Predictions from computational models of associative learning on the effect of stimulus uncertainty

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◈Introduction

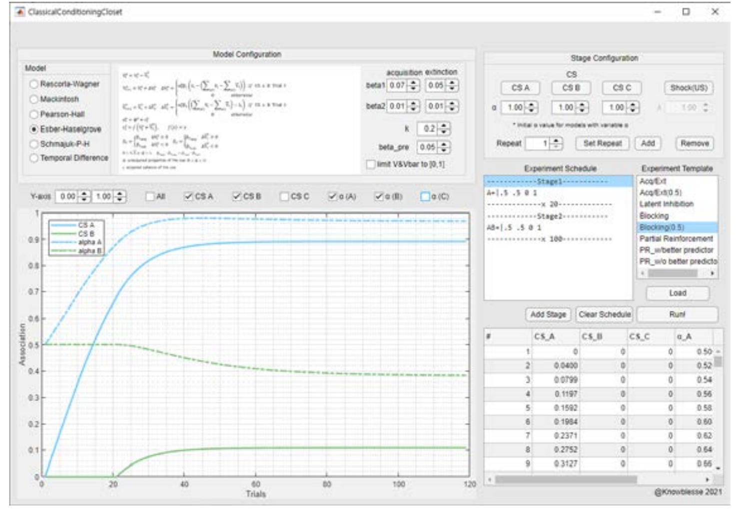
Computational models of associative learning have been instrumental in guiding experimental research in psychology and neuroscience. In computational learning, important ideas are expressed in formalized logic or clear mathematical expressions, preventing guesswork and misunderstanding and generating testable predictions. However, it is often difficult to compare different models as each require time-consuming computer programming to express its key algorithms. Here we developed an integrated user environment where five influential models of associative learning can be tested under various learning protocols of Pavlovian conditioning. In particular, we have generated novel predictions from conditioning with stimulus uncertainty and compared them with the experimental results.

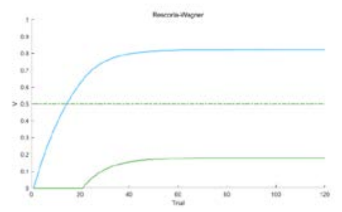
◈Summary of the models

Models	Learning rule	Key features	Key algorithm in mathematical notations
Rescorla-Wagner model (R-W)	Delta rule	• US-expectancy violation • Stimulus competition	$\lambda_t - \sum_{s\in t} V_t$
Mackintosh model (Mac)	Delta rule	• US-predictability • Selective attention toward a better predictor of the US	$\Delta\alpha > 0 \quad \left\|\lambda_t - \left(\left(\sum_{s\in t} V_t\right) - V_t^s\right)\right\| > \|\lambda_t - V_t^s\|$ $\Delta\alpha < 0 \quad \left\|\lambda_t - \left(\left(\sum_{s\in t} V_t\right) - V_t^s\right)\right\| \le \|\lambda_t - V_t^s\|$
Pearce-Hall model (P-H)	Alpha rule	• US-predictability • Selective attention toward a uncertain predictor of the US	$\dot{V}_t^s = V_t^s - \overline{V}_t^s$ $\Delta V_t^s = S^s\alpha_t^s\lambda_t, \Delta\overline{V}_t^s = S^s\alpha_t^s\overline{\lambda}_t$ $\overline{\lambda}_t = \left(\sum_{s\in t} \dot{V}_t\right) - \lambda_t$
Esber-Haselgrove model (E-H)	Hybrid rule	• Hybrid model : Predictiveness-driven & Uncertainty-driven	$\alpha_t^s = \phi^s + \epsilon_t^s$ ΔV_t^s $= \alpha_t^s\beta_1\left(\lambda_t - \left(\sum_{s\in t} V_t - \sum_{s\in t} \overline{V}_t\right)\right)$
Temporal Difference model (TD)	Temporal difference	• Neuron-like adaptive element • Eligibility trace • Time feature	Δw_t^s $= c\left(w_t^{US}x_t^{US} + \gamma\max\left(\mathbf{w}_t^{\mathbf{T}}\mathbf{x}_t, 0\right)\right)$

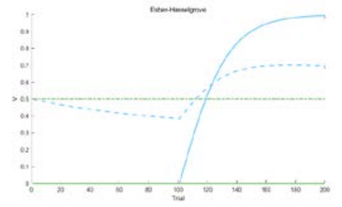
◈Simulator description

<Classical Conditioning Closet> (CCC) is a ©MATLAB-based multi-model simulator capable of simulating six different associative learning models with adjustable parameters. A user-friendly interface allows a researcher to test several traditional conditioning protocols, such as latent inhibition, blocking, and partial reinforcement, and also custom protocols can be applied. With few clicks, the researcher can compare different models with different parameters.



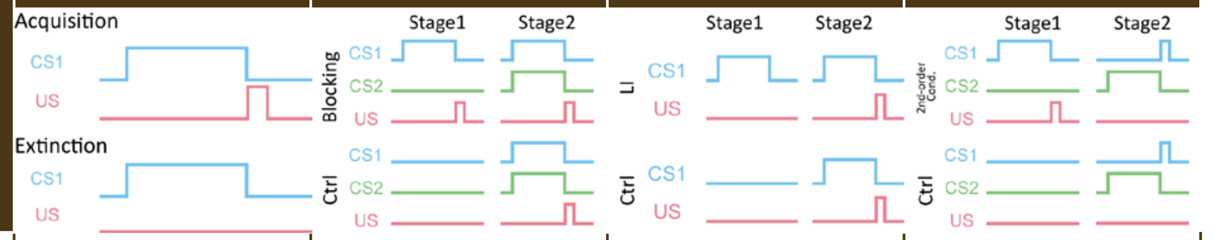


RW Model - Blocking



EH Model – Latent Inhibition

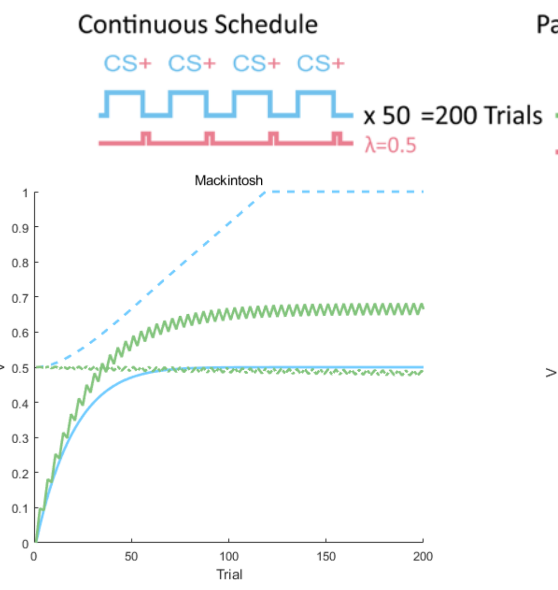
◈Predictions from different models under basic conditioning protocols



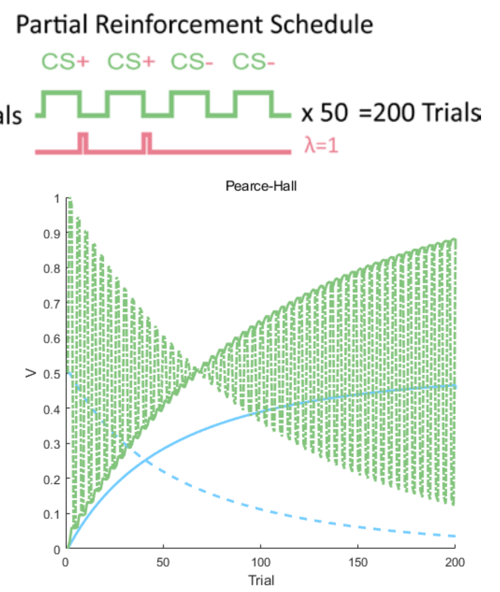
Models	Conditioning & Extinction	Blocking	Latent inhibition	Second order conditioning
R-W model	Y	Y	N	N
Mac model	Y	Y	Y	N
P-H model	Y	Y	Y	N
E-H model	Y	Y	Y	N
TD model	Y	Y	Y	Y

◈Predictions from the simulations

1.Partial reinforcement



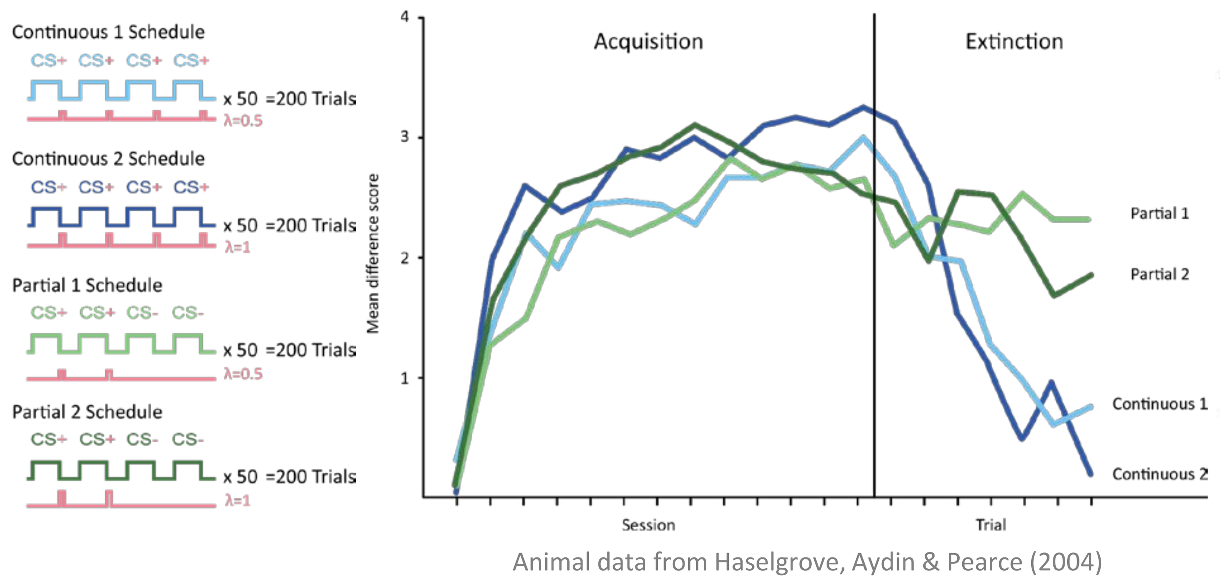
Mackintosh Model



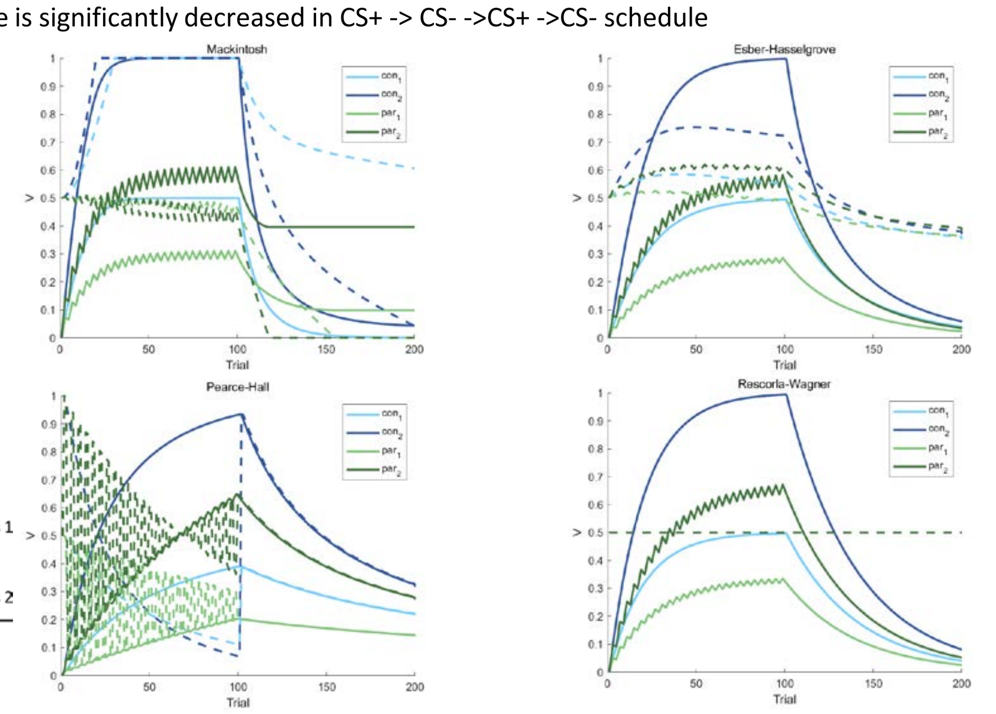
Pearce-Hall Model*

Models	Asymptotic V value compared to continuous reinforcement	Time to reach the asymptotic value	Saliency compared to continuous reinforcement
R-W model	Higher	Similar	Unchanged
Mac model	Higher	Similar	Decreased
P-H model	Higher*	Longer	Fluctuate/Increase
E-H model	Higher	Similar	Increased
TD model	Same	Same	N/A

*Asymptote is significantly decreased in CS+ -> CS- ->CS+ ->CS- schedule

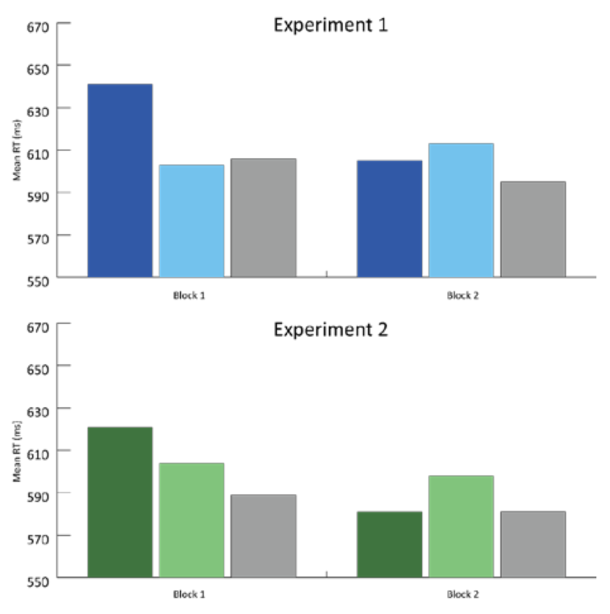


Animal data from Haselgrove, Aydin & Pearce (2004)



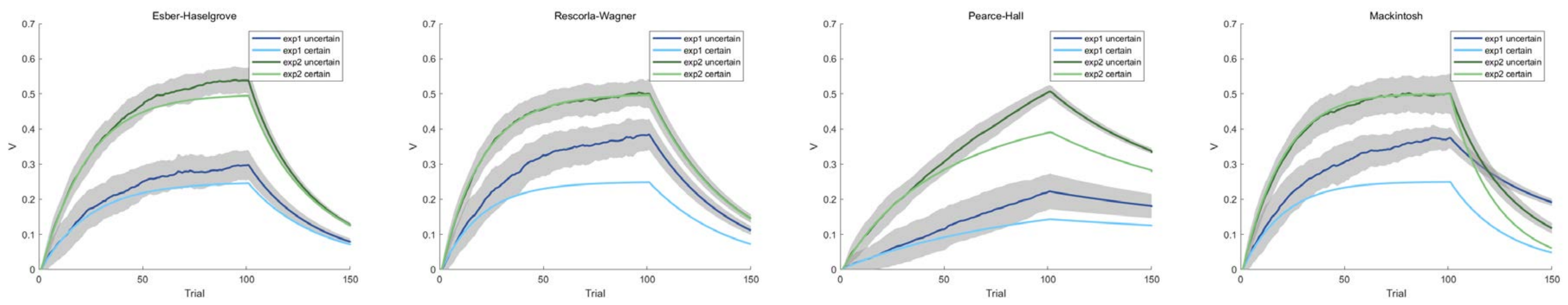
Haselgrove et al. (2004) performed 4 Pavlovian conditioning experiments with rats to assess the Partial Reinforcement Extinction Effect(PREE). The primary purpose of the paper was to identify which aspect of the reinforcement schedule produces the PREE. We simulated the second experiment of the paper, which compared the effect of the partial reinforcement and the total number of the received unconditioned stimulus. In the thesis, they calculated animal's behavior by subtracting the duration of magazine activity during the pre-CS period from the CS period. We used this value as V and compared it with all five computational models. Although animals showed similar asymptote values across all conditions, all models showed different asymptote according to the lambda value. These different starting points in the extinction session lead conditions with higher lambda to maintain a greater V value after following the CS presentation. However, the Mackintosh model showed a steep decrease in continuous schedules which resulted in lower V values than the partial conditions.

2.Value-driven attentional capture under reward uncertainty



Human data from Choi & Choi (2021)

Models	Experiment 1		Experiment 2	
	Uncertain	Certain	Uncertain	Certain
	1 : 25% | 0 : 75%	.25 : 100%	.10 : 25% | .25 : 25% .75 : 25% | .90 : 25%	.50 : 100%
R-W model	>		=	
Mac model	>		=	
P-H model	>		>	
E-H model	>		>	
TD model	=		=	



Value-Driven Attentional Capture (VDAC) is an involuntary attentional bias toward a high value-associated stimulus. This phenomenon can be measure through a simple visual search task, where a high value-associated distractor increases the overall response time (RT). Choi & Choi (2021) tested whether uncertainty-associated, not high value, distractor evoke similar interference-effect. In two experiments, the uncertain distractor increased RT compared to the certain distractor showing that reward uncertainty modulated the existence of VDAC successfully. However, these effects only appeared during the first block of the testing session, and the amount of the effect varied through reward probability conditions. We simulated this experiment with 100 trials of acquisition followed by 50 extinction trials. The Pearce-Hall model and the Esber-Haselgrove model successfully simulated the uncertainty-driven attentional capture shown by the higher V value of uncertain CS. Moreover, the Esber-Haselgrove model showed that the discrepancy between the uncertain and the certain cue decreases during the extinction phase. Interestingly, this discrepancy is much higher in the experiment 1, and this result accord with human data that the RT difference is higher in the first experiment.

For detailed explanation and questions,
please attend the Zoom E-poster session on **May 20 (Thr), 2pm-5pm**
or
send us an email to **j-schoi@korea.ac.kr**

◈Computational details

*** Rescorla-Wagner Model**

$$V_{t+1}^s = V_t^s + \Delta V_t^s$$

$$\Delta V_t^s = \begin{cases} \alpha^s\beta\left(\lambda_t - \sum_{s\in t} V_t\right) & if\ \text{CS s} \exists \text{ Trial t} \\ 0 & otherwise \end{cases}$$

$$\beta = \begin{cases} \beta_{acq} & \lambda > 0 \\ \beta_{ext} & \lambda = 0 \end{cases}$$

α^s : saliency of CS s
β : learning rate
λ_t : US saliency in the Trial t
$\sum_{s\in t} V_t$: sum of V of all CSs exist in the Trial t

*** Pearson-Hall Model (Uncertainty)**

$$\dot{V}_t^s = V_t^s - \overline{V}_t^s$$

$$V_{t+1}^s = V_t^s + \Delta V_t^s \quad \Delta V_t^s = \begin{cases} S^s\alpha_t^s\lambda_t & if\ \text{CS s} \exists \text{ Trial t} \\ 0 & otherwise \end{cases}$$

$$\overline{V}_{t+1}^s = \overline{V}_t^s + \Delta\overline{V}_t^s \quad \Delta\overline{V}_t^s = \begin{cases} S^s\alpha_t^s\overline{\lambda}_t & if\ \text{CS s} \exists \text{ Trial t} \\ 0 & otherwise \end{cases}$$

$$\overline{\lambda}_t = \left(\sum_{s\in t} \dot{V}_t\right) - \lambda_t$$

$$\alpha_{t+1} = \left|\lambda_t - \left(\sum_{s\in t}(V_t) - \sum_{s\in t}(\overline{V}_t)\right)\right|$$

$\dot{V}_t^s$: net prediction of US (=CR strength)
$\overline{V}_t^s$: association strength between CS s and no US in trial t
S^s : Saliency of CS s

*** Temporal Difference Model**

$$w_{t+1}^s = w_t^s + \Delta w_t^s$$

$$\Delta w_t^s = c\left(w_t^{US}x_t^{US} + \gamma\max\left(\mathbf{w}_t^{\mathbf{T}}\mathbf{x}_t, 0\right) - \max\left(\mathbf{w}_t^{\mathbf{T}}\mathbf{x}_{t-1}, 0\right)\right)\overline{x}_t^s$$

$$\overline{x}_t^s = \beta\overline{x}_{t-1}^s + (1-\beta)x_{t-1}^s$$

$\overline{x}_t^s$: eligibility trace of CS s in trial t
w_t^s : weight of CS s in trial t. analogue of V_t^s
$\mathbf{w}_t$: weight vector of all CS in trial t
$\mathbf{x}_t$: CS vector of all CS in trial t
c : learning rate
β : eligibility trace parameter
γ : relative importance between presence and onset/offset of the CS

*** Mackintosh Model (Explainability)**

$$V_{t+1}^s = V_t^s + \Delta V_t^s$$

$$\Delta V_t^s = \begin{cases} \alpha^s\beta_t(\lambda_t - V_t^s) & if\ \text{CS s} \exists \text{ Trial t} \\ 0 & otherwise \end{cases}$$

$$\beta_t = \begin{cases} \beta_{acq} & \lambda_t > 0 \\ \beta_{ext} & \lambda_t = 0 \end{cases}$$

$$\alpha_{t+1}^s = \min(\max(\alpha_t^s + \Delta\alpha_t^s, 0), 1)$$

$$\Delta\alpha_t^s = \begin{cases} k\cdot\left(\left|\lambda_t - \left(\left(\sum_{s\in t} V_t\right) - V_t^s\right)\right| - |\lambda_t - V_t^s + \epsilon|\right) & if\ \text{CS s} \exists \text{ in Trial t} \\ 0 & otherwise \end{cases}$$

k : proportional parameter ($0 \le k$)

ϵ : small value to make $\Delta\alpha_t^s$ negative when $\left|\lambda_t - \left(\left(\sum_{s\in t} V_t\right) - V_t^s\right)\right| = |\lambda_t - V_t^s|$

*** Esber-Haselgrove Model (Mackintosh + Pearson-Hall)**

$$\dot{V}_t^s = V_t^s - \overline{V}_t^s$$

$$V_{t+1}^s = V_t^s + \Delta V_t^s \quad \Delta V_t^s = \begin{cases} \alpha_t^s\beta_1\left(\lambda_t - \left(\sum_{s\in t} V_t - \sum_{s\in t} \overline{V}_t\right)\right) & if\ \text{CS s} \exists \text{ Trial t} \\ 0 & otherwise \end{cases}$$

$$\overline{V}_{t+1}^s = \overline{V}_t^s + \Delta\overline{V}_t^s \quad \Delta\overline{V}_t^s = \begin{cases} \alpha_t^s\beta_2\left(\left(\sum_{s\in t} V_t - \sum_{s\in t} \overline{V}_t\right) - \lambda_t\right) & if\ \text{CS s} \exists \text{ Trial t} \\ 0 & otherwise \end{cases}$$

$$\alpha_t^s = \phi^s + \epsilon_t^s - k\sum V_t^{pre\to s}$$

$$\epsilon_t^s = f\left(V_t^s + \overline{V}_t^s\right), \quad f(x) = \frac{1}{2}x, \quad \Delta V_t^{pre\to s} = \begin{cases} \alpha_t^s\beta_{pre}\left(1 - \sum_{s\in t} V_t\right) & if\ \text{CS s} \exists \text{ Trial t} \\ 0 & otherwise \end{cases}$$

$$\beta_1 = \begin{cases} \beta_{1acq} & \Delta V_t^s \ge 0 \\ \beta_{1ext} & \Delta V_t^s < 0 \end{cases} \quad \beta_2 = \begin{cases} \beta_{2acq} & \Delta\overline{V}_t^s \ge 0 \\ \beta_{2ext} & \Delta\overline{V}_t^s < 0 \end{cases}$$

$0 \le V, \overline{V}, \alpha\cdot\beta \le 1, \quad \beta_{1acq}\cdot\beta_{2acq} > \beta_{1ext}\cdot\beta_{2ext}$
ϕ: unacquired properties of the cue ($0 \le \phi \le 1$)
ϵ: acquired salience of the cue

◈Reference

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