uOttawa

CSI 3105 - Design and Analysis of Algorithms

Fall 2022 Section A

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Assignment 3

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1. **Answer**

Below is the recursion definition that is filled in,

$$
\mathrm{opt}[i,j] = \begin{cases} 0 & \text{(1) if } i=0, \\ 0 & \text{(2) if } j=0, \\ \max \begin{cases} v_i + \mathrm{opt}[i-1, j-w_i], \\ \mathrm{opt}[i-1, j] \end{cases} & \text{(3) if } j \geqslant w_i,\ i \neq 0,\ j \neq 0, \\ \mathrm{opt}[i-1, j] & \text{(4) otherwise.} \end{cases}
$$

Below is the justification (for my own reference).

1. No object, therefore the optimal value is 0.
2. Max weight is 0, and since $w_i > 0$, no such w_i can satisfy the condition.
3. The optimal value is the maximum of the following two cases:
— The value of the current item v_i _plus the existing optimal value for the remaining weight after including item i, and
— the existing optimal value from the previous calculation.
4. The optimal value is simply the existing optimal value from the previous calculation.

2. **Answer**

Following the recursive algorithm proposed in question 1, we can solve the knapsack problem with the given values using the matrix method of dynamic programming.

We are given,

$$W = 11,\, n = 6$$

as well as the weight of each item,

$$w_1 = 2, w_2 = 3, w_3 = 2, w_4 = 9, w_5 = 3, w_6 = 2$$

and the value of each item,

$$v_1 = 7, v_2 = 1, v_3 = 6, v_4 = 18, v_5 = 22, v_6 = 28$$

The following matrix can be produced.

		\W \ i \	0	1	2	3	4	5	6	7	8	9	10	11
p_i	w_i	0	0	0	0	0	0	0	0	0	0	0	0	0
7	2	1	0	0	7	7	**7**	7	7	7	7	7	7	7
1	3	2	0	0	7	7	7	8	8	8	8	8	8	8
6	2	3	0	0	7	7	13	13	**13**	14	14	14	14	14
18	9	4	0	0	7	7	13	13	13	14	14	18	18	25
22	3	5	0	0	7	22	22	29	29	35	35	**35**	36	36
28	2	6	0	0	28	28	35	50	50	57	57	63	63	**63**

Green represents the backtracking (equivalent to arrows).
Bold represents the item that is selected to be in the knapsack.

Result:

— The items included in the knapsack are items 6, 5, 3, and 1.
— The total weight is 2 + 3 + 2 + 2 = 9, which is less than the maximum weight, W = 11.
— The total value is 63.