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# Fault Estimation and Fault Tolerant Control Strategies Applied to VTOL Aerial Vehicles With Soft and Aggressive Actuator Faults

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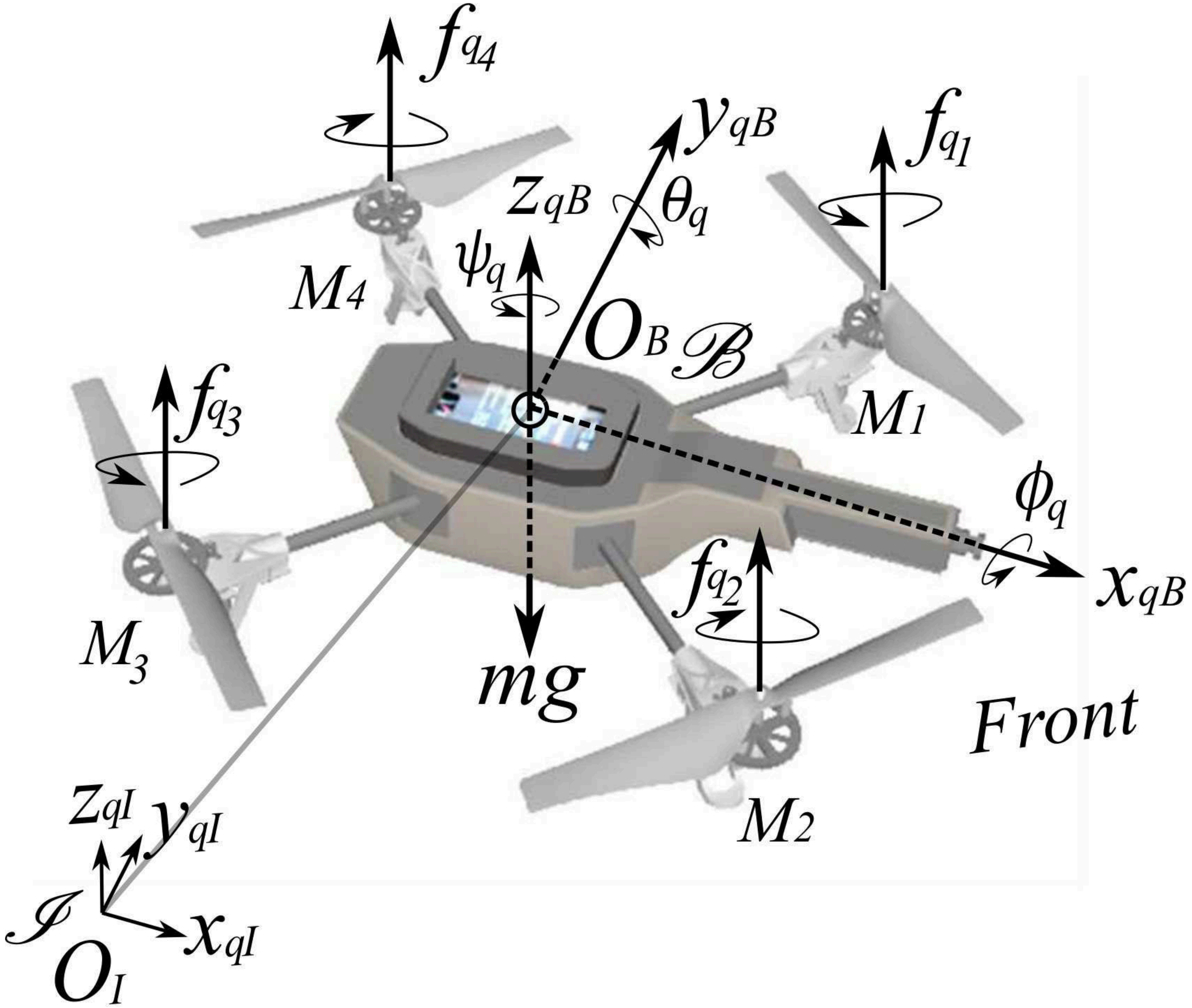
# VTOL Importance And Most Familiar Types

- VTOL best known for vertical take off and landing
  - Takes less space
  - Easier to design and construct
- Fault estimation and diagnosis is vital to VTOL aircrafts
- VTOL types:
  - Quadcopter
  - PVTOL (planar VTOL)

# Quadcopter (Parrot AR 2.0) Scheme

- 4 motors (actuators) (M1, M2, M3, M4)
- 3 translational states (xq,yq,zq)
- 3 speeds of translational states
- 3 Euler angles (phi,theta,psi)(roll,pitch,yaw)
- 3 speeds of Euler angles
- I is the base frame, B is the aircraft frame with center on its COM
- All thrust forces pointing up



# Quadcopter Dynamics

- Newton-Euler gives:
- 12 states:
  - 6 positional states & 6 velocity states
  - 6 translational states & 6 angular states
  - (1-2) is rotational dynamics
  - (2-2) is altitude dynamics
- s is sine and c is cosine

$\Omega$M(t) is the overall residual propeller angular speed

$$\begin{cases} \ddot{\phi}_q(t) = \dot{\theta}_q(t)\dot{\psi}_q(t)\left(\frac{J_y - J_z}{J_x}\right) \\ -\frac{J_{rz}}{J_x}\dot{\theta}_q(t)\Omega_M(t) + \frac{l}{J_x}u_{q_{\text{۲}}}(t) \\ \ddot{\theta}_q(t) = \dot{\phi}_q(t)\dot{\psi}_q(t)\left(\frac{J_z - J_x}{J_y}\right) \\ +\frac{J_{rz}}{J_y}\dot{\phi}_q(t)\Omega_M(t) + \frac{l}{J_y}u_{q_{\text{۳}}}(t) \\ \ddot{\psi}_q(t) = \dot{\phi}_q(t)\dot{\theta}_q(t)\left(\frac{J_x - J_y}{J_z}\right) + \frac{d}{J_z}u_{q_{\text{۴}}}(t) \end{cases} \qquad (\text{۱ـ۲})$$

$$\begin{cases} \ddot{z}_q(t) = -g + c\theta_q(t)c\phi_q(t)\frac{\text{۱}}{m}u_{q_{\text{۱}}}(t) \end{cases} \qquad (\text{۲ـ۲})$$

$$\begin{cases} \ddot{x}_q(t) = (c\psi_q(t)s\theta_q(t)c\phi_q(t) \\ +s\psi_q(t)s\phi_q(t))\,\frac{\text{۱}}{m}u_{q_{\text{۱}}}(t) \\ \ddot{y}_q(t) = (s\psi_q(t)s\theta_q(t)c\phi_q(t) \\ -c\psi_q(t)s\phi_q(t))\,\frac{\text{۱}}{m}u_{q_{\text{۱}}}(t) \end{cases} \qquad (\text{۳ـ۲})$$

# Quadcopter Dynamics

$$O(t) = [\phi_q(t), \theta_q(t), \psi_q(t)]^{\top}$$

$$V(t) = [\dot{x}_q(t), \dot{y}_q(t), \dot{z}_q(t)]^{\top}$$

$$\Omega(t) = \left[\dot{\phi}_q(t), \dot{\theta}_q(t), \dot{\psi}_q(t)\right]^{\top}$$

$$\mathbf{u}_q(t) = [u_{q_۱}(t), u_{q_۲}(t), u_{q_۳}(t), u_{q_۴}(t)]^{\top}$$

$$\mathbf{u}_r(t) = [u_{q_۲}(t), u_{q_۳}(t), u_{q_۴}(t)]'$$

$$\begin{cases} \ddot{\phi}_q(t) = \dot{\theta}_q(t)\dot{\psi}_q(t)\left(\frac{J_y-J_z}{J_x}\right) \\ -\frac{J_{rz}}{J_x}\dot{\theta}_q(t)\Omega_M(t) + \frac{l}{J_x}u_{q_۲}(t) \\ \ddot{\theta}_q(t) = \dot{\phi}_q(t)\dot{\psi}_q(t)\left(\frac{J_z-J_x}{J_y}\right) \\ +\frac{J_{rz}}{J_y}\dot{\phi}_q(t)\Omega_M(t) + \frac{l}{J_y}u_{q_۳}(t) \\ \ddot{\psi}_q(t) = \dot{\phi}_q(t)\dot{\theta}_q(t)\left(\frac{J_x-J_y}{J_z}\right) + \frac{d}{J_z}u_{q_۴}(t) \end{cases}$$

(۱ـ۲)

$$\begin{cases} \ddot{z}_q(t) = -g + c\theta_q(t)c\phi_q(t)\frac{۱}{m}u_{q_۱}(t) \end{cases}$$

(۲ـ۲)

$$\begin{cases} \ddot{x}_q(t) = (c\psi_q(t)s\theta_q(t)c\phi_q(t) \\ +s\psi_q(t)s\phi_q(t))\,\frac{۱}{m}u_{q_۱}(t) \\ \ddot{y}_q(t) = (s\psi_q(t)s\theta_q(t)c\phi_q(t) \\ -c\psi_q(t)s\phi_q(t))\,\frac{۱}{m}u_{q_۱}(t) \end{cases}$$

(۳ـ۲)

$$\mathbf{u}_q(t) = \begin{bmatrix} u_{q_۱}(t) \\ \mathbf{u}_r(t) \end{bmatrix} = \underbrace{\begin{bmatrix} ۱ & ۱ & ۱ & ۱ \\ ۱ & -۱ & -۱ & ۱ \\ ۱ & ۱ & -۱ & -۱ \\ -۱ & ۱ & -۱ & ۱ \end{bmatrix}}_{\Delta_q} \underbrace{\begin{bmatrix} f_{q_۱}(t) \\ f_{q_۲}(t) \\ f_{q_۳}(t) \\ f_{q_۴}(t) \end{bmatrix}}_{\bar{f}_q(t)}$$

(۴ـ۲)

# PVTOL Dynamics

$$f_{p_۲}(t) = f_{q_۲}(t) + f_{q_۳}(t) \text{ و } f_{p_۱}(t) = f_{q_۱}(t) + f_{q_۴}(t)$$

- Simplifying (1-1),(1-2),(1-3) gives:
- 6 states:
  - 3 positional states & 3 velocity states
  - 4 translational states & 2 angular states
  - (5-2) is rotational dynamics
  - (6-2) is altitude dynamics
- forces are combined

$$\ddot{\phi}_p(t) = \frac{l}{J_x} u_{p_۲}(t) \qquad (۵\_۲)$$

$$\ddot{z}_p(t) = -g + c\phi_p(t)\frac{۱}{m} u_{p_۱}(t) \qquad (۶\_۲)$$

$$\ddot{y}_p(t) = -s\phi_p(t)\frac{۱}{m} u_{p_۱}(t) \qquad (۷\_۲)$$

$$\mathbf{u}_p(t) = \begin{bmatrix} u_{p_۱}(t) \\ u_{p_۲}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} ۱ & ۱ \\ ۱ & -۱ \end{bmatrix}}_{\Delta_p} \underbrace{\begin{bmatrix} f_{p_۱}(t) \\ f_{p_۲}(t) \end{bmatrix}}_{\bar{f}_p(t)} . \qquad (۸\_۲)$$

# Problem Formulation

- LoE
- Multiplicative actuator fault
- Additive actuator fault
- v = p for PVTOL
- v = q for quadcopter

$$\mathbf{u}_{\vartheta f}(t) = \Delta_\vartheta \left(I_w - \bar{\lambda}_\vartheta(t)\right) \bar{f}_\vartheta(t) \qquad (۹\text{–}۲)$$

$$\bar{\lambda}_\vartheta(t) = diag(\lambda_{\vartheta_۱}(t), \lambda_{\vartheta_۲}(t), ..., \lambda_{\vartheta_w}(t)) \qquad (۱۰\text{–}۲)$$

$$\mathbf{u}_{\vartheta f}(t) = \mathbf{u}_\vartheta(t) + \bar{\eta}_\vartheta(t) \qquad (۱۱\text{–}۲)$$

$$\bar{\eta}_\vartheta(t) = -\Delta_\vartheta \bar{\lambda}_\vartheta(t) \bar{f}_\vartheta(t) \qquad (۱۲\text{–}۲)$$

# VTOL Fault Controllability

$$\dot{\mathbf{x}}_c(t) = A_c\mathbf{x}_c(t) + B_c\mathbf{u}_c(t)$$

- Linear dynamic system near hover mode

$$\mathcal{M} = \left\{\mathbf{u}_{qf}(t) \mid \mathbf{u}_{qf}(t) = \Delta_q \left(I_w - \bar{\lambda}_q(t)\right) \bar{f}_q(t), \bar{f}_q(t) \in \mathcal{F}\right\},$$

$$\mathcal{U} = \{\mathbf{u}_c(t) \mid \mathbf{u}_c(t) = \mathbf{u}_{qf}(t) - G_c \in \mathcal{M}\}$$

- Theorem 2-1: system is controllable if and only if:
  - 1) $$RANK\,[B_c, A_cB_c, \ldots, A_c^{\vee}B_c] = \wedge$$
  - 2) ACAI: $$\rho\left(G_c, \partial\mathcal{U}\right) > \cdot$$ $$\rho\left(G_c, \partial\mathcal{U}\right) \triangleq \min\left\{\left\|G_c - \mathbf{u}_{qf}(t)\right\|, \mathbf{u}_{qf}(t) \in \partial\mathcal{U}\right\}$$

# VTOL Fault Controllability

- LoE = [0.1 , 0.4] is soft fault
- LoE = [0.5 , 0.9] is aggressive fault
- CN us controllable
- UCN is uncontrollable
- WY is without Yaw
- both systems are UCN with aggressive fault
  - but quadcopter can become CN
  - by giving up the control of yaw angle

| Quadcopter | | PVTOL and Quadcopter | | LoE | عیب |
|---|---|---|---|---|---|
| CY | ACAI – WY | CY | ACAI | | |
| – | – | CN | ۱/۴۷ | ۰/۱ | نرم |
| – | – | CN | ۱/۰۸ | ۰/۲ | |
| – | – | CN | ۰/۷۰ | ۰/۳ | |
| – | – | CN | ۰/۳۱ | ۰/۴ | |
| CN | ۱/۹۹ | UCN | −۰/۰۶ | ۰/۵ | شدید |
| CN | ۱/۲۶ | UCN | −۰/۴۵ | ۰/۶ | |
| CN | ۰/۷۲ | UCN | −۰/۸۴ | ۰/۷ | |
| CN | ۰/۴۴ | UCN | −۱/۲۲ | ۰/۸ | |
| CN | ۰/۱۷ | UCN | −۱/۶۱ | ۰/۹ | |

جدول ۲ـ۲: آنالیز کنترل پذیری برای سیستم های VTOL

# PVTOL FE using Nonlinear AO

- Using only roll and altitude dynamics
- defining the actuator fault vector
- for a nonlinear system like this

$$\mathbf{x}_p(t) = \left[\mathbf{x}_{p_۱}(t), \mathbf{x}_{p_۲}(t), \mathbf{x}_{p_۳}(t), \mathbf{x}_{p_۴}(t)\right]^\top = \left[z_p(t), \phi_p(t), \dot{z}_p(t), \dot{\phi}_p(t)\right]^\top$$

$$\bar{\eta}_p(t) = \left[\eta_{p_۱}(t), \eta_{p_۲}(t)\right]^\top$$

$$\dot{\mathbf{x}}_{\phi f}(t) = \alpha_۱ + \beta_۱ \eta_{p_۲}(t)$$

$$\dot{\mathbf{x}}_{zf}(t) = \alpha_۲ + \beta_۲ \eta_{p_۱}(t)$$

$$\mathbf{x}_{\phi f}(t) = \left[\mathbf{x}_{pf_۲}(t), \mathbf{x}_{pf_۴}(t)\right]^\top,$$

$$\mathbf{x}_{zf}(t) = \left[\mathbf{x}_{pf_۱}(t), \mathbf{x}_{pf_۳}(t)\right]^\top,$$

$$\mathbf{y}_{pf}(t) = \mathbf{x}_{pf}(t)$$

$$\alpha_۱ = \begin{bmatrix} \mathbf{x}_{pf_۴}(t) \\ l u_{p_۲}(t)/J_x \end{bmatrix}, \quad \beta_۱ = \begin{bmatrix} ۰ \\ l/J_x \end{bmatrix}$$

$$\alpha_۲ = \begin{bmatrix} \mathbf{x}_{pf_۳}(t) \\ c\left(\mathbf{x}_{pf_۲}(t)\right) u_{p_۱}(t)/m - g \end{bmatrix}, \quad \beta_۲ = \begin{bmatrix} ۰ \\ c\left(\mathbf{x}_{pf_۲}(t)\right)/m \end{bmatrix}$$

# PVTOL FE using Nonlinear AO

- exists a nonlinear adaptive observer such that
- Ly1, Ly2, Lf1, Lf2 are observer gains
- Theorem 3-1 gives the existence of this observer

$$\dot{\hat{\mathbf{x}}}_{\phi f}(t) = \alpha_1 + \beta_1 \hat{\eta}_{p_2}(t) - L_{y_1} \mathbf{e}_{\phi y}(t)$$

$$\dot{\hat{\eta}}_{p_2}(t) = -L_{f_1} \beta_1^\top \mathbf{e}_{\phi y}(t)$$

$$\dot{\hat{\mathbf{x}}}_{z f}(t) = \alpha_2 + \beta_2 \hat{\eta}_{p_1}(t) - L_{y_2} \mathbf{e}_{z y}(t)$$

$$\dot{\hat{\eta}}_{p_1}(t) = -L_{f_2} \beta_2^\top \mathbf{e}_{z y}(t)$$

$$\mathbf{e}_{\phi y}(t) = [\mathbf{y}_{pf_2}(t) - \hat{\mathbf{y}}_{pf_2}(t), \mathbf{y}_{pf_4}(t) - \hat{\mathbf{y}}_{pf_4}(t)]^\top$$

$$\mathbf{e}_{py}(t) = \mathbf{y}_{pf}(t) - \hat{\mathbf{y}}_{pf}(t)$$

$$\mathbf{e}_{zy}(t) = [\mathbf{y}_{pf_1}(t) - \hat{\mathbf{y}}_{pf_1}(t), \mathbf{y}_{pf_3}(t) - \hat{\mathbf{y}}_{pf_3}(t)]^\top$$

# PVTOL FE using Linear PIO

- Using only roll and altitude dynamics
- Augmented faulty linear dynamic system is

$$\dot{\mathbf{x}}_{\breve{a}}(t) = \bar{\breve{A}}\mathbf{x}_{\breve{a}}(t) + \bar{\breve{B}}\mathbf{u}_p(t) + \bar{\breve{\Gamma}}\mathbf{w}_p(t) + \bar{\mathbf{g}}$$

$$\mathbf{y}_{pf}(t) = \bar{\breve{C}}\mathbf{x}_{\breve{a}}(t)$$

$$\mathbf{x}_{\breve{a}}(t) = \begin{bmatrix} \mathbf{x}_{pf}(t) \\ \bar{\eta}_p(t) \end{bmatrix}, \quad \bar{\breve{A}} = \begin{bmatrix} \breve{A} & \breve{E} \\ ۰_{۲\times۴} & ۰_{۲\times۲} \end{bmatrix}, \bar{\breve{B}} = \begin{bmatrix} \breve{B} \\ ۰_{۲\times۲} \end{bmatrix}$$

$$\bar{\breve{\Gamma}} = \begin{bmatrix} \breve{W} \\ ۰_{۲\times۱} \end{bmatrix}, \quad \bar{\mathbf{g}} = \begin{bmatrix} \mathbf{g} \\ ۰_{۲\times۱} \end{bmatrix}, \bar{\breve{C}} = \begin{bmatrix} \breve{C} \\ ۰_{۲\times۴} \end{bmatrix}^{\top}$$

$$\breve{A} = \begin{bmatrix} ۰_{۲\times۲} & I_۲ \\ ۰_{۲\times۲} & ۰_{۲\times۲} \end{bmatrix}, \quad \breve{B} = \begin{bmatrix} ۰_{۲\times۲} \\ J_p \end{bmatrix}, \breve{C} = I_۴,$$

# PVTOL FE using Linear PIO

- Then the augmented linear proportional-integral observer is

$$\dot{\hat{\mathbf{x}}}_{\breve{a}}(t) = \bar{\breve{A}}\hat{\mathbf{x}}_{\breve{a}}(t) + \bar{\breve{B}}\mathbf{u}_p(t) + \bar{\breve{L}}_{PI}\mathbf{e}_{py}(t) + \bar{\mathbf{g}}$$

$$\hat{\mathbf{y}}_{pf}(t) = \bar{\breve{C}}\hat{\mathbf{x}}_{\breve{a}}(t)$$

$$\dot{\mathbf{e}}_{\breve{a}}(t) = \left(\bar{\breve{A}} - \bar{\breve{L}}_{PI}\bar{\breve{C}}\right)\mathbf{e}_{\breve{a}}(t) + \bar{\breve{\Gamma}}\mathbf{w}_p(t)$$

$$\mathbf{e}_{\breve{a}}(t) = \mathbf{x}_{\breve{a}}(t) - \hat{\mathbf{x}}_{\breve{a}}(t)$$

- LPI is the observer gain which can be found from
- Theorem 3-2:

$$\bar{\breve{L}}_{PI} = P^{-۱}M$$

$$\min_{P,M} \gamma,$$

$$\textit{subject to}$$

$$\begin{bmatrix} \mathrm{He}\{P\bar{\breve{A}} - M\bar{\breve{C}}\} + I & P\bar{\breve{\Gamma}} \\ \bar{\breve{\Gamma}}^{\top}P & -\gamma^{۲}I \end{bmatrix} < ۰$$

$$\mathrm{He}\{P\bar{\breve{A}} - M\bar{\breve{C}}\} = (P\bar{\breve{A}} - M\bar{\breve{C}}) + (P\bar{\breve{A}} - M\bar{\breve{C}})^{\top}$$

# Quadcopter FE using qLPV PIO

- Using the sector method to turn nonlinear dynamic system to sectors of local linear systems
- rewritten in this nonlinear state space system:

$$\begin{aligned}\dot{\mathbf{x}}_r(t) &= A\left(\dot{\phi}_q(t), \dot{\theta}_q(t)\right)\mathbf{x}_r(t) + B\mathbf{u}_r(t)\\ &\quad + W\left(\dot{\phi}_q(t), \dot{\theta}_q(t)\right)\mathbf{w}_r(t)\\ \mathbf{y}_r(t) &= C\mathbf{x}_r(t)\end{aligned}$$

- given n nonlinear elements
- we can have 2^n sectors of linear systems

$$A\left(\dot{\phi}_q(t), \dot{\theta}_q(t)\right) = \begin{bmatrix} ۰_{۳\times۳} & I_۳ \\ ۰_{۳\times۳} & A_r \end{bmatrix}, \quad C = I_۶$$

$$W\left(\dot{\phi}_q(t), \dot{\theta}_q(t)\right) = \left[۰_{۳\times۱}, b_۱\dot{\theta}_q(t), b_۲\dot{\phi}_q(t), ۰\right]^\top$$

$$A_r = \begin{bmatrix} ۰ & ۰ & h_۱\dot{\theta}_q(t) \\ ۰ & ۰ & h_۲\dot{\phi}_q(t) \\ h_۳\dot{\theta}_q(t) & ۰ & ۰ \end{bmatrix}, \quad B = \begin{bmatrix} ۰_{۳\times۳} \\ J_r \end{bmatrix}$$

$$\mathbf{x}_r(t) = \left[\phi_q(t), \theta_q(t), \dot{\psi}_q(t), \dot{\phi}_q(t), \dot{\theta}_q(t), \dot{\psi}_q(t)\right]^\top$$

# Quadcopter FE using qLPV PIO

$$\zeta(t) = [\zeta_{۱}(t), \zeta_{۲}(t)]^{\top}$$

- Scheduling variables: $\zeta_{۱}(t) = \dot{\phi}_q(t) \in [-\pi/۲, \pi/۲](\mathrm{rad/s})$ $\zeta_{۲}(t) = \dot{\theta}_q(t) \in [-\pi/۲, \pi/۲](rad/s)$
- weighting functions:
- 2 corresponding to each variable
- Scheduling functions:

$$\mu_{۰}^{۱} = \left(\pi - ۲\dot{\phi}_q(t)\right)/۲\pi, \quad \mu_{۱}^{۱} = ۱ - \mu_{۰}^{۱}$$

$$\mu_{۰}^{۲} = \left(\pi - ۲\dot{\theta}_q(t)\right)/۲\pi, \quad \mu_{۱}^{۲} = ۱ - \mu_{۰}^{۲}$$

$$\rho_{۱}(\zeta(t)) = \mu_{۰}^{۱}\mu_{۰}^{۲}, \quad \rho_{۲}(\zeta(t)) = \mu_{۰}^{۱}\mu_{۱}^{۲}$$

$$\rho_{۳}(\zeta(t)) = \mu_{۱}^{۱}\mu_{۰}^{۲}, \quad \rho_{۴}(\zeta(t)) = \mu_{۱}^{۱}\mu_{۱}^{۲}$$

- convex set conditions on SF:

$$\rho_i(\zeta(t)) \geqslant ۰, \quad \sum_{i=۱}^{۴} \rho_i(\zeta(t)) = ۱, \forall t, \forall i = ۱, \ldots, ۴$$

# Quadcopter FE using qLPV PIO

- Ai and Wi: use the maximum and minimum values of scheduling functions to substitute in A & W
- 4 matrices for $A_i = A\left(\zeta_1^{\Lambda_i^1}, \zeta_2^{\Lambda_i^2}\right), \quad W_i = W\left(\zeta_1^{\Lambda_i^1}, \zeta_2^{\Lambda_i^2}\right)$
- augmented qLPV faulty system:

$$\dot{\mathbf{x}}_a(t) = \sum_{i=1}^{4} \rho_i(\zeta(t)) \left(\bar{A}_i \mathbf{x}_a(t) + \bar{B}\mathbf{u}_r(t) + \bar{\Gamma}_i \mathbf{w}_r(t)\right)$$

$$\mathbf{y}_{rf}(t) = \bar{C}\mathbf{x}_a(t)$$

$$\mathbf{x}_a(t) = \begin{bmatrix} \mathbf{x}_{rf}(t) \\ \bar{\eta}_r(t) \end{bmatrix}, \quad \bar{A}_i = \begin{bmatrix} A_i & E \\ 0_{3\times6} & 0_{3\times3} \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0_{3\times3} \end{bmatrix}$$

$$\bar{\Gamma}_i = \begin{bmatrix} W_i \\ 0_{3\times1} \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} C \\ 0_{3\times6} \end{bmatrix}^\top, \bar{\eta}_r(t) = \begin{bmatrix} \eta_{q_2}(t) \\ \eta_{q_3}(t) \\ \eta_{q_4}(t) \end{bmatrix}$$

# Quadcopter FE using qLPV PIO

- augmented qLPV PIO observer:

$$\dot{\hat{\mathbf{x}}}_a(t) = \sum_{i=۱}^{۴} \rho_i(\zeta(t)) \left(\bar{A}_i \hat{\mathbf{x}}_a(t) + \bar{B}\mathbf{u}_r(t) + \bar{L}_{PIi}\mathbf{e}_{ry}(t)\right)$$

$$\hat{\mathbf{y}}_{rf}(t) = \bar{C}\hat{\mathbf{x}}_a(t) \qquad \mathbf{e}_{ry}(t) = \mathbf{y}_{rf}(t) - \hat{\mathbf{y}}_{rf}(t)$$

- LPIi matrices are the observer gains

$$\mathbf{e}_a(t) = \mathbf{x}_a(t) - \hat{\mathbf{x}}_a(t) \qquad \bar{L}_{PIi} = [L_{Pi}, L_{Ii}]^\top$$

$$\dot{\mathbf{e}}_a(t) = \sum_{i=۱}^{۴} \rho_i(\zeta(t)) \left(\left(\bar{A}_i - \bar{L}_{PIi}\bar{C}\right)\mathbf{e}_a(t) + \bar{\Gamma}_i \mathbf{w}_r(t)\right)$$

- solving the error dynamics like
- theorem-3-2 but with 4 Optimization problems

# Fault Detection and Isolation System (FDI)

- Alarm goes off if estimate fault larger than threshold

$$\left|\bar{\eta}_{\vartheta_j}(t)\right| \geqslant T_{\vartheta_j} \quad \Rightarrow \quad \text{in faulty case ( Alarm } = \text{۱)}$$

$$\left|\bar{\eta}_{\vartheta_j}(t)\right| < T_{\vartheta_j} \quad \Rightarrow \quad \text{in fault–free case ( Alarm } = \text{۰)}$$

# Fault Detection and Isolation System (FDI)

- + and - are the positive and negative signs for fault signal
- NC means no change and it has no effect
- The table can be found like this: $\hat{\bar{\eta}}_q(t) = \left[-\left|\hat{\eta}_{q_۲}(t)\right|, \hat{\bar{\eta}}_r(t)\right]^\top$
- example: fault in M2
- using Delta_q $\bar{\eta}_q(t) = [\eta_{q_۱}(t), \eta_{q_۲}(t), \eta_{q_۳}(t), \eta_{q_۴}(t)]^\top$
- The signs are revealed

$$= [-\lambda_{q_۲}(t) f_{q_۲}(t), \lambda_{q_۲}(t) f_{q_۲}(t)$$
$$-\lambda_{q_۲}(t) f_{q_۲}(t), -\lambda_{q_۲}(t) f_{q_۲}(t)]^\top$$

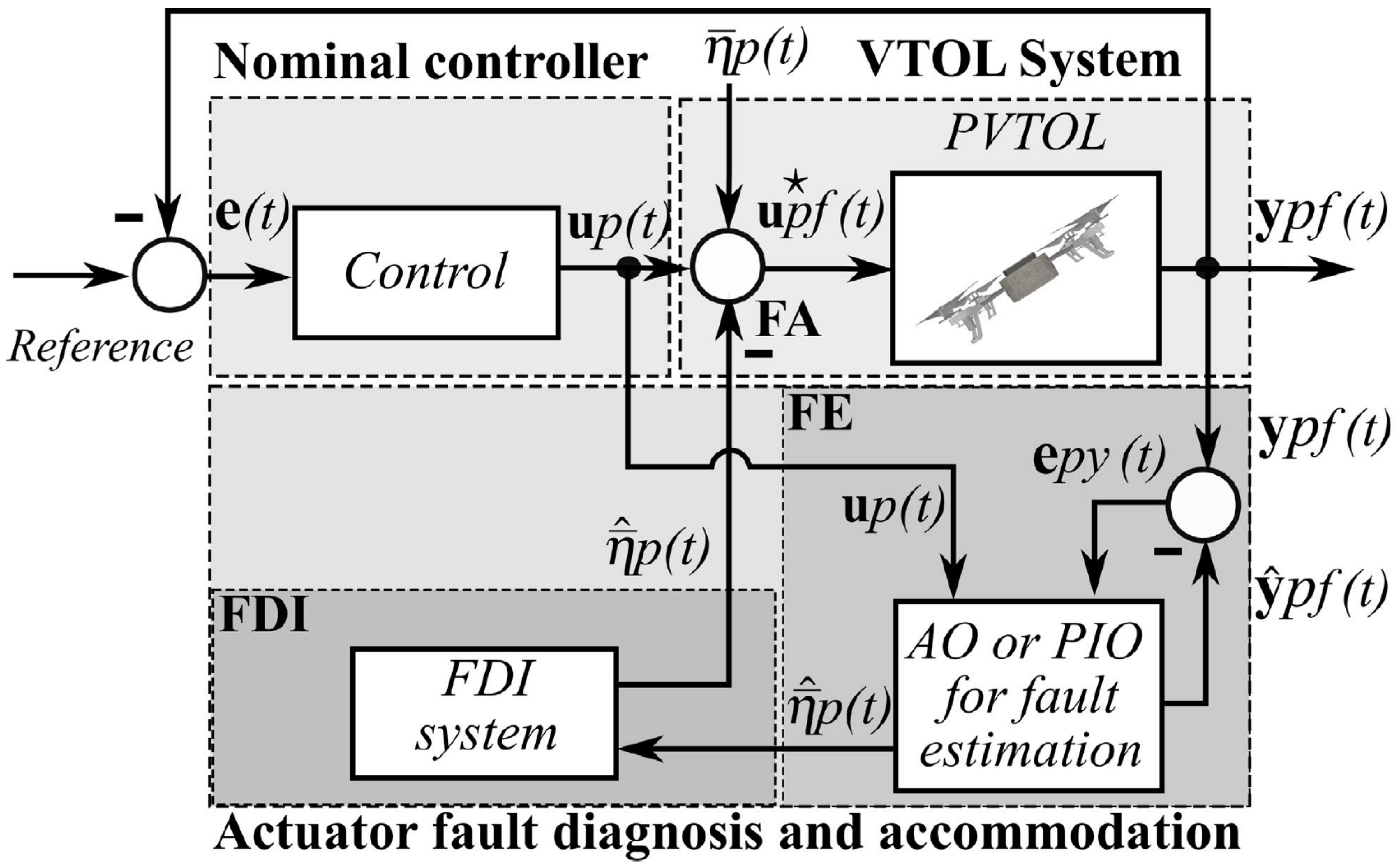
PVTOL

| عیب ها | $\hat{\eta}_{p_۱}(t)$ | $\hat{\eta}_{p_۲}(t)$ |
|---|---|---|
| $\{M_۱, M_۴\}$ | $-$ | $-$ |
| $\{M_۲, M_۳\}$ | $-$ | $+$ |
| $\{M_۱, M_۲, M_۳, M_۴\}$ | $+$ | NC |

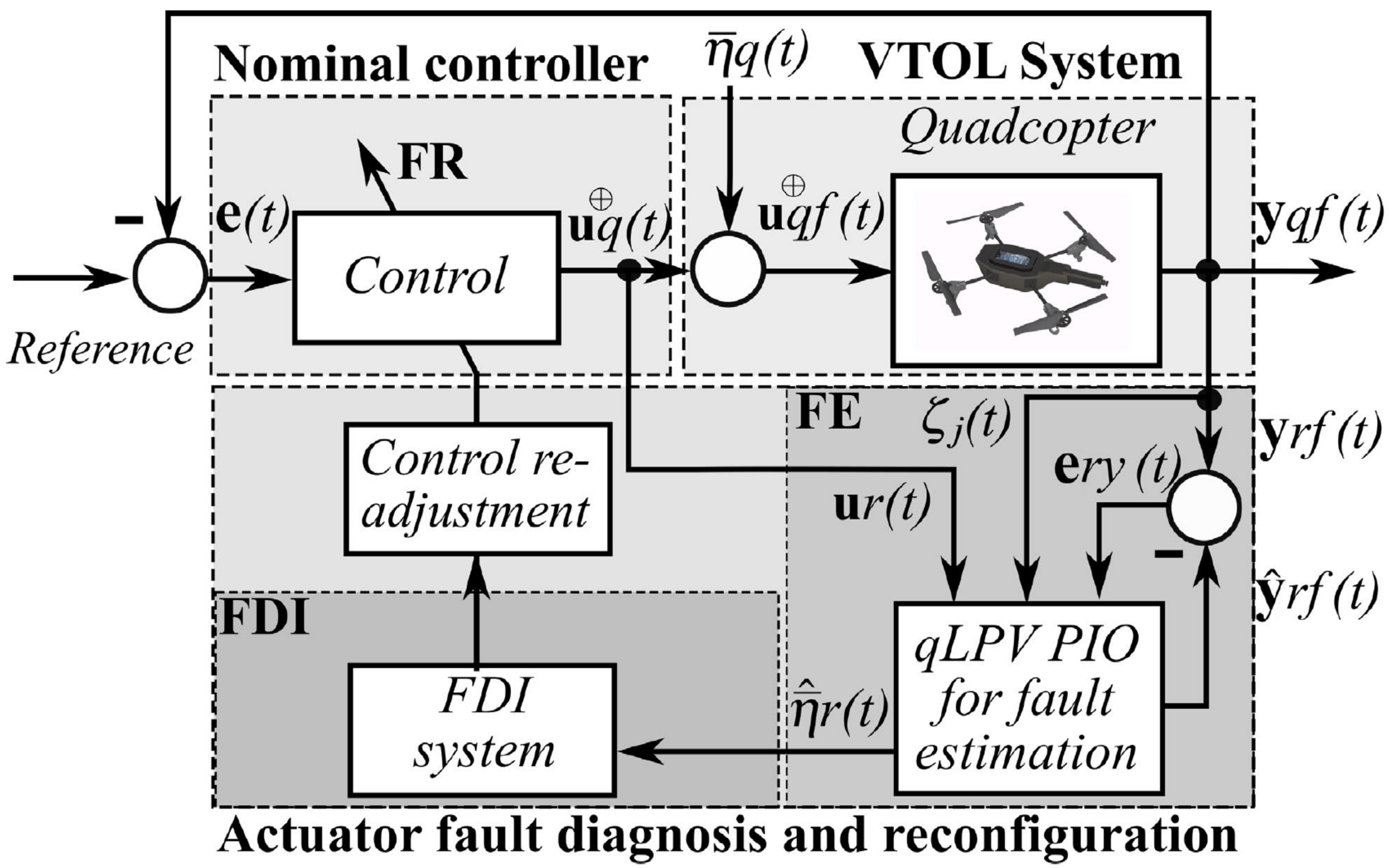
Quadcopter

| عیب ها | $\hat{\eta}_{q_۲}(t)$ | $\hat{\eta}_{q_۳}(t)$ | $\hat{\eta}_{q_۴}(t)$ |
|---|---|---|---|
| $M_۱$ | $-$ | $-$ | $+$ |
| $M_۲$ | $+$ | $-$ | $-$ |
| $M_۳$ | $+$ | $+$ | $+$ |
| $M_۴$ | $-$ | $+$ | $-$ |

# Fault Tolerant Control System (FTC)




شکل ۴ـ۱: ساختار کنترل تطبیق با عیب در PVTOL




شکل ۴ـ۲: ساختار کنترل پیکربندی دوباره با عیب در Quadcopter

# Nominal Controller

- Control law using unit quaternions:
- quaternions let you use an
- under-actuated system like VTOL as a fully-actuated system.

$$\mathbf{u}_{\vartheta}(t)=\begin{bmatrix} \left\|-K_{P\xi}\left(\xi(t)-\xi_d(t)\right)-K_{D\xi}\left(\bar{v}(t)-\bar{v}_d(t)\right)\right\| \\ -۲K_{P\Omega}\ln\left(\mathrm{q}_e(t)\right)-K_{D\Omega}\left(\Omega(t)-\Omega_d(t)\right) \end{bmatrix}$$

$$\boldsymbol{q}=q_{۰}+\bar{q}=q_{۰}+\begin{bmatrix} q_{۱} \\ q_{۲} \\ q_{۳} \end{bmatrix}$$

$$K_{P\Omega}=\operatorname{diag}\left(k_{P\bar{\Omega}}\right)،K_{D\xi}=\operatorname{diag}\left(k_{D\bar{\xi}}\right)،K_{P\xi}=\operatorname{diag}\left(k_{P\bar{\xi}}\right)$$

$$\bar{v}(t)=\left[\dot{x}_q(t),\dot{y}_q(t),\dot{z}_q(t)\right]^{\top}،\Omega(t)=\left[\dot{\phi}_q(t),\dot{\theta}_q(t),\dot{\psi}_q(t)\right]،\xi(t)=\left[x_q(t),y_q(t),z_q(t)\right]^{\top}$$

$$\boldsymbol{q}_e=\boldsymbol{q}\otimes\boldsymbol{q}_d^{*} \qquad \boldsymbol{q}^{*}=q_{۰}-\bar{q} \qquad \boldsymbol{q}\otimes\boldsymbol{r}=\left(q_{۰}r_{۰}-\bar{q}\cdot\bar{r}\right)+\left(r_{۰}\bar{q}+q_{۰}\bar{r}+\bar{q}\times\bar{r}\right) \qquad \ln\boldsymbol{q}=\begin{cases} \ln\|\boldsymbol{q}\|+\frac{\bar{q}}{\|\bar{q}\|}\arccos\frac{q_{۰}}{\|\boldsymbol{q}\|}, & \|\bar{q}\|\neq ۰ \\ \ln\|\boldsymbol{q}\|, & \|\bar{q}\|=۰ \end{cases}$$

- In PVTOL :

$$\dot{\psi}_q(t),\dot{\theta}_q(t),\dot{x}_q(t)=۰,\forall t \text{ و } \psi_q(t),\theta_q(t),x_q(t)=۰,\forall t$$

# Fault Accommodation Control Rule

- Both PVTOL and quadcopter with soft actuator fault:
- this will lower the effect of the fault on the system

$$\mathbf{u}^{\star}_{\vartheta f}(t) = \mathbf{u}_{\vartheta f} - \hat{\bar{\eta}}_{\vartheta}(t)$$

# Fault Reconfiguration Control Rule

- IF fault detected : orientation control parameters should change and

  $k_{P\psi}$ . should be set to zero

  - Do not control yaw.

- IF fault isolated :

  $$\dot{\psi}_d(t) = \bar{h}\dot{\bar{\psi}} \text{ ،}$$

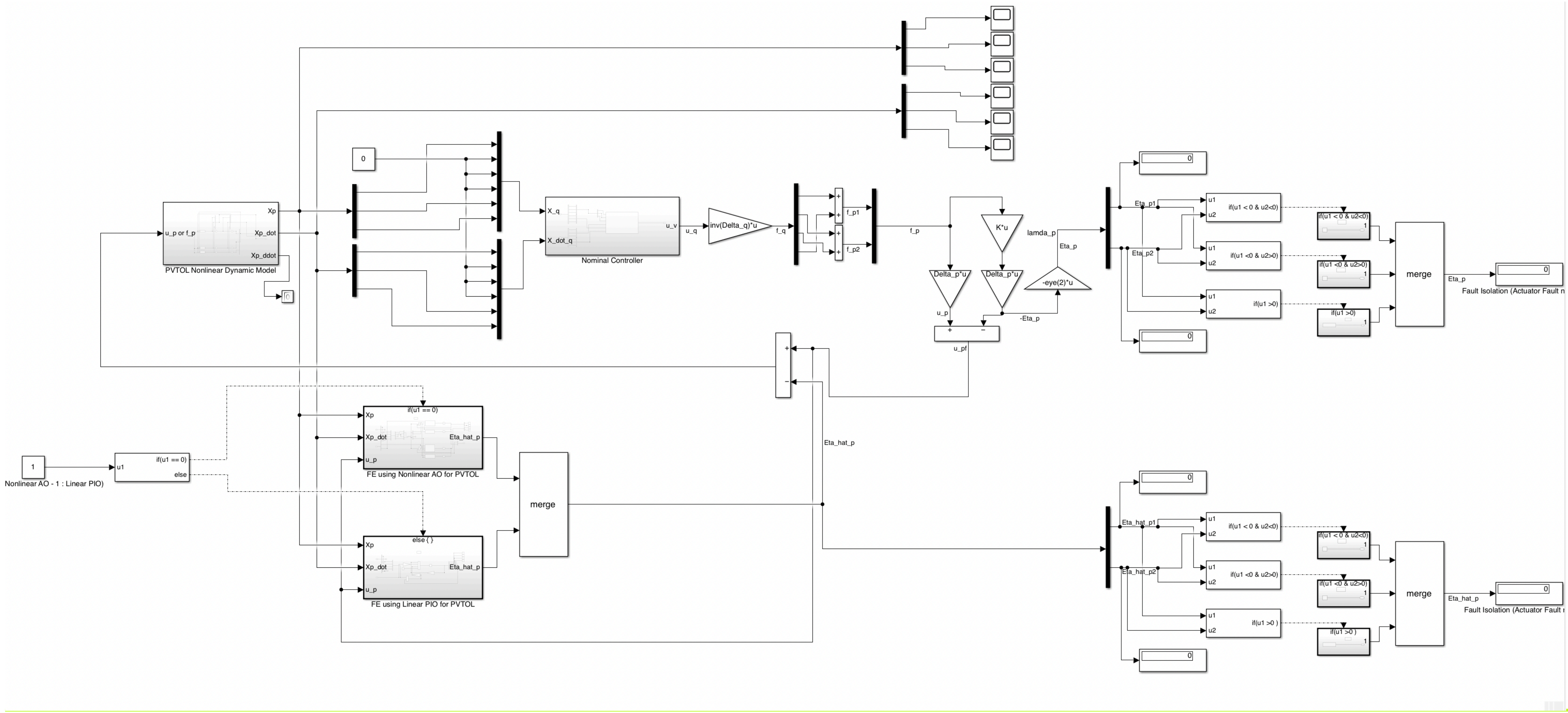
  - for fault in motors M1 and M3 : $\bar{h} = \text{١} \text{ ،}$
  - for fault in motors M2 and M4 : $\bar{h} = -\text{١}$ .
  - yaw constant speed depends on yaw damping coefficient

# Simulations

- predefine parameters:

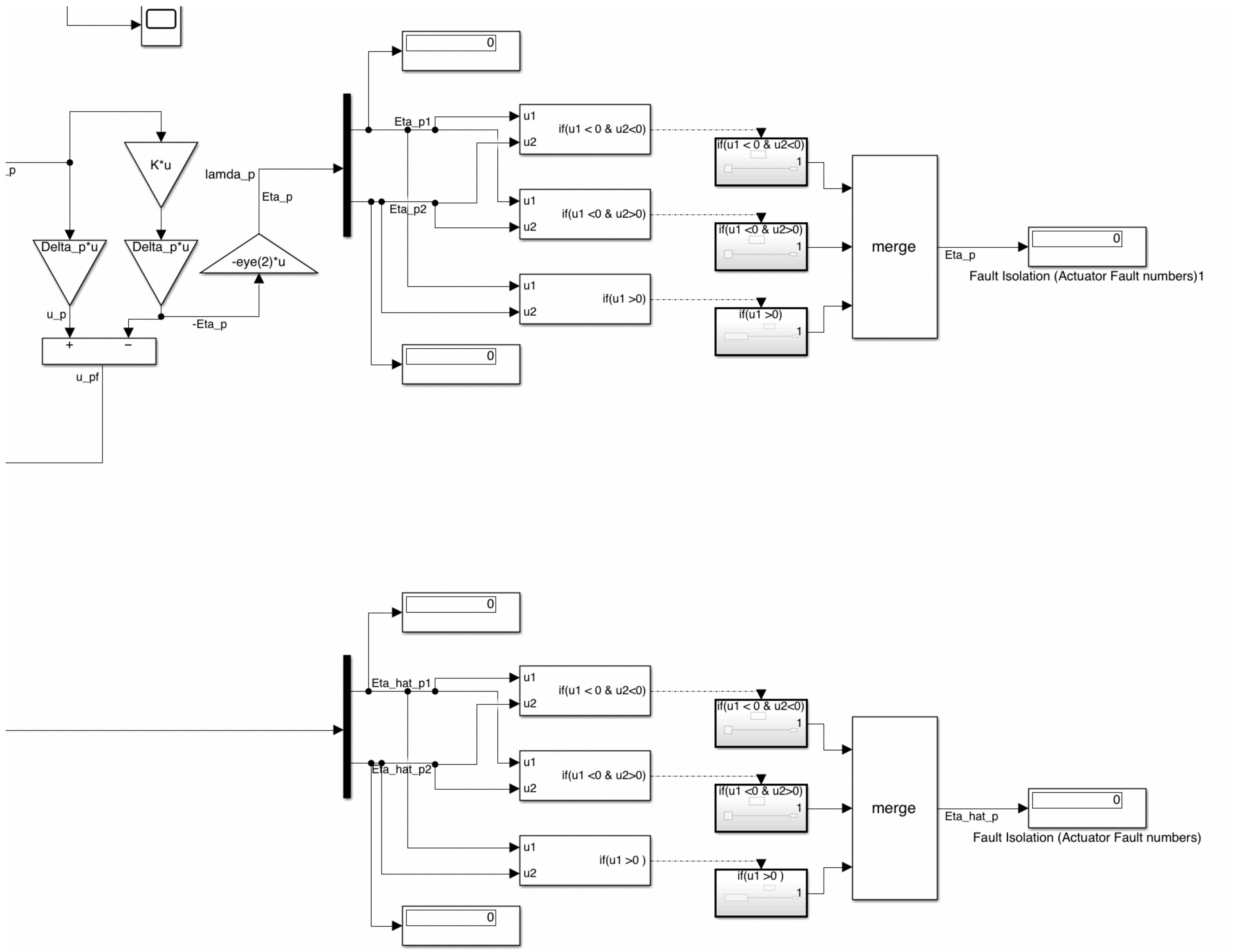
| | |
|---|---|
| $L_{f_۱} = ۵۰$ | $L_{y_۱} = ۰/۰۰۹$ |
| $L_{f_۲} = ۵۰$ | $L_{y_۲} = ۵$ |
| $K_{D\xi} = \mathrm{diag}(۰/۰۸, ۰/۰۸, ۰/۲۲)$ | $K_{P\xi} = \mathrm{diag}(۰/۱۸, ۰/۱۸, ۰/۵۲)$ |
| $K_{D\Omega} = \mathrm{diag}(۰/۲, ۰/۲, ۰/۵)$ | $K_{P\Omega} = \mathrm{diag}(۱/۵, ۱/۵, ۲)$ |

# PVTOL FTC Schematics

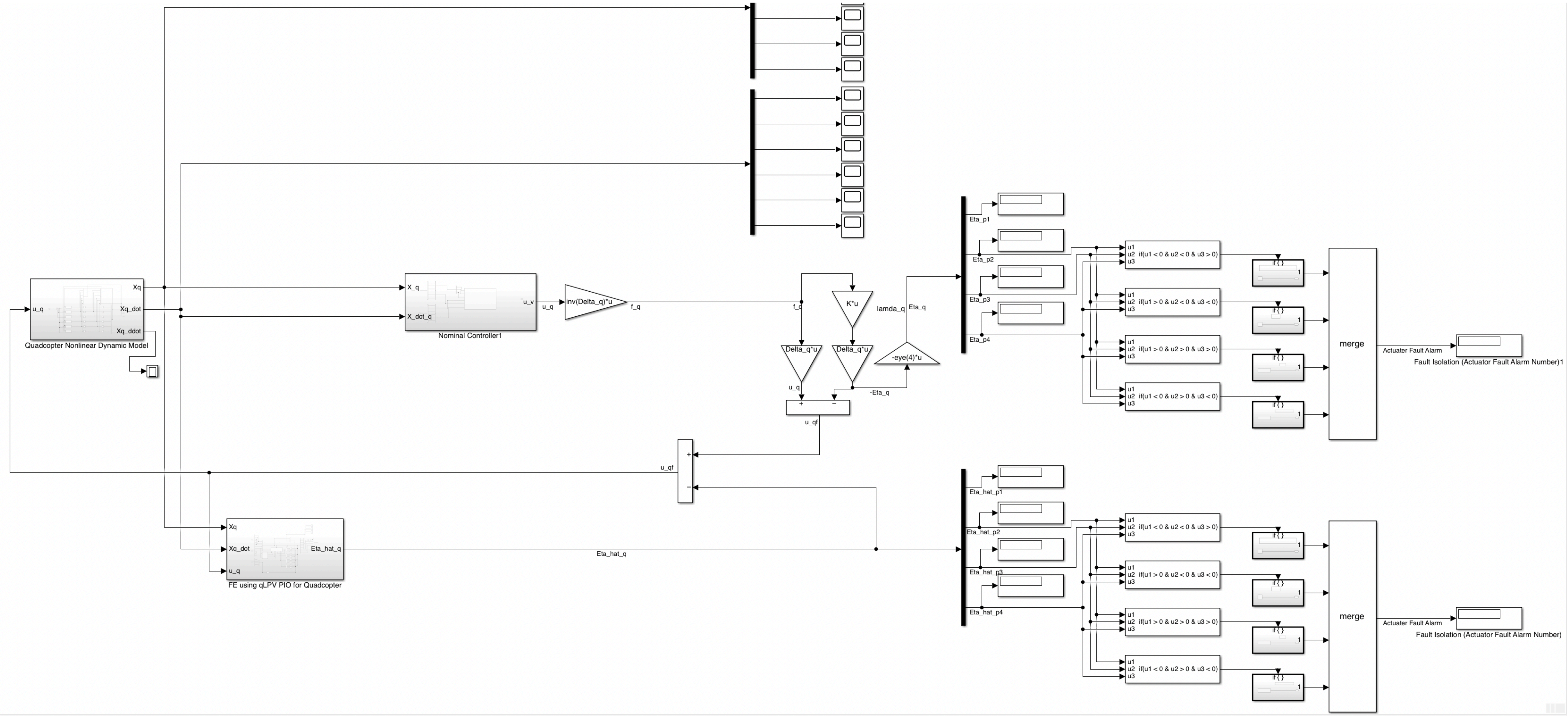


# PVTOL FTC Schematics

- PVTOL FDI:
- based on the sign table

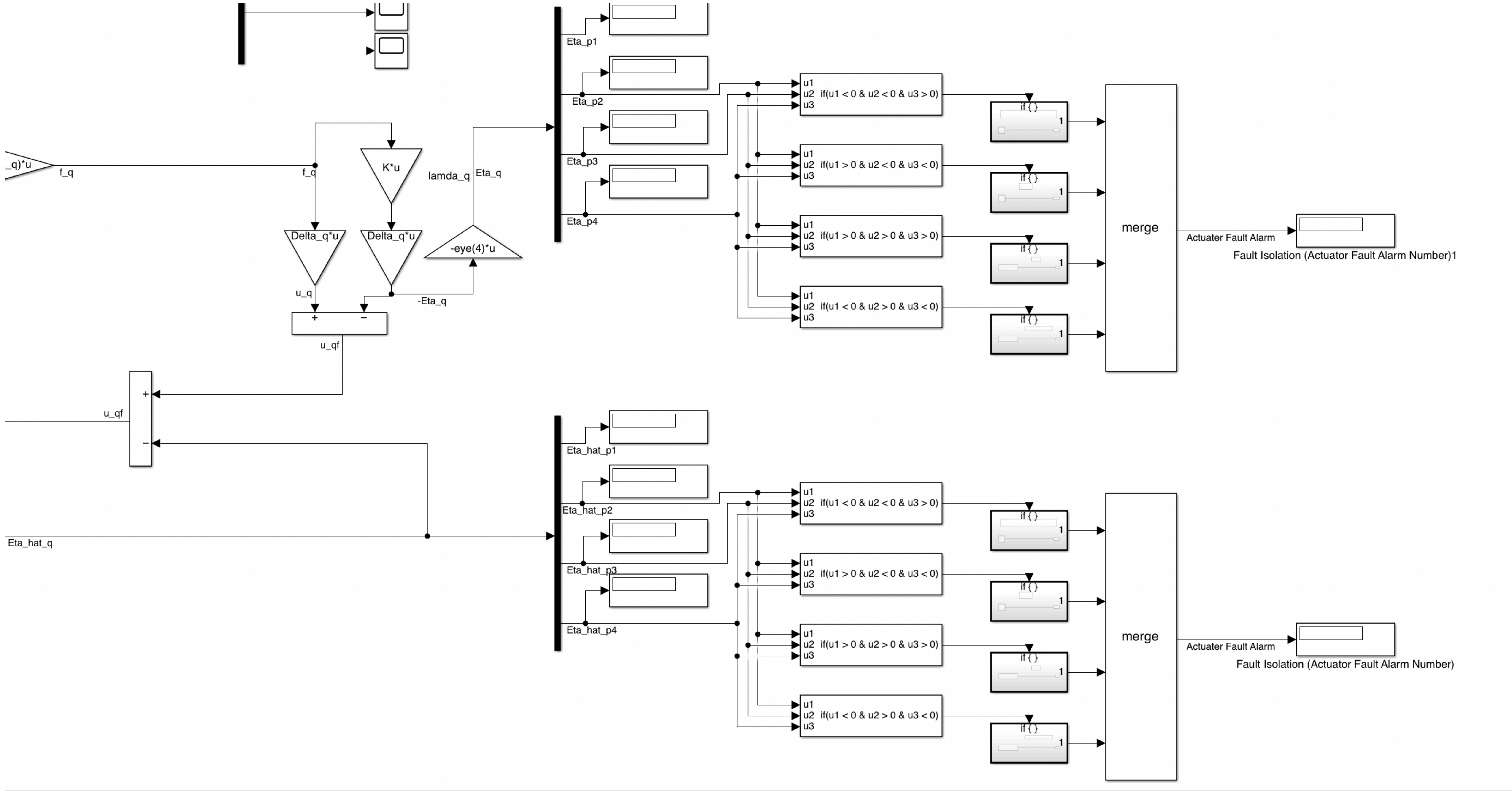


# Quadcopter FTC Schematics



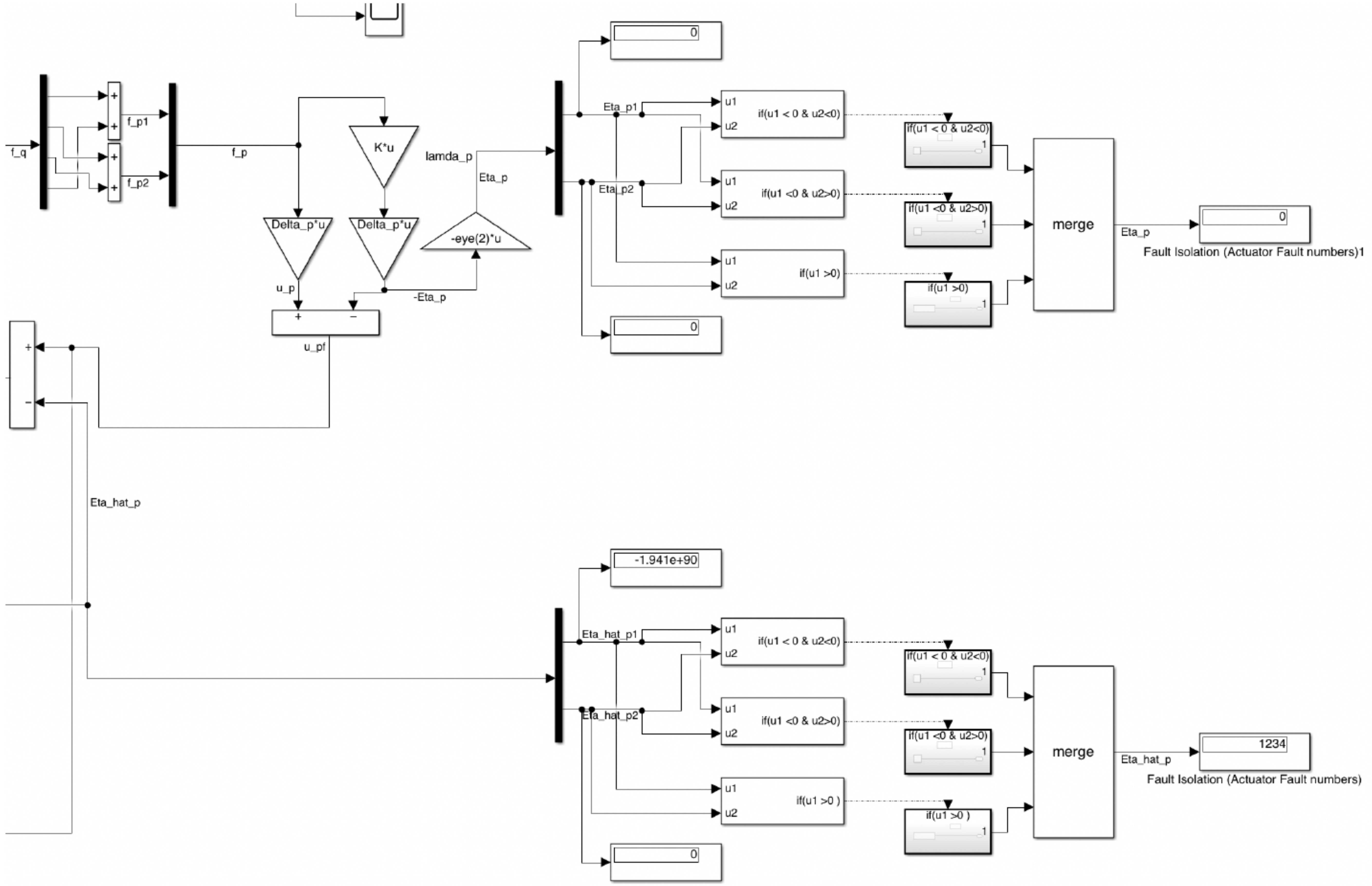
# Quadcopter FTC Schematics

- Quadcopter FDI:
- based on the sign table



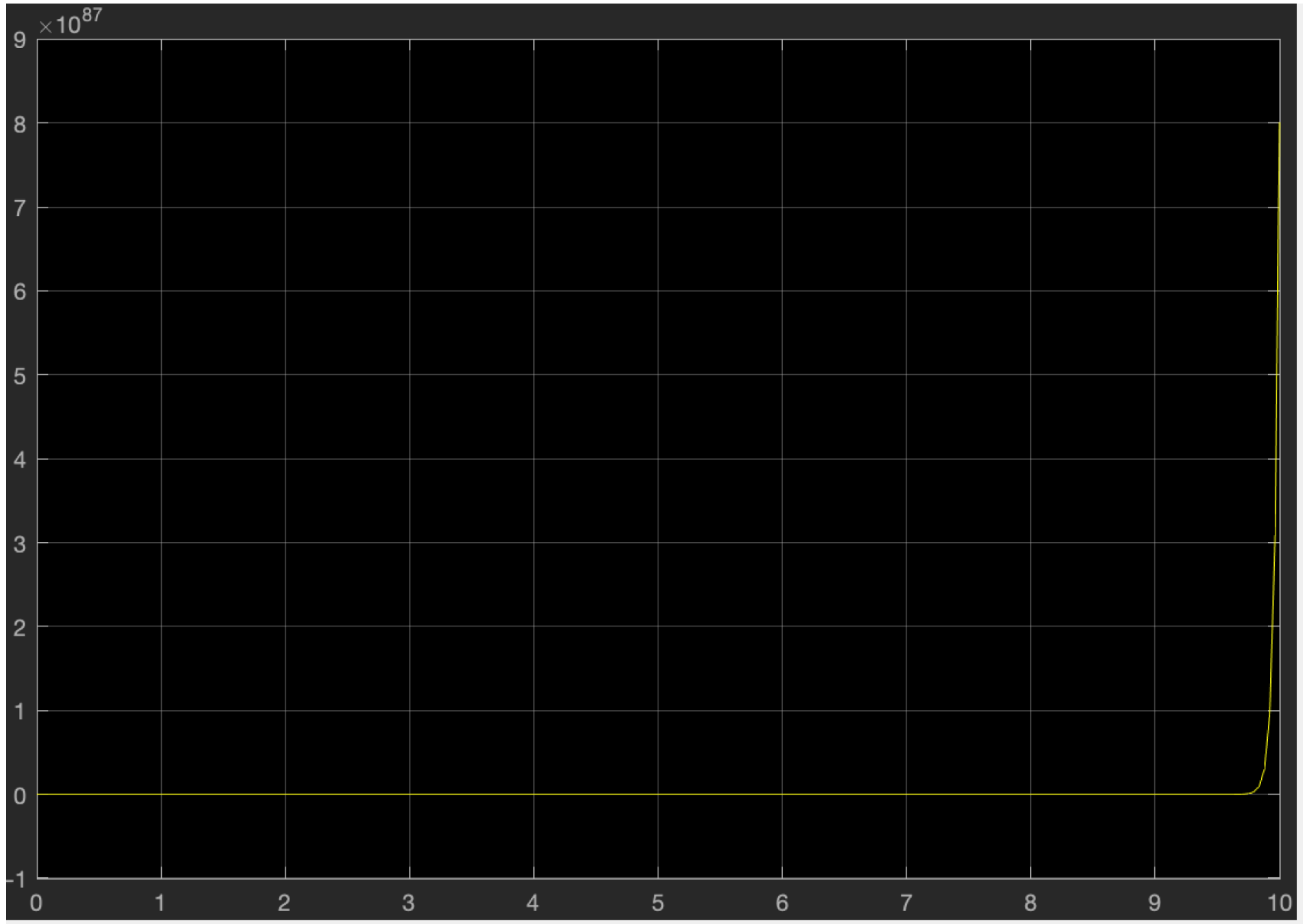
# Simulations

- fault detected
- unstable

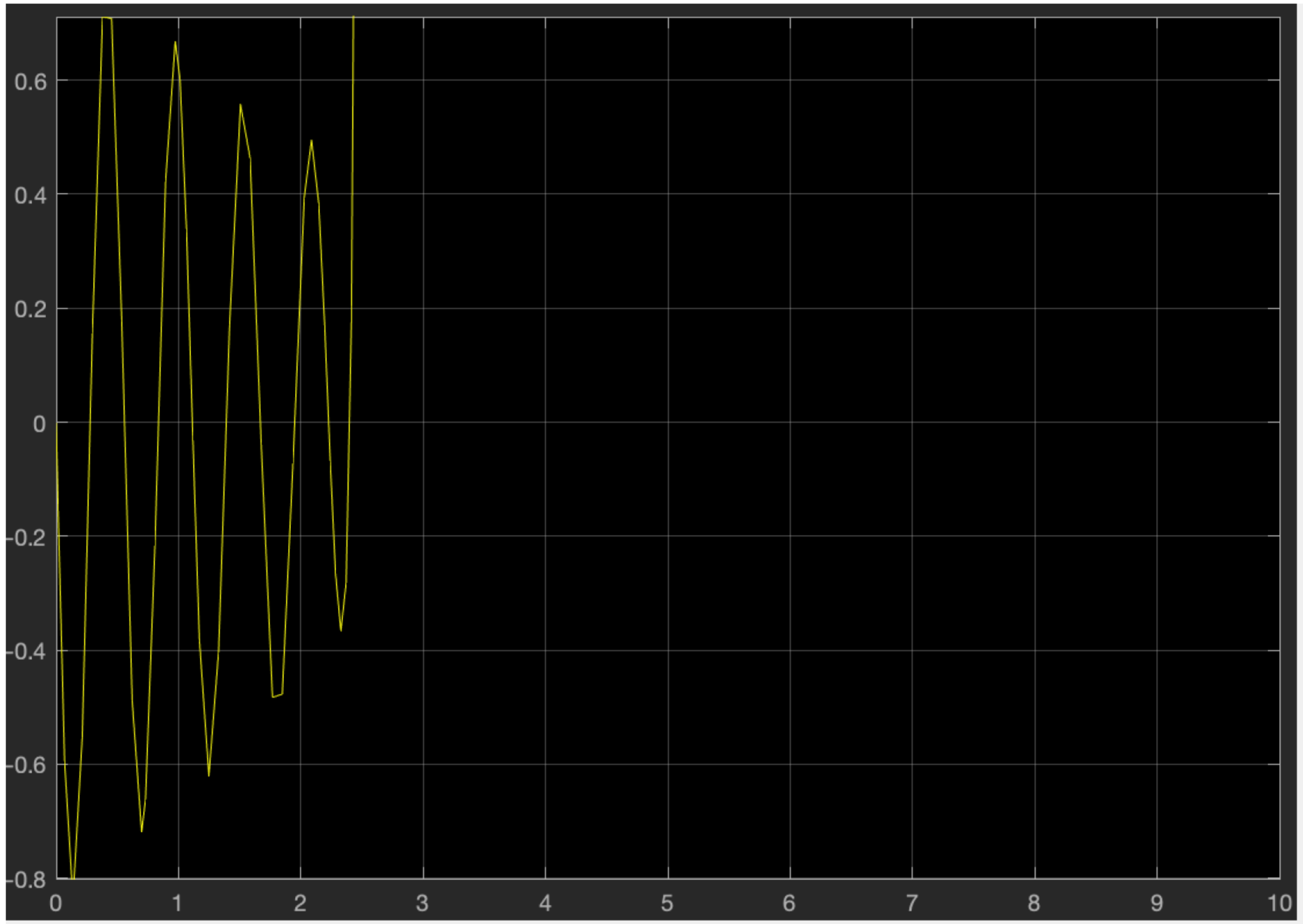



شکل ۵ـ۱۱: روش رویتگر غیر خطی تطبیقی در PVTOL اگر عیبی وجود نداشته باشد

# Simulations



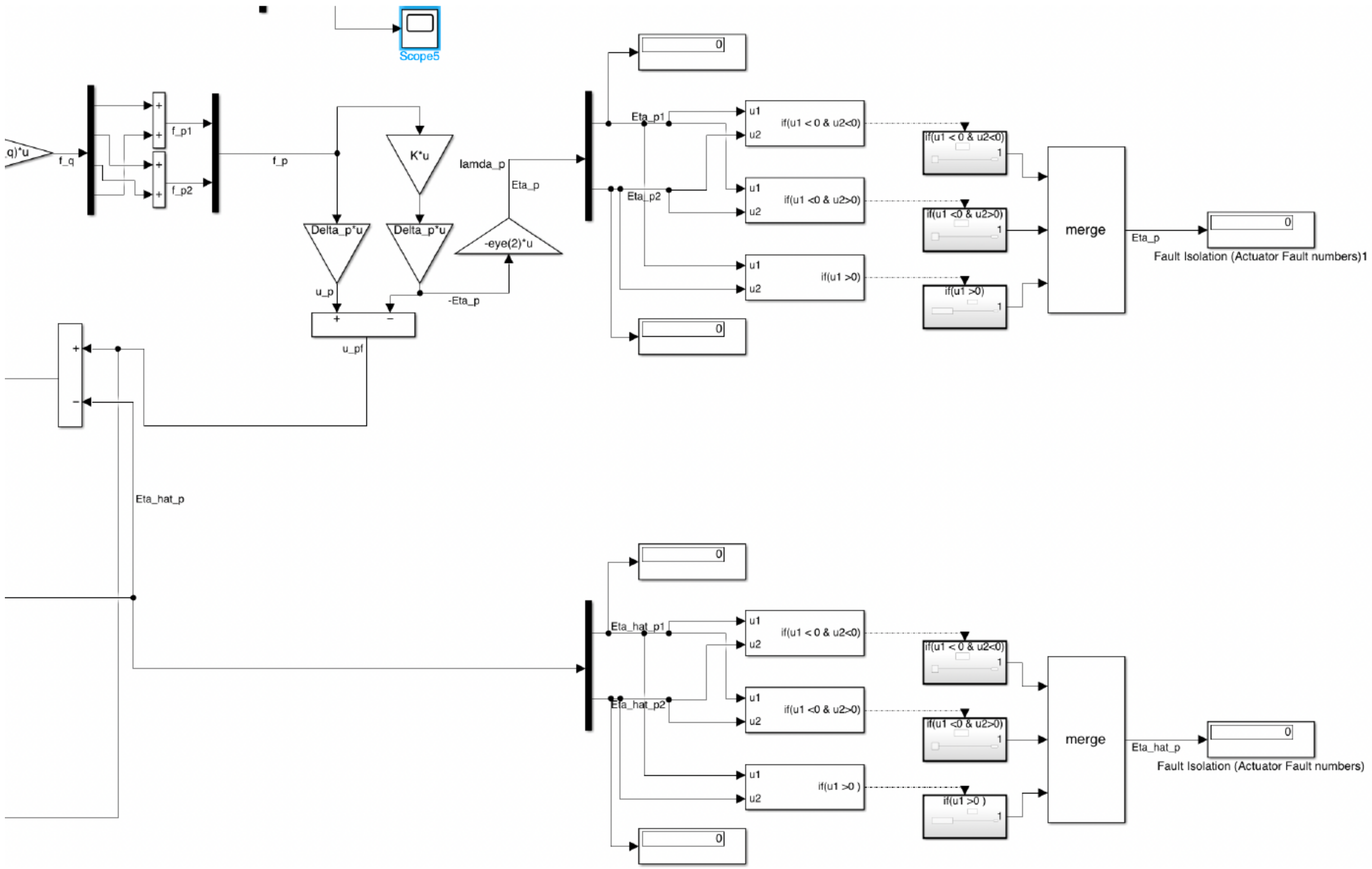

شکل ۵ـ۱۲: نمودار z برای رویتگر غیر خطی تطبیقی در حالت بدون عیب برای PVTOL




شکل ۵ـ۱۳: نمودار zdot برای رویتگر غیر خطی تطبیقی در حالت بدون عیب برای PVTOL

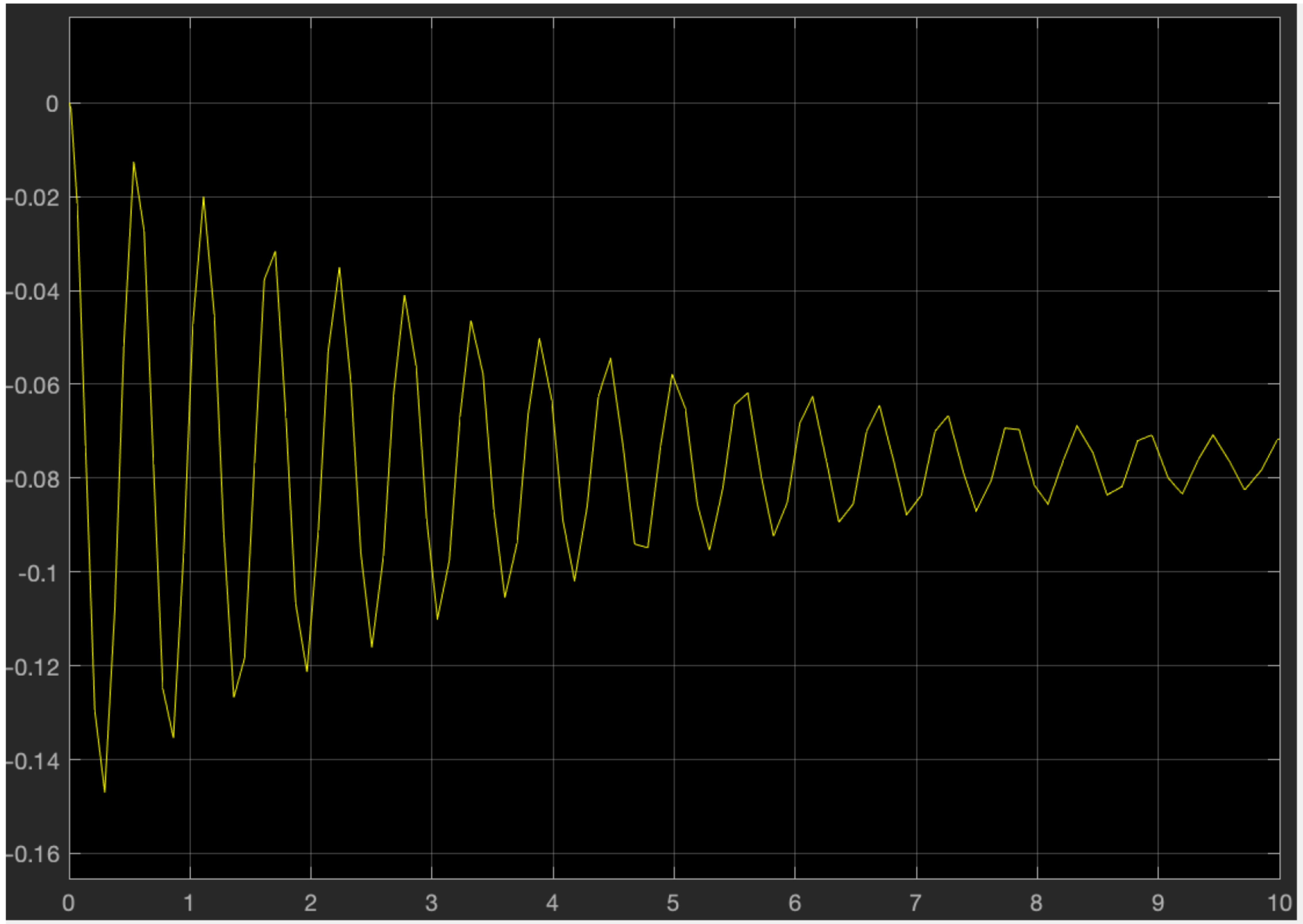
# Simulations

- no fault detected = OK
- stable

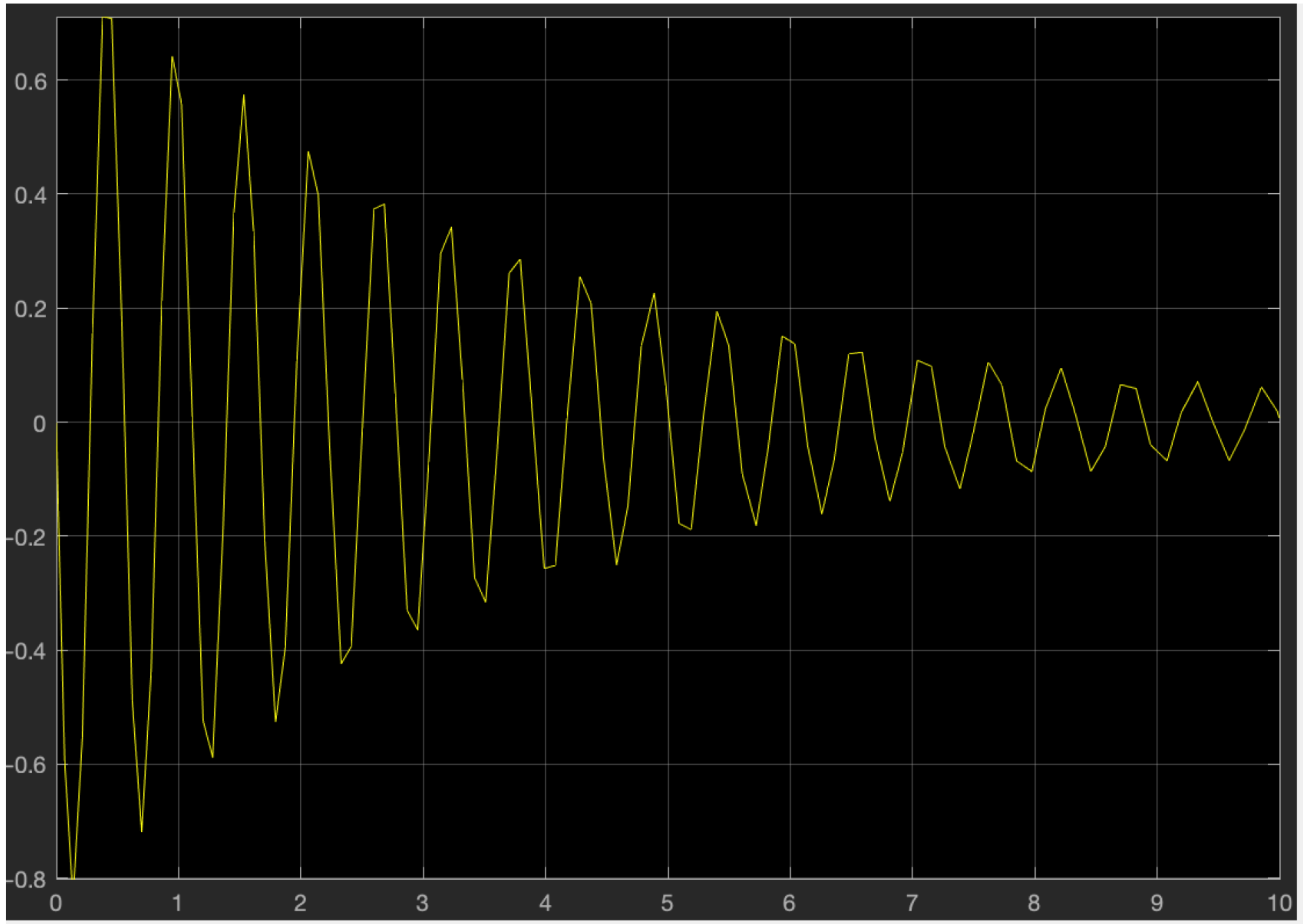



شکل ۵ـ۱۴: روش رویتگر PIO خطی در PVTOL اگر عیبی وجود نداشته باشد

# Simulations



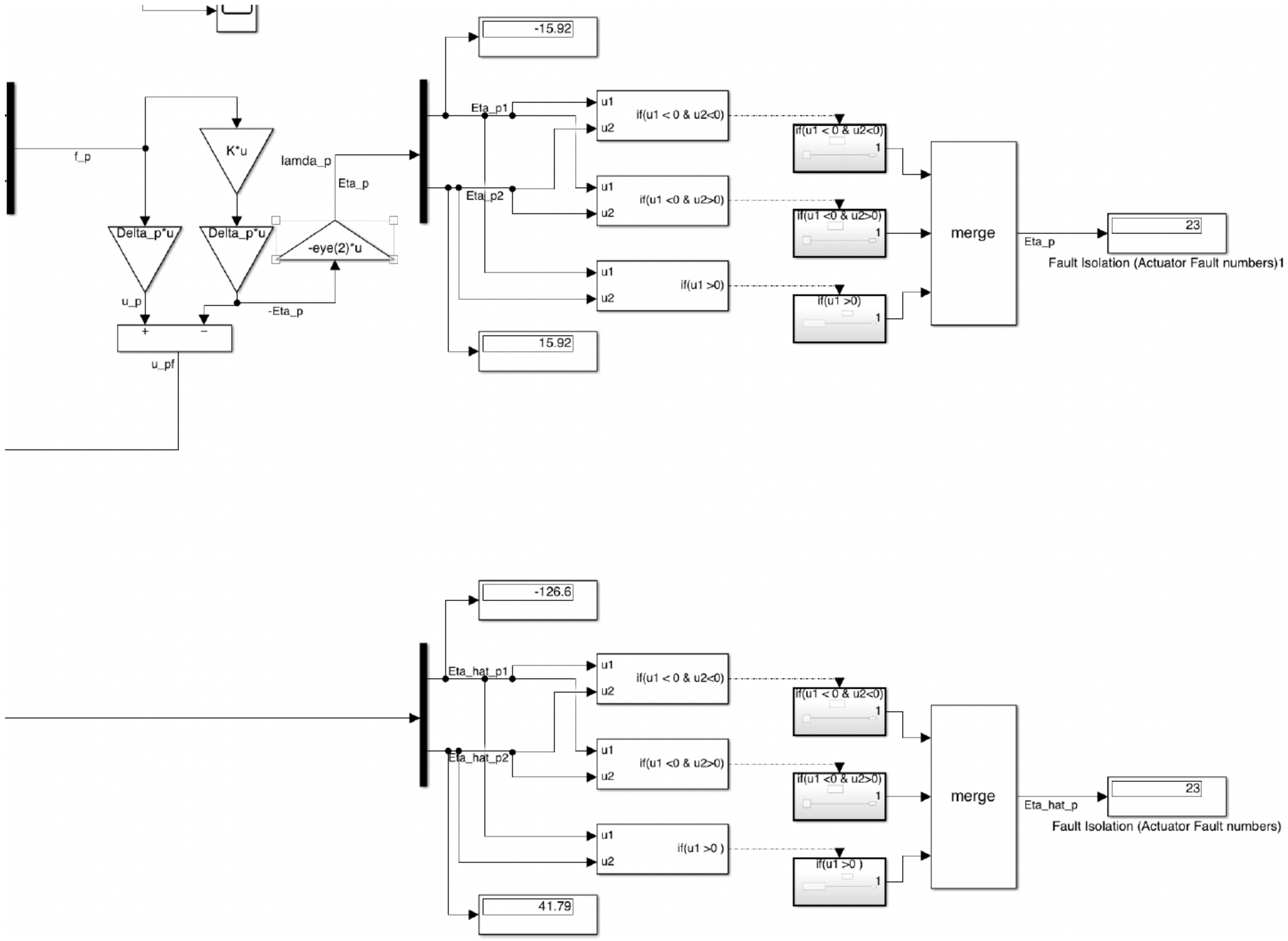

شکل ۵ــ۱۵: نمودار z برای رویتگر PIO خطی در حالت بدون عیب برای PVTOL




شکل ۵ــ۱۶: نمودار zdot برای رویتگر PIO خطی در حالت بدون عیب برای PVTOL

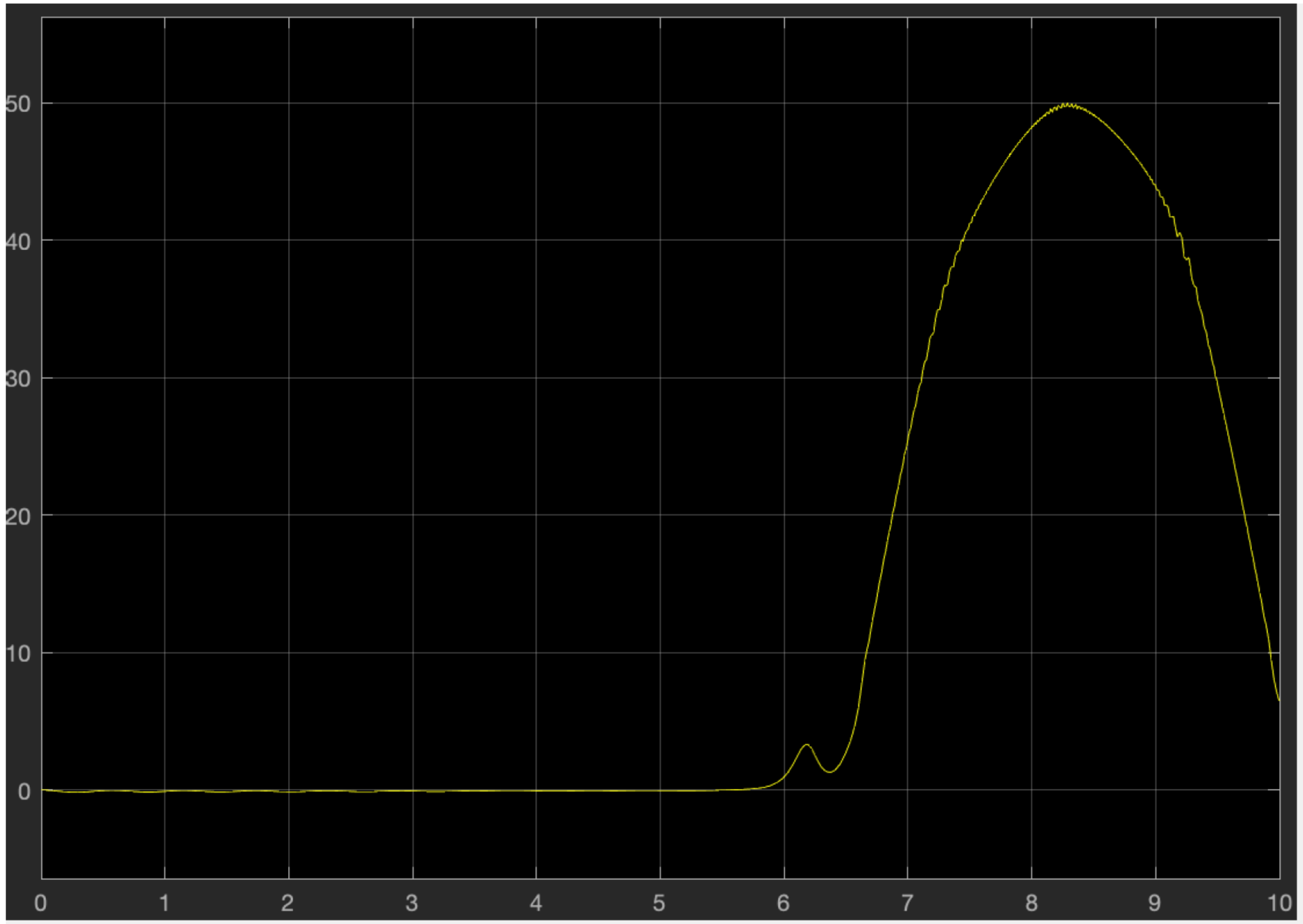
# Simulations

- fault detected: correct only this fault
- a little unstable
- estimates not accurate

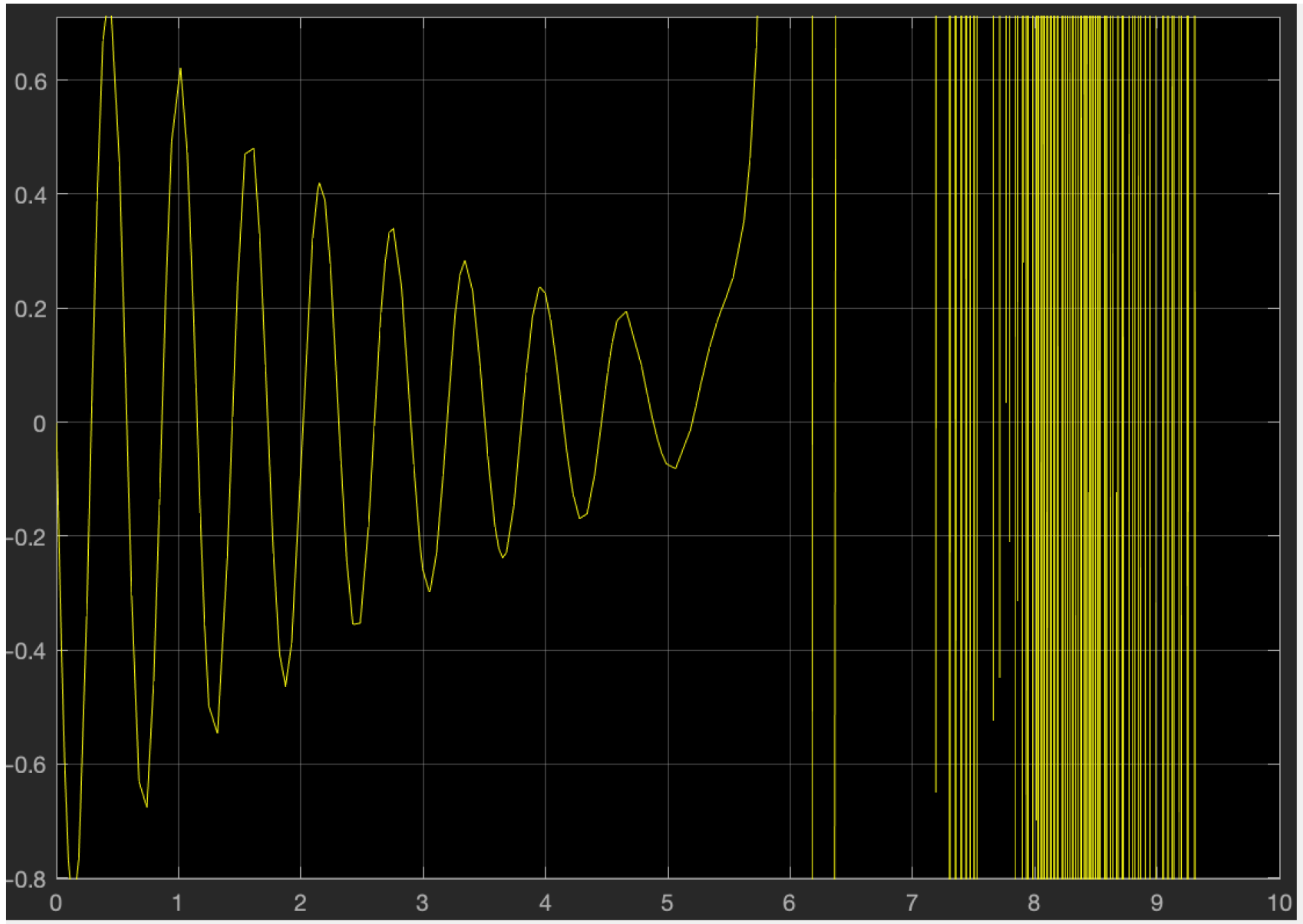



شکل ۵ــ۱۷: روش رویتگر PIO خطی در PVTOL با عیب عملگر ۲

# Simulations




شکل ۵ـ۱۸: نمودار z برای رویتگر PIO خطی در حالت با عیب عملگر ۲ برای PVTOL




شکل ۵ـ۱۹: نمودار zdot برای رویتگر PIO خطی در حالت با عیب عملگر ۲ برای PVTOL

# Simulations

- Quadcopter system was stable without faults just like linear PIO
- Quadcopter system was very unstable with that lead to errors and therefore simulations were not carried out

# Simulations

در این قسمت نتایج شبیه سازی و درستی آنها بررسی می شود. متاسفانه مقاله ی اصلی در خیلی از جاها از جمله ارائه ی مدل دینامیک سیستم های VTOL و پارامتر های آزمایشی رویتگر ها و روش کامل محاسبه ی کواترنیون کنترل کننده ی نامی نقص داشته و برای همین ناپایداری و کراندار نبودن سیگنال ها به ازای عیب در سیستم های شبیه سازی شده دید می شود و نتایج شبیه سازی خیلی مورد قبول نیست. در صورتی که اگر مقاله ی اصلی پارامتر ها و بعضی سیگنال ها را دقیق تر تعریف می کرد، نتایج مطلوب هم داشتیم چرا که شماتیک کلی سیستم شبیه سازی شده درست و دقیقاً مطابق مقاله است. به علاوه چون نتایج آزمایش های مقاله شبیه سازی نبوده و نتایج آزمایش های واقعی بلادرنگ از دستگاه quadcopter با مدل Parrot AR ۲.۰ می باشد، بیشتر آزمایش هایش در شبیه سازی قابل انجام نیست و نیاز به اطلاعات سنسور ها و انکودر ها است و همین طور مدل حالت بلادرنگ دقیق می باشد در صورتی که مدل استفاده شده در این شبیه سازی غیردقیق و حتی در قسمت هایی ناقص و اشتباه است مثلاً مقاله نحوه ی محاسبه ی سیگنال مانده ی کلی سرعت زاویه ای برای این پهباد را نگفته و اصلاً حساب نکرده است چرا که با توجه به داشتن سیستم واقعی نیازی به مدل سازی آن ندارد و در طراحی ها هم به عنوان اغتشاش به آن نگاه کرده است.

# Simulations

در سیستم PVTOL به ازای روش رویتگر غیر خطی تطبیقی اگر عیبی وجود نداشته باشد، سیستم ناپایدار است و یکی از مولفه های سیگنال عیب عملگر خیلی بزرگ است و در نتیجه طبق شکل ۵ـ۱۱ تشخیص عیب اشتباه انجام شده است. در شکل های ۵ـ۱۲ و ۵ـ۱۳ هم نمودار سیگنال z و zdot نشان داده شده است که خود نشان دهنده ی ناپایداری و سقوط پهباد می باشد. بقیه ی حالت ها پایدار می باشند. دلیلی که می توان برای این ناپایداری حدس زد هم به احتمال زیاد طراحی ناقص کنترل کننده ی نامی و پارامتر های اشتباه رویتگر می باشد.

چون سیستم ناپایدار است بررسی حالت های با عیب هم بدون معنی است چرا که تخمین ها با مقدار های بزرگ و اشتباه خواهد بود و MATLAB هم در این حالت ها معمولاً ارور می دهد.

# Conclusions

- nonlinear dynamic systems for both quadcopter and PVTOL
- additive actuator fault controllability
- FA using nonlinear AO
- FA using linear PIO
- FA using qLPV PIO
- Nominal Controller using unit quaternions
- Fault accommodation and Fault reconfiguration
- simulations had undesirable results but linear PIO had the best results

# Thank You

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