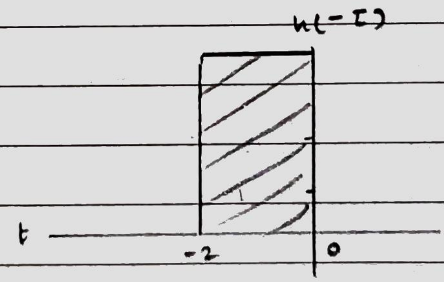


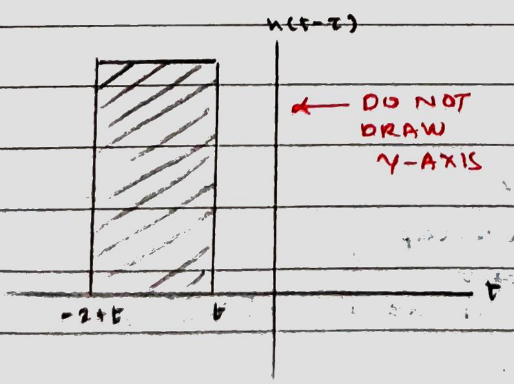
replacing t by τ, we get,

$x(t) = x(\tau)$ and $h(t) = h(\tau)$

now,







← DO NOT DRAW Y-AXIS

~~[illegible]~~

$x(t) \rightarrow \{-2, 2\}$

$h(t) \rightarrow \{0, 2\}$

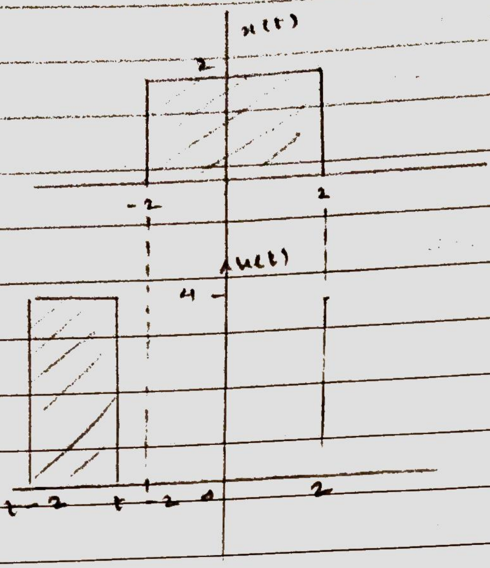
on adding,

cases: $\{-2, 0, 2, 4\}$

case 1: $t < 2$,

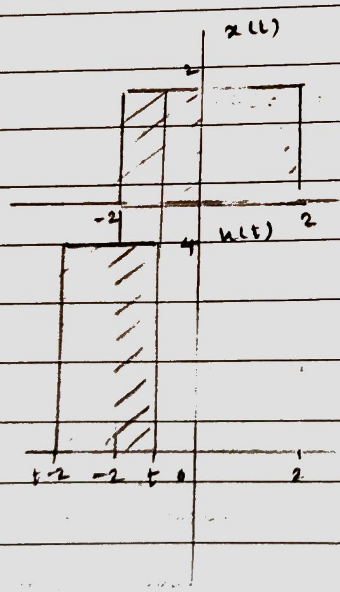
they don't overlap.

$\therefore \; y(t) = 0$



case 2: $-2 \leq t \leq 0$

$\therefore$ x(t) and h(t) overlap.



$$y(t) = \int_{-2}^{t} 2.4 \; d\tau$$

$$= 8\,[\tau]_{-2}^{t}$$

$$= 8t + 16$$

$$\therefore \; y(t) = 8(t+2)$$

case 3: $0 \leq t \leq 2$

x(t) and h(t) overlap.

$$y(t) = \int_{t-2}^{t} 2.4. \; d\tau = 8\,[\tau]_{t-2}^{t}$$

$$= 8t - 8t + 16 = 16$$

case 4: $2 \le t \le 4$

$$y(t) = \int_{t-2}^{2} 8 \, d\tau = 8[\tau]_{t-2}^{2}$$

$$= 8(2) - 8(t-2)$$

$$= 16 - 8t + 16$$

$$= 32 - 8t$$

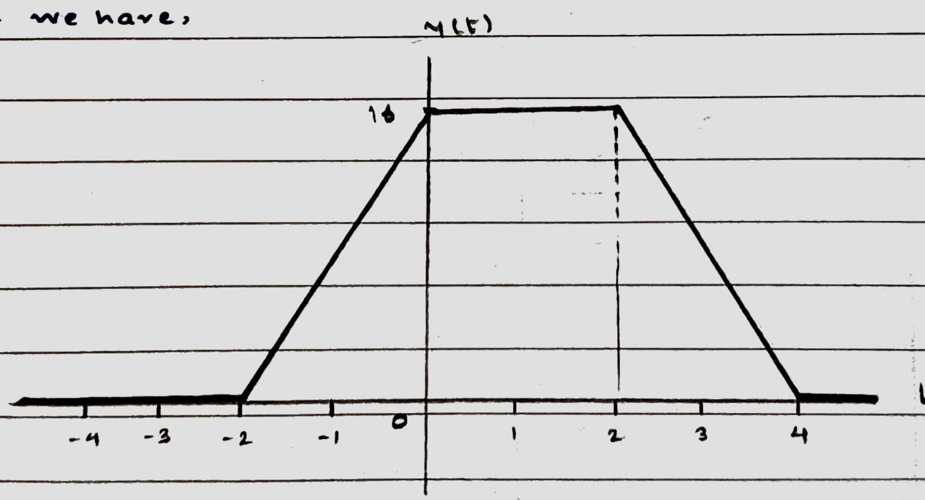
case 5: $t \geq 4$

$y(t) = 0$ – no overlap.

final figure:

$$y(t) = \begin{cases} 0, & t \geq 4 \\ 32 - 8t, & 2 \le t \le 4 \\ 16, & 0 \le t \le 2 \\ 8(t+2), & -2 \le t \le 0 \\ 0, & t \le -2 \end{cases}$$

$\therefore$ we have,



– answer

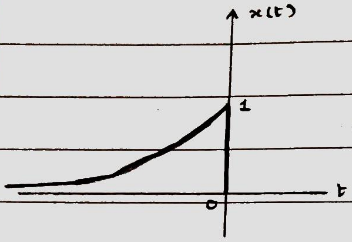
q: perform convolution if:

$x(t) = e^{t} u(-t)$, $h(t) = 2u(t) - u(t-1) - u(t-2)$

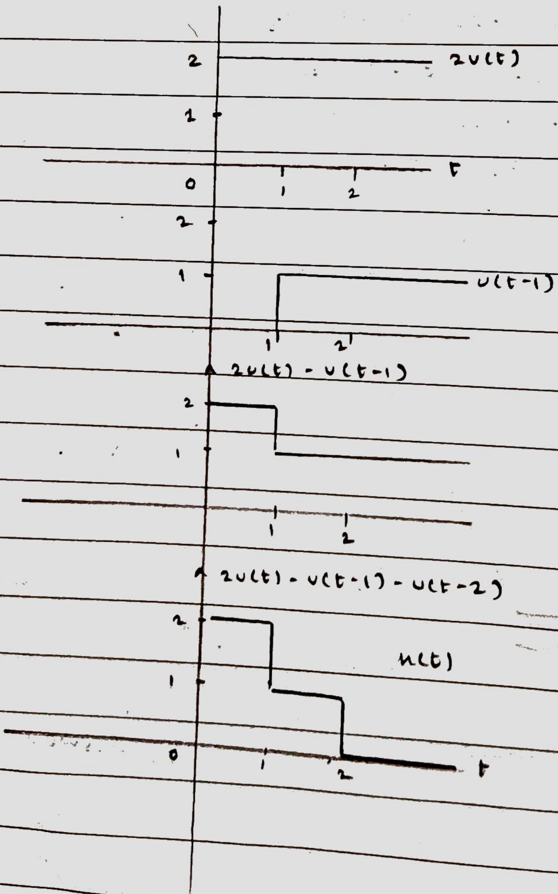
→ convolution integral:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

we have, $x(t) = e^{t} u(-t)$ =



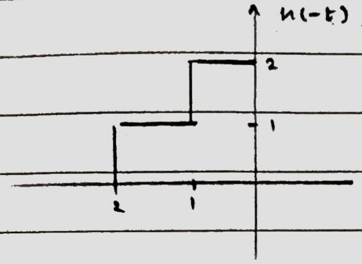
for h(t),

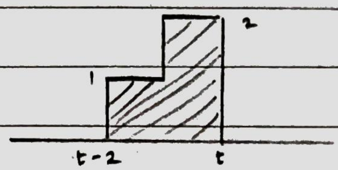


replacing t by τ,

$x(t) = x(\tau)$ and $h(t) = h(\tau)$

now,





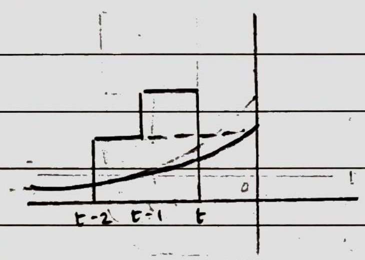
we have, $x(t) = (-\infty, 0]$

$h(t) = \{ 0, \overset{1}{\underset{\wedge}{,}} 2 \}$ → signal change.

cases : $\{-\infty, 0, 1, 2\}$

now,

case 1 : $t < 0$,

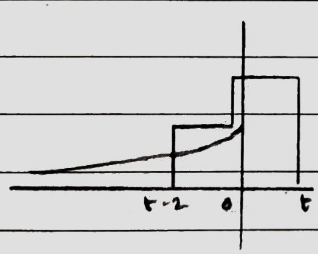


$$y(t) = \int_{t-2}^{t-1} e^{\tau}\, d\tau + \int_{t-1}^{t} 2e^{\tau}\, d\tau$$

$$= e^{t-1} - e^{t-2} + 2e^{t} - 2e^{t-1}$$

$$= 2e^{t} - e^{t-2} - e^{t-1}$$

case 2 : $0 \le t \le 1$



$$y(t) = \int_{t-2}^{t-1} e^{\tau}\, d\tau + \int_{t-1}^{0} 2e^{\tau}\, d\tau$$

$$= e^{t-1} - e^{t-2} + 2 - 2e^{t-1}$$

$$= -e^{t-1} - e^{t-2} + 2$$

case 3 : $1 \le t \le 2$

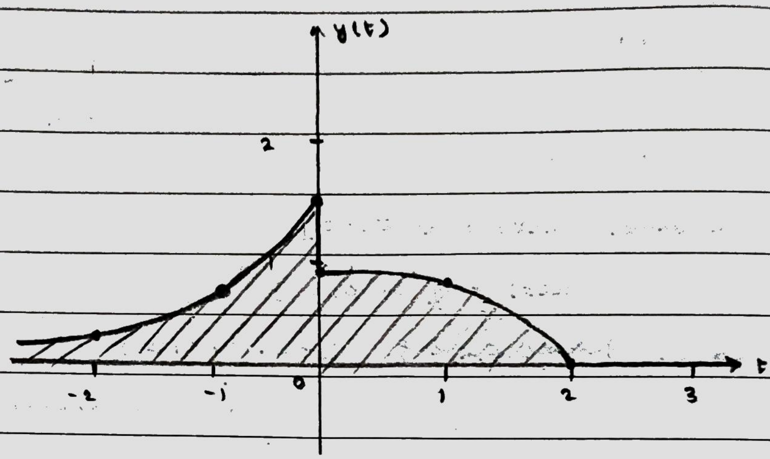
$$y(t) = \int_{t-2}^{0} e^{\tau}\, d\tau = 1 - e^{t-2}$$

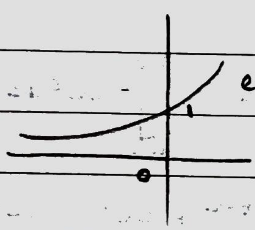
case 4 : $t \geqslant 2$

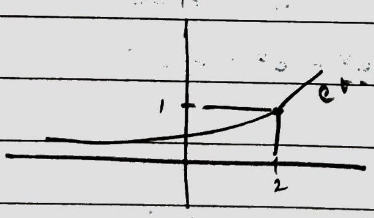
$y(t) = 0$

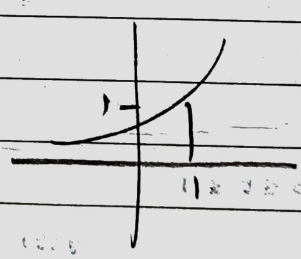
combining all cases, we have,

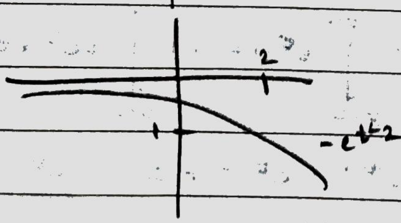
$$y(t) = \begin{cases} 2e^{t} - e^{t-1} - e^{t-2}, & -\infty < t \le 0 \\ 2 - e^{t-1} - e^{t-2}, & 0 \le t \le 1 \\ 1 - e^{t-2}, & 1 \le t \le 2 \\ 0, & t \ge 2 \end{cases}$$



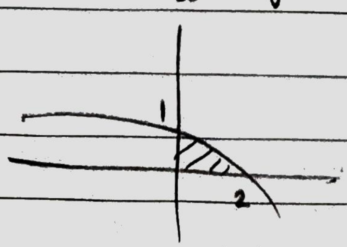








adding 1

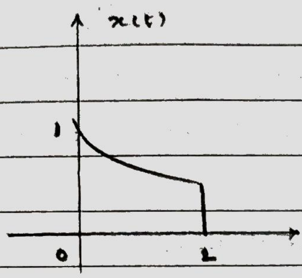


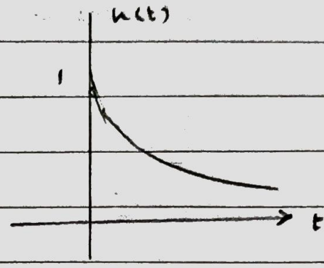
q: the impulse response of the circuit is given by,

$h(t) = e^{-t} u(t)$. the circuit is excited by an input of

$x(t) = e^{-3t}[u(t) - u(t-2)]$. determine response of system.

→ we have,

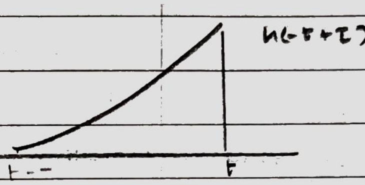




$x(t) = \{0, 2\}$

$h(t) = \{0, \infty\}$

$\therefore$ cases $= \{0, 2, \infty\}$



case 1: $t < 0$

$y(t) = 0$ — show graphs in each case.

case 2: $0 \leq t \leq 2$

$$y(t) = \int_0^{\infty} e^{-3\tau} \cdot e^{-(t-\tau)}\, d\tau$$

$$= \int_0^{t} e^{-t-2\tau}\, d\tau$$

$$= \left[\frac{e^{-t-2\tau}}{-2}\right]_0^{t}$$

$$= -\frac{e^{-3t}}{2} + \frac{e^{-t}}{2}$$

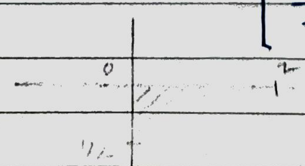
case 3: $t \geq 2$

$$y(t) = \int_0^{2} e^{-3\tau} \cdot e^{-(t-\tau)}$$

~~final graph,~~

$$= \left[\frac{e^{-t-2\tau}}{-2}\right]_0^{2} = \frac{e^{-t-4}}{-2} + \frac{e^{-t}}{2}$$

$$= \frac{-e^{-t}}{2}(e^{-4} - 1)$$



combine them for final graph.

correlation:

it is a mathematical operation that is similar to convolution

two signals are involved but they never fold.

types — auto correlation ⟶ when signal is correlated with itself to form another signal

— cross correlation

↳ when one signal is correlated with other signal to form another signal.

vignesh palande in universe 45

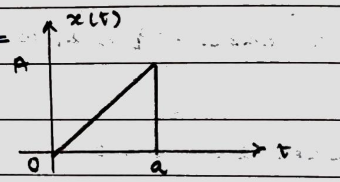
using convolution, correlation

$$r_{xh}(t) = x(t) ** h(t)$$

$$= x(t) * h(-t)$$

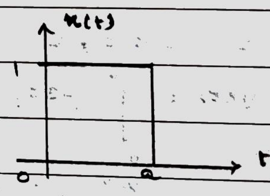
convolution

q: find correlation if $x(t) =$



$**$

$h(t) =$



→ $r_{xh}(t) = x(t) ** h(t)$

$= x(t) * h(-t)$

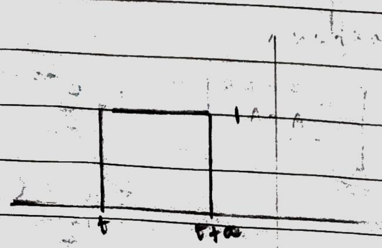
replacing t by τ,

$x(t) = x(\tau)$ and $h(-t) = h(\tau)$

$x(\tau) = \{0, a\}$

$h(-\tau) = \{-a, 0\}$

cases $= \{-a, 0, a\}$



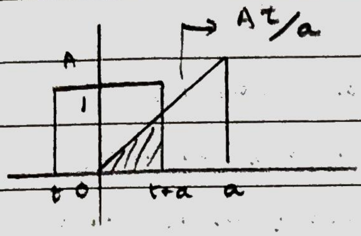
case 1 : $t < -a$

$y(t) = 0$

~~case 2 : $-a < t < 0$~~

~~$y(t) = 0$~~

case $\cancel{3}$ 2 : ~~$0 < t < a$~~ $-a < t < 0$



$$y(t) = \int_0^{t+a} \frac{A\tau}{a}\, d\tau$$

$$= \frac{A}{2a}(t+a)^2$$

case 3: $0 < t < a$

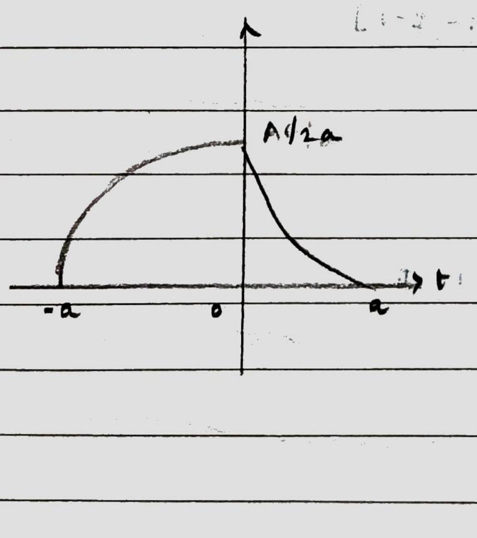
$$y(t) = \int_t^a \frac{A\tau}{a}\, d\tau = \frac{A}{2a}(a^2 - t^2)$$

case 4: $t > a$

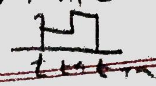
$y(t) = 0$

combining all cases, we get,

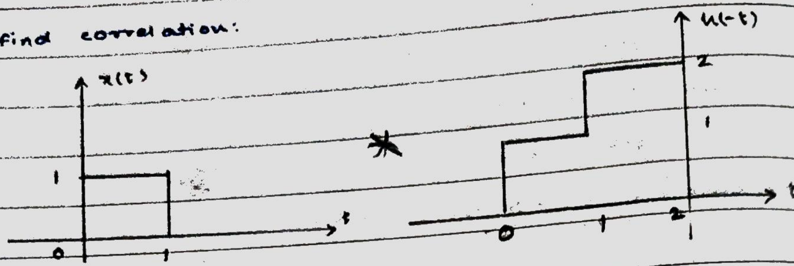
$$y(t) = \begin{cases} 0 & , \quad t < -a \\ (t+a)^2 A/2a & , \quad -a < t < 0 \\ A(a^2 - t^2)/2a & , \quad 0 < t < a \\ 0 & , \quad t > a \end{cases}$$

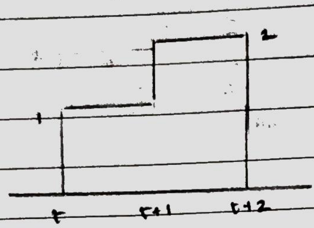


- final graph



q: find correlation:





$x(t) = \{0, 1\}$

$h(-t) = \{-2, -1, 0\}$

cases: $\{-2, -1, 0, 1\}$

replacing t by τ,

case 1: $t \leq -2$

$y(t) = 0$

case 2: $-2 \leq t < -1$

$$y(t) = \int_{0}^{t+2} 2 \cdot 1 \cdot d\tau = 2(t+2)$$

case 3: $-1 \leq t < 0$

$$y(t) = \int_{0}^{t+1} 1 \, d\tau + \int_{t+1}^{1} 2 \, d\tau$$

$$= t+1 + 2[1 - t - 1]$$

$$= t+1-2t$$

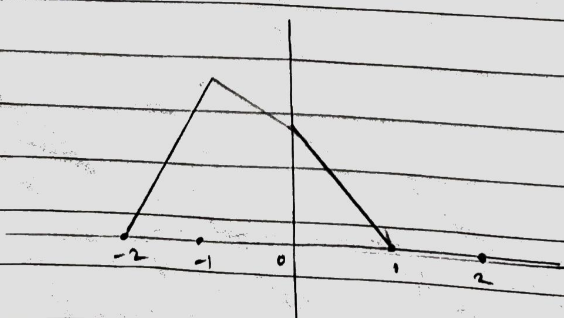
$$= 1-t$$

case 4: $0 \leq t < 1$

$$y(t) = \int_{t}^{1} d\tau = 1-t$$

case 5: $y \geq 1$

$y(t) = 0$



analytical evaluation of convolution:

q: perform convolution on:

$u(t) = x(t)$ and $h(t) = u(t+3)$

→ $y(t) = \int_{-\infty}^{\infty} x(\tau).\, h(t-\tau)\, d\tau$ - convolution integral

replacing t by τ,

$x(\tau) = u(\tau)$ → lower limit changes to 0

$h(\tau) = u(\tau+3)$

$h(t-\tau) = u(t-\tau+3)$

→ upper limit changes to $t+3$

$y(t) = \int_{0}^{t+3} (1).(1)\, d\tau = t+3$

$\therefore y(t) = (t+3)\, u(t+3)$

q: perform convolution on:

$x(t) = e^{-2t} u(t)$ and $h(t) = e^{-t} u(t)$

→ replacing t by τ, → lower limit = 0

$x(\tau) = e^{-2\tau} u(\tau)$, $h(\tau) = e^{-\tau} u(\tau)$ equate to 0

$h(t-\tau) = e^{\tau-t} u(t-\tau)$

→ higher limit = t

$\therefore y(t) = \int_{0}^{t} e^{-2\tau} e^{-t+\tau}\, d\tau$

$= \int_{0}^{t} e^{-\tau-t}\, d\tau = -\left[e^{-\tau-t} \right]_{0}^{t}$

$\therefore y(t) = -e^{-2t} + e^{-t}$ + 3

$\int_{-3}^{t} e^{-2\tau}\, d\tau$

Q: perform convolution on:

$x(t) = u(t+1) - u(t-1)$ with itself.

$x(\tau) = u(\tau+1) - u(\tau-1)$

$h(-\tau+t) = u(t-\tau+1) - u(t-\tau-1)$

$$y(t) = \int_{-\infty}^{\infty} [u(\tau+1) - u(\tau-1)][u(t-\tau+1) - u(t-\tau-1)]\, d\tau$$

$$= \int_{-\infty}^{\infty} \Big\{ [\underbrace{u(\tau+1)u(t-\tau+1)}_{(-1,\ t+1)} - \underbrace{u(\tau+1)u(t-\tau-1)}_{(-1,\ t-1)}]$$

$$- [u(\tau-1)u(t-\tau+1)] + u(\tau-1)u(t-\tau-1) \Big\}$$

$$= \int_{-1}^{t+1} d\tau - \int_{-1}^{(1,\ t+1)} d\tau - \int_{1}^{t+1} d\tau + \int_{1}^{(1,\ t-1)} d\tau$$

$$= (t+2)u(t+2) - t\,u(t) - t\,u(t) + (t-2)u(t-2)$$

$$= (t+2)u(t+2) - 2t\,u(t) + (t-2)u(t-2)$$

— answer

Systems:

- defined as set of elements or functional blocks which are connected together and produce an output in response to an input signal.
- classification:
  1. static and dynamic systems
  2. time invariant and time variant
  3. linear and ~~or~~ non linear systems