Skewness

- In everyday language, the terms “skewed” and “askew” are used to refer to something that is out of line or distorted on one side.
- When referring to the shape of frequency or probability distributions, “skewness” refers to asymmetry of the distribution.
- A distribution with an asymmetric tail extending out to the right is referred to as “**positively skewed**” or “**skewed to the right**”.
- A distribution with an asymmetric tail extending out to the left is referred to as “**negatively skewed**” or “**skewed to the left**.”
- Skewness can range from minus infinity to positive infinity.

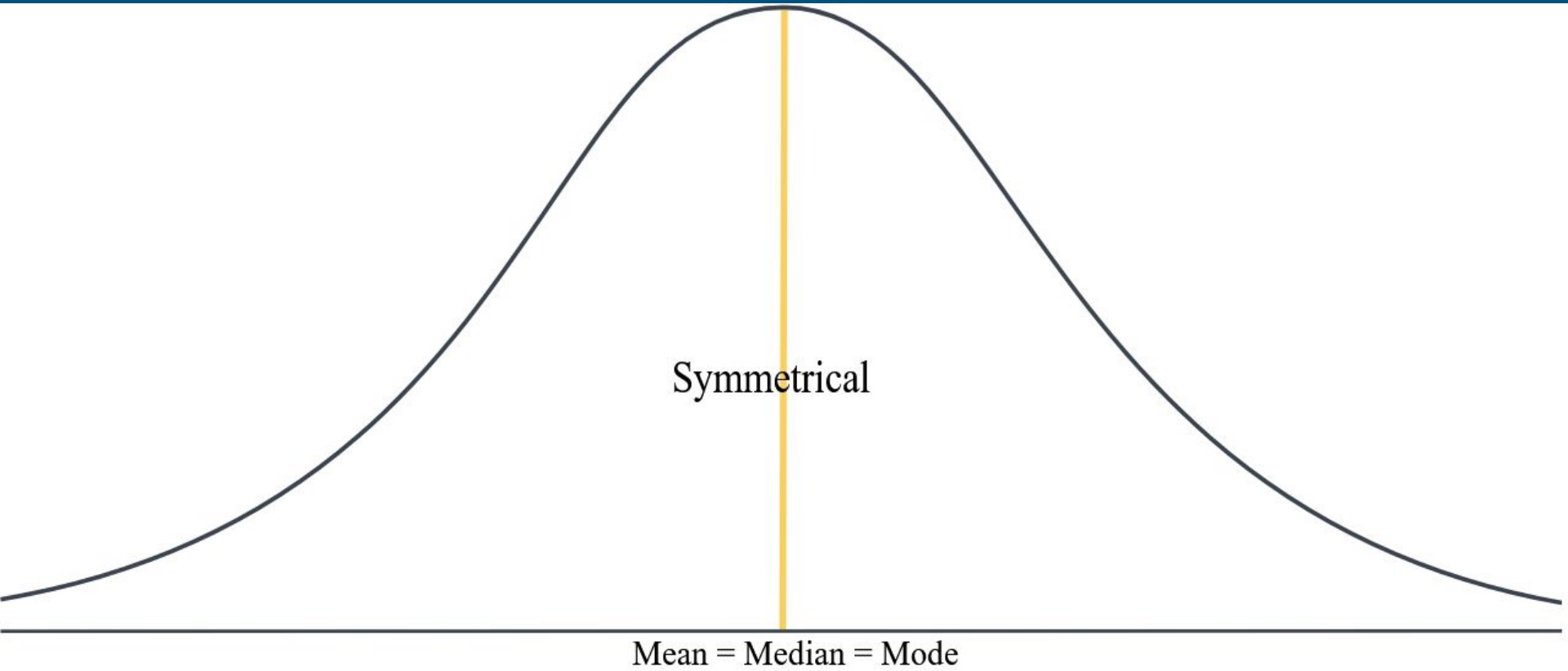
Skewness Formula

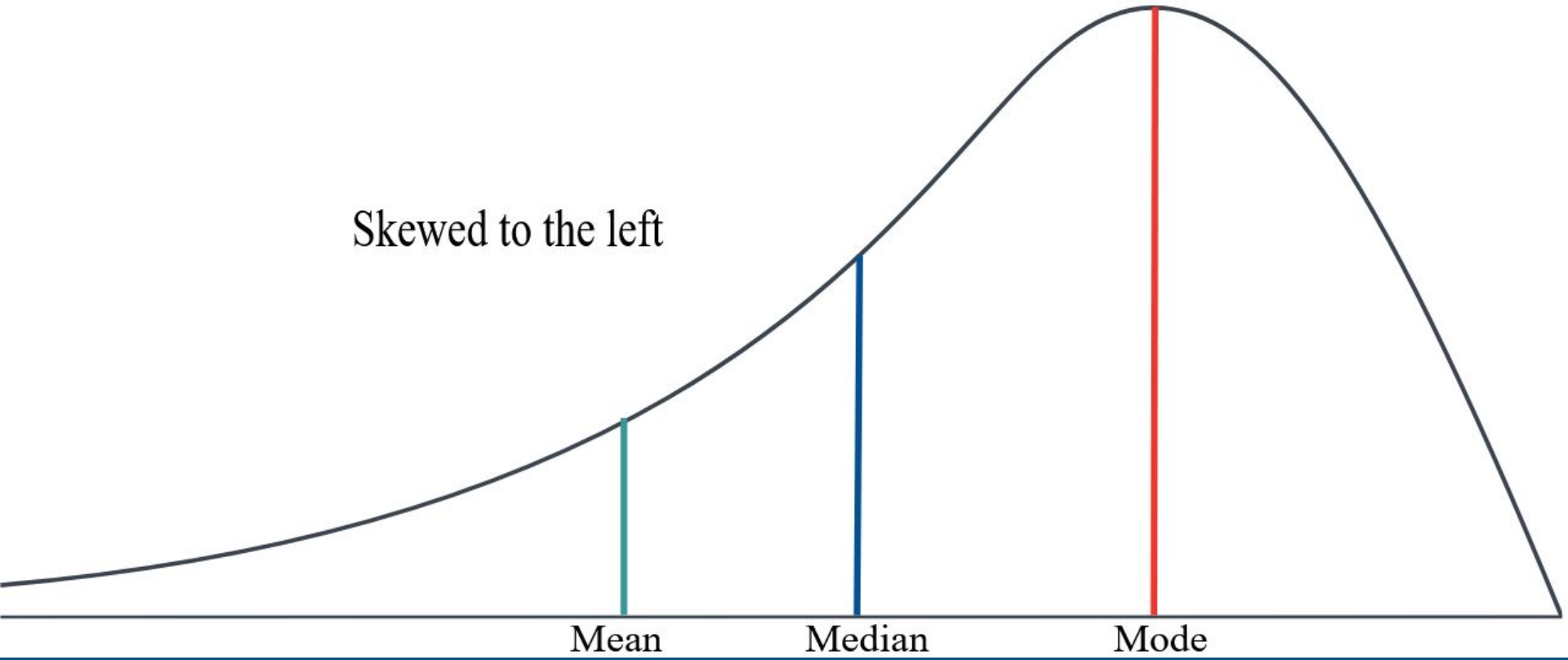
- Karl Pearson (1895) first suggested measuring skewness by standardizing the difference between the mean and the mode:

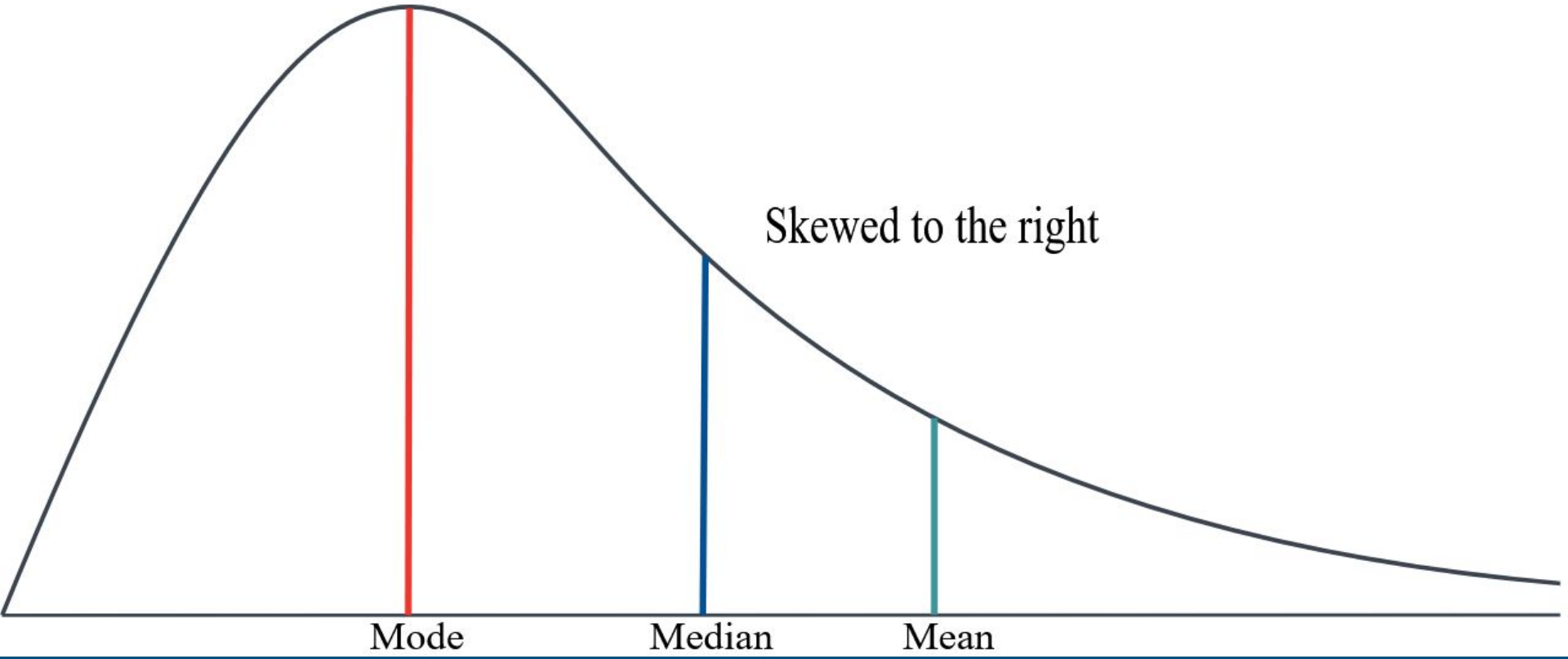
$$sk = \frac{\mu - \text{mode}}{\sigma}$$

- Population modes are not well estimated from sample modes
- Difference between the mean and the mode as being three times the difference between the mean and the median (Stuart & Ord, 1994):

$$sk_{est} = \frac{3(M - \text{median})}{s}$$


Symmetrical
Mean = Median = Mode


Skewed to the left
Mean
Median
Mode


Skewed to the right
Mode
Median
Mean

Kurtosis

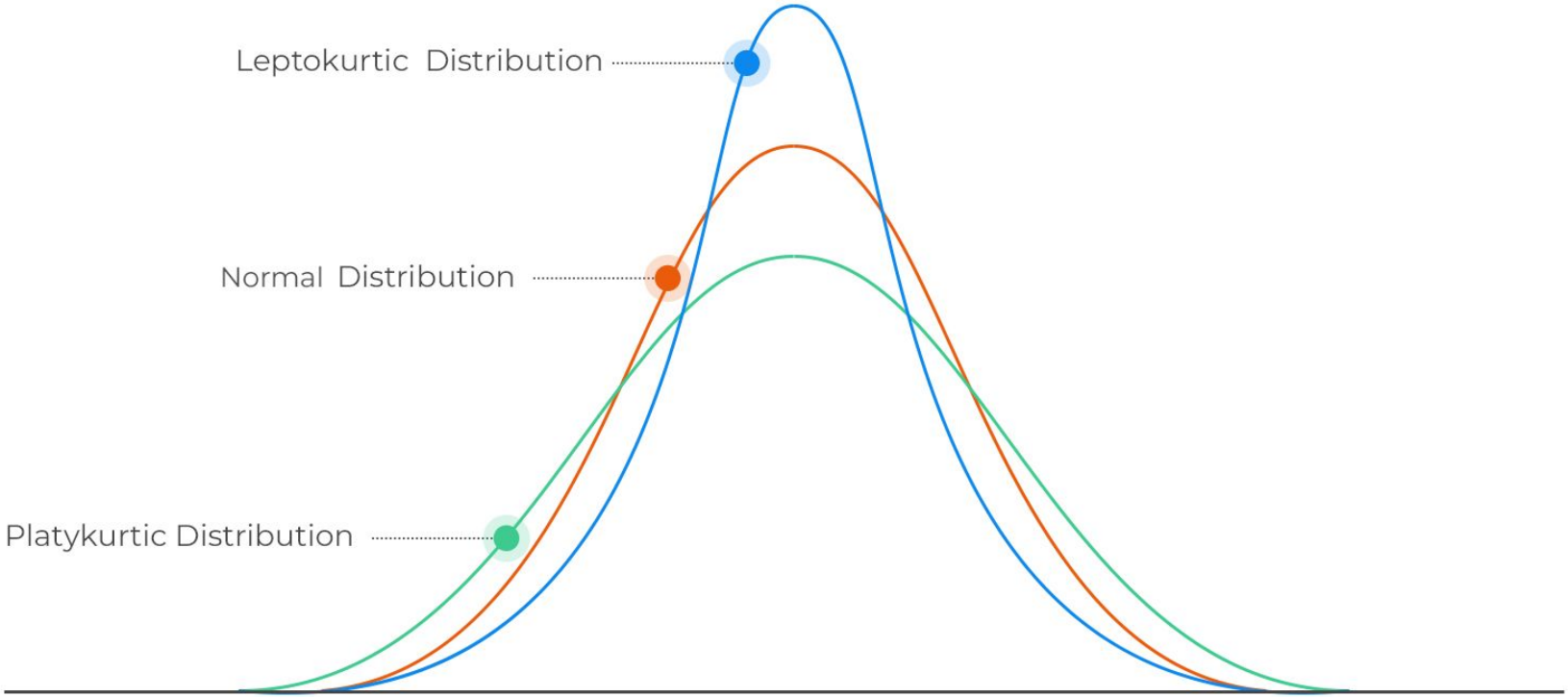
Kurtosis refers to measuring the degree to which a given distribution is more or less ‘peaked’ relative to the normal distribution.

The concept of kurtosis is very useful in decision-making.

In this regard, we have 3 categories of distributions:

1. Leptokurtic
2. Mesokurtic
3. Platykurtic

Kurtosis


Leptokurtic Distribution
Normal Distribution
Platykurtic Distribution

Leptokurtic

- A leptokurtic distribution is more peaked than the normal distribution.
- The higher peak results from the clustering of data points along the X-axis.
- “Lepto” means thin.
- The tails are also fatter than those of a normal distribution.
- The coefficient of kurtosis is usually found to be more than 3.

Interpretation

- When analyzing historical returns, a leptokurtic distribution means that changes are less frequent since historical values are clustered around the mean.
- However, there are also large fluctuations represented by the fat tails.

Platykurtic

- A platykurtic distribution has extremely dispersed points along the X-axis, resulting in a lower peak when compared to the normal distribution.
- “Platy” means broad.
- Hence, the prefix fits the distribution’s shape, which is wide and flat.
- The points are less clustered around the mean compared to the leptokurtic distribution.

Platykurtic

- Returns that follow this type of distribution have fewer major fluctuations compared to leptokurtic returns.
- However, you should note that fluctuations represent the riskiness of an asset.
- More fluctuations represent more risk and vice versa.
- Therefore, platykurtic returns are less risky than leptokurtic returns.

Mesokurtic

- Meso = middle; intermediate.
- This means the distribution is a normal distribution.

Sample Skewness

$$S_k = \frac{1}{n} \frac{\sum_{i=1}^{n} (X_i - \bar{X})^3}{S^3}$$

Where: $\bar{X}$ is the sample mean;

S is the sample standard deviation; and

n is the number of observations.

Sample Kurtosis

Sample kurtosis is always measured relative to the kurtosis of a normal distribution, which is 3. Therefore, we are always interested in the “excess“ kurtosis, i.e.,

Excess kurtosis = sample kurtosis – 3, where:

$$S_{kr} = \frac{1}{n} \frac{\sum_{i=1}^{n} (X_i - \bar{X})^4}{S^4}$$