CSCI 6470 Quiz #9 Questions Answers

Monday December 4, 2023 (12:40pm-1:10pm EST)

Student Name ______________________________ **Student ID**____________________________

Rules. Violation will result in **zero credit** for the exam and possibly the final grade.
1. Closed book/note/electronics/neighborhood.
2. Surrender your cell phone to the podium before using the restroom.

There are 5 questions and 60 points in total. Good luck!

1. (**10 points**) Based on the definition of class $\mathcal{NP}$, for any problem $D \in \mathcal{NP}$, there is a polynomial-time verifier (algorithm) V_D, such that for every input x,

$$D(x) = \text{"}yes\text{"} \text{ if and only if } \exists y, V_D(x, y) = \text{"}yes\text{"}$$

(1) What is y called? certificate or witness;

(2) If for some y, $V_D(x, y) =$ "no", then $D(x) =$ answer to $D(x)$ is unclear

(3) If for some y, $V_D(x, y) =$ "yes", then $D(x) =$ "yes"

(4) Does **Hamiltonian Path** have a polynomial-time verifier? yes

(5) Does **Reachability** have a polynomial-time verifier? yes

Each of the above questions has $\boxed{2\,points}$

2. (**10 points**) To prove that a polynomial-time reduction $A \leq_p B$ exists between problems A and B, we need to prove a few things. List them.

(1) there is a mapping $f : \Sigma^* \longrightarrow \Sigma^*$; $\boxed{2\,points}$

(2) on any x, $A(x) =$ "yes" if and only if $B(f(x)) =$ "yes"; $\boxed{4\,points}$

(3) f can be computed in polynomial time; $\boxed{4\,points}$;

3. (**10 points**) There are two different methods to prove that a problem is $\mathcal{NP}$-hard. What are they? Please give succinct answers.

Let the problem be A.

(1) to prove $\forall D \in \mathcal{NP}$, $D \leq_p A$; $\boxed{5\,points}$

(2) to pick an $\mathcal{NP}$-hard problem B and to prove $B \leq_p A$ $\boxed{5\,points}$

4. (**15 points**) Let $A \le_p B$ be a polynomial-time reduction between problems A and B. Mark the following individual statements with TRUE/FALSE.

   (1) If A is $\mathcal{NP}$-hard, then B is also $\mathcal{NP}$-hard. T

   (2) If A is $\mathcal{NP}$-complete, then B is also $\mathcal{NP}$-complete. F

   (3) If B is $\mathcal{NP}$-complete, then A is $\mathcal{NP}$-hard. F

   (4) If B is $\mathcal{NP}$-complete, then $A \in \mathcal{NP}$. T

   (5) If $B \in P$, then $\mathcal{P} = \mathcal{NP}$. F

   (6) If $B \in \mathcal{P}$, then $A \in \mathcal{P}$. T

   (7) If $A \le_p$ **SAT**, then $B \le_p$ **SAT**. F

   (8) If **SAT** $\le_p A$, then B is $\mathcal{NP}$-hard. T

   Each of the above questions has $\boxed{2\,\mathit{points}}$

5. (**15 points**) We know that there is a polynomial-time reduction **SAT** $\le_p$ **Clique**.

   (1) State reasons (other than they are both $\mathcal{NP}$-complete) making the reduction hold:

   SAT $\le_p$ **IndependentSet**; $\boxed{2\,\mathit{points}}$

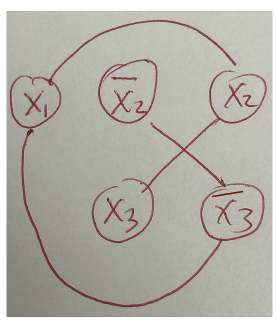
   IndependentSet $\le_p$ **Clique**; $\boxed{2\,\mathit{points}}$

   transitivity holds for $\le_p$; $\boxed{2\,\mathit{points}}$

   (2) What will the following formula be transformed to by reduction **SAT** $\le_p$ **Clique**?

$$\phi = (x_1 \vee \neg x_2 \vee x_3) \wedge (x_2 \vee \neg x_3)$$

   Your answer: the following graph G_ϕ $\boxed{7\,\mathit{points}}$ and $k_\phi = 2$. $\boxed{9\,\mathit{points}}$



[The following space will not be graded.]