

# 컴퓨터그래픽스

2017학년 1학기  
김준호

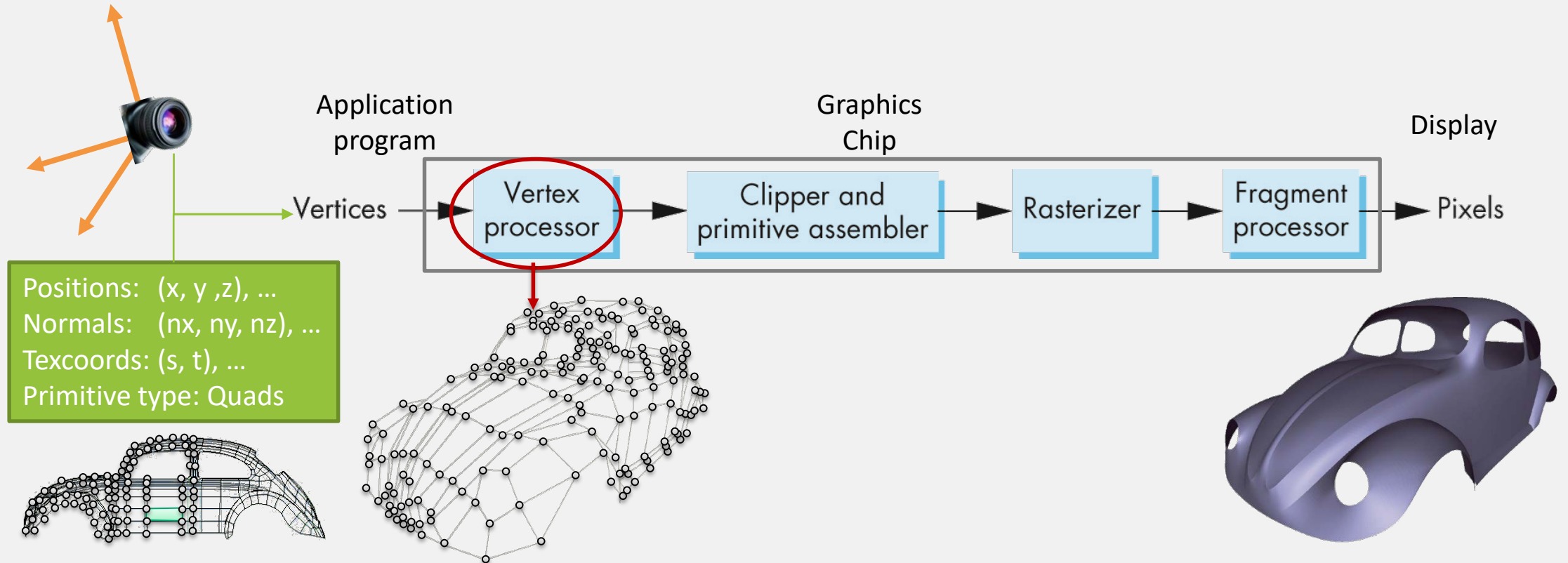
국민대학교 소프트웨어학부

- Overview of Vertex Processor
- Coordinate System & Coordinate Values
- ModelView matrix
- Projection matrix
- Viewport

# Vertex Processor

# Overview of Vertex Processor

- Vertex processor
  - Converting object representations from one coordinate system to another
    - Object coordinates  $\rightarrow$  Camera coordinates  $\rightarrow$  Screen coordinates



# Objectives

- We are interested in an image captured from the camera
  - First of all, we should know the coordinate of a 3D point, from camera's viewpoint
  - It means, we have to understand the change of coordinates
    - Coordinate values in object space → Coordinate values in camera space

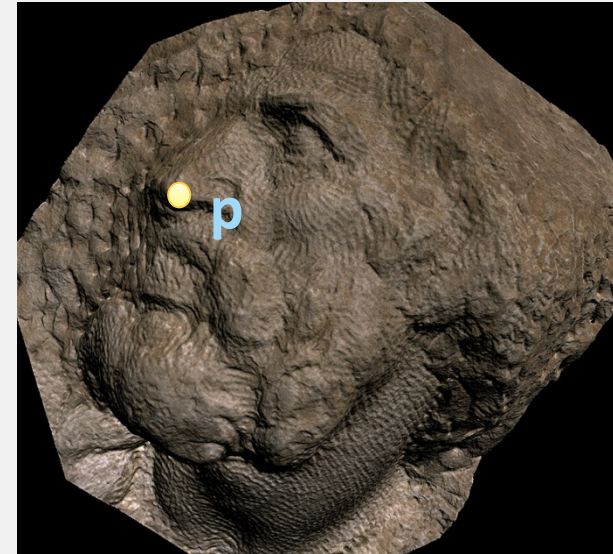
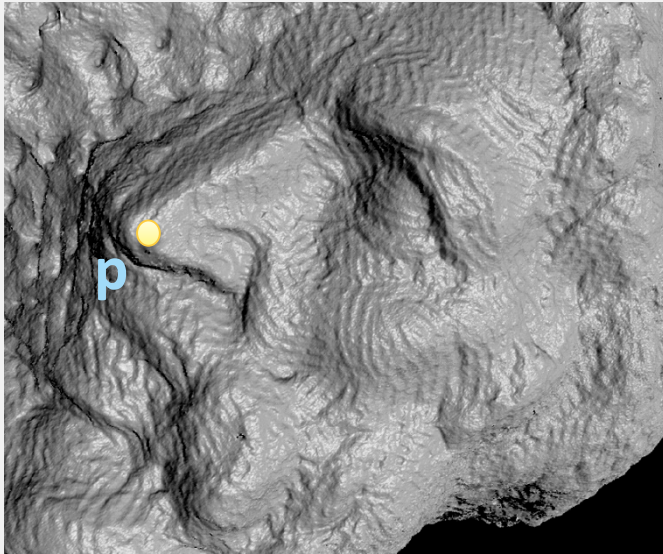


- Coordinate system & Coordinate Values
- MoveView matrix

# Coordinate System and Coordinate Values

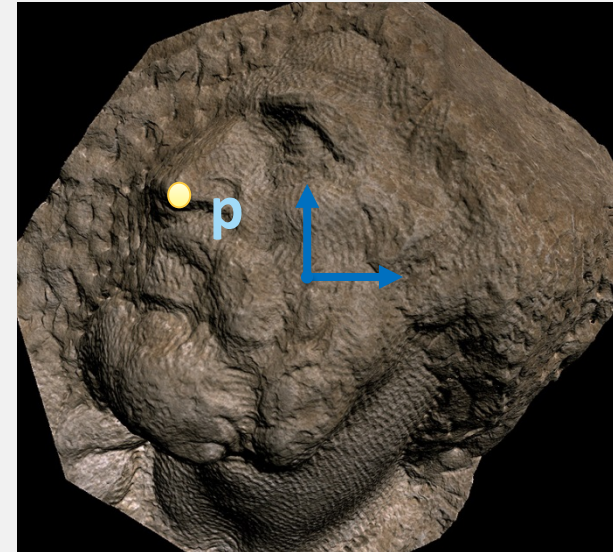
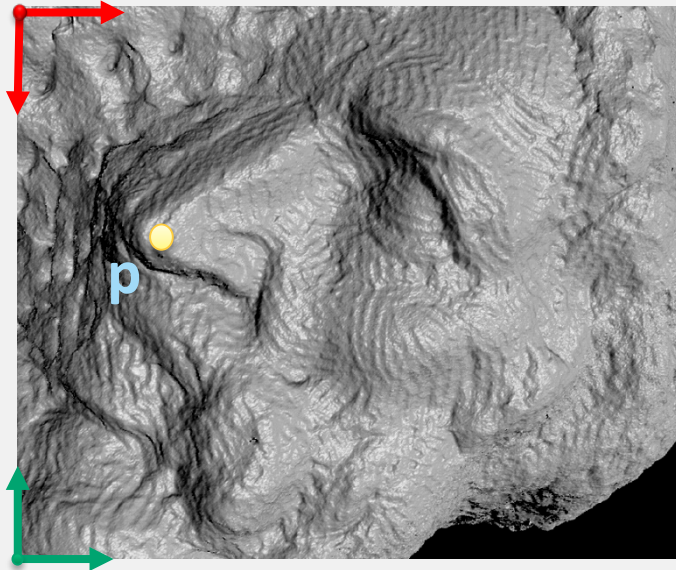
# Coordinate Value – Representation of a Point

- Where is a point **p**?
  - For the same point, we can represent it with different coordinates



# Coordinate Value – Representation of a Point

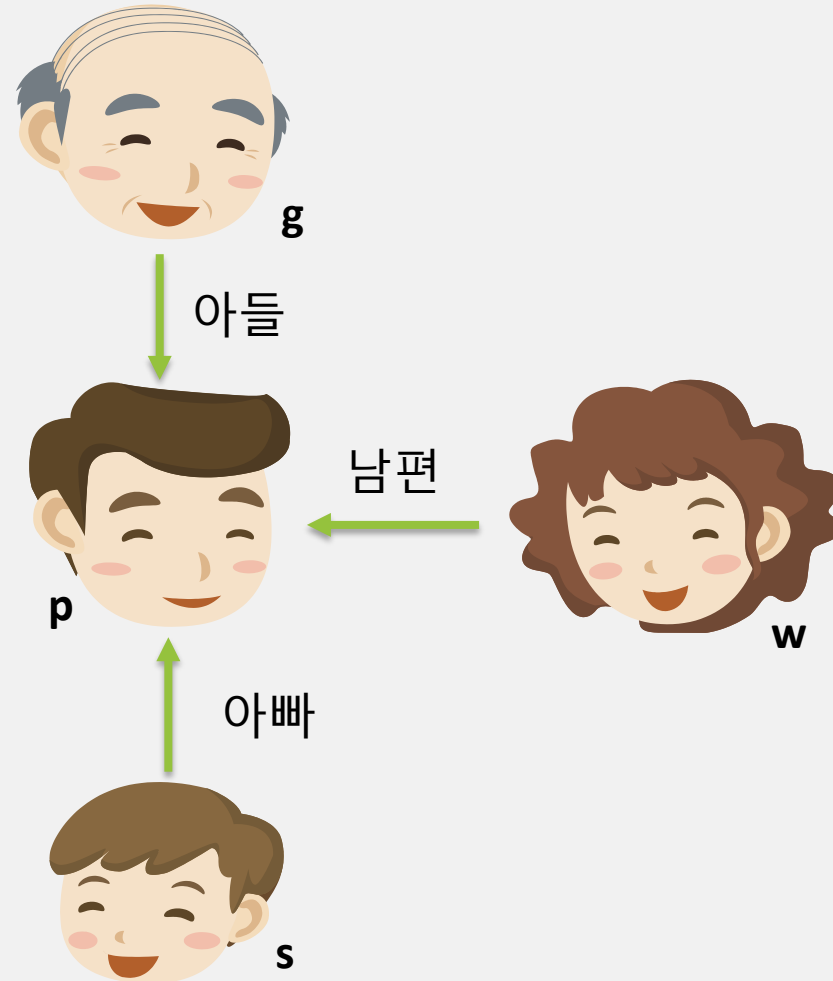
- The coordinate value of a point is meaningful, only when we specify a coordinate system
  - The same point can be represented with different coordinate values!
    - $p = (1.5, 3) = (1.5, 2.5) = (-1.2, 1)$





# Coordinate Value – Representation of a Point

- Analogy in real-world
  - A point  $p$ 
    - 존재
  - Coordinate system (or Frame)
    - 관점
  - Coordinate value of  $p$ 
    - 특정 관점에서 해당 존재를 부르는 호칭 (representation)
    - 동일한 존재는 여러가지 호칭으로 불릴 수 있음
    - $p_{[g]} = \text{아들}$
    - $p_{[w]} = \text{남편}$
    - $p_{[s]} = \text{아빠}$





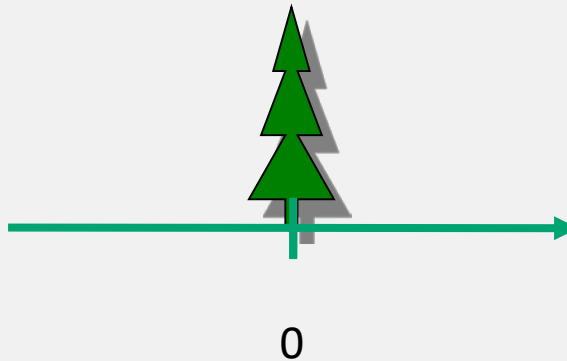
# ModelView matrix

# What is ModelView Matrix?

- The composition of a model matrix and a view matrix
  - OpenGL manages the model matrix and the view matrix together
    - c.f.) Direct3D separates the model matrix and the view matrix
  - Model matrix
    - 3D transformation of an object (or model) in the world coordinate system
  - View matrix
    - 3D transformation of a camera in the world coordinate system
    - This is the extrinsic parameters of the camera!
- We can obtain the camera coordinates by multiplying the ModelView matrix to the object coordinates
  - $\mathbf{x}_{view} = \mathbf{V}^{-1}\mathbf{M}\mathbf{x}_{obj}$
  - $\mathbf{V}^{-1}\mathbf{M}$  is called the modelview matrix

# 1D Case

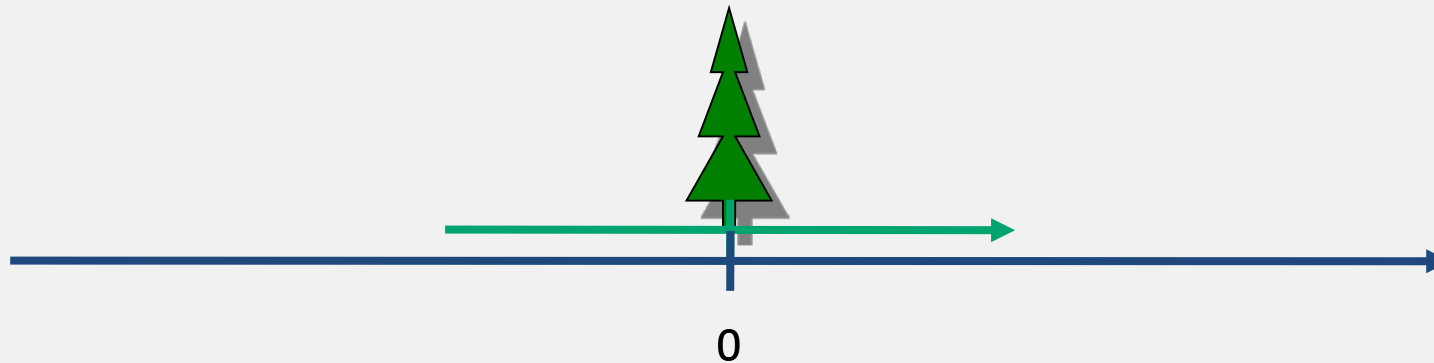
- You model an object in the object-space coordinate system
  - Every point is represented with object-space coordinates  $\mathbf{x}_{obj}$
  - 3D positions specified in [glVertexPointer\(\)](#) are in the objects-space coordinate system



# 1D Case

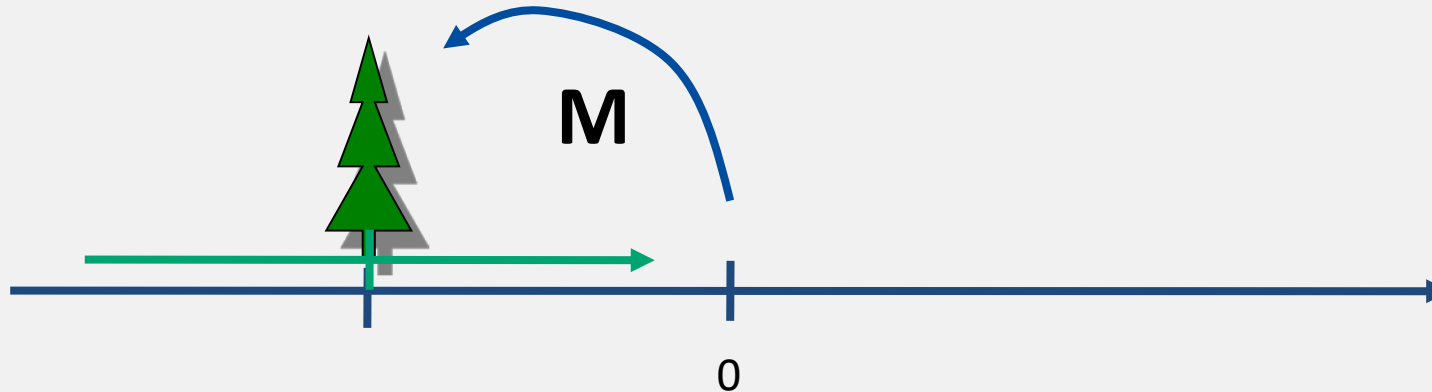
- You may place the object in the origin of the world

- $\mathbf{x}_{world} = \mathbf{x}_{obj}$



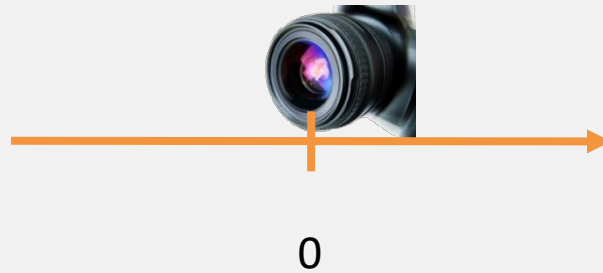
# 1D Case

- You move the object to somewhere in the world
  - **M**: world-transform matrix
    - You set **M** by using the composition of [glTranslate\(\)](#), [glRotate\(\)](#), [glScale\(\)](#)
  - $\mathbf{x}_{world} = \mathbf{M}\mathbf{x}_{obj}$



# 1D Case

- Now, let's consider a camera



# 1D Case

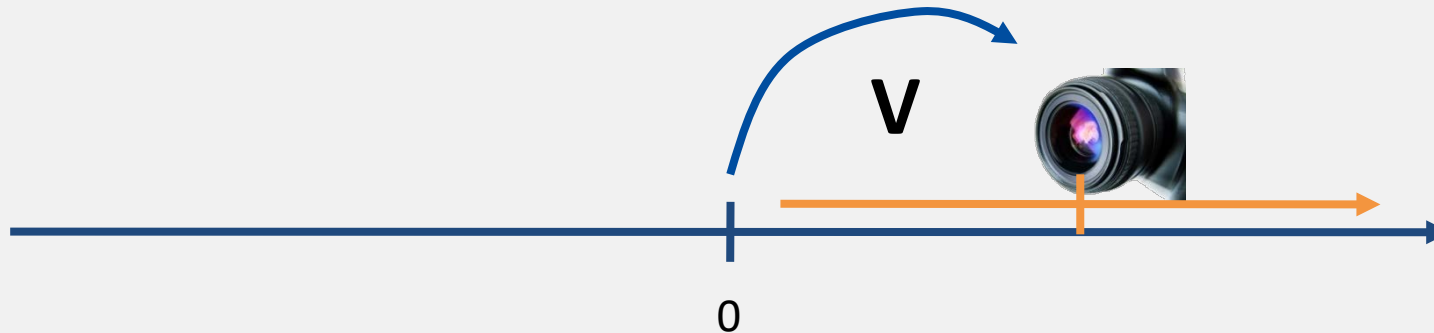
- You may place the camera in the origin of the world
  - $\mathbf{x}_{world} = \mathbf{x}_{view}$





# 1D Case

- You move the camera to somewhere in the world
  - $\mathbf{V}$ : view-transform matrix
    - You set  $\mathbf{V}$  by using [gluLookAt\(\)](#)
  - $\mathbf{x}_{world} = \mathbf{V}\mathbf{x}_{view}$

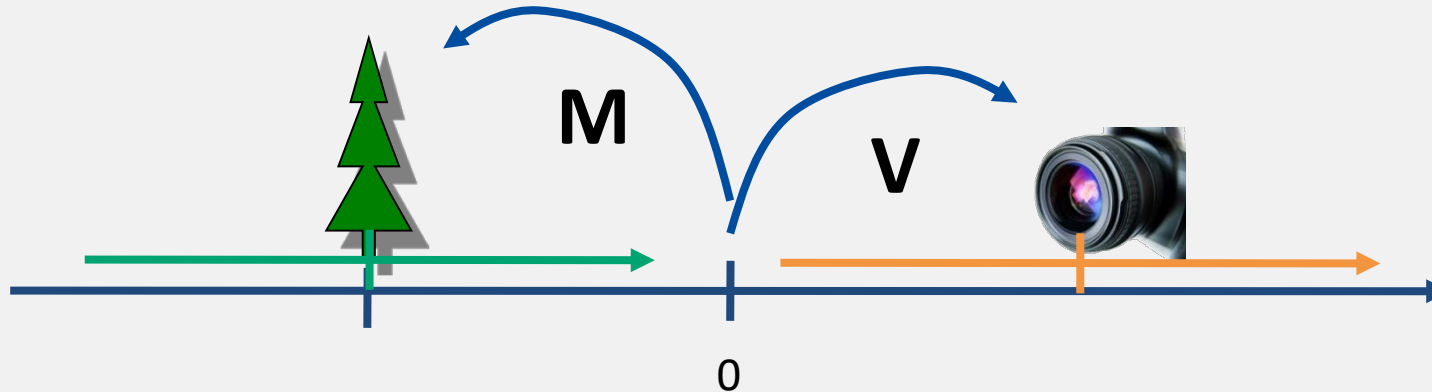


# 1D Case

- Let's consider both of camera & object

- $\mathbf{x}_{world} = \mathbf{M}\mathbf{x}_{obj}$

- $\mathbf{x}_{world} = \mathbf{V}\mathbf{x}_{view}$



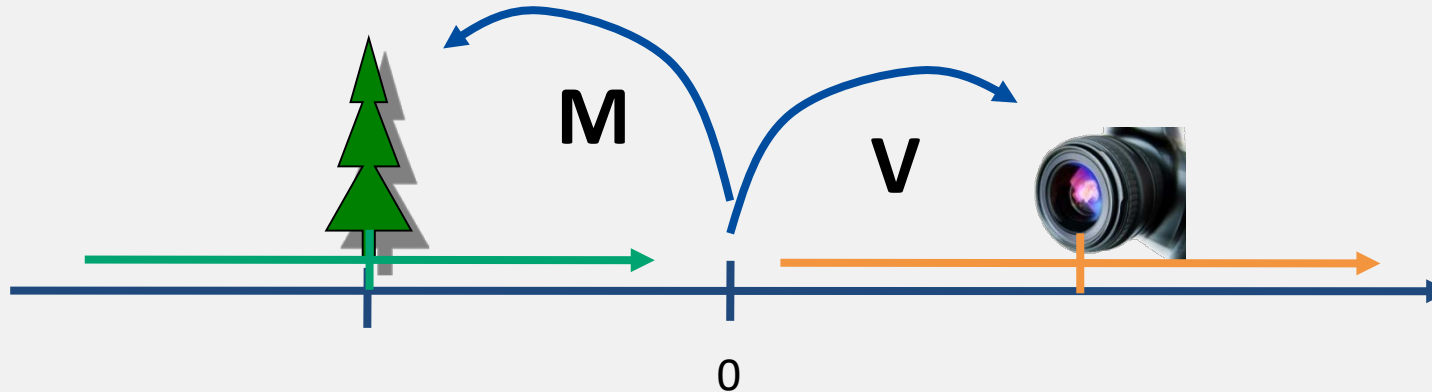
# 1D Case

- How can we obtain the camera-space coordinates of the object?

- $\mathbf{x}_{world} = \mathbf{M}\mathbf{x}_{obj}$
- $\mathbf{x}_{world} = \mathbf{V}\mathbf{x}_{view}$

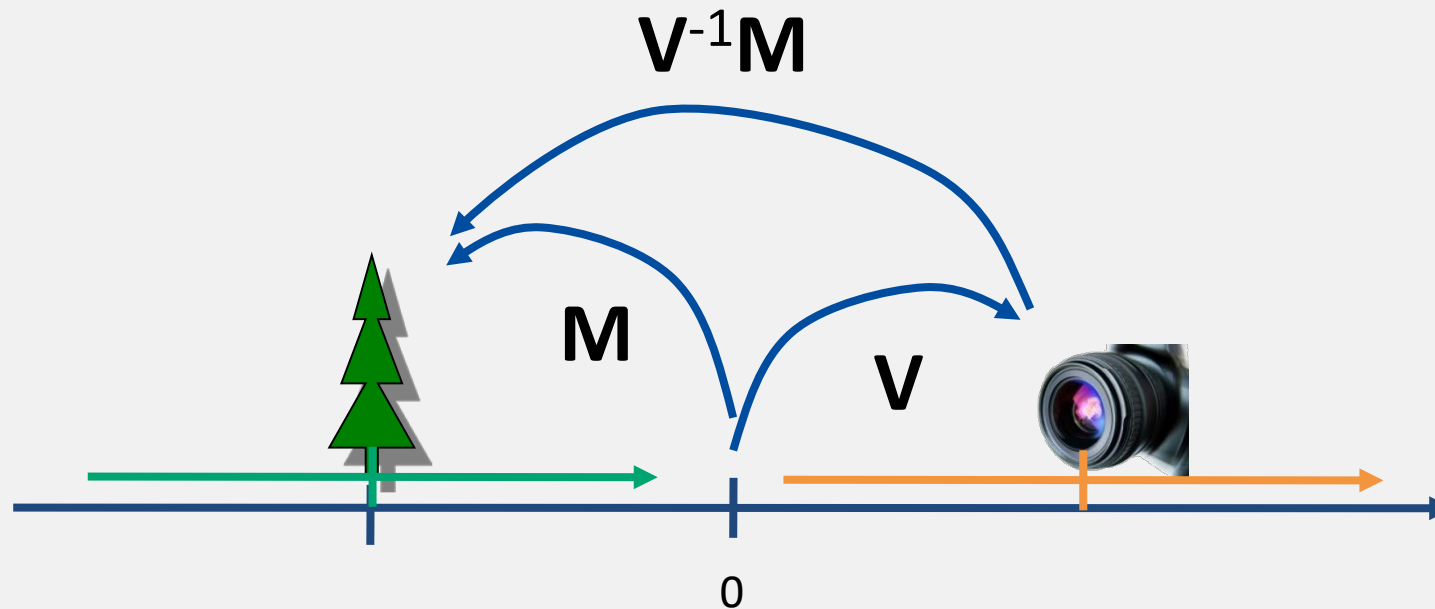
$$\} \rightarrow \mathbf{M}\mathbf{x}_{obj} = \mathbf{V}\mathbf{x}_{view}$$

$$\mathbf{x}_{view} = \mathbf{V}^{-1}\mathbf{M}\mathbf{x}_{obj}$$



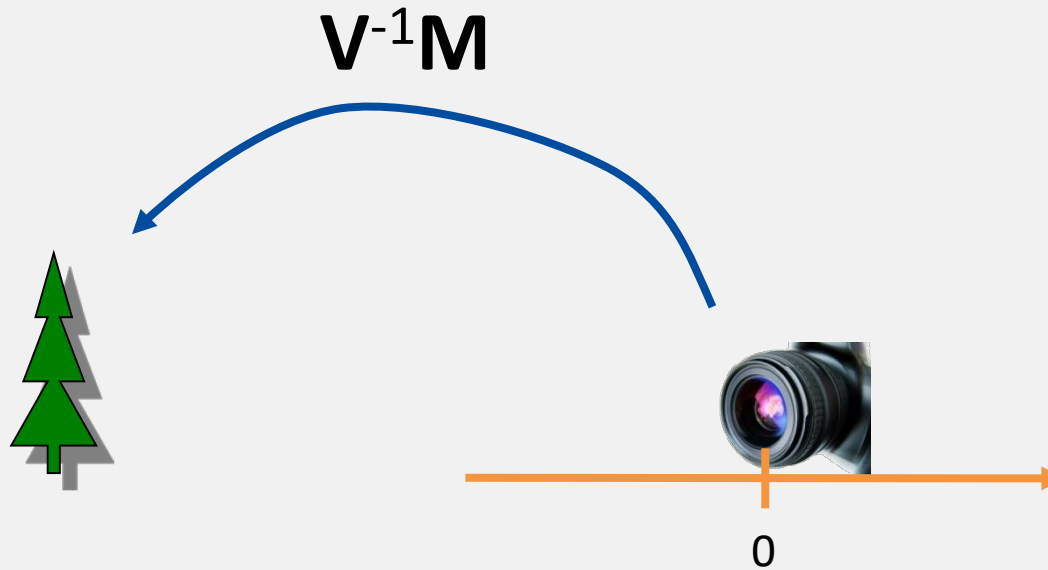
# 1D Case

- What does “ $\mathbf{x}_{view} = \mathbf{V}^{-1}\mathbf{M}\mathbf{x}_{obj}$ ” mean?



# 1D Case

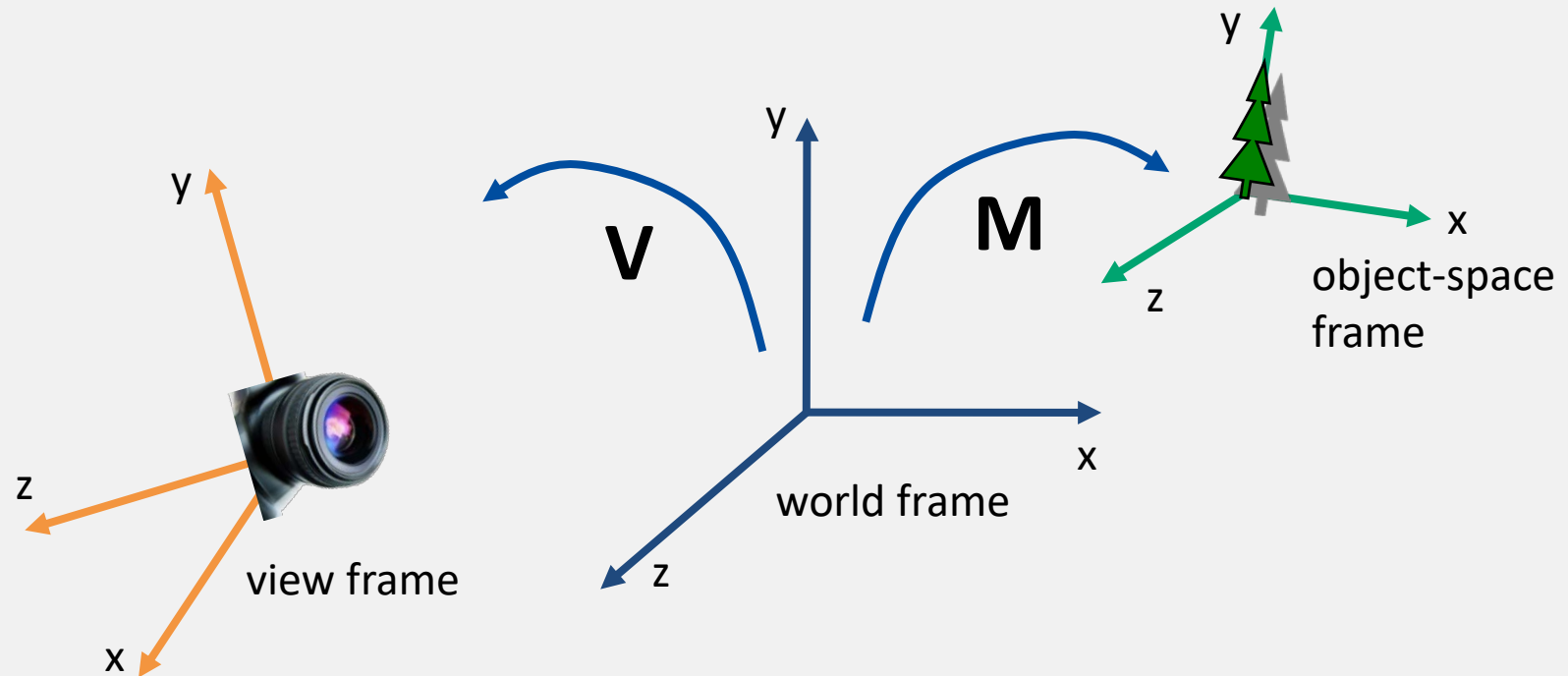
- What does “ $\mathbf{x}_{view} = \mathbf{V}^{-1}\mathbf{M}\mathbf{x}_{obj}$ ” mean?
  - 3D Position of  $\mathbf{x}$ , measured from the coordinate system of the camera
  - World-frame-independent representation
  - Now, you may think the world frame as an illusion.



# 3D Case

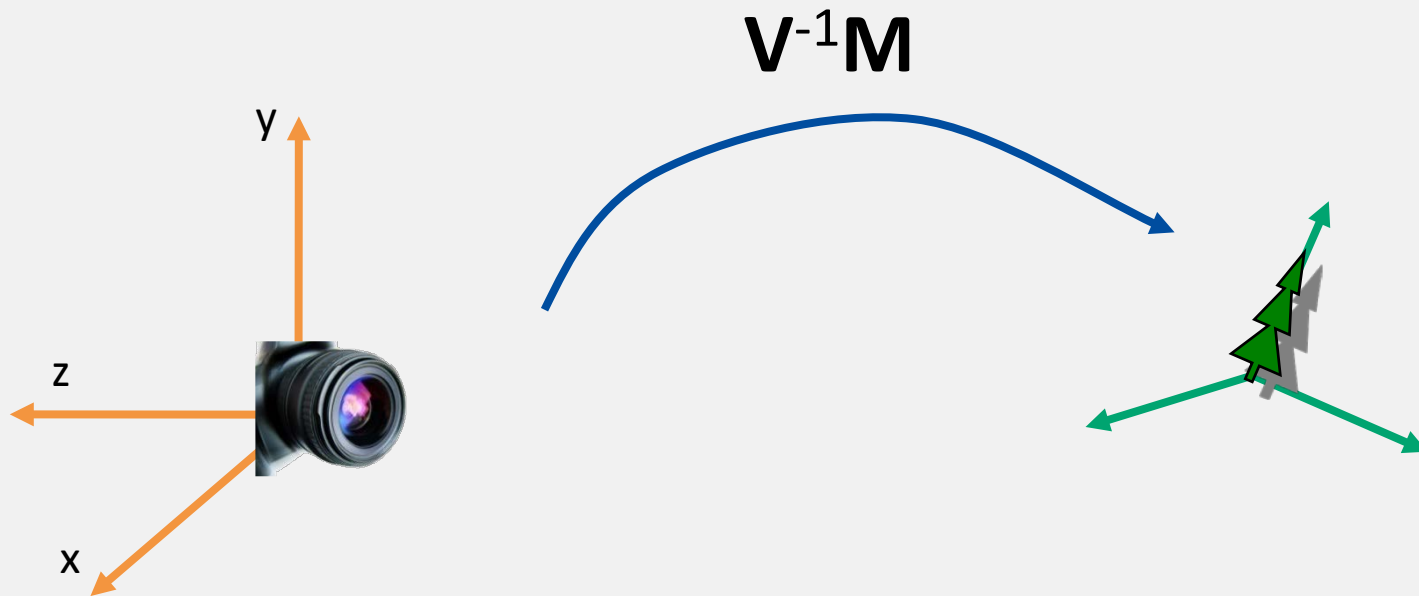
- Exactly same!
  - $\mathbf{x}_{world} = \mathbf{M}\mathbf{x}_{obj}$
  - $\mathbf{x}_{world} = \mathbf{V}\mathbf{x}_{view}$

$$\mathbf{x}_{view} = \mathbf{V}^{-1}\mathbf{M}\mathbf{x}_{obj}$$



# 3D Case

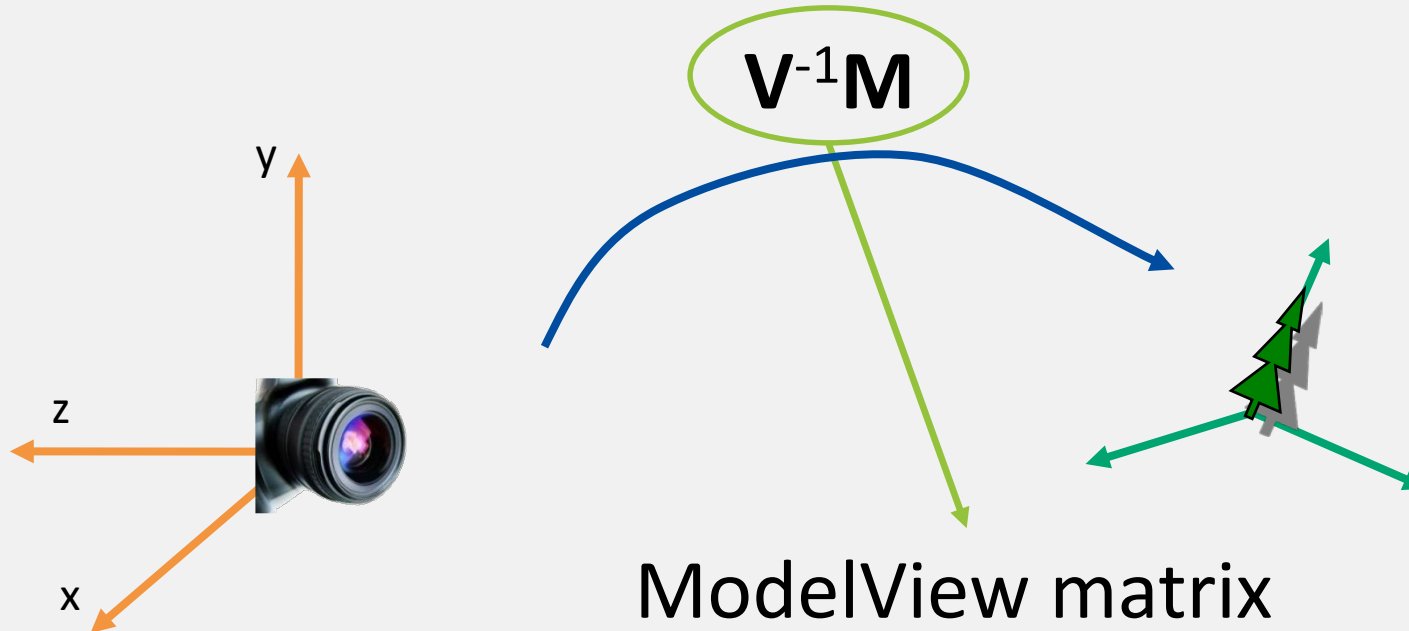
- $\mathbf{x}_{view} = \mathbf{V}^{-1}\mathbf{M} \mathbf{x}_{obj}$





# 3D Case

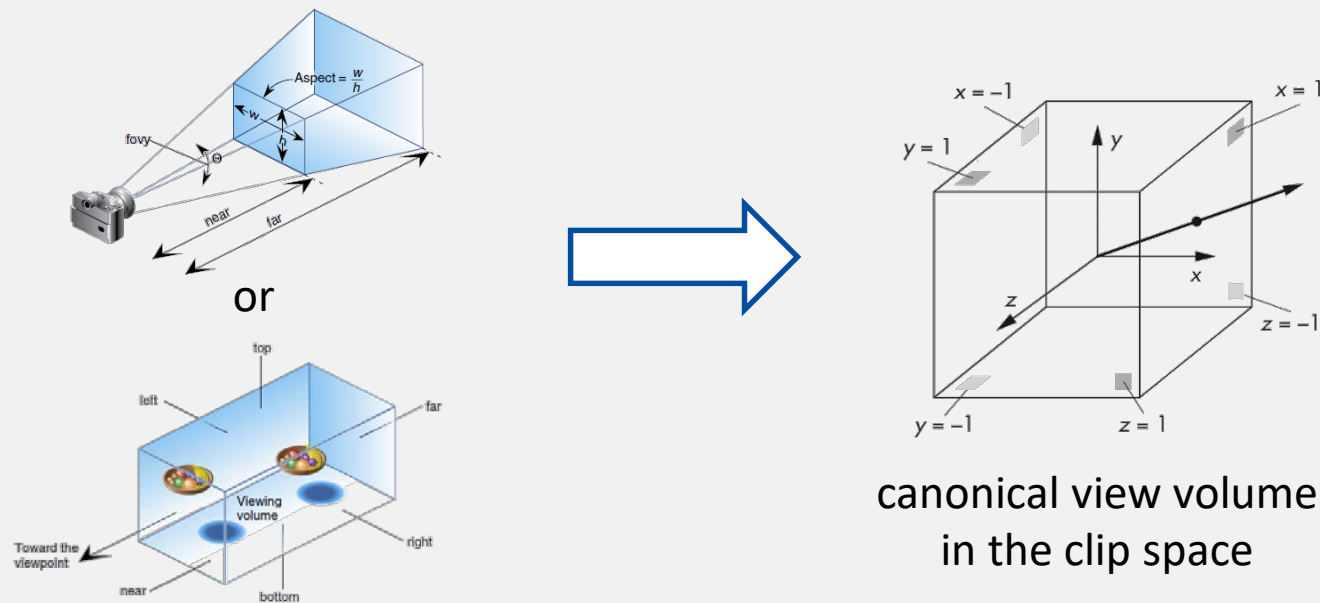
- In OpenGL,  $V^{-1}M$  is called as the ModelView matrix
  - GL\_MODELVIEW\_MATRIX



# Projection matrix

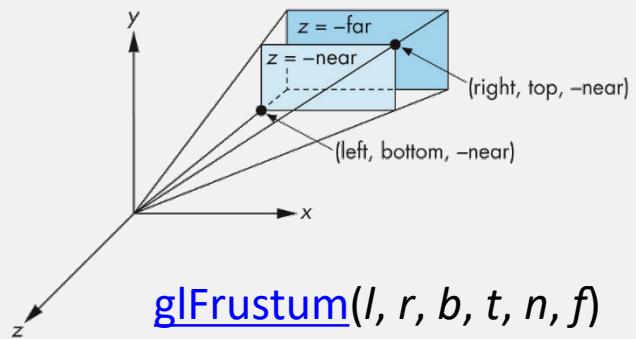
# What is Projection Matrix?

- Projection matrix  $\mathbf{P}$  transforms camera coordinates into clip coordinates
  - $\mathbf{x}_{clip} = \mathbf{P}\mathbf{x}_{view}$   
 $= \mathbf{P}\mathbf{V}^{-1}\mathbf{M}\mathbf{x}_{obj}$
- The canonical view volume is defined in the clip space



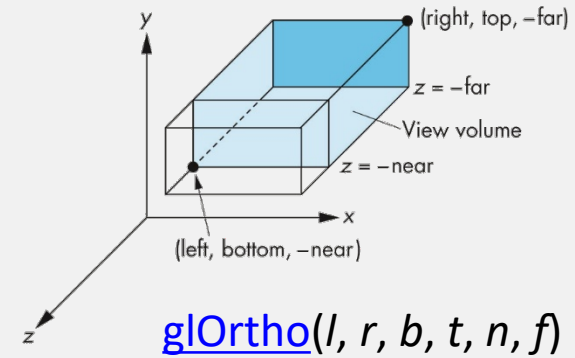
# Projection Matrix

- Perspective projection



$$P = \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

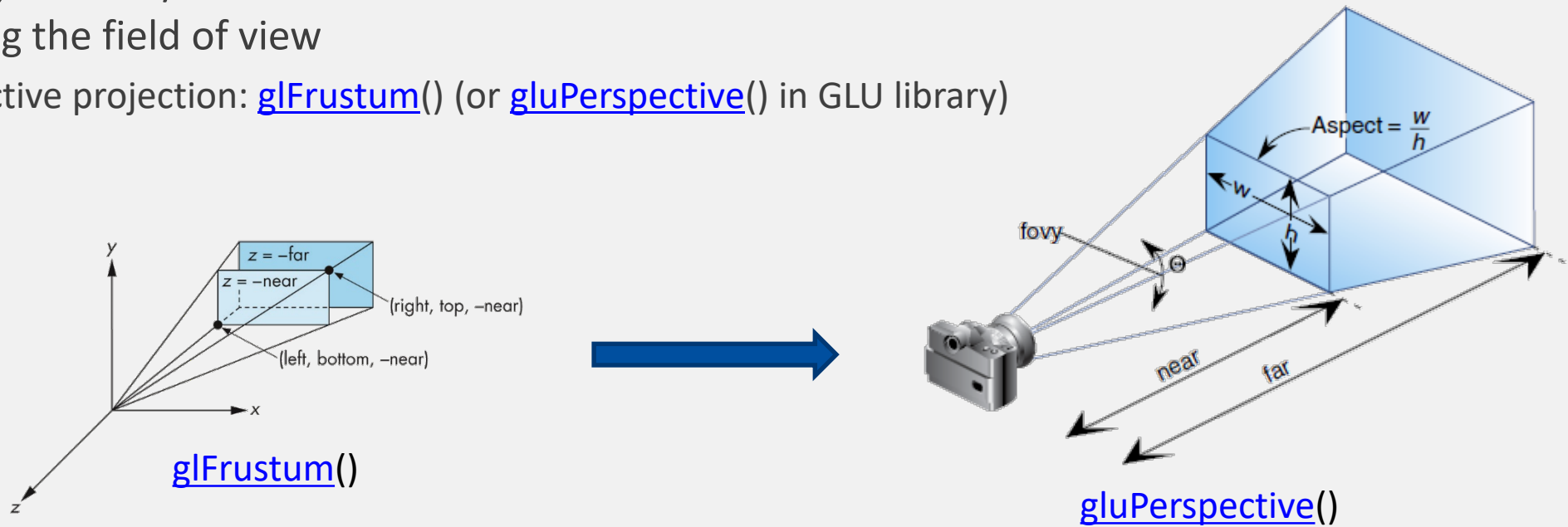
- Orthographic projection



$$P = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Perspective Projection

- Focal length
  - In OpenGL, there is no physical meaning
- Field of view (FOV)
  - In OpenGL, zoom-in/-out is handled by changing the field of view
    - Perspective projection: [glFrustum\(\)](#) (or [gluPerspective\(\)](#) in GLU library)



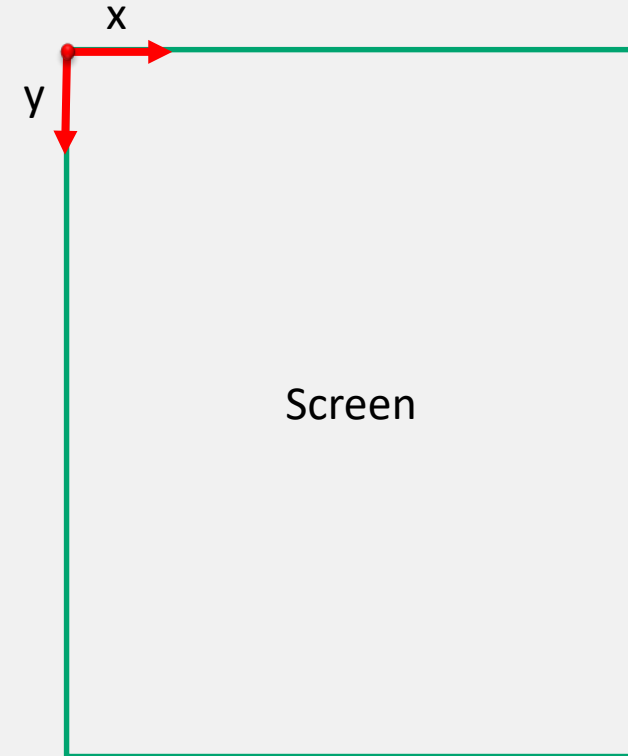
# Viewport

# What is Viewport?

- Viewport matrix **W** transforms clip coordinates into screen-space coordinates

$$\begin{aligned} - \mathbf{x}_{screen} &= \mathbf{W}\mathbf{x}_{clip} \\ &= \mathbf{WP}\mathbf{x}_{view} \\ &= \mathbf{WPV}^{-1}\mathbf{M}\mathbf{x}_{obj} \end{aligned} \quad = \begin{bmatrix} win_x \\ win_y \\ win_z \end{bmatrix}$$

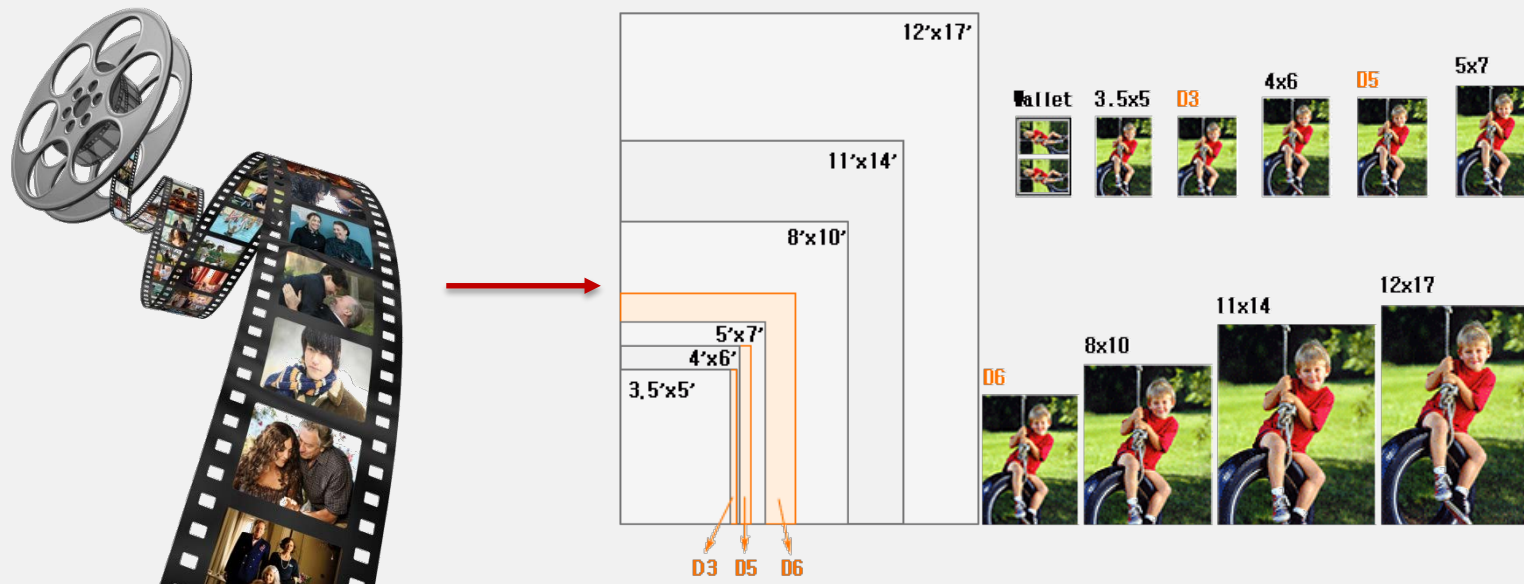
- $(win_x, win_y)$  are screen-space coordinates
  - $(win_x, win_y)$  units are in pixel (with fractions)
- $win_z$  is depth coordinate
  - $win_z$  is in range of 0.0 to 1.0, or depth range





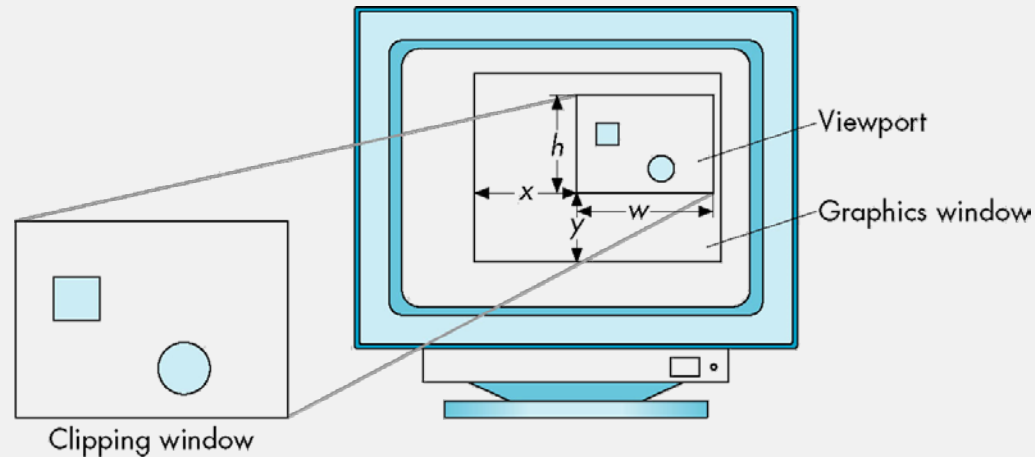
# Camera Specification – Viewport

- Viewport
  - Similar to the size of photo printing
    - A film → Photos of different sizes
  - A rectangular area of the display window



# Camera Specification – Viewport

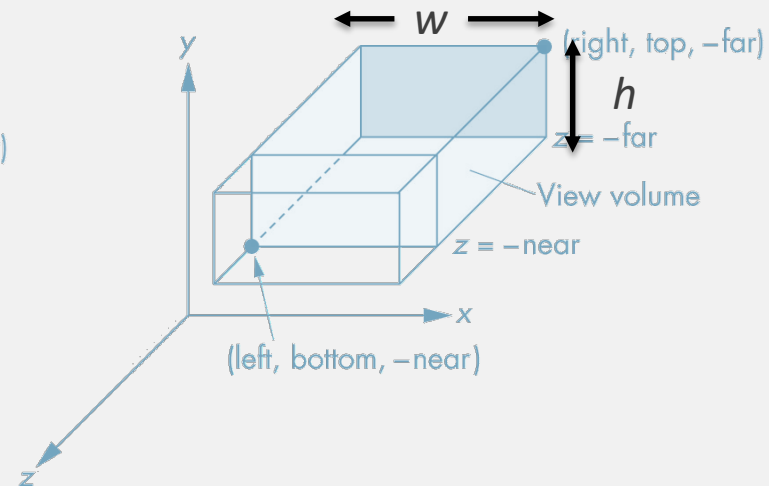
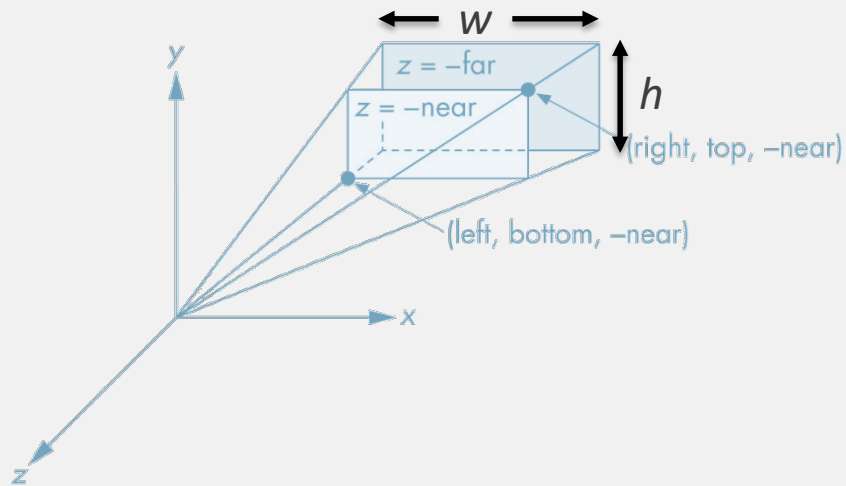
- Viewport
  - Similar to the size of photo printing
    - A film  $\rightarrow$  Photos of different sizes
  - A rectangular area of the display window:  $x, y, w, h$ 
    - $(x, y)$ : the lower-left corner of the viewport
    - $w, h$ : the width and height of the viewport



A mapping to the viewport

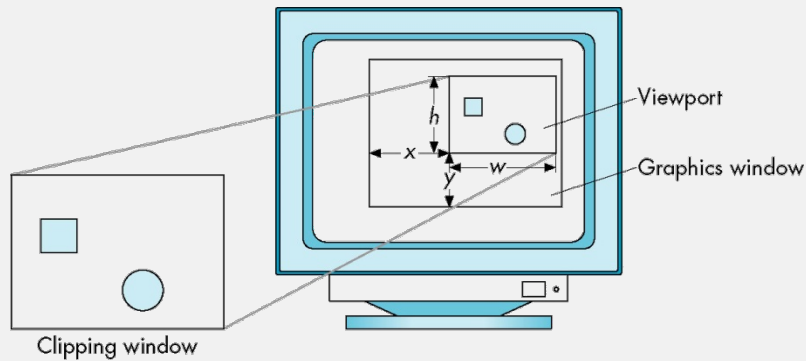
# Camera Specification – Aspect ratio

- Aspect ratio
  - width / height
    - For aspect ratio, absolute sizes of width & height are meaningless
    - Aspect ratio of display window (i.e., device screen) is important

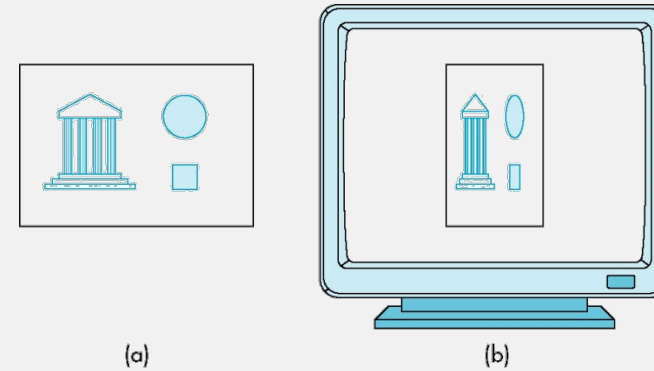


# Camera Specification – Aspect ratio

- Aspect ratio
  - width / height
    - For aspect ratio, absolute sizes of width & height are meaningless
    - Aspect ratio of display window (e.g., device screen) is important



A mapping to the viewport



Aspect-ratio mismatch.  
(a) viewing rectangle, (b) display window

# 감사합니다

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