Report of manuscript "Exponential methods for solving hyperbolic problems with application to kinetic equations"

By Nicolas Crouseilles, Lukas Einkemmer, and Josselin Massot

This paper compares the exponential integrators and the Lawson methods for the Vlasov-Poisson and drift-kinetic equations. For these equations, the (linear) transport term is usually stiff hence imposing stringent CFL condition, while the (nonlinear) forcing term is less stiff. Both exponential integrators and Lawson methods apply to these equations in a natural way that the linear term is solved exactly by exponential methods and the nonlinear term is treated explicitly. The paper conducts a linear stability analysis for these methods and derives the CFL conditions needed. The general conclusion is that the Lawson methods have larger stability domain and are better suited for these kinds of equations. I think the paper is interesting and deserves publication in JCP provided the following questions/comments are addressed.

1. I would suggest to add "collisionless kinetic equations" in the title and in the main text when the kinetic equations are first mentioned. This is to distinguish the focus from the collisional kinetic equations (such as Boltzmann or Landau equations).

2. Section 2, I found the introduction of the exponential Euler method a bit confusing. It is mentioned that "use the rectangular rule at the left endpoint ..." to obtain the method. However, if the term $\exp((\Delta t - s)A)$ is also approximated by the same quadrature rather than integrated exactly, one obtains nothing but the Lawson method presented below. Hence a clearer explanation is needed.

3. Section 3.3, what happens if the Fourier method is used in the velocity space as well? I understand for some problems the WENO method is a better choice but some comments/remarks about the Fourier method would be helpful.

4. Section 4.1, here the results are only for Lawson methods. How do they compare with exponential integrators?

5. Section 4.2, Figure 8 mainly demonstrates that WENO is better than centered difference. What is the comparison of time integrators?

6. Section 4.2, it seems to me that the energy conservation property cannot be revealed by the stability analysis. Then what is the conclusion of Figure 9? Can something more be said regarding energy conservation?