NEUR2RO: NEURAL TWO-STAGE ROBUST OPTIMIZATION

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ABSTRACT

Robust optimization provides a mathematical framework for modeling and solving decision-making problems under worst-case uncertainty. This work addresses two-stage robust optimization (2RO) problems (also called *adjustable robust optimization*), wherein first-stage and second-stage decisions are made before and after uncertainty is realized, respectively. This results in a nested min-max-min optimization problem which is extremely challenging computationally, especially when the decisions are discrete. We propose `Neur2RO`, an efficient machine learning-driven instantiation of column-and-constraint generation (CCG), a classical iterative algorithm for 2RO. Specifically, we learn to estimate the value function of the second-stage problem via a novel neural network architecture that is easy to optimize over by design. Embedding our neural network into CCG yields high-quality solutions quickly as evidenced by experiments on two 2RO benchmarks, knapsack and capital budgeting. For knapsack, `Neur2RO` finds solutions that are within roughly 2% of the best-known values in a few seconds compared to the three hours of the state-of-the-art exact branch-and-price algorithm; for larger and more complex instances, `Neur2RO` finds even better solutions. For capital budgeting, `Neur2RO` outperforms three variants of the k-adaptability algorithm, particularly on the largest instances, with a 10 to 100-fold reduction in solution time. Our code and data are available at `https://github.com/khalil-research/Neur2RO`.

1 INTRODUCTION

A wide range of real-world optimization problems in logistics, finance, and healthcare, among others, can be modeled by discrete optimization models (Petropoulos et al., 2023). While such mixed-integer (linear) problems (MILP) can still be challenging to solve, the problem size that can be tackled with modern solvers has increased significantly thanks to algorithmic developments (Wolsey, 2020; Achterberg & Wunderling, 2013). In recent years, the incorporation of Machine Learning (ML) models into established algorithmic frameworks has received increasing attention (Zhang et al., 2023; Bengio et al., 2021).

While most of ML for discrete optimization has focused on deterministic problems, in many cases, decision-makers face uncertainty in the problem parameters, e.g., due to forecasting or measurement errors in quantities of interest such as customer demand in inventory management. Besides the stochastic optimization approach, for which learning-based heuristics have been proposed recently (Dumouchelle et al., 2022), another popular approach to incorporate uncertainty into optimization models is *robust optimization*, where the goal is to find solutions which are optimal considering the worst realization of the uncertain parameters in a pre-defined uncertainty set (Ben-Tal et al., 2009). This more conservative approach has been extended to two-stage robust problems (2RO) where some of the decisions can be made on the fly after the uncertain parameters are realized (Ben-Tal et al., 2004); see Yanıkoğlu et al. (2019) for a survey.

Example (Capital Budgeting). As a classical example of a two-stage robust problem, consider the capital budgeting problem as defined in Subramanyam et al. (2020) where a company decides to invest in a subset of n projects. Each project i has an uncertain cost $c_i(\boldsymbol{\xi})$ and an uncertain profit

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